As-You-Go Deployment of a 2-Connected Wireless Relay Network for Sensor-Sink Interconnection

Avishek Ghosh*, Arpan Chattopadhyay*, Anish Arora†, and Anurag Kumar*
*Dept. of ECE, Indian Institute of Science
Bangalore 560012, India
{avishek, arpan, anurag}@ece.iisc.ernet.in
†Dept. of CSE, The Ohio State University
Columbus, OH 43210, USA
anish@cse.ohio-state.edu

***Abstract*—A person walks along a line (which could be an idealisation of a forest trail, for example), placing relays as he walks, in order to create a multihop network for connecting a sensor at a point along the line to a sink at the start of the line. The potential placement points are equally spaced along the line, and at each such location the decision to place or not to place a relay is based on link quality measurements to the previously placed relays. The location of the sensor is unknown apriori, and is discovered as the deployment agent walks. In this paper, we extend our earlier work on this class of problems to include the objective of achieving a 2-connected multihop network. We propose a network cost objective that is additive over the deployed relays, and accounts for possible alternate routing over the multiple available paths. As in our earlier work, the problem is formulated as a Markov decision process. Placement algorithms are obtained for two source location models, which yield a discounted cost MDP and an average cost MDP. In each case we obtain structural results for an optimal policy, and perform a numerical study that provides insights into the advantages and disadvantages of multi-connectivity. We validate the results obtained from numerical study experimentally in a forest-like environment.**

I. Introduction

There are situations in which deployment of a wireless relay network (for connecting wireless sensors to a base-station) needs to be carried out in an "as-you-go" fashion (e.g., emergency situation monitoring networks set up by first responders [1],[2] or impromptu deployment over a large terrain, such as a forest). Motivated by as-you-go deployment of a wireless relay network over a forest trail, in earlier work [3], [4], we focussed on developing algorithms for the deployment of relay nodes, and setting their powers, so as to provide a 1 node-connected path from the source to the sink, the deployment being done as an agent walks from the sink to the source. Such a network, however, will not be *reliable*, as the network will cease to work if any one node fails, or if on any one link the radio propagation deteriorates.

In this work, we retain many of the assumptions we made in our earlier work [3]: (i) a single deployment agent walks along a line, away from a sink at the start of the line. (ii) there are potential relay placement points at multiples of a fixed, given, distance δ (say, 10 meters). (iii) based on link quality measurements to the already placed relays, the agent must decide whether to place a relay at a potential placement location or move on. (iv) a sensor has to be placed at an a priori unknown location that is discovered as the agent walks over the line. (v) assuming a light packet rate regime, the objective of the deployment is to minimise an expected additive cost over the deployed nodes, where the cost at each node placement is a linear combination of the node power and the cost of placing a node. However, we now seek deployment algorithms that place relays in such a way that the network is K node-connected, with $K > 1$. The choice of K could be determined by a statistical characterization of the long term variations in the links. The goal, in this paper, is for the deployment agent to place nodes as he walks along a line, so as to ensure K (node disjoint) paths from the sensor (source) to the sink (destination).

In this paper, we focus on the case of $K = 2$. In the forest monitoring application, the source to sink distance can be several hundreds of meters. In order to ensure a reasonable probability of delivery, we need a network with a small number of hops (up to 5, say). Hence the hop lengths will be relatively large, and with typical transmit power levels of the radios used in these systems, it is unlikely that good links will exist between nodes that are more than two hops apart. Thus, in practice, $K = 2$ would suffice.

In the $K = 2$ case, while formulating the sequential decision problem, extending the earlier work [3], [4], we need to define the cost of placing a relay at a potential location. We do this by taking a linear combination of the costs of the two downstream links so created and provide a method for determining the combining coefficients. Then the problem is formulated as a discounted cost or average cost Markov decision process, and structural results for the optimal policy are obtained. The techniques that we use easily extend to $K > 2$, albeit with the need to take more measurements at each decision step, and the increased computational complexity of determining the optimal policy.

Related Work: Howard et al. [2] provide heuristic algorithms for the incremental deployment problem. Souryal et al. [5], study the problem with an experimental study of RF link variation in an indoor setting. Their algorithms [5] consider links to several of the previously placed relays and the decision of placing relays is done heuristically approach. Liu et al. ([6]) describe a *bread-crumb* system to aid fire fighters inside buildings. The problem of relay placement is rigorously formulated in [4] and extented to a measurement-based approach in [3].

II. System Model

Starting from the sink node, the deployment agent stops at multiples of a fixed "step-length" δ (e.g., 10 meters), makes measurements, and given certain state variables and

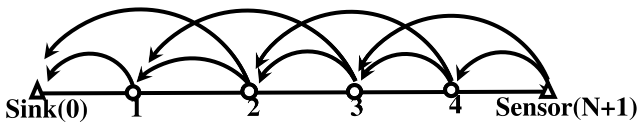

Fig. 1. A topology in which each relay, except the first, has a link to two (immediately) previous neighbours.

the measurements, decides whether or not to place a relay at that point. Thus, starting from the sink, we can think of points along the line at multiples of δ as being *potential node locations*. We assume that the source has to be placed at one of these locations. Since the deployment is based on on-line measurements of the channel qualities, the locations of the deployed relays and their number N, are random. As shown in Figure 1, the sink is called Node 0, the relay closest to the sink is called Node 1, and the source is called Node $(N+1)$.

***Length of the Line**:* We consider two cases : (1) We assume that the source (i.e., the sensor) is at an unknown distance $L\times\delta$ away, where $L \geq 1$ is an integer valued random variable with mean $\overline{L}$. The choice of the step length δ will be discussed in Section VII. It is well known that the geometric distribution is the maximum entropy discrete probability mass function with a given mean. With this motivation, one model is to take L to be geometrically distributed with "continuation" probability θ; i.e., $Prob(L > k) = \theta^{(k-1)}, k \geq 1$. Then θ is obtained by setting $\frac{1}{\theta} = \overline{L}$. By using the geometric distribution, we are leaving the length of the line as uncertain as we can, given the prior knowledge of $\overline{L}$. In the analysis part of this paper, we assume $\delta = 1$ for simplicity.

(2) An alternate model is to take the line to be of infinite length. The agent aims to deploy a string of relays so that the average cost of the network per unit distance is small. Such a deployment of relays could then be used to connect pairs of sources and sinks placed along the trail, provided the aggregate traffic over the network is very light. This model would be useful in a situation (such as a forest trail) where the line is long and there is *no additional information* about the location along it at which the sensor needs to be placed. Indeed, the optimal deployment policy for this case is obtained from the previous formulation by taking $\theta \to 0$.

***Channel Model**:* The received signal power for a particular link (i.e., a transmitter-receiver pair) of length r is given by:

$$P_{\text{rcv}} = P_{\text{xmt}} c \left(\frac{r}{r_0}\right)^{-\eta} HW \tag{1}$$

where P_{xmt} is the transmit power, c corresponds to the path-loss at the reference distance r_0, η is the path-loss exponent, H denotes the marginal random variable of the fading random process, and W denotes the shadowing. For a given link the value of W is fixed, whereas the fading is a random process. Thus, over a link, for a given transmitter power, due to fading, there is a positive probability that the receive power (RSSI) falls below a given target; we call this the *outage* probability. For a link of length r steps, we denote the transmit power required (for a preset target outage) by Γ_r. Owing to shadowing, this is modeled as a random variable over the various links of length r. Γ_r takes values from a discrete set, $\mathcal{S}$, as practical radios can transmit only a finite set of power levels. The cumulative distribution function and the probability mass function of Γ_r are denoted by $G_r(\cdot)$ and $g(r,\cdot)$ respectively. $g(r,\gamma)$ denotes the probability that to establish a link of length r (with the target outage probability), the least transmit power level is γ.

***Traffic Model**:* We consider a traffic model where the traffic intensity is very low. We assume that there is only one packet in the network at a time. We call this the "lone packet model". As the traffic is very low, the transmit power over a link only depends on losses in the propagation environment. This is because there are no simultaneous transmissions and hence no interference. Very light traffic is a practical assumption for ad-hoc networks that carry occasional alarm packets.

III. 2-Connected Topologies

Given any deployment environment, radio links are infeasible between locations that are very far apart. Let us denote the set of potential locations by $V_p := \{0, 1, 2, \cdots\}$, with the sink at location 0. We assume that there is a *given* positive integer parameter B, such that there is a *potential link* between a pair of potential node locations only if the two locations are no more than B steps apart, i.e., the set of potential edges is $E_p := \{(i,j) : j < i, i-j \leq B, i \in V_p, j \in V_p\}$. The corresponding directed graph is denoted by $G_p = (V_p, E_p)$.

Given a deployment of N relays, indexed $1, 2, \cdots, N$, at the potential locations $\{\ell_1, \ell_2, \cdots, \ell_N\}$, we denote $V := \{0, \ell_1, \ell_2, \cdots, \ell_N, L\}$. Let $E \subset E_p$ denote the set of edges (on V) selected by the deployment algorithm. Consider the directed acyclic graph $G = (V, E)$. The deployment should be such that there are two node disjoint *and* edge disjoint directed paths on this graph, connecting the sensor to the sink, such that the paths have acceptable end-to-end performance. After the deployment is over, the link whose transmitter is Node m (at location ℓ_m) and receiver is Node n (at location ℓ_n) is called link (m,n). Let $\Gamma^{(m,n)}$ denote the (random) power required to establish one such link (m,n).

We assume that the powers required to establish any two different *potential* links in the network are independent; it holds if δ is chosen to be greater than the shadowing de-correlation distance.

Two Neighbour (2N) Topologies: Consider a subgraph in which for each $j, 2 \leq j \leq N$, we retain the links (j, i_1) and (j, i_2), such that $0 \leq i_2 < i_1 < j$, i.e., every node has a link with two of the earlier placed nodes. It is easy to see, and will be proved in Theorem 1, that each node $j, 2 \leq j \leq N+1$, has two node disjoint and edge disjoint directed paths to the sink. The special case in which, with $j \geq 2$, it holds that $i_1 = j-1$, and $i_2 = j-2$ will be called *Two Nearest Neighbour (2NN) Topologies.* Figure 1 shows a 2NN topology with $N = 4$.

Definition 1. *In a directed graph, a pair of nodes (s,t) is said to be K edge connected (resp., K relay connected) if the* *removal of any $K-1$ arbitrary edges (resp., relays) ensures the existence of a directed (s,t) path.*

Theorem 1. *In a 2N topology with number of relays $N \geq 1$, the (source, sink) pair is 2 edge-connected as well as 2 relay-connected.*

Proof: The proof is straightforward, and, due to lack of space, we provide a brief sketch. Two edge connectivity follows from the max-flow min-cut theorem as the minimum edge cut is of size 2. Also, due the 2NN structure, if any single node is removed, it can easily be shown that there is a path that by-passes the removed node. ■

IV. Formulation as an MDP

As described previously in [3], we will formulate the measurement-based optimal deployment problem as a Markov decision problem. At multiples of the step-length δ, the agent stops, makes link quality measurements to the previously placed nodes, and then decides whether to place a relay at that location or not, and, if a relay is placed, the power levels to be used over the links to each of the previous relays.

Step Costs: The Markov decision process formulation requires the notion of a *step* cost, i.e., an evaluation of the cost of placing a relay at a potential placement point. The cost of the entire network is then evaluated in terms of this sequence of step costs: as a discounted sum of the step costs, or a per step average of the step costs. In our previous work, where the step cost involves only the immediately previous relay, if a power γ is used with some preset outage probability, then the step cost for placing a relay at this point, at power γ was evaluated as $\gamma+\xi$, where ξ is the cost of placing a relay. In the present problem at each potential placement point, the previous two relays are probed with power, say, γ (the immediate previous relay) and γ_1 (the node (sink, or relay) before the immediate previous relay). In order to specify the step cost, we need to somehow combine the two costs $\gamma+\xi$ and $\gamma_1+\xi$. We propose to combine them by the linear combination, $c\gamma + c_1\gamma_1 + \xi$, thus leaving us with the task of coming up with reasonable choices for $c \geq 0$ and $c_1 \geq 0$. We discuss this next.

An approach for choosing c and c_1: It would be reasonable to base the choice of c and c_1 on the *probability* of the one-hop or the two-hop link being used to forward a packet. We, therefore, conclude that the choice of c and c_1 should be governed by the *routing* protocol over the realised network. We consider probabilistic routing, i.e. during network operation a relay uses the one hop previous neighbour with probability p, and the two hop previous neighbour with probability $1-p$.

With probabilistic routing, in order to develop expressions for c and c_1 in terms of p, we consider an infinitely long network with a 2NN topology, and trace the path of a packet from the source to the sink. In this set up, consider the kth relay from the source, and define η_k to be the probability that the packet traverses this node. It can then be shown (Lemma 2 in the Appendix) that $\lim_{k\to\infty}\eta_k = \frac{1}{2-p}$. Thus, for large k, the probability that the link to the immediate neighbour towards

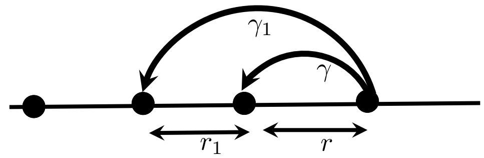

Fig. 2. Illustration of the MDP state.

the sink is used is $\frac{p}{2-p}$, whereas the probability that the other link is used is $\frac{1-p}{2-p}$. Based on this analysis we take $c=\frac{p}{2-p}$ and $c_1 = \frac{1-p}{2-p}$.

V. Optimal Deployment

A. Geometrically Distributed L: Discounted cost MDP

The problem is to place the relay nodes sequentially such that the expected sum of the total power cost in all links and the relay cost is minimized. We formulate this problem as a Markov Decision Process with state $(r, r_1, \gamma, \gamma_1)$, where r is the distance of the current location from the previous node, γ is the transmit power required to establish a link to the previous node from the current location (with a preset target outage probability), r_1 is the distance between the previous placed relay from the next to previous placed relay, and γ_1 is the transmit power required for link establishment to that node (Figure 2). Based on $(r, r_1, \gamma, \gamma_1)$ a decision is made whether to place a relay at the current position or not. $\mathbf{0}$ denotes the state at the beginning of the process (at the sink node), and $(\mathbf{0}; r)$ denotes the state where a relay has just been placed at the current location and the distance between the current location and the previous relay location is r. When the source is placed, the deployment process terminates. The action space is $\{$*place*, *do not place*$\}$.

Recall the definition of $\Gamma^{(m,n)}$ from Section III. The problem we seek to solve is:

$$\min_{\pi\in\Pi} \mathbb{E}_\pi \left(c\sum_{i=1}^{N+1} \Gamma^{(i,i-1)} + c_1 \sum_{i=2}^{N+1} \Gamma^{(i,i-2)} + \xi N \right) \tag{2}$$

where Π is the set of all stationary deterministic Markov placement policies, since by Proposition 1.1.1 of Bertsekas [7], we can restrict ourselves to this class of policies. Let us define $J_\xi(r, r_1, \gamma, \gamma_1)$ and $J_\xi(\mathbf{0})$ to be the optimal cost-to-go starting from state $(r, r_1, \gamma, \gamma_1)$ and $\mathbf{0}$ respectively.

Bellman Equation: We have an infinite horizon total cost MDP with finite state space, finite action space and nonnegative single-stage costs. Hence, by Proposition 3.1.1 of [7], the optimal value function $J_\xi(\cdot)$ satisfies the following Bellman equation (the explanation is provided after the expressions):

$$J_\xi(r, r_1, \gamma, \gamma_1) = \min\{c_p, c_{np}\}; r + r_1 \leq B-1 \tag{3}$$

$$J_\xi(r, B-r, \gamma, \gamma_1) = c_p(r, B-r, \gamma, \gamma_1) \tag{4}$$

Where c_p and c_{np} are given by,

$$c_p(r, r_1, \gamma, \gamma_1) = c\gamma + c_1\gamma_1 + \xi + J_\xi(\mathbf{0}; r) \tag{5}$$

$$\begin{aligned} c_{np}(r, r_1, \gamma, \gamma_1) \quad &= \quad \theta\mathbb{E}(c\Gamma_{r+1} + c_1\Gamma_{r+r_1+1}) + (1-\theta) \\ & \qquad \mathbb{E}J_\xi(r+1, r_1, \Gamma_{r+1}, \Gamma_{r+r_1+1}) \end{aligned} \tag{6}$$

$$J_\xi(\mathbf{0};r) = \theta[c\mathbb{E}(\Gamma_1) + c_1\mathbb{E}(\Gamma_{r+1})] + (1-\theta) \\ \mathbb{E}J_\xi(1,r,\Gamma_1,\Gamma_{r+1}); r \le B-1 \quad (7)$$

Consider the current state is $(r, r_1, \gamma, \gamma_1)$ and the line has not ended. If a relay is placed, a cost of $c\gamma + c_1\gamma_1 + \xi$ and an additional cost of $J_\xi(\mathbf{0}; r)$ is incurred. If the relay is not placed and if the line does not end at the next step, the expected cost-to-go from there is $\mathbb{E}J_\xi(r+1, r_1, \Gamma_{r+1}, \Gamma_{r+r_1+1})$. If the line ends (with probability θ), a cost of $\theta\mathbb{E}(c\Gamma_{r+1} + c_1\Gamma_{r+r_1+1})$ is incurred. Unless the first relay is placed, there is only one downstream neighbour with respect to the current location and hence, the typical state in this situation is denoted by (r, γ).

$$\begin{aligned} J_\xi(r,\gamma) &= \min\{\xi + \gamma + J(\mathbf{0}; r), \\ &\quad \theta\mathbb{E}(\Gamma_{r+1}) + (1-\theta)\mathbb{E}J_\xi(r+1, \Gamma_{r+1})\};\ r \le B-1 \\ J_\xi(B,\gamma) &= \xi + \gamma + J(\mathbf{0}; B) \end{aligned} \quad (8)$$

The optimal cost-to-go from state $\mathbf{0}$ (sink) is given by:

$$J_\xi(\mathbf{0}) = \theta\mathbb{E}(\Gamma_1) + (1-\theta)\mathbb{E}J_\xi(1,\Gamma_1) \quad (9)$$

Value Iteration: The value iteration for (2) is given by the same set of equations (3) to (9), where $J_\xi(\cdot)$ in the L.H.S of each equation is replaced by $J_\xi^{(k+1)}(\cdot)$ and $J_\xi(\cdot)$ in the R.H.S of each equation is replaced by $J_\xi^{(k)}(\cdot)$. We must initialize $J_\xi^{(0)}(\cdot) = 0$ for all states.

Lemma 1. *The value iteration provides a nondecreasing sequence of iterates that converges to the optimal value function, i.e.,* $J_\xi^{(k)}(\cdot) \uparrow J_\xi(\cdot)$ *for all states as* $k \uparrow \infty$.

Theorem 2. *(**Policy Structure**) The optimal policy for Problem (2) is a threshold policy with a threshold* $\gamma_{th}(r, r_1)$ *such that at a state* $(r, r_1, \gamma, \gamma_1)$ *it is optimal to place a relay if and only if* $c\gamma + c_1\gamma_1 \le \gamma_{th}(r, r_1)$. *This corresponds to the condition* $c_p \le c_{np}$.

Proof: See Appendix A. ■

B. Formulation via an Average Cost MDP: Infinite length line

Let us denote the number of relays placed within x steps by N_x. In this subsection, we seek to minimize the average cost per step as follows:

$$\inf_{\mu \in \Pi} \limsup_{x\to\infty} \frac{c\sum_{i=1}^{N_x+1}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x+1}\Gamma^{(i,i-2)} + \xi N_x}{x} \quad (10)$$

where Π is the set of stationary, deterministic policies.

For any ξ, let the optimal value function of the problem (2) be denoted by $J_{\xi,\theta}(\mathbf{0})$. By Proposition 4.1.7 of Bertsekas [7], the optimal policy for (10) is the same as that of (2) where θ is sufficiently close to 0 since problem (2) can be considered as infinite horizon discounted cost problem with discount factor $(1-\theta)$ and the state and action spaces are finite. Also, the optimal per-step cost λ^* of problem (10) is equal to $\lim_{\theta\to 0}\theta J_{\xi,\theta}(\mathbf{0})$ (by Section 4.1.1 of Bertsekas [7]).

VI. Computational Examples (Deployment for Minimum Average Cost Per Step)

We take the path loss factor $\eta = 4.7$, the shadowing random variable, W, to be log-normally distributed with $\sigma = 7.7$dB,

	$\xi = 0.001$	$\xi = 0.005$	$\xi = 0.01$
$\overline{u}$ (in steps)	2.8	3.2	3.5
$\overline{\gamma}$ (in mW)	0.0394	0.0439	0.0473
λ^*	0.01121	0.01195	0.01269

TABLE I
Components of network cost, and average network cost: $K = 2$, $p = 0.5$, for various values of relay cost ξ.

	$\xi = 0.001$	$\xi = 0.005$	$\xi = 0.01$
$\overline{u}$ (in steps)	4.2	4.8	5.1
$\overline{\gamma}$ (in mW)	0.0372	0.0417	0.0421
λ^*	0.00982	0.00996	0.01021

TABLE II
Components of network cost, and average network cost: $K = 1$, for various values of relay cost ξ.

$\delta = 6$ meters and $B = 10$ (i.e., the maximum length of a link is 10 steps, i.e., 60 meters). The preset target outage probability is 1%. Relay cost ξ is varied and mean power cost per relay, $\overline{\gamma}$, mean placement distance (in steps of δ), $\overline{u}$, and average cost per step, λ^*, is computed for $p = 0.5$ and $p = 1$. The results are tabulated in Tables I, II.

Discussion: (i) As would be expected, in both cases ($K = 1$ and $K = 2$), the relays are placed farther apart as the relay cost ξ increases. (ii) In both cases ($K = 1$ and $K = 2$), the mean network power cost per link increases as ξ increases. As the relays are placed farther apart, and a target outage needs to be maintained over each selected link, we would expect that the power for each link increases as ξ increases. (iii) Comparing across the two cases ($K = 1$ and $K = 2$), we observe that with $K = 2$, the relays are closer, in order to enable workable links to two previously placed nodes. (iv) A further comparison across the two cases is with respect to the mean power cost per link. With $K = 2$, there is an increase in mean power cost. With $c = c_1$, an equal weight is given to the link with the immediate neighbour and the two hop neighbour. In order to make the two hop link workable more power is needed, thus raising the average power cost for $K = 2$ above that for $K = 1$.

VII. Experimental Results

A total of 22 TelosB motes were deployed in the forest-like Jubilee Park of the Indian Institute of Science (see Figure 3). The length of the trail is 300m and 11 motes were placed on each side of the trail. The distance between successive motes along the trail edge (i.e., step size δ) is 11m. Each relay broadcasts 2000 packets, at each power level, while the others are quiet and collect measurements to assess their link qualities *from* the transmitting node. In this manner, each relay gets a turn to broadcast 2000 packets. At each power level, the average received power, and link outage at each other node is measured. A maximum likelihood approach gave the path-loss exponent, η, as 4.7, the standard deviation of W, σ, as 7.7 dB and the spatial de-correlation distance of W as 6m. We take the step size $\delta = 11$m and B is taken to be 5. The set of possible power transmit levels is $\mathcal{S} = \{-25, -15, -10, -5, 0\}$ (in dBm). A link is said to be in outage if the received signal power (RSSI) is less than -88 dBm. The cost of a link consists of two components, power cost and relay cost (with a preset

(a) (b)

Fig. 3. A segment of the trail, motes were mounted on the trees at a height of about 2 meters. The right panel shows as depiction of the deployment of 22 motes along a stretch of the trail

K	p	Placement Locations (Location no.)	No. of relays	Total Power Cost (in mW)	Total Cost (in mW)
1	1	3,5,7,8,10	5	0.463	0.963
2	0.5	2,3,5,7,8,9,10	7	0.513	1.213

TABLE III
NETWORK REALIZATION FOR THE LEFT SIDE OF THE TRAIL UNDER CONSIDERATION FOR DIFFERENT VALUES OF p

K	p	Placement Locations (Location no.)	No. of relays	Total Power Cost (in mW)	Total Cost (in mW)
1	1	2,3,5,7,9	5	0.536	1.036
2	0.5	3,4,5,6,7,9,10	7	0.606	1.306

TABLE IV
NETWORK REALIZATION FOR THE RIGHT SIDE OF THE TRAIL UNDER CONSIDERATION FOR DIFFERENT VALUES OF p

target outage of 1%). We assume average cost formulation as described in Section V.

Since our experimental set up is quite small, we take $\xi = 0.1$, i.e., a large node cost, in order to prevent the algorithm from placing relays at all potential locations. Given the measurements described above, we have all possible measurements that can be possibly made during an actual deployment. Thus, we can use the measurements to determine the actual network that will be deployed if an agent was to walk along the trail starting from sink at location 1 (Figure 3) and the source at location 11. In Tables III, IV, we report the "virtual" deployment results obtained. In Table III, we see that with 2 connectivity, a total of 7 relays are being placed over a 110m trail whereas with 1 connectivity, the agent places only 5 relays. The "Total Power Cost" columns show the sum of the weighted transmitter powers over all the deployed nodes. We note that this measure is proportional to the rate at which batteries will need to be replaced in the network ([4]). Comparing the $K = 1$ with the $K = 2$ deployment, we notice that the number of deployed relays increased by 40% (from 5 to 7), whereas the total power increased by 10% to 12%. This occurs because, with the nodes closer together, the power required to the nearest neighbours decreases. That, in essence, is the additional operational cost we pay for the increase in path redundancy.

VIII. CONCLUSION AND FUTURE WORK

We have provided an approach for measurement-based as-you-go deployment of a 2-connected wireless relay network along a line, to connect a sensor with a sink, so as to carry very light traffic. The problem was formulated as a Markov decision process and policy structures were obtained. Computational and experimental experience was reported. We found that for a small increase in network cost, path redundancy can be incorporated in the deployed network, thus rendering the network robust to node failures and the inevitable long term variations in link quality.

Our formulation in this paper assumed a given target outage probability for each link. It will be interesting to extend the formulation so that end-to-end outage is itself a part of the cost. Also, we assumed the deployed network uses probabilistic routing, thereby obtaining a simple characterisation of the cost multipliers c and c_1. In practice, however, the routing would be adaptive (using a protocol such as RPL); determining appropriate values for c and c_1 in such a setting will be another important item of future work. Innovative ways to use the redundant downlink neighbours, perhaps using *physical layer* techniques would also be of interest.

APPENDIX A

Lemma 2. *For probabilistic routing in an infinite node 2NN network,* $\lim_{k\to\infty} \eta_k = \frac{1}{2-p}$.

Proof: It is easily seen that

$$\eta_k = p\eta_{k+1} + (1-p)\eta_{k+2} \tag{11}$$

Clearly, $\eta_0 = 1$, $\eta_1 = p$. We take z-transform on both sides of (11). After rearranging, taking the inverse z-transform and taking the limit $k \to \infty$, $\lim_{k\to\infty} \eta_k = \frac{1}{2-p}$. ■

Proof of Theorem 2 By Proposition 3.1.3 of [7], when the state is $(r, r_1, \gamma, \gamma_1)$ with $r+r_1 \leq B-1$, it is optimal to place the relay if $c_p \leq c_{np}$, i.e.,

$$\begin{aligned} & c\gamma + c_1\gamma_1 \leq \theta\mathbb{E}(c\Gamma_{r+1} + c_1\Gamma_{r_1+1}) \\ & +(1-\theta)\mathbb{E}J_\xi(r+1, r_1+1, \Gamma_{r+1}, \Gamma_{r_1+1}) - (\xi + J_\xi(\mathbf{0}; r)) \\ & =: \gamma_{th}(r, r_1) \end{aligned}$$

REFERENCES

[1] T. Aurisch and J. Tölle. Relay placement for ad-hoc networks in crisis and emergency scenarios. In *Proc. of the Information Systems and Technology Panel (IST) Symposium*. NATO Science and Technology Organization, 2009.

[2] A. Howard, M.J. Matarić, and G. S. Sukhatme. An incremental self-deployment algorithm for mobile sensor networks. *Kluwer Autonomous Robots*, 13(2):113–126, 2002.

[3] A. Chattopadhyay, M. Coupechoux, and A. Kumar. Measurement based impromptu deployment of a multi-hop wireless relay network. In *Proc. of the 11th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*. IEEE, 2013.

[4] P. Mondal, K.P. Naveen, and A. Kumar. Optimal deployment of impromptu wireless sensor networks. In *Proc. of the IEEE National Conference on Communications (NCC),2012*. IEEE, 2012.

[5] M.R. Souryal, J. Geissbuehler, L.E. Miller, and N. Moayeri. Real-time deployment of multihop relays for range extension. In *Proc. of the ACM International Conference on Mobile Systems, Applications and Services (MobiSys), San Juan, Puerto Rico, June 2007*, pages 85–98. ACM, 2007.

[6] H. Liu, J. Li, Z. Xie, S. Lin, K. Whitehouse, J.A. Stankovic, and D. Siu. Automatic and robust breadcrumb system deployment for indoor firefighter applications. In *Proc. of the ACM International Conference on Mobile Systems, Applications and Services (MobiSys)*, 2010.

[7] D.P. Bertsekas. *Dynamic Programming and Optimal Control, Vol. II*. Athena Scientific, 2007.