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Delay Optimal Event Detection on Ad Hoc Wireless Sensor Networks

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Extended Abstract

The classical *sequential change detection problem* [1] aims at detecting a change in the state of a system from the *normal state* to the *abnormal state* at a random time T. The state of the system can not be observed directly and is observed only partially through the noisy observations $\mathbf{X}_k$ which are obtained at discrete time instants $k = 1, 2, 3, \cdots$. Given the state of the system, $\mathbf{X}_k$s are conditionally i.i.d. (over time and across sensors) and the distribution of $\mathbf{X}_k$ before change is given by F_0 and that after change is given by F_1. At each time k, after having observed $\mathbf{X}_k$, the decision maker (DM) has to make a decision to stop and raise an alarm or continue sampling at time $k+1$. The problem is to obtain a detection–delay optimal stopping rule with respect to the sequence $\mathbf{X}_1, \mathbf{X}_2, \cdots$ which stops at time τ that minimises the expected detection delay $\mathsf{E}\left[(\tau - T)^+\right]$ while the probability of false alarm $\mathsf{P}\{\tau < T\}$ does not exceed a desired quantity α. It is to be noted that at each time k, the DM has the knowledge of the observations $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_k$, the distribution functions F_0, F_1, and the distribution of the change time T.

In practical change detection applications in wireless sensor networks, a number of sensor nodes (say N) are deployed that sample a physical quantity (for e.g., temperature, pressure, vibrations, thermal energy etc) periodically at a sampling rate of r samples/slot (i.e., the sampling time instants are $1/r, 2/r, 3/r, \cdots$), and communicate the observations to a fusion center. *The communication between the sensor nodes and the fusion center is facilitated by a wireless network based on a random access mechanism, such as the CSMA standards codified in IEEE 802.11 or IEEE 802.15.4.* An event occurs at a random time and causes a change in the statistics of the sensor measurements, and hence, can be detected by sequential change detection procedures ([1], [2]). However, the medium access control (MAC) protocol of the network between the sensor nodes and the fusion center poses the following challenges:

1) The samples of different sensors of a batch b are generated at the same time instant $t_b = b/r$, but arrive at the fusion center at different time instants $t_b + D_b^{(1)}, t_b + D_b^{(2)}, \cdots, t_b + D_b^{(N)}$ (where the network delays $D_b^{(i)} \geq 1$), i.e., different samples of the same batch reach the fusion center after different network delays
2) An observation that is generated earlier may arrive at the fusion center later, i.e., for some sensor nodes i, j, and for some batch indices $b' < b$, the reception time at the fusion center of the sample $X_b^{(j)}$ can be earlier than that of $X_{b'}^{(i)}$ (i.e., $t_b > t_{b'}$ and $t_b + D_b^{(j)} < t_{b'} + D_{b'}^{(i)}$).

In this problem, we are interested in obtaining an optimal detection procedure that minimises the detection delay between the change time and the stopping time while the probability of false alarm does not exceed a desired quantity α. Furthermore, we are also interested in obtaining an optimal sampling rate r which minimises the mean detection delay.

In order to process the samples that reach the fusion center with different (random) delays, the fusion center is equipped with sequencer queues, one corresponding to each sensor node. We consider two strategies at the fusion center to process the received observations:

- **Network Oblivious Decision Making** (NODM) [3] : The sequencer at the fusion center queues the samples as they arrive until all the samples of a "batch" (batch b is the vector of samples $[X_b^{(1)}, X_b^{(2)}, \cdots, X_b^{(N)}]$ generated at the sampling instant $t_b = b/r$) are accumulated. The DM processes the batches at those time

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instants ($U_b := t_b + D_b := \max\{t_b + D_b^{(i)} : i = 1, 2, \cdots, N\}$, $b = 1, 2, \cdots$) when a complete batch is received. Here, the reception times at the fusion center and the number of outstanding packets in the sequencer queues are not used in making a decision. Hence, we call this, *Network Oblivious Decision Making (*NODM*)*. Let $\widetilde{U}$ be the time instant at which the DM stops and declares an alarm. It is to be noted here that the detection delay $\widetilde{U} - T$ is the sum of i) *sampling delay* $t_K - T$, where K is the index of the first batch after the change point, ii) *network delay* due to Fork–Join queueing (FJQ) in the network $D_{\widetilde{K}}$, where $\widetilde{K}$ is the index of the batch at which the DM stops, and iii) *decision delay* due to the DM $\frac{1}{r}(\widetilde{K} - K)$. We show, under periodic sampling, that the delay components (sampling delay, network delay, decision delay) decouple additively.

Theorem 1 *([3], [4]) If the sampling is periodic at rate r and the batch sojourn time process D_b, $b \geqslant 1$, is stationary with mean $d(r)$, then*

$$\min_{\Pi_\alpha} \mathsf{E}\left[(\widetilde{U} - T)\mathbf{1}_{\{\widetilde{T} \geqslant T\}}\right] = (d(r) + l(r))(1 - \alpha) - \rho \cdot l(r) + \frac{1}{r} \min_{\Pi_\alpha} \mathsf{E}\left[\left(\widetilde{K} - K\right)^+\right]$$

where $l(r)$ is the delay due to (coarse) sampling, α is the probability of false alarm constraint, and $\rho :=$ $\mathsf{P}\{T \leq 0\}$.

From Theorem 1, we infer that under periodic sampling, the optimum NODM procedure is the same as the Shiryayev rule [1]. Thus, in NODM, the optimal decision maker continues to have the same structure as in the classical problem without the network.

- **Network Aware Decision Making (NADM)** [4]: In this case, the fusion center contains a sequencer between the arriving samples and the DM. The sequencer releases samples to the DM only in the order in which they were generated at the sensors, but does not wait for an entire batch to accumulate. The decision maker processes samples as they arrive. We call this, *Network Aware Decision Making* (NADM). The out–of–time sequence samples (the samples that are generated later compared to that of the batch that is currently under processing) that the fusion center receives are queued in the sequencer queues and are released to the DM at the earliest time instant when all the samples from previous batches are processed.
The service model at the MAC between the sensor nodes and the fusion center is the generalized processing sharing (GPS) which is explained as follows. As long as there are packets in any of the queues, successes are assumed to occur at the constant rate of σ $(0 < \sigma < 1)$ per slot, with the intervals between the successes being i.i.d., geometrically distributed random variables, with mean $1/\sigma$. If, at the time a success occurs, there are n nodes contending (i.e., n queues are nonempty) then the success is ascribed to any one of the n nodes with equal probability. For the queues to be stable, we require the sampling rate $r < \sigma/N$.
Because of MAC, at time k, the samples that are processed by the DM suffered a delay of Δ_k in the network. Thus, the samples that are processed carry a delayed information about the (normal/abnormal) state of the system. Thus, we define a discrete–time dynamical system with the state being the vector $\mathbf{\Theta}_k = [\Theta_{k-\Delta_k}, \Theta_{k-\Delta_k+1}, \Theta_k]$, and we are interested in detecting the change as early as possible subject to a false alarm constraint, i.e.,

$$\begin{aligned} \min \quad & \mathsf{E}\left[(\tau - T)^+\right] \\ \text{subject to} \quad & \mathsf{P}\left\{\tau < T\right\} \leqslant \alpha. \end{aligned} \tag{1}$$

We formulate the event detection problem as a partially observable Markov decision process. Even though the state of the dynamical system is $[\Theta_{k-\Delta_k}, \Theta_{k-\Delta_k+1}, \Theta_k]$, we show that a sufficient statistic at time k is $[\mathbf{Q}_k, \Pi_k]$, where $\mathbf{Q}_k$ is the network–state and $\Pi_k := \mathsf{P}\left\{T \leq k | \mathbf{I}_k\right\}$ is the a posteriori probability of change having occurred at or before time slot k given that the DM has received the information $\mathbf{I}_k$. By using Bellman's equation, we obtain the optimal event detection rule. We show that the optimal policy is a network–state dependent threshold rule.

Theorem 2 *The optimal stopping rule τ^* is a network–state dependent threshold rule on the a posteriori probability Π_k, i.e., there exists thresholds $\gamma(\mathbf{q})$ such that*

$$\tau^* = \inf\{k \geqslant 0 : \Pi_k \geqslant \gamma(\mathbf{Q}_k)\} \tag{2}$$

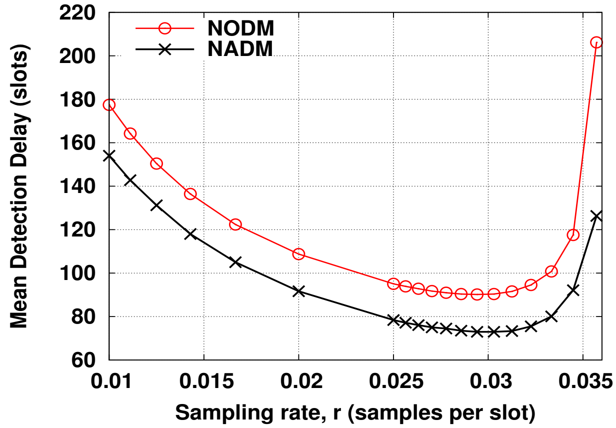

Fig. 1. Mean detection delay for $N = 10$ nodes is plotted against the sampling rate r for both NODM and NADM. This was obtained with the prior probability $\rho = 0$, $p = 0.0005$, probability of false alarm target $\alpha = 0.01$, $\sigma = 0.3636$ and with the sensor observations being $\mathcal{N}(0,1)$ and $\mathcal{N}(1,1)$, before and after the change respectively.

It is to be noted that the set of network–states is large, and hence, we numerically study only a suboptimal detection procedure which is a simple threshold rule.

$$\tau^{\mathsf{sub}} = \inf\{k \geqslant 0 : \Pi_k \geqslant \gamma\}. \tag{3}$$

The threshold γ is chosen such that the probability of false alarm criterion is met with equality.

Numerical Results: Based on the optimal detection rules thus obtained, we study the detection delay performance of the NODM and the suboptimal NADM given in Eqn. 3 as a function of the sampling rate r. The distribution of sensor observations are taken to be $\mathcal{N}(0,1)$ and $\mathcal{N}(1,1)$[1], before and after the change respectively for all the 10 nodes. We choose the probability of occurrence of change in a slot to be $p = 0.0005$, i.e., the mean time until change is 2000 slots. $\min_{\Pi_\alpha} \mathsf{E}\left[(\widetilde{K} - K)^+\right]$ and $d(r)$ are obtained from simulation for $\alpha = 0.01$ and $\sigma = 0.3636$ and the mean detection delay for the NODM and the sub–optimal NADM is plotted against r in Figure 1. Note that in both the procedures, for a sampling rate r, we compute the thresholds that satisfies the probability of false alarm constraint (with equality) and use the thresholds thus obtained to obtain the mean detection delay.

As would have been expected, we see from Figure 1 that the NADM procedure has a better mean detection delay performance than the NODM procedure. Note that $\sigma/N = 0.03636$ and hence for the queues to be stable, the sampling rate has to be less that $\sigma/N = 0.03636$ ($1/28 < 0.03636 < 1/27$). When the sampling rate r is small, the mean detection delay is large which occurs due to large sampling and decision delays. When the sampling rate r is large, the mean detection delay is large due to congestion in the network (large network delay) There is a tradeoff between the sampling rate and the detection delay and that there exits an optimal sampling rate r^* for which the detection delay is minimum. We observe from Figure 1 that as the sampling rate r increases to 1/28 (the maximum allowed sampling rate), the queueing delay increases rapidly. Also, we see from Figure 1 that operating at a sampling rate around $1/34 (\approx 0.0294)$ samples/slot would be optimum. The optimal sampling rate is found to be approximately the same for NODM and NADM. At the optimal sampling rate, the mean detection delay of NODM is 90 slots and that of NADM is 73 slots.

In this Extended Abstract we have provided a summary of our work in [3] and [4]. Further details can be found in those publications and in the PhD thesis [5].

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[1]As usual, $\mathcal{N}(a, v)$ denotes a normal distribution with mean a and variance v

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