Optimum Association of Mobile Wireless Devices with a WLAN–3G Access Network

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***Abstract*— In this paper, we consider the problem of association of wireless stations (STAs) with an access network served by a wireless local area network (WLAN) and a 3G cellular network. There is a set of WLAN Access Points (APs) and a set of 3G Base Stations (BSs) and a number of STAs each of which needs to be associated with one of the APs or one of the BSs. We concentrate on downlink bulk elastic transfers. Each association provides each STA with a certain transfer rate. We evaluate an association on the basis of the sum log utility of the transfer rates and seek the utility maximizing association. We also obtain the optimal time scheduling of service from a 3G BS to the associated STAs. We propose a fast iterative heuristic algorithm to compute an association. Numerical results show that our algorithm converges in a few steps yielding an association that is within 1% (in objective value) of the optimal (obtained through exhaustive search); in most cases the algorithm yields an optimal solution.**

I. Introduction

In third generation (3G) wireless cellular networks, mobile multimedia services requiring data rates of the order of 100s of Kbps are envisaged. The sophisticated, wide coverage infrastructure of wireless cellular networks is expensive and it is difficult to meet the ever increasing traffic demands. Wireless Local Area Networks (WLANs), on the other hand, are easy to install and can coexist with a cellular network. WLANs are being widely deployed in public areas, such as campuses, hotels and offices. Thus, a seamless integration of WLANs and 3G cellular networks is imminent. In this paper, we focus on the problem of optimal association of a number of wireless stations (STAs) with an access network comprising WLAN Access Points (APs) and 3G Base Stations (BSs).

In many practical wireless systems including the IS-856 (HDR) system, an STA is associated with the BS from which it can receive the highest signal to interference plus noise ratio (SINR). In WLANs, each STA detects its nearby APs and associates itself with the AP from which it has the strongest received signal strength, while ignoring load considerations. In this paper, we formulate an association problem in which STAs associate with BSs or APs so as to maximize a network objective function.

Recently, there has been considerable work done in the area of optimal association and resource allocation ([1],[10],[14],[3]). Hanly [10] and Yates and Huang [14] studied the base station assignment problems for a CDMA cellular system. They provided a combined power control and cell site selection algorithm that minimizes the transmit power while meeting the uplink target SINR. Lee, Mazumdar and Shroff [11] studied the downlink rate and power allocation and base station assignment problem, again for a CDMA cellular system. They formulated the power allocation and base station assignment problem based on a pricing approach. Kumar and Kumar [1] formulate the optimal association problem for WLANs as a log sum utility maximization problem and obtain solutions for some simple situations. Bejerano et. al. [3] also consider the same problem. They propose the idea of fractional association, formulate a linear programming (LP) problem and relate the result of the LP problem to the integer solution using graph theoretic ideas.

In this paper, we study the association and scheduling problem when there is a heterogenous access infrastructure comprising 3G BSs and WLAN APs. Several STAs seek to connect to a 3G cellular network or to a WLAN with the objective of performing elastic downloads. We formulate the association problem as a cooperative sum utility maximization problem. The utility obtained by each STA is the log of the TCP bulk download rate obtained by the STA in an association. Our work differs from [10] and [14] since these papers are concerned with fixed rate services each with a target E_b/N_0; this yields the problem of setting transmitter power so as to achieve target carrier-to-interference ratio (CIR). In [11] the problem of joint power and rate control which maximizes the downlink expected throughput is studied. Our present work is an extension of the work reported in [1] where the problem of optimal association with WLAN APs was considered, while here we consider optimal association in a heterogenous network of WLAN and 3G cellular network. We formulate the problem and propose an iterative "greedy search" algorithm for obtaining a good association in a few iterations. We also give a sufficient condition to check if the association is optimal. An extensive numerical study of our algorithm is provided.

The rest of the paper is organized as follows. In Section II, we describe the WLAN and 3G system with respect to the association, physical rates, and the throughput. We define the system utility and state the optimal association problem based on the system utility in Section III. We analyze the optimum association problem in Section IV. We present the algorithm to determine a good association in Section V. A sufficient condition on the optimality of the solution given by the algorithm is also discussed. We provide numerical results in Section VI. Section VII concludes the paper.

II. Association, Physical rates, and Throughput

There are l WLAN APs indexed by the set $\mathcal{L} = \{1, 2, \ldots, l\}$ and b BSs indexed by the set $\mathcal{B} = \{l+1, l+2, \ldots, l+b\}$. Let the set $\mathcal{N}$ be $\mathcal{L} \cup \mathcal{B}$. Let $n = l + b$. There are m STAs indexed by the set $\mathcal{M} = \{1, 2, \ldots, m\}$. Let $a_i \in \mathcal{N}$ be the AP or BS to which STA i is associated. Let $A = (a_1, a_2, \ldots, a_m)$ denote an association vector, i.e., $A \in \mathcal{A} = \mathcal{N}^m = \{(a_1, a_2, \ldots, a_m) : a_i \in \mathcal{N}, \forall i \in \mathcal{M}\}$. Let $\mathcal{S}_j$ be the set of STAs associated with AP/BS j, $j \in \mathcal{N}$ and let $m_j = |\mathcal{S}_j|$, be the number of STAs associated with AP/BS j, $j \in \mathcal{N}$. We focus on downlink elastic bulk data transfers to the STAs (e.g., TCP controlled file transfers). We assume the channel gains to be static over the time scale of the optimization. Given an association $A = (a_1, a_2, \ldots, a_m)$, let r_{ia_i} be the raw physical data rate with which STA i is associated with a_i, $a_i \in \mathcal{N}$ ($a_i \in \mathcal{L}$ or $a_i \in \mathcal{B}$). Since we are considering a CDMA cellular network, if $a_i \in \mathcal{B}$, then in general r_{ia_i} will depend on the interference and hence the associations of the other STAs to BSs.

A. Physical rates: WLAN AP to a STA

Let STA i be associated with AP j at the raw physical data rate $r_{ij} \in \mathcal{C}$. For example, IEEE 802.11b supports the set of physical data rates $\mathcal{C} = \{1, 2, 5.5, 11\}$ Mbps. These rates are achieved depending on the distance between the AP and the STA. For simplicity, we assume that the WLAN coverage is such that an STA that can be associated with a WLAN AP does so at the maximum physical data rate. We also assume that the cochannel APs are placed sufficiently far apart such that there is no intercell cochannel interference [12].

B. Physical rates: 3G BS to a STA

Let G_{ij} be the channel gain (which includes distance loss, shadowing and fading) between STA i and BS j. Let P_{ij} be the power transmitted from BS j to STA i and let $P^{(j)}$ be the total power transmitted from BS j to STAs $i \in \mathcal{S}_j$. Let P_T be the maximum transmit power of any BS. If N_0 is the one sided noise power spectral density and W is the system bandwidth, then the received downlink SINR of STA i when associated with BS j is given by

$$\Gamma_{ij} = \frac{G_{ij} P_{ij}}{\sum_{k \neq j} G_{ik} P^{(k)} + N_0 W}. \tag{1}$$

assuming that the users of the same cell use orthogonal spreading codes, and ignoring intracell interference caused by multipath.

We follow the model given in [13] for computing the transmission rate as a function of SINR and system bandwidth. If the physical rate r_{ij} is used from BS j to STA i then the energy-per-bit to noise power spectral density ratio of STA i when associated with BS j is given by

$$\frac{W}{r_{ij}} \cdot \Gamma_{ij} \tag{2}$$

In order for STA i to be able to decode the BS's signal with an acceptable probability of error ϵ, it is necessary that the energy-per-bit to noise power spectral density ratio exceeds γ, where, for a given digital modulation scheme, $\gamma = \gamma(\epsilon)$ is a threshold which is determined by the probability of error ϵ. Then from Eqn. 2, the achievable physical rate is given by

$$\begin{aligned} r_{ij} &= \frac{W}{\gamma} \cdot \Gamma_{ij} \\ &= \alpha \Gamma_{ij} \end{aligned} \tag{3}$$

where $\alpha = W/\gamma$. It should be noted from Eqn. 3 that, with γ fixed, the achievable physical rate r_{ij} between STA i and BS j is linearly proportional to the SINR Γ_{ij}.

C. TCP Bulk Throughput

We consider persistent, TCP controlled (local area) elastic data transfers to all m users. Let θ_{ij} be the TCP throughput to STA i when associated with AP or BS j from server in the local area.

Assuming that when an STA associates with an AP it does so at a fixed rate (e.g., 11 Mbps) and the packet error rate is negligible, it can be shown that the TCP bulk transfer download throughput is of the form $\theta_{ij} = \Theta_0/m_j$ if m_j STAs are associated with the AP with which STA i is associated, where Θ_0 is a number that depends on the IEEE 802.11 MAC parameters and the TCP packet length (see [8] & [9]). For example, with a physical data rate of 11 Mbps, TCP packet size of 1500 B, RTS/CTS for data packets and basic access for TCP acks, Θ_0 = 4.3 Mbps.

In a pedestrian or static scenario, when an STA i is associated with a BS j we assume that the target E_b/N_0, γ is achieved and hence the packet error rate is sufficiently low (e.g., 10^{-2}). Then $\theta_{ij} = r_{ij}$. In the case of a 3G cellular network, it is shown in [2] that the optimal sum physical rate can be achieved if each BS transmits to only one data user at a time (regardless of the topology of the network,the presence of voice users in the cell, and user locations) and with maximum power (when there are only data users and no voice users), i.e., the multiple users associated with BS j are served in a time division multiple access (TDMA) mode. It is to be noted that HDR uses TDMA. We thus take $P_{ij} = P_T, \forall i \in \mathcal{M}, \forall j \in \mathcal{B}$ and $P^{(j)} = P_T, \forall j \in \mathcal{B}$ and a slotted system for 3G cellular network where the users are served one at a time ([2]). Let $\phi_i^{(j)}$ be the fraction of time STA i is serviced by BS j. Since, we consider local area bulk TCP throughput for low packet error rates, $\theta_{ij} = \phi_i^{(j)} r_{ij}$ is the average TCP throughput that STA i gets on association with BS j. Note that under this model r_{ij} does not depend on the association.

We define an $m \times n$ matrix $\mathbf{R}$, where $r_{ij},\ \forall i \in \mathcal{M},\ \forall j \in \mathcal{B}$, represents the physical rate that STA i gets from BS j and $r_{ij} = \Theta_0,\ \forall i \in \mathcal{M},\ \forall j \in \mathcal{L}$ is the bulk TCP throughput that STA i gets when it is the only STA associated with an AP.

III. The Downlink Optimal Association Problem

When user i is associated with AP or BS j and obtains throughput θ_{ij} we evaluate the utility obtained by user i as

$U(\theta_{ij})$ where $U(.)$ is an increasing concave function. We define the system utility as the sum of the individual utilities of STAs each of which is associated with either a BS or an AP. We are interested in finding the optimal association A^* that maximizes the system utility.

$$A^* = \arg\max_{A\in\mathcal{A}} \sum_{i\in\mathcal{M}} U(\theta_{ia_i}) \tag{4}$$

Based on the discussion in Section II-C, Eqn. 4 can be written as

$$\begin{aligned} A^* &= \arg\max_{A\in\mathcal{A}} \left(\sum_{j\in\mathcal{L}} \sum_{i\in\mathcal{S}_j} U(\theta_{ij}) + \sum_{j\in\mathcal{B}} \sum_{i\in\mathcal{S}_j} U(\theta_{ij}) \right) \\ &= \arg\max_{A\in\mathcal{A}} \left(\sum_{j\in\mathcal{L}} m_j U\left(\frac{\Theta_0}{m_j}\right) + \sum_{j\in\mathcal{B}} \sum_{i\in\mathcal{S}_j} U\left(\phi_i^{(j)} r_{ij}\right) \right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_j} \phi_i^{(j)} = 1, \forall j \in \mathcal{B} \end{aligned} \tag{5}$$

The variables $\phi_i^{(j)}$, $\forall j \in \mathcal{B}$, model the TDM scheduling of STAs associated with BS j.

In this paper, we consider $U(\cdot) = \log(\cdot)$ (see [1]). Thus, we have the optimum association problem

$$\begin{aligned} A^* &= \arg\max_{A\in\mathcal{A}} \left(\sum_{j\in\mathcal{L}} m_j \log\left(\frac{\Theta_0}{m_j}\right) + \sum_{j\in\mathcal{B}} \sum_{i\in\mathcal{S}_j} \log\left(\phi_i^{(j)} r_{ij}\right) \right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_j} \phi_i^{(j)} = 1, \forall j \in \mathcal{B} \end{aligned} \tag{6}$$

Remarks 1:

(i) It should be noted that for computing the optimal association, the rates $r_{ij}, \forall i \in \mathcal{M}$, $\forall j \in \mathcal{N}$ should be known at a central device.

(ii) Note also that the solution of this problem will also yield $\phi_i^{(j)}, j \in \mathcal{B}, i \in \mathcal{S}_j$.

IV. Analysis of the Optimum Association Problem

In this section, we study the optimization problem (Eqn. 6) we posed in Section III. We divide the above problem into two sub-problems. The first sub-problem is finding the optimum partition of m STAs into two groups, one comprising STAs associated with the 3G cellular network, and the other comprising those STAs associated with the WLAN. The second subproblem is finding the optimum association in each of the partitions. Let the subset of STAs to be associated with the WLAN be $\mathcal{S}_W$ and let $|\mathcal{S}_W| = m_W$. Also, let the subset of STAs to be associated with the 3G cellular network be $\mathcal{S}_C$ (where $\mathcal{S}_C = \mathcal{M} - \mathcal{S}_W$). The first sub-problem will yield $\mathcal{S}_W$ and $\mathcal{S}_C$. The second sub-problem requires only m_W for the WLAN. But, $\mathcal{S}_C$ is required as a whole for the association with the 3G cellular network.

A. Sub-Problem: Optimum Association among APs

Given m_W, we need to find the optimal $m_1, m_2, \ldots, m_l$ such that $m_1 + m_2 + \ldots + m_l = m_W$. For the log utility function, we have

$$\begin{aligned} & \max_{m_1,m_2,\ldots,m_l} \sum_{j\in\mathcal{L}} m_j \log\left(\frac{\Theta_0}{m_j}\right) \\ \text{s.t.} \quad & \sum_{j\in\mathcal{L}} m_j = m_W \\ & m_j \in \{0, 1, 2, \ldots, m_W\}, \ \forall j \in \mathcal{L} \end{aligned} \tag{7}$$

We have an integer programming problem. Let us replace m_j by m_j/m_W. Notice that this does not change the optimization problem. We relax the integer constraint by setting $x_j = m_j/m_W$ and letting $x_j \in [0, 1]$. This yields the following *relaxed problem* whose solution will provide an upper bound to the maximization problem defined in Eqn. 7.

$$\begin{aligned} & \max_{x_1,x_2,\ldots,x_l} \sum_{j\in\mathcal{L}} x_j \log\left(\frac{\Theta_0}{x_j}\right) \\ \text{s.t.} \quad & \sum_{j\in\mathcal{L}} x_j = 1 \\ & x_j \geq 0, \ \forall j \in \mathcal{L} \end{aligned} \tag{8}$$

The above problem is a concave maximization problem with linear constraints. Solving this problem gives $x_j^* = 1/l$. Hence, $m_j^* = m_W/l, \forall j \in \mathcal{L}$ solves the original problem, defined in Eqn. 7, *whenever these are integers*. In such a case, the optimum is to equally divide the STAs over the APs.

B. Sub-Problem: Optimum Association among BSs

Given $\mathcal{S}_C$, define $m_C = |\mathcal{S}_C|$. We need to find the optimal $\mathcal{S}_j, \ j \in \mathcal{B}$, such that $\cup_{j\in\mathcal{B}}\mathcal{S}_j = \mathcal{S}_C$. For the log utility function, we have

$$\begin{aligned} & \max_{\mathbf{\Phi},\mathcal{S}_j,\forall j\in\mathcal{B}} \sum_{j\in\mathcal{B}} \sum_{i\in\mathcal{S}_j} \log\left(\phi_i^{(j)} r_{ij}\right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_j} \phi_i^{(j)} = 1, \ \forall j \in \mathcal{B} \\ & \phi_i^{(j)} \geq 0, \ \forall i \in \mathcal{S}_j, \ \forall j \in \mathcal{B} \\ & \cup_{j\in\mathcal{B}}\mathcal{S}_j = \mathcal{S}_C \\ & \mathcal{S}_j \cap \mathcal{S}_k = \emptyset, \forall j \neq k, j \in \mathcal{B}, k \in \mathcal{B} \end{aligned} \tag{9}$$

where $\mathbf{\Phi} = [\phi_i^{(j)}], \forall i \in \mathcal{S}_j, \forall j \in \mathcal{B}$.

The above maximization problem can be resolved into two maximization problems one over $\mathbf{\Phi}$ and the other over the partitions $\mathcal{S}_j, \forall j \in \mathcal{B}$ of $\mathcal{S}_C$. Given a partition of $\mathcal{S}_C$ into $\mathcal{S}_j$, $j \in \mathcal{B}$, the maximization problem over $\mathbf{\Phi}$,

$$\begin{aligned} & \max_{\mathbf{\Phi}} \sum_{j\in\mathcal{B}} \sum_{i\in\mathcal{S}_j} \log\left(\phi_i^{(j)} r_{ij}\right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_j} \phi_i^{(j)} = 1, \ \forall j \in \mathcal{B} \\ & \phi_i^{(j)} \geq 0, \ \forall i \in \mathcal{S}_j, \ \forall j \in \mathcal{B} \end{aligned} \tag{10}$$

is a concave maximization problem in $\phi_i^{(j)}$s with linear constraints. It is easily seen that solving this problem yields $\phi_i^{(j)} = 1/|\mathcal{S}_j|, \forall i \in \mathcal{S}_j, \forall j \in \mathcal{B}$.

To solve the second maximization problem, we can now write Eqn. 9 as

$$\begin{aligned} & \max_{\mathcal{S}_j, \forall j \in \mathcal{B}} \sum_{j \in \mathcal{B}} \sum_{i \in \mathcal{S}_j} \log\left(\frac{r_{ij}}{m_j}\right) \\ \text{s.t.} \quad & \cup_{j \in \mathcal{B}} \mathcal{S}_j = \mathcal{S}_C \\ & \mathcal{S}_j \cap \mathcal{S}_k = \emptyset, \forall j \neq k, j, k \in \mathcal{B} \end{aligned} \tag{11}$$

The solution to the above problem yields the optimal $\mathcal{S}_j$, $\forall j \in \mathcal{B}$. Note that this is a nonlinear combinatorial optimization problem, since $\mathcal{S}_j$s are discrete sets.

C. Optimal Association among APs and BSs: A Special Case

Motivated by the above discussion, in this subsection we consider a special scenario of a simplified formulation. We assume that all the STAs are clustered (e.g., as in an auditorium). In this scenario, we assume that all STAs see the same channel to BS j. Thus, for $j \in \mathcal{B}$, we denote r_{ij}, $i \in \mathcal{M}$ by the common value Θ_j. We simplify the formulation as follows. Motivated by the result in Section IV-A, we seek a number $m_0 \geq 0$ of STAs that will be associated with each AP, and a number m_j of STAs that will be associated with BS j. This yields the following problem.

$$\begin{aligned} U^* &= \max_{m_0, m_j, \forall j \in \mathcal{B}} l m_0 \log\left(\frac{\Theta_0}{m_0}\right) + \sum_{j \in \mathcal{B}} m_j \log\left(\frac{\Theta_j}{m_j}\right) \\ \text{s.t.} \quad & l m_0 + \sum_{j \in \mathcal{B}} m_j = m \\ & m_0, m_j, \ \forall j \in \mathcal{B} \text{ are non negative integers.} \end{aligned} \tag{12}$$

We relax the above integer constraints by taking $m_j/m = x_j$. The solution of the relaxed problem will provide an upper bound to the maximization problem defined in Eqn. 12. Thus, the above optimization problem can be written as

$$\begin{aligned} U^* &= \min_{x_0, x_j, \forall j \in \mathcal{B}} \Bigg[l x_0 \log(x_0) + \sum_{j \in \mathcal{B}} x_j \log(x_j) \\ & \quad - l x_0 \log(\Theta_0) - \sum_{j \in \mathcal{B}} x_j \log(\Theta_j) \Bigg] \\ \text{s.t.} \quad & l x_0 + \sum_{j \in \mathcal{B}} x_j = 1 \\ & x_0 \geq 0, x_j \geq 0, \ \forall j \in \mathcal{B} \end{aligned} \tag{13}$$

The above objective is a convex function of x_js and the constraints are linear. Solving the above problem gives the optimal solution,

$$x_0^* = \frac{\Theta_0}{l\Theta_0 + \sum_{k \in \mathcal{B}} \Theta_k} \tag{14}$$

$$x_j^* = \frac{\Theta_j}{l\Theta_0 + \sum_{k \in \mathcal{B}} \Theta_k}. \tag{15}$$

Hence consider, for $j \in \mathcal{B}$,

$$m_0^* = \frac{m}{l\Theta_0 + \sum_{k \in \mathcal{B}} \Theta_k} \cdot \Theta_0 \tag{16}$$

$$m_j^* = \frac{m}{l\Theta_0 + \sum_{k \in \mathcal{B}} \Theta_k} \cdot \Theta_j. \tag{17}$$

If m_0^* and m_j^*, $\forall j \in \mathcal{B}$, are integers then the above solution is optimal and the STAs are distributed over the APs and the BSs in proportion to the Θ_j values. Even though Θ_0 is quite large, (4.3 Mbps for 11 Mbps physical data rate [9]), we see that a positive number of STAs are associated with the BSs.

We look at the following examples to understand the above scenario.

Example 1: We take $m = 6$, $l = 2$, and $b = 2$. Let us consider the scenario where Θ_0 (= Θ_1 = Θ_2) = 4 Mbps, Θ_3 = 2Mbps, and Θ_4 = 2 Mbps. We obtain the optimal association vector, A^* by enumerating all possible associations. The optimal utility based association, A^* is found to be [1 1 2 2 3 4]. It is noted that under optimal association A^*, AP1 and AP2 service two STAs and, BS3 and BS4 service one STA each. This result is in conformance with that of Eqns. 16 and 17. It is to be noted that the resulting m_0^* and m_j^* (given by Eqns. 16 and 17) are integers in this case. If the STAs are associated with APs/BSs based on the rates r_{ij} (as in HDR) then the resulting association A_1 assigns three STAs with AP1 and the remaining three with AP2. Though the rates that individual STAs get when associating with APs seem to be more, it is beneficial to associate some STAs with BSs as this increases both the individual throughput of the STAs and the system utility. In association A_1, the throughput of an STA is 1.33 Mbps whereas in association A^*, the throughput of an STA is 2.0 Mbps. The system utility for association A_1 is 84.62 whereas the utility corresponding to the optimal association, A^* is 87.05.

Example 2: We take $m = 9$, $l = 2$, and $b = 2$. Let us consider the scenario where Θ_0 (= Θ_1 = Θ_2) = 4 Mbps, Θ_3 = 2 Mbps, and Θ_4 = 1 Mbps. The optimal association vector, A^* (obtained through enumeration) for this case is [1 1 1 2 2 2 3 3 4]. Under optimal association A^*, AP1 and AP2 service three STAs and, BS3 service two STAs and BS4 service one STA. In this case, the m_0^* and m_j^* given by Eqns. 16 and 17 are not integers and the optimal number of STAs associated among the APs and BSs are distributed in the ratio of 3:3:2:1 instead of 4:4:2:1. This solution is obtained by exhaustive search. We consider the association A_1 which associates (based on the rates r_{ij}) four STAs with AP1 and the remaining five with AP2 and compare it with that of utility maximizing optimal association, A^*. This example also shows that it is beneficial to associate with BSs as this increases individual STA's throughput and the overall system utility. In association A_1, the throughput of an STA is either 0.8 Mbps or 1.0 Mbps whereas in association A^*, the throughput of an STA is 1.0 Mbps or 1.33 Mbps. The system utility for association A_1 is 123.22 whereas the utility corresponding to the optimal association, A^* is 126.07.

V. A Greedy Search Algorithm

Here, we provide an algorithm to compute a *near optimal association* of STAs with the access network comprising 3G BSs and WLAN APs. We provide the following theorem which gives a sufficient condition for an association to be optimal.

Theorem 1: For a given rate matrix, $\mathbf{R}$, if the association vector A satisfies

$$\theta_{ia_i} \quad \geq \quad \theta_{ij},\ \forall i \in \mathcal{M}, \forall j \in \mathcal{N}, \tag{18}$$

where $\theta_{ij} = \frac{r_{ij}}{m_j},\ \forall i \in \mathcal{M}, \forall j \in \mathcal{N}$, then the association vector A is a utility maximizing (optimal) association vector.

Proof: See Appendix. ■

Remarks 2: This result says that an association is optimal if the throughput an STA gets from the device with which it is associated is at least as large as the throughput it would otherwise get if its association is exchanged with the association of a device associated with any other AP/BS. This can be used as a way to check if an association is optimal. This condition is similar to the *Nash equilibrium* of a pure strategy game.

We now consider a criterion for looking for the best association among a particular set of associations. Let $U(A)$ denote the system utility for the association vector $A = [a_1, a_2, \ldots, a_m]$. Let us denote the association vector $[a_1, a_2, \ldots, a_{i-1}, k, a_{i+1}, \ldots, a_m]$ by A_{ik}. It is to be noted that A_{ik} differs from A only in the ith position. We thus have a collection of associations, $\mathcal{C} = \{A_{ik},\ \forall i \in \mathcal{M},\ \forall k \in \mathcal{L} \cup \mathcal{B}\}$. Let the change in system utility when STA i's association is changed from a_i to k be given by $\Delta(i,k) = U(A_{ik}) - U(A)$.

Lemma 1: The best association in $\mathcal{C}$ is $A_{i^*k^*}$ if and only if

$$\begin{aligned} (i^*, k^*) &= \arg\max_{i \in \mathcal{M}, k \in \mathcal{N}} \Delta(i,k) \\ &= \arg\max_{i \in \mathcal{M}, k \in \mathcal{N}} \Big[-(m_k+1)\log(m_k+1) \\ &\quad + m_k \log(m_k) - (m_{a_i}-1)\log(m_{a_i}-1) \\ &\quad + m_{a_i}\log(m_{a_i}) + \log\left(\frac{r_{ik}}{r_{ia_i}}\right)\Big] \end{aligned} \tag{19}$$

where m_{a_i} is the number of STAs associated with each AP/BS a_i under association A.

Proof: See Appendix. ■

We now display a greedy algorithm that computes a *good association*.

Algorithm:

1) Let h = 0. $\forall i \in \mathcal{M}$, associate STA i with the AP/BS a_i = $\arg\max_{k \in \mathcal{N}} \{r_{ik}\}$. Let the association vector be $A^{(0)}$.
2) Find the number of STAs m_j associated with each AP/BS j.
3) Check if *Theorem* 1 is satisfied. If *Theorem* 1 is satisfied, then the association $A^{(h)}$ is optimal. Go to step 5. If *Theorem* 1 is not satisfied, then the association $A^{(h)}$ may or may not be optimal.
4) Using *Lemma* 1, compute (i^*, k^*). If k^* = a_{i^*} go to step 5. Else, associate STA i^* with k^* keeping the association of the rest of the STAs same. Let the new association be $A^{(h+1)}$. Let $h = h + 1$. Go to step 2.
5) $A^{(h)}$ is the association yielded by the algorithm.

VI. Numerical Evaluation of the Greedy Search Algorithm

In this section, we study the algorithm for various scenarios. For various values of m, l, and b, we generate a random rate matrix $\mathbf{R}$. The first l columns are fixed at Θ_0 and the remaining b columns of $\mathbf{R}$ are generated using uniformly distributed channel gains G_{ij}s. We find the optimal association which maximizes the utility by enumerating all possible associations. We also run our algorithm and the result of our algorithm is compared against the optimal association got by enumeration. Though the enumeration technique guarantees optimal association, its computational complexity is of the order $(l+b)^m = n^m$ which is prohibitively high.

We consider 6 different cases of (m, l, b). In each case, we compute the association for 50 different rate matrices $\mathbf{R}$ (12 in the case of (9,2,5)). For each instance of the rate matrix $\mathbf{R}$, we observe the utilities of all n^m possible associations and thus note down the maximum and the minimum utilities obtained for each rate matrix $\mathbf{R}$. Thus from the 50 instances analyzed, we tabulate the range of the maximum and the minimum utilities obtained. We also observe the number of computations required by the enumeration technique, the average time taken by the enumeration technique, the average number of iterations taken by our algorithm (averaged over the number of instances analyzed), the average time taken by our algorithm and the average percentage of difference between the utilities of the optimal association computed by the enumeration technique and that given by our algorithm. The observations are shown in Table I.

We note from Table I that there is a large gap between the minimum and the maximum utility of associations, in every case, thus emphasizing the importance of seeking an optimal association. For a given n, the average number of iterations increases with m. We observe that the number of iterations taken by the algorithm to converge is significantly lower than the number of searches to be evaluated in the enumeration technique. The time taken for the algorithm to converge is negligible, whereas the time taken for the enumeration technique is significantly higher.

We observe from Table I that for large number of STAs m (cases 1,3,4, and 6) our algorithm converge to the optimum association vector and for small number of STAs (cases 2 and 5) our algorithm is found to converge to a suboptimal association vector. It is also noted that in Case 2 (m = 7, l = 2, b = 4) the average difference in the system utility between the optimal association vector and that obtained by our algorithm is 0.1% and that in Case 5 (m = 9, l = 2, b = 5), it is 0.15%. This could be explained as follows. When m is small, the gradient search method does a coarse exploration of the objective function and hence the algorithm converges to a local maxima. On the other hand for large m, the gradient search method carries out a finer exploration and hence the algorithm converges to a better solution.

TABLE I

COMPARISON OF OPTIMAL ASSOCIATION BY ENUMERATION AND THE SOLUTION OF OUR ALGORITHM.

Case	(m,l,b)	# of instances Analyzed	System Utility Range		Enumeration		Algorithm		Average % error from optimal value.
			Maximum	Minimum	No. of evaluations = n^m	Time (ms)	Average # Iterations	Time (ms)	
1	(10,1,2)	50	131.2–136.9	102.3–113.3	59049	154.0	3.36	0.1029	0
2	(7,2,4)	50	95.2– 98.0	60.9– 73.5	279936	612.6	6.08	0.254	0.1
3	(10,1,3)	50	130.5–134.9	96.3–107.0	1048576	3166.0	5.94	0.1456	0
4	(15,1,2)	50	191.7–199.4	148.9–163.5	14348907	65939.8	4.82	0.1286	0
5	(9,2,5)	12	120.6–123.5	79.5– 88.8	40353607	164845.0	8.42	0.3125	0.15
6	(19,1,2)	50	238.3–247.1	187.0–203.2	1162261467	475364.0	5.74	0.1548	0

It is to be noted if *Theorem* 1 is satisfied then the solution is optimal. On the other hand, if *Theorem* 1 is not satisfied then the solution of the algorithm may not be optimal. We found that in Case 4 of Table I, *Theorem* 1 is satisfied for 7 out of 50 rate matrices and in cases 1 and 6, *Theorem* 1 is satisfied for 4 out of 50 rate matrices. In cases 2, 3, and 5, *Theorem* 1 is not satisfied for any rate matrix.

VII. CONCLUSION

In this paper, we studied the optimal association of STAs in a heterogenous wireless access network comprising WLAN APs and 3G BSs. We formulated the problem as one of maximizing the total utility provided to the users, where the utility of a user is evaluated as the log of the elastic download throughput that the user gets. We studied some characteristics of the general solution, and derived the solution for a simple special case. Then we provided a heuristic greedy search algorithm that yields an optimal or near optimal association for a large number of problems for which we could obtain the exact solution by exhaustive search. In our ongoing work we are studying other algorithmic techniques, such as the genetic algorithm. Our future work will include optimal association under user mobility, the effect of network pricing, and decentralized algorithms for optimal association.

APPENDIX

PROOF OF THEOREM 1

Let us rewrite Eqn. 4 (motivated by [3]) for the log utility as follows. Let $f_i^{(j)}$ be the fraction of time STA i is associated with AP/BS j. Let the matrix $\mathbf{F}$ be $[f_i^{(j)}]$. This yields the following problem.

$$\begin{aligned} U^* &= \max_{\mathbf{F}} \sum_{i\in\mathcal{M}} \log\left(\sum_{j\in\mathcal{N}} f_i^{(j)} r_{ij}\right) \\ \text{s.t.} & \quad \sum_{i\in\mathcal{M}} f_i^{(j)} = 1,\ \forall j\in\mathcal{N} \\ & \quad f_i^{(j)} \geq 0,\ \forall i\in\mathcal{M}, \forall j\in\mathcal{N}. \end{aligned} \tag{20}$$

If the matrix $\mathbf{F}$ that solves this problem is such that $f_i^{(j)} = 0$, $\forall j \neq a_i$, $\forall i\in\mathcal{M}$, then we have solved Eqn. 4, i.e., $\exists A\in\mathcal{A}$ which is the optimal association.

The above problem is a concave maximization problem with linear constraints. The Lagrangian of the above problem is

$$\begin{aligned} L(\mathbf{F},\underline{\lambda},\underline{\nu}) &= \sum_{i\in\mathcal{M}} \log\left(\sum_{j\in\mathcal{N}} f_i^{(j)} r_{ij}\right) - \sum_{j\in\mathcal{N}} \lambda_j \sum_{i\in\mathcal{M}} f_i^{(j)} \\ & \quad + \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} \nu_{ij} f_i^{(j)} \end{aligned} \tag{21}$$

Differentiating w.r.t. $f_i^{(j)}$ gives

$$\frac{\partial L}{\partial f_i^{(j)}} = \frac{r_{ij}}{\sum_{k\in\mathcal{N}} f_i^{(k)} r_{ik}} - \lambda_j + \nu_{ij}, \forall i\in\mathcal{M},\ \forall j\in\mathcal{N} \tag{22}$$

Since we are maximizing a strictly concave function over linear constraints, $\mathbf{F}$ is the optimizer if there exists $(\underline{\lambda},\underline{\nu})$ that satisfies the following *KKT* conditions

$$\frac{r_{ij}}{\sum_{k\in\mathcal{N}} f_i^{(k)} r_{ik}} - \lambda_j + \nu_{ij} = 0,\ \forall i\in\mathcal{M},\ \forall j\in\mathcal{N} \tag{23}$$

$$\nu_{ij} \geq 0,\ \forall i\in\mathcal{M},\ \forall j\in\mathcal{N} \tag{24}$$

$$\nu_{ij}\cdot f_i^{(j)} = 0,\ \forall i\in\mathcal{M},\ \forall j\in\mathcal{N}. \tag{25}$$

Now consider an association $A = (a_1, a_2, \ldots, a_m)$ and examine $\mathbf{F}$ such that $f_i^{(a_i)} > 0$, and $f_i^{(j)} = 0$, $\forall j \neq a_i$, $\forall i\in\mathcal{M}$. For such a solution to be a *KKT* point we need

$$\nu_{ij} = \begin{cases} 0 & \text{if } j = a_i \\ \geq 0 & \text{if } j \neq a_i \end{cases},\ \forall i\in\mathcal{M},\ \forall j\in\mathcal{N} \tag{26}$$

Eqn. 23 gives

$$\sum_{k\in\mathcal{N}} f_i^{(k)} r_{ik} = \frac{r_{ij}}{\lambda_j - \nu_{ij}},\ \forall i\in\mathcal{M},\ \forall j\in\mathcal{N} \tag{27}$$

For the set of STAs, $i\in\mathcal{S}_p$, associated with AP/BS $p\in\mathcal{N}$, we have from Eqn. 27,

$$f_i^{(p)} r_{ip} = \frac{r_{ij}}{\lambda_j - \nu_{ij}},\ \forall i\in\mathcal{S}_p,\ \forall j\in\mathcal{N} \tag{28}$$

When $j = p$, we need $\nu_{ip} = 0$, $\forall i\in\mathcal{S}_p$. This gives

$$\lambda_p = \frac{1}{f_i^{(p)}},\ \forall i\in\mathcal{S}_p,\ \forall p\in\mathcal{N}$$

Thus,

$$\lambda_p = (f_i^{(p)})^{-1} = |\mathcal{S}_p|, \ \forall i \in \mathcal{S}_p, \ \forall p \in \mathcal{N} \quad (29)$$

Then from Eqn. 23, it is clear that

$$\begin{aligned}\nu_{ij} &= \lambda_j - \frac{r_{ij}}{f_i^{(a_i)} r_{ia_i}} \\ &= \frac{m_{a_i} m_j}{r_{ia_i}} \left(\frac{r_{ia_i}}{m_{a_i}} - \frac{r_{ij}}{m_j} \right) \quad (30)\end{aligned}$$

To ensure $\nu_{ij} \geq 0$, we need $\frac{r_{ia_i}}{m_{a_i}} \geq \frac{r_{ij}}{m_j}$ $\forall i \in \mathcal{M}$, $\forall j \in \mathcal{N}$. Thus we have a *KKT* point if $\frac{r_{ia_i}}{m_{a_i}} \geq \frac{r_{ij}}{m_j}$ $\forall i \in \mathcal{M}$, $\forall j \in \mathcal{N}$.

■

PROOF OF LEMMA 1

The system utilities corresponding to the associations A and A_{ik} are given by

$$\begin{aligned}U(A) &= \sum_{p \in \mathcal{N}} \sum_{q \in \mathcal{S}_p} \log \left(\frac{r_{qp}}{m_p} \right) \\ U(A_{ik}) &= \sum_{p \in \mathcal{N}, p \neq a_i, p \neq k} \sum_{q \in \mathcal{S}_p} \log \left(\frac{r_{qp}}{m_p} \right) \\ &\quad + \sum_{q \in \mathcal{S}_{a_i}, q \neq i} \log \left(\frac{r_{qa_i}}{m_{a_i} - 1} \right) \\ &\quad + \sum_{q \in \mathcal{S}_k \cup \{i\}} \log \left(\frac{r_{qk}}{m_k + 1} \right)\end{aligned}$$

Hence,

$$\begin{aligned}U(A_{ik}) - U(A) &= -(m_k + 1) \log (m_k + 1) \\ &\quad + m_k \log (m_k) - (m_{a_i} - 1) \log (m_{a_i} - 1) \\ &\quad + m_{a_i} \log (m_{a_i}) + \log \left(\frac{r_{ik}}{r_{ia_i}} \right)\end{aligned}$$

Thus, in a collection of associations $\mathcal{C}$, $A_{i^*k^*}$ is one of the best associations if $U(A_{i^*k^*}) - U(A) \geq U(B) - U(A), \forall B \in \mathcal{C}$. Thus the maximizer of Eqn. 19 gives the best association in $\mathcal{C}$.

■

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