Fixed Point Analysis of Single Cell IEEE 802.11e WLANs: Uniqueness and Multistability*

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***Abstract*— We consider the vector fixed point equations arising out of the analysis of the saturation throughput of a single cell IEEE 802.11e (EDCA) wireless local area network with nodes that have different backoff parameters, including different Arbitration InterFrame Space (AIFS) values. We consider balanced and unbalanced solutions of the fixed point equations arising in homogeneous (i.e., one with the same backoff parameters) and nonhomogeneous networks. By a balanced fixed point, we mean one where all coordinates are equal. We are concerned, in particular, with (i) whether the fixed point is balanced within a class, and (ii) whether the fixed point is unique. Our simulations show that when multiple unbalanced fixed points exist in a homogeneous system then the time behaviour of the system demonstrates severe short term unfairness (or *multistability*). We provide a condition for the fixed point solution to be balanced, and also a condition for uniqueness. We then extend our general fixed point analysis to capture AIFS based differentiation and the concept of virtual collision when there are multiple queues per station; again a condition for uniqueness is established. For the case of multiple queues per node, we find that a model with as many nodes as there are queues, with one queue per node, provides an excellent approximation. Implications for the use of the fixed point formulation for performance analysis are also discussed.**

***Index Terms*— Performance of Wireless LANs, Short term Unfairness, Saturation Throughput Analysis of EDCA**

I. Introduction

A new component of the IEEE 802.11e medium access control (MAC) is an enhanced distributed channel access (EDCA), which provides differentiated channel access to packets by allowing different backoff parameters (see [3]). Several traffic classes are supported, the classes being distinguished by different backoff parameters. Thus, whereas in the legacy DCF all nodes have a single queue, and a single backoff "state machine", all with the same backoff parameters (we say that the nodes are *homogeneous*), in EDCA the nodes can have multiple queues with separate backoff state machines per queue with different parameters, and hence are permitted to be *nonhomogeneous*.

This paper is concerned with the saturation throughput analysis of IEEE 802.11e (EDCA) wireless LANs. We consider a single cell network of IEEE 802.11e nodes (single cell meaning that all nodes are within control channel range of each other), with an ideal channel (without capture, fading or frame error) and assume that packets are lost only due to collision of simultaneous transmissions. For ease of understanding, much of our presentation is for the case in which each node has only one EDCA queue of some access category. The analysis for the general case of multiple EDCA queues (of different access categories) per node is provided in Section VII.

Much work has been reported on the performance evaluation of EDCA to support differentiated service. Most of the analytical work reported has been based on a decoupling approximation proposed initially by Bianchi ([4]). While keeping this basic decoupling approximation, in [2] Kumar et al. presented a significant simplification and generalisation of the analysis of the IEEE 802.11 backoff mechanism. This analysis led to a certain one dimensional fixed point equation for the collision probability experienced by the nodes in a homogeneous system. In this paper we consider *multidimensional fixed point equations* for a homogeneous system of nodes, and also for a nonhomogeneous system of nodes. The nonhomogeneity arises due to different initial backoffs, or different backoff multipliers, or different amounts of time that nodes wait after a transmission before restarting their backoff counters (i.e., the AIFS (Arbitration InterFrame Space) mechanism of IEEE 802.11e), or different number of access categories per node.

Our approach in this paper builds upon the one provided in [2]. The main contributions of this paper are the following:

1) We provide examples of homogeneous systems in which, even though a unique balanced fixed point exists (i.e., a solution in which all the coordinates are equal), there can be multiple unbalanced fixed points, thus suggesting *multistability*. We demonstrate by simulation that, in such cases, significant short term unfairness can be observed and the unique balanced fixed point fails to capture the system performance.
2) Next, in the case where the backoff increases multiplicatively (as in IEEE 802.11 and IEEE 802.11e access categories AC_BE, AC_BK), we establish a simple sufficient condition for the uniqueness of the solution of the multidimensional fixed point equation in the homogeneous and the nonhomogeneous cases. In particular, we do this for the case of the AIFS mechanism with multiple access categories per node. The case of multiple access categories per node presented here extends the material provided in [1].
3) Further, the fixed point approach as developed in this work provides an elegant and easy way to study the performance differentiation provided by the different

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backoff mechanisms in EDCA (see the section on throughput differentiation in [20]).

A survey of the literature: There has been much research activity on modeling the performance of IEEE 802.11 and in particular of IEEE 802.11e medium access standards. The general approach has been to extend the decoupling approximation introduced by Bianchi ([4]). Without modeling the AIFS mechanism, the extension is straightforward. Only the initial backoff, and the backoff multiplier (*persistence factor*) are modeled. In [5], [6] and [7], such a scheme is studied by extending Bianchi's Markov model per access category. In this paper, in Section III, we will provide a generalisation and simplification of this approach. We will then provide examples of homogeneous systems where nonunique fixed points can exist, demonstrate the consequences of such nonuniqueness, and also obtain conditions that guarantee uniqueness.

The AIFS technique is a further enhancement in IEEE 802.11e that provides a sort of priority to queues that have smaller values of AIFS. After any transmission activity in the channel, whereas high priority queues (with AIFS = DIFS) wait only for DIFS (DCF Interframe Space) to resume counting down their backoff counters, low priority queues (with AIFS $>$ DIFS) defer the initiation of countdown for an additional AIFS$-$DIFS slots. Hence a high priority queue decrements its backoff counter earlier than a low priority queue and also has fewer collisions.

Among the approaches that have been proposed for modeling the AIFS mechanism (for example, [8], [9], [10], [11], [12] and [13]) the ones in [11], [12] and [13] come much closer to capturing the service differentiation provided by the AIFS feature. In [11] the authors propose a Markov model to capture both the backoff window expansion approach and AIFS. AIFS is modeled by expanding the state-space of the Markov chain to include the number of slots elapsed since the previous transmission attempt on the channel. [12] uses a Markov chain on the number of slots elapsed since the previous transmission to model AIFS based service differentiation. In [13] the authors observe that the system exists in states in which only nodes of certain access categories can attempt transmission. The approach is to model the evolution of these states as a Markov chain. The transition probabilities of this Markov chain are obtained from the assumed, decoupled attempt probabilities. This approach yields a fixed point formulation. This is the approach we will discuss in Section VI. [10] extends the Bianchi's analysis to multiple access categories per node case using the Markov chain approach.

We note that the analyses in [10], [11] and [13] are based on Bianchi's approach to modeling the residual backoff by a Markov chain. In this paper, we have extended the simplification reported in [2] (which was for a homogeneous system of nodes) to nonhomogeneous nodes with different backoff parameters and AIFS based priority schemes. Also, we model the case of multiple queues (of different access categories) per node. Thus, in our work, we have provided a simplified and integrated model to capture all the essential backoff based service differentiation mechanisms of IEEE 802.11e.

In the previous literature on IEEE 802.11 and IEEE 802.11e, it is assumed that the collision rate experienced by a queue of any access category is constant over time. There appears to have been no attempt to study the phenomenon of short term unfairness in the fixed point framework. A related work on Ethernet ([19]) identifies short-term unfairness in the system by experimentation and simulation, and suggests modifications to the protocol to eliminate it. Also, all the existing work assumes that the collision probabilities of all the queues with identical access parameters are the same. Thus there appears to have been no earlier work on studying the possibility of unbalanced solutions of the fixed point equations. In addition, the possibility of nonuniqueness of the solution of the fixed point equations arising in the analyses seems to have been missed in the earlier literature. In our work, we study the fixed point equations for IEEE 802.11e networks and take into account all these possibilities.

Outline of the paper: In Section II we review the generalised backoff model that was first presented in [2]. In Section III we develop the multidimensional fixed point equations for the homogeneous and nonhomogeneous cases (without AIFS), and obtain the necessary and sufficient conditions satisfied by the solutions to the fixed point equations. We provide examples in Section IV to show that even in the homogeneous case there can exist multiple unbalanced fixed points and show the consequence of this. In Section V-A, we analyse the fixed point equations for a homogeneous system of nodes and obtain a condition for the existence of only one fixed point. In Sections V-B and VI, we extend the analysis to nonhomogeneous system of nodes, with different backoff parameters (including AIFS). In Section VII we analyse the case of multiple EDCA queues per node. Section VIII concludes the paper and discusses future work. *The proofs of all lemmas and theorems, if not in the paper, are provided in [20].*

II. The Generalised Back-Off Model

There are n nodes, indexed by $i, 1 \leq i \leq n$. We begin with considering the case in which each node has one EDCA queue. We adopt the notation in [2], whose authors consider a generalisation of the backoff behaviour of the nodes, and define the following backoff parameters (for node i)

$K_i :=$ At the $(K_i + 1)$th attempt either the packet being attempted by node i succeeds or is discarded

$b_{i,k} :=$ The *mean* backoff (in slots) at the kth attempt for a packet being attempted by node i, $0 \leq k \leq K_i$

Definition 2.1: A system of n nodes is said to be **homogeneous**, if all the backoff parameters of the nodes, like, K_i, $b_{i,k}, 0 \leq k \leq K_i$ are the same for all $i, 1 \leq i \leq n$. A system of nodes is called **nonhomogeneous** if the backoff parameters of the nodes are not identical. ■

Remark: IEEE 802.11e permits different backoff parameters to differentiate channel access obtained by the nodes in an attempt to provide QoS. The above definitions capture the possibility of having different CW_{min} and CW_{max} values, different exponential backoff multiplier values and even different number of permitted attempts. For ease of discussion and understanding, we will postpone the topic of AIFS until Section VI. Hence in the discussions up to Section V-B, all the nodes wait only for a DIFS after a busy channel. ■

It has been shown in [2] (and later in [18]) that under the decoupling assumption, introduced by Bianchi in [4], the attempt probability of node i (in a backoff slot, and conditioned on being in backoff) for given collision probability γ_i is given by,

$$G_i(\gamma_i) := \frac{1 + \gamma_i + \cdots + \gamma_i^{K_i}}{b_{i,0} + \gamma_i b_{i,1} + \cdots + \gamma_i^{K_i} b_{i,K_i}} \tag{1}$$

Remarks 2.1:

1) We will assume that $b_{i,\cdot}$ are such that $0 \leq G_i(\gamma_i) \leq 1$ for all $\gamma_i, 0 \leq \gamma_i \leq 1$ and $G_i(\gamma_i) < 1$ whenever $\gamma_i > 0$.
2) When the system is homogeneous then we will drop the subscript i from $G_i(\cdot)$, and write the function simply as $G(\cdot)$.

III. The Fixed Point Equation

It is important to note that in the present discussion all rates are conditioned on being in the backoff periods; i.e., we have eliminated all durations other than those in which nodes are counting down their backoff counters, in order to obtain the collision probability γ_i of node i and its attempt probability β_i ($= G_i(\gamma_i)$). Later one brings back the channel activity periods in order to compute the throughput in terms of the attempt probabilities (see [2]). Now consider a nonhomogeneous system of n nodes. Let $\boldsymbol{\gamma}$ be the vector of collision probabilities of the nodes. With the slotted model for the backoff process and the decoupling assumption, the natural mapping of the attempt probabilities of other nodes to the collision probability of a node is given by

$$\gamma_i = \Gamma_i(\beta_1, \beta_2, \ldots, \beta_n) = 1 - \prod_{j=1, j\neq i}^{n} (1 - \beta_j)$$

where $\beta_j = G_j(\gamma_j)$. We could now expect that the equilibrium behaviour of the system will be characterised by the solutions of the following system of equations. For $1 \leq i \leq n$,

$$\gamma_i = \Gamma_i(G_1(\gamma_1), \cdots, G_n(\gamma_n))$$

We write these n equations compactly in the form of the following multidimensional fixed point equation.

$$\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma})) \tag{2}$$

Since $\boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ is a composition of continuous functions it is continuous. We thus have a continuous mapping from $[0,1]^n$ to $[0,1]^n$. Hence by Brouwer's fixed point theorem there exists a fixed point in $[0,1]^n$ for the equation $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$.

Consider the i^{th} component of the fixed point equation, i.e.,

$$\gamma_i = 1 - \prod_{1\leq j\leq n, j\neq i} (1 - G_j(\gamma_j))$$

or equivalently,

$$(1 - \gamma_i) = \prod_{1\leq j\leq n, j\neq i} (1 - G_j(\gamma_j))$$

Multiplying both sides by $(1 - G_i(\gamma_i))$, we get,

$$(1 - \gamma_i)(1 - G_i(\gamma_i)) = \prod_{1\leq j\leq n} (1 - G_j(\gamma_j))$$

Thus a *necessary and sufficient condition* for a vector of collision probabilities $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_n)$ to be a fixed point solution is that, for all $1 \leq i \leq n$,

$$(1 - \gamma_i)(1 - G_i(\gamma_i)) = \prod_{j=1}^{n} (1 - G_j(\gamma_j)) \tag{3}$$

where the right-hand side is seen to be independent of i.

Define $F_i(\gamma) := (1-\gamma)(1 - G_i(\gamma))$. From (3) we see that if $\boldsymbol{\gamma}$ is a solution of (2), then for all $i, j, 1 \leq i, j \leq n$,

$$F_i(\gamma_i) = F_j(\gamma_j) \tag{4}$$

Notice that this is only a *necessary condition*. For example, in a homogeneous system of nodes, the vector $\boldsymbol{\gamma}$ such that $\gamma_i = \gamma$ for all $1 \leq i \leq n$, satisfies (4) for any $0 \leq \gamma \leq 1$, but not all such points are solutions of the fixed point equation (2).

Definition 3.1: We say that a fixed point $\boldsymbol{\gamma}$ (i.e., a solution of $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$) is **balanced** if $\gamma_i = \gamma_j$ for all $1 \leq i, j \leq n$; otherwise, $\boldsymbol{\gamma}$ is said to be an **unbalanced fixed point**. ■

Remarks 3.1:

1) It is clear that if there exists an unbalanced fixed point for a homogeneous system, then every permutation is also a fixed point and hence, in such cases, we do not have a unique fixed point.
2) In the homogeneous case, by symmetry, the average collision probability must be the same for every node. If the collision probabilities correspond to a fixed point (see 3, next), then this fixed point will be of the form $(\gamma, \gamma, \cdots, \gamma)$ where γ solves $\gamma = \Gamma(G(\gamma))$ (since $\Gamma_i(\cdot) = \Gamma(\cdot)$ and $G_i(\cdot) = G(\cdot)$ for all $1 \leq i \leq n$). Such a fixed point of $\gamma = \Gamma(G(\gamma))$ is guaranteed by Brouwer's fixed point theorem. The uniqueness of such a balanced fixed point was studied in [2]. We reproduce this result in Theorem 5.1.
3) There is, however, the possibility that even in the homogeneous case, there is an unbalanced solution of $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$. By simulation examples we observe in Section IV that when there exist unbalanced fixed points, the balanced fixed point of the system does not characterise the average performance, even if there exists only one balanced fixed point. In Section V-A, we provide a condition for homogeneous IEEE 802.11 and IEEE 802.11e type nodes (with exponential backoff) under which there is a unique balanced fixed point and no unbalanced fixed point. In such cases, it is now well established, that the unique balanced fixed point accurately predicts the saturation throughput of the system.
4) For the homogeneous case the backoff process can be exactly modeled by a positive recurrent Markov chain (see [2]). Hence the attempt and collision processes will be ergodic and, by symmetry, the nodes will have equal attempt and collision probabilities. In such a situation the existence of multiple unbalanced fixed points will suggest short term unfairness or multistability. We will observe this phenomenon in Section IV.
5) Consider a system of homogeneous nodes having unbalanced solutions for the fixed point equation $\boldsymbol{\gamma} =$

$\mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ (i.e., there exists i, j such that $\gamma_i \neq \gamma_j$), then from (4), we see that $F(\gamma_i) = F(\gamma_j)$, or the function F is many-to-one. Hence for a homogeneous system of nodes, if the function F is one-to-one then there cannot exist unbalanced fixed points. In Section V-B we use this observation to obtain a sufficient condition for the uniqueness of the fixed point for the nonhomogeneous case.

IV. Nonunique Fixed Points and MultiStability: Simulation Examples

A. Example 1

Consider a homogeneous system (let us call it System-I) with $n = 10$ nodes. The function $G(\cdot)$ of the nodes is given by,

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \gamma^3 + \ldots}{1 + \gamma + \gamma^2 + \gamma^3 + 64(\gamma^4 + \gamma^5 + \ldots)}$$

The system corresponds to the case where $K = \infty$, $b_0 = b_1 = b_2 = b_3 = 1$ and $b_4 = b_5 = b_6 = \ldots = 64$ (b_i are distributed uniformly over the integers in [1, CW_i] for appropriate CW_i). From the form of function $G(\cdot)$, we can see that a node which is currently at backoff stage 0 is more likely to remain at that stage as it *takes* 4 *successive collisions* to make the attempt rate of the node < 1. Likewise, a node that is in the larger backoff stages $b_4 = b_5 = \cdots = 64$, will retry continuously with mean inter-attempt slots of 64 until it succeeds. Observe that only one node can be at backoff stage 0 at any time. This leads to the apparent multistability of the system.

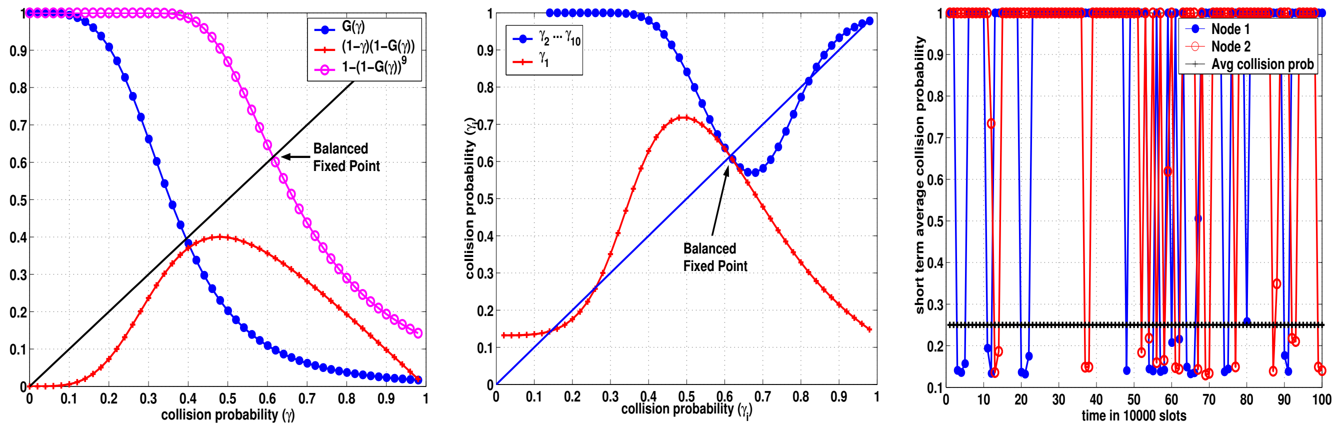
Figure 1(a) plots $G(\gamma)$, the corresponding $F(\gamma) = (1 - \gamma)(1 - G(\gamma))$ and shows the balanced fixed point of the system for $n = 10$ nodes. The balanced fixed point of the system shown in the figure is obtained using the fixed point equation $\gamma = 1 - (1 - G(\gamma))^9$. Observe that the function $F(\cdot)$ is not one-to-one (the function $F(\cdot)$ not being one-to-one does not imply that there exist multiple fixed point solutions; see Remarks 3.1, 5).

Figure 1(b) shows the existence of unbalanced fixed points for System-I. These fixed points are obtained as follows. Assume that we are interested in fixed points such that $\gamma_1 \neq \gamma_2 = \cdots = \gamma_n$. Given $\gamma_2 = \cdots = \gamma_n$, the attempt probability of the nodes $2, \cdots, n$ is given by $G(\gamma_2)$. Hence, the collision probability of node 1 is given by $\gamma_1 = 1 - (1 - G(\gamma_2))^{n-1}$. The attempt probability of node 1 would then be $G(\gamma_1)$. Using the decoupling assumption, the collision probability of any of the other $n - 1$ nodes would then be, $1 - (1 - G(\gamma_2))^{n-2}(1 - G(\gamma_1)) = \gamma_2$. Thus we obtain a fixed point equation for γ_2 (and hence for all the other $\gamma_j, 3 \leq j \leq n$). In Figure 1(b) we plot $1 - (1 - G(\gamma))^8(1 - G(1 - (1 - G(\gamma))^9))$ (plotted as the line marked with dots), the intersection of which with the "y=x" line shows the solutions for $\gamma_2 (= \cdots = \gamma_n)$. In the same way, we obtain the fixed point equation for γ_1 by eliminating $\gamma_2, \cdots, \gamma_n$ from the multidimensional system of equations. This functiton is plotted in Figure 1(b) using pluses and lines and the intersection of this curve with the "y=x" line shows the corresponding solutions for γ_1. We see that there are three solutions in each case. The smallest values of γ_1 (approx. 0.14) pairs up with the largest value of $\gamma_2 = \cdots = \gamma_n$ (approx. 0.97). Notice that the balanced fixed point of the system is also a fixed point in the plot (compare with Figure 1(a)). Then there is one remaining unbalanced fixed point whose values can be read off the plot. We note that there could exist many other unbalanced fixed points for this system of equations, as we have considered only a particular variety of fixed points that have the property that $\gamma_1 \neq \gamma_2 = \cdots = \gamma_n$.

In order to examine the consequences of multiple unbalanced fixed points we simulated the backoff process with the backoff parameters of System-I. The following remarks summarise our simulation approach in this paper.

Remarks 4.1 (On the Simulation Approach used):

1) We have developed an event-driven simulator written in the "C" language based on the coupled multidimensional backoff process of the various nodes, to compare with the analytical results. In this simulator, we do not simulate the detailed wireless LAN system (as is done in an ns-2 simulator), but only the backoff slots. We will refer to this as the CMP (Coupled Markov Process) simulator. The main aim of the CMP simulator is to understand the backoff behaviour of the nodes and its dependence on the different backoff parameters. From the point of view of performance analysis, it may also be noted that once the backoff behaviour is correctly modelled the channel activity can easily be added analytically, and thus throughput results can be obtained (see [4] and [2]). Note that, for IEEE 802.11 type networks, a good match between analysis that uses a decoupled Markov model for the backoff process and ns-2 simulations has already been reported in earlier works (see the literature survey in Section I). In addition, in Section VI, ns-2 simulation results have also been provided in comparison with the CMP simulator and the analytical results.
2) Our CMP simulator is programmed as follows. The system evolves over backoff slots. All the nodes are assumed to be in perfect slot synchronisation. The actual coupled evolution of the backoff process is modeled. The backoff distribution is uniform and the residual backoff time is the state for each node. At every slot, depending on the state of the backoff process, there are three possibilities: the slot is idle, there is a successful transmission, or there is a collision. This causes further evolution of the backoff process.
3) Our CMP simulator, which we primarily use to study the backoff behaviour of the nodes, takes a few seconds to complete a simulation run, in comparison with the *ns-2* simulations which takes any time between few minutes to an hour depending on the number of nodes in the system. The coupled backoff evolution approach we use captures all the essential features of a single cell system with ideal channel (no capture, fading or frame error) and where there is perfect synchronisation among the nodes (typical for single cell systems). The simulation provides the attempt rates and collision probabilities directly, which can be used with the throughput formula provided in [2] to obtain the throughput of the nodes.



(a) (b) (c)

Fig. 1. Example System-I: 1(a) The balanced fixed point. Plots of $G(\gamma)$, $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and $1-(1-G(\gamma))^9$ vs. the collision probability γ; we also show the "y=x" line. 1(b) Demonstration of unbalanced fixed points. Plots of $\gamma_2 = 1-(1-G(\gamma))^8(1-G(1-(1-G(\gamma))^9))$ (the curve drawn with dots and lines) and the function for the fixed point equation for γ_1 (see text) using pluses and lines. 1(c) Snap-shot of short term average collision probability of 2 of the 10 nodes. Also plotted is the average collision probability of the nodes (averaged *over all frames and nodes*). The 95% confidence interval for the average collision probability lies within 0.7% of the mean value.

4) In all our simulations, b_i are distributed uniformly over the integers in $[1, CW_i]$ for appropriate CW_i. We note here that the backoff behaviour of IEEE 802.11e EDCA with the backoff range $[0, CW]$ can be modeled in the same way as IEEE 802.11 DCF with the backoff range $[1, CW+1]$ and the value of AIFS reduced by 1 (see [13], [17]). Thus, the "0 sampling problem" found in IEEE 802.11 DCF is not observed in IEEE 802.11e EDCA, see the technical report [20] for further details.
5) In Figures 1(c), 2(c) and 3(b), for the purpose of reporting the short term unfairness results, the entire duration of simulation is divided into k frames, where the size of each frame is 10,000 slots. The short-term average of the collision probability of each node $j, 1 \leq j \leq n$, is calculated as $\frac{C_j(i)}{A_j(i)}$ where $C_j(i)$ and $A_j(i)$ correspond to the number of collisions and attempts in frame $i, 1 \leq i \leq k$, for node j. The long-term average is similarly calculated as $\frac{1}{n}\sum_{j=1}^{n}\frac{\sum_{i=1}^{k} C_j(i)}{\sum_{i=1}^{k} A_j(i)}$ where n is the number of nodes. Notice that the long-term average collision rate is a batch biased average of the short-term collision rates. Hence, when looking at the graphs, it will be incorrect to visually average the short-term collision rate plots in an attempt to obtain the long-term average collision rate. This is because when a node is shown to have a low collision probability, it is the one that is attempting every slot (while the other nodes attempt with a mean gap of 64 slots), and hence it sees a low probability of collision. In this case $A_j(\cdot)$ is large and $C_j(\cdot) \ll A_j(\cdot)$. On the other hand, when a node is shown to have a high collision probability it is attempting at an average rate of $\frac{1}{64}$ and almost all its attempts collide with the node that is then attempting in every slot. In this case $A_j(\cdot)$ is small and $C_j(\cdot) \approx 1$. Thus, in obtaining the overall average, it is essential to account for the large variation in $A_j(\cdot)$ between the two cases.

■

In Figure 1(c) we plot a (simulation) snap shot of the short term average collision probability of 2 of the 10 nodes of System-I and the average collision probability of the nodes (The average is calculated over all frames and all nodes. Since the nodes are identical, the average collision probability is the same for all the nodes). Observe that the short term average has a huge variance around the long term average. It is evident that over 1000's of slots one node or the other monopolises the channel (and the remaining nodes see a collision probability of 1 during those slots). This could be described as multistability. A look into the fairness index (see Figure 3(c)) plotted as a function of the frame size used to calculate throughput suggests that System-I exhibits significant unfairness in service even over reasonably large time intervals.

Implication for the use of the balanced fixed point: Notice also that the average collision rate shown in Figure 1(c) is about 0.25, whereas the balanced fixed point shown in Figure 1(a) shows a collision probability of about 0.62. *Hence we see that in this case, where there are multiple fixed points, the balanced fixed point does not capture the actual system performance.*

B. Example 2

Let us now consider yet another homogeneous example (let us call it System-II) with $n = 20$ nodes. The function $G(\cdot)$ of the nodes is given by,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\cdots+\gamma^7}{1+3\gamma+9\gamma^2+27\gamma^3+\cdots+2187\gamma^7}$$

The system corresponds to the case where $K = 7$, $b_0 = 1$, $p = 3$ and $b_k = p^k b_0$ for all $0 \leq k \leq K$ (b_i are uniformly distributed in [1, CW_i] for appropriate CW_i). We notice that in this example the way the backoff expands is similar to the way it expands in the IEEE 802.11 standard, except that the initial backoff is very small (1 slot) and the multiplier is 3, rather than 2. Figure 2(a) plots $G(\gamma)$, the corresponding $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and the balanced fixed point of the system for $n = 20$ nodes. The balanced fixed point of the system shown in the figure is obtained using the fixed point equation $\gamma = 1-(1-G(\gamma))^{19}$.

As in the case of System-I, Figure 2(b) shows the existence of multiple unbalanced fixed points for System-II. The fixed points we have shown correspond to the case where $\gamma_1 \neq \gamma_2 = \cdots = \gamma_n$ and are obtained just as discussed for System-I.

Figure 2(c) plots a snap shot of the short term average collision probability (from simulation) of 2 of the 20 nodes and the average collision probability of the nodes (same for all the nodes). Observe that the short term averages vary a lot as compared to the long term average, suggesting multistability. Again, as in the case of System-I, comparing the average collision probability with the balanced fixed point of the system in Figure 2(a), we see that the fixed point does not capture the actual system performance.

Discussion of Examples 1 and 2: From the simulation examples, we can make the following inferences.

1) When there are multiple unbalanced fixed points in a homogeneous system then the system can display multistability, which manifests itself as significant short term unfairness in channel access.
2) When there are multiple unbalanced fixed points in a homogeneous system then the collision probability obtained from the balanced fixed point may be a poor approximation to the long term average collision probability.

Similar conclusions can be drawn for nonhomogeneous systems when the system of fixed point equations have multiple solutions. ■

It appears that the existence of multiple-fixed points is a consequence of the form of the $G(\cdot)$ function in the above examples, where $G(\cdot)$ is similar to a switching curve; see, for example, Figure 1(a) where there is a very high attempt probability at low collision probabilities and a very low attempt probability at high collision probabilities.

C. Example 3

Consider a homogeneous system in which backoff increases multiplicatively as in IEEE 802.11 DCF (let us call it System-III), with $n = 10$ nodes. The function $G(\cdot)$ is given by,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\ldots+\gamma^7}{16+32\gamma+64\gamma^2+\ldots+2048\gamma^7}$$

The system corresponds to the case where $K = 7$, $p = 2$ and $b_0 = 16$ and $b_k = p^k b_0$ for all $0 \leq k \leq K$ (b_i are uniformly distributed in [1, CW_i] for appropriate CW_i). These parameters are similar to those used in the IEEE 802.11 standard. Figure 3(a) plots $G(\cdot)$, the corresponding $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and the unique balanced fixed point of the system. (Notice that F is one-to-one and uniqueness of the fixed point will be proved in Section V-A.) The balanced fixed point of the system is obtained using the fixed point equation $\gamma = 1-(1-G(\gamma))^9$. The balanced fixed point yields a collision probability of approximately 0.29.

Figure 3(b) plots a snap shot of the short term average collision probability (from simulation) of 2 of the 10 nodes and the average collision probability of the nodes of the Example System-III. Notice that the short term average collision rate is close to the average collision rate (the vertical scale in this figure is much finer than in the corresponding figures for System-I and System-II). Also, the average collision rate matches well with the balanced fixed point solution obtained in Figure 3(a).

Remark: Thus we see that in a situation in which there is a unique fixed point not only is there lack of multistability, but also the fixed point solution yields a good approximation to the long run average behaviour. ■

D. Short Term Fairness in Examples 1, 2 and 3

Figure 3(c) plots the throughput fairness index $\frac{1}{n}\frac{(\sum_{i=1}^{n}\tau_i)^2}{\sum_{i=1}^{n}\tau_i^2}$ (where τ_i is the average throughput of node i over the measurement frame, see [16]) against the frame size used to measure throughput. The fairness index is obtained for each frame and is averaged over the duration of the simulation. Also plotted in the figure is the 95% confidence interval. We note that values of this index will lie in the interval $[0,1]$, and smaller values of the index correspond to greater unfairness between the nodes. The performance of all the three example systems are compared. Notice that the Example System-III (similar to IEEE 802.11 DCF) has the best fairness properties. The system achieves fairness of 0.9 over 1000's of slots. However, for Example System-I and II, similar performance is achieved only over 1,000,000 and 100,000 slots. The unfairness of Example Systems-I and II can be attributed to their apparent multistability.

In Section V we establish conditions for the uniqueness of the solutions to the multidimensional fixed point equation.

V. Analysis of the Fixed Point

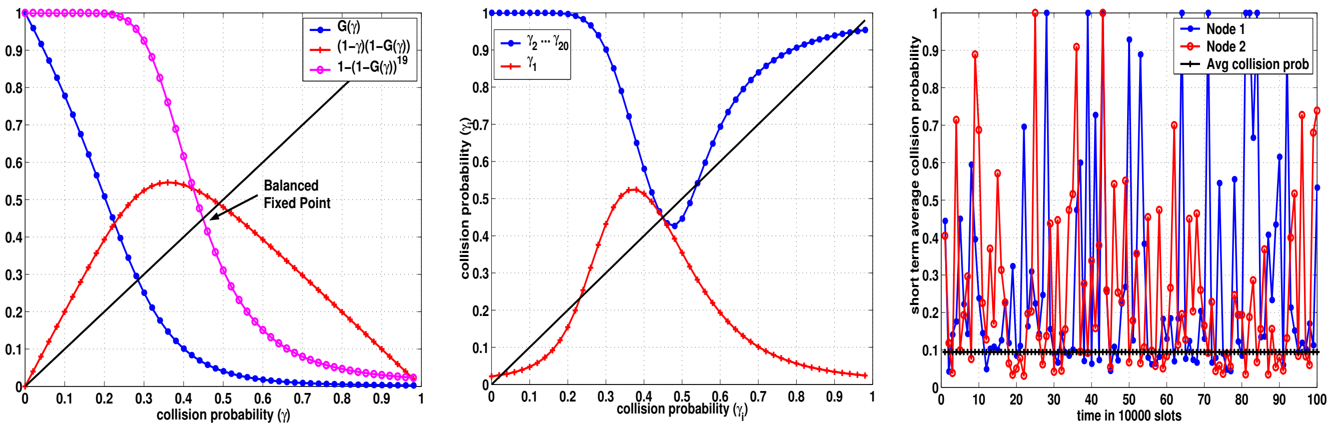
A. The Homogeneous Case

The following two results are adopted from [2].

Lemma 5.1: $G(\gamma)$ is nonincreasing in γ if $b_k, k \geq 0$, is a nondecreasing sequence. In that case, unless $b_k = b_0$ for all k, $G(\gamma)$ is strictly decreasing in γ. ■

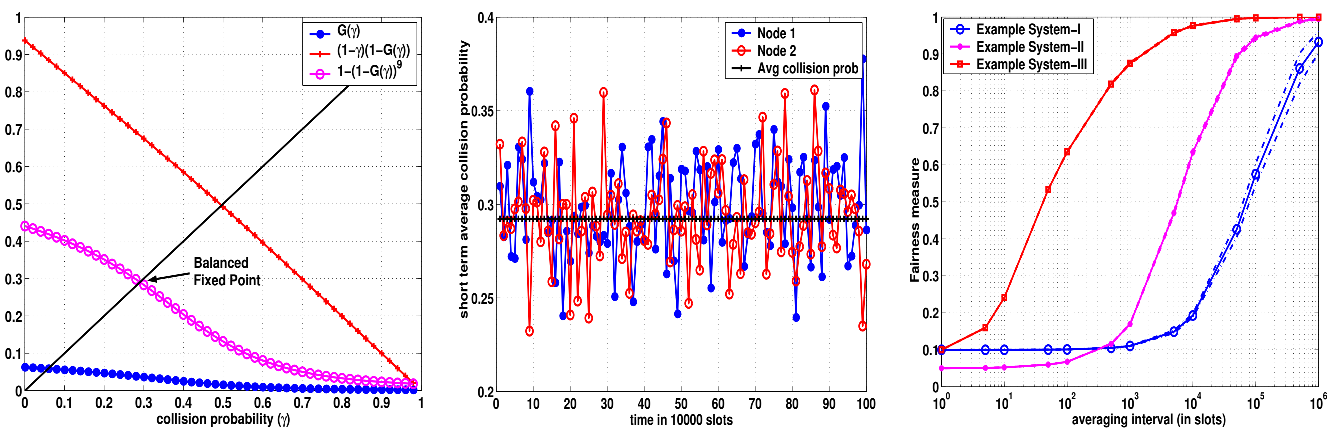
Theorem 5.1: For a homogeneous system of nodes, $\Gamma(G(\gamma)) : [0,1] \rightarrow [0,1]$, has a unique fixed point if $b_k, k \geq 0$, is a nondecreasing sequence. ■

Remark: The fixed point $(\gamma, \gamma, \cdots, \gamma)$ (where $\gamma = \Gamma(G(\gamma))$) is the unique balanced fixed point for $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$. From (4), we see that a *necessary* condition for the existence of unbalanced fixed points in a homogeneous system of nodes is that the function $F(\gamma) = (1-\gamma)(1-G(\gamma))$ needs to be many-to-one. In other words, if the function $(1-\gamma)(1-G(\gamma))$ is one-to-one and if $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ is a solution of the system $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$, then $\gamma_i = \gamma_j$ for all i, j. ■



(a) (b) (c)

Fig. 2. Example System-II: 2(a) The balanced fixed point. Plots of $G(\gamma)$, $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and $1-(1-G(\gamma))^{19}$ vs. the collision probability γ; the line "y=x" is also shown. Notice that the function F is not one-to-one. 2(b) Demonstration of unbalanced fixed points. Plots of $\gamma_2 = 1-(1-G(\gamma))^{18}(1-G(1-(1-G(\gamma))^{19}))$ (the curve drawn with dots and lines) and the function for the fixed point equation for γ_1 (see text) using pluses and lines. 2(c) Snap-shot of short term average collision probability of 2 of the 20 nodes. The average collision probability is also plotted in the figure (averaged *over all slots and nodes*). The 95% confidence interval for the average collision rate lies within 0.7% of the mean value.



(a) (b) (c)

Fig. 3. Example System-III: 3(a) Plots of $G(\gamma)$, $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and $1-(1-G(\gamma))^9$ vs. the collision probability γ; the line "y=x" is also shown. 3(b) Snap-shot of short term average collision probability of 2 of the 10 nodes. Also plotted is the average collision probabilty obtained by the nodes. The 95% confidence interval of the average collision rate lies within 0.2% of the mean value. 3(c) Throughput fairness index of Example III compared with Examples I and II, plotted against the number of slots used to measure throughput. The dotted lines mark the 95% confidence interval for all the three example systems.

Consider the exponentially increasing backoff case for which $G(\cdot)$ is given by,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\ldots+\gamma^K}{b_0(1+p\gamma+p^2\gamma^2+\ldots+p^K\gamma^K)} \quad (5)$$

Clearly, $G(\gamma)$ is a continuously differentiable function and so is $F(\gamma) = (1-\gamma)(1-G(\gamma))$. The following simple lemma is a consequence of the mean value theorem.

Lemma 5.2: $F(\gamma)$ is one-to-one in $0 \le \gamma \le 1$ *if* $F^{'}(\gamma) \neq 0$ for all $0 \le \gamma \le 1$. ∎

Remarks 5.1:
When $F(\cdot)$ is one-to-one in $0 \le \gamma \le 1$ and $G(\cdot)$ is such that $0 \le G(\gamma) \le 1$ for all $0 \le \gamma \le 1$, the following hold

(i) $F(\gamma) = 0$ iff $\gamma = 1$,
(ii) $F(0) > 0$, and
(iii) $F(\gamma)$ is a decreasing function of γ. ∎

Now the derivative of F is

$$F^{'}(\gamma) = -1 + G(\gamma) - G^{'}(\gamma)(1-\gamma)$$

Lemma 5.3: If $K \ge 1, p \ge 2$ and $G(\cdot)$ is as in (5), then $G^{'}(\gamma) < 0$ and $|G^{'}(\gamma)| \le \frac{2p}{b_0}$ for all $0 \le \gamma \le 1$.

Proof: See Technical Report [20]. ∎

Clearly, $G(\gamma) \le \frac{1}{b_0}$ and $1 \ge (1-\gamma) \ge 0$ for all $0 \le \gamma \le 1$. Substituting into the expression for $F^{'}(\gamma)$, we get,

$$F^{'}(\gamma) \le -1 + \frac{1+2p}{b_0}$$

Thus, if in addition to the other condition in Lemma 5.3, if $b_0 > 1 + 2p$, then $F^{'}(\gamma) < 0$ and the following result holds by virtue of the remark following Theorem 5.1.

Theorem 5.2: For a function $G(\cdot)$ defined as in (5) if $K \geq 1, p \geq 2$ and $b_0 > 2p + 1$, then the system $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ has a unique fixed point which is balanced. ■

Remark: It can be shown that if Lemma 5.3 holds for $G(\cdot)$ as in (5) it also holds for any case in which $b_k = p^k b_0$ for $0 \leq k \leq m \leq K$ and $b_k = p^m b_0$ for $m < k \leq K$. The latter situation closely matches the IEEE 802.11 standard (with $b_0 = 16, p = 2, K = 7, m = 5$). Hence a homogeneous IEEE 802.11 WLAN has a unique fixed point which is also balanced. In general, if the function $G(\cdot)$ is arbitrary (as in (1)) but monotone decreasing, then there exists a unique balanced fixed point for the system whenever the function $(1-\gamma)(1-G(\gamma))$ is one-to-one.

B. The Nonhomogeneous Case

In this section, we will extend our results to systems with nonhomogeneous nodes. AIFS will be introduced in Section VI. Nonhomogeneity is introduced by using different values of b_0, p and K in different nodes.

Consider a nonhomogeneous system of n nodes, with $G_i(\cdot)$ a monotonically decreasing function and $F_i(\gamma) := (1-\gamma)(1-G_i(\gamma))$ being one-to-one for all i. Let there be two fixed point solutions $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ and $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ for the above system (see Section III for the fixed point equations), and there exists $k, 1 \leq k \leq n$, such that $\gamma_k \neq \lambda_k$. From the necessary condition (4) we require that, for all i, and for some $J_1 > 0$ and $J_2 > 0$ (clearly, $J_1, J_2 \neq 0$, see Remarks 5.1),

$$(1-\gamma_i)(1-G_i(\gamma_i)) = J_1$$
$$(1-\lambda_i)(1-G_i(\lambda_i)) = J_2$$

Since $(1-\gamma)(1-G_i(\gamma))$ is one-to-one, applying this to γ_k and λ_k, we require $J_1 \neq J_2$. Without loss of generality, assume $J_1 < J_2$. Hence, $\gamma_i > \lambda_i$ for all i (see Remarks 5.1). Using (3) we have,

$$\begin{aligned} \lambda_i &= 1 - \prod_{j \neq i}(1 - G_j(\lambda_j)) \\ &\geq 1 - \prod_{j \neq i}(1 - G_j(\gamma_j)) \\ &= \gamma_i \end{aligned}$$

a contradiction. Hence, it must be that $J_1 = J_2$ or there exists a unique fixed point.

Notice that the arguments above immediately imply the following result.

Theorem 5.3: If $G_i(\gamma)$ is a decreasing function of γ for all i and $(1-\gamma)(1-G_i(\gamma))$ is a strictly monotone function on $[0,1]$, then the system of equations $\beta_i = G_i(\gamma_i)$ and $\gamma_i = \Gamma_i(\beta_1, \ldots, \beta_i, \ldots, \beta_n)$ has a unique fixed point. ■

Where nodes use exponentially increasing backoff, the next result then follows.

Theorem 5.4: For a system of nodes $1 \leq i \leq n$, with $G_i(\cdot)$ as in (5), that satisfy $K_i \geq 1, \ p_i \geq 2$ and $b_{0_i} > 2p_i + 1$, there exists a unique fixed point for the system of equations, $\gamma_i = 1 - \prod_{j \neq i}(1 - G_j(\gamma_j))$ for $1 \leq i \leq n$. ■

Remark: The above result has relevance in the context of the IEEE 802.11e standard where the proposal is to use differences in backoff parameters to differentiate the throughputs obtained by the various nodes. While Theorem 5.4 only states a sufficient condition, it does point to a caution in choosing the backoff parameters of the nodes.

VI. ANALYSIS OF THE AIFS MECHANISM

Our approach for obtaining the fixed point equations when the AIFS mechanism is included is the same as the one developed in [13]. However, we develop the analysis in the more general framework introduced in [2] and extended here in Section III. We show that under the condition that $F(\cdot)$ is one-to-one there exists a unique fixed point for this problem as well. The analysis is presented here for two different AIFS class case, but can be extended to any number of classes. Also in this section, we consider only the case in which there is one queue (of an AIFS class) in each node. Extension to the case of multiple queues per node is done in Section VII.

Let us begin by recalling the basic idea of AIFS based service differentiation (see [3]). In legacy DCF, a node decrements its backoff counter, and then attempts to transmit only after it senses an idle medium for more than a DCF interframe space (DIFS). However, in EDCA (Enhanced Distributed Channel Access), based on the access category of a node (and its AIFS value), a node attempts to transmit only after it senses the medium idle for more than its AIFS. Higher priority nodes have smaller values of AIFS, and hence obtain a lower average collision probability, since these nodes can decrement their backoff counters, and even transmit, in slots in which lower priority nodes (waiting to complete their AIFSs) cannot. Thus, *nodes of higher priority (lower AIFS) not only tend to transmit more often but also have fewer collisions compared to nodes of lower priority (larger AIFS).* The model we use to analyze the AIFS mechanism is quite general and accomodates the actual nuances of AIFS implementations (see [14] for how AIFS and DIFS differs) when the AIFS parameter value and the sampled backoff value is suitably adjusted (see technical report [20] for details).

A. The Fixed Point Equations

Let us consider two classes of nodes of two different priorities. The priority for a class is supported by using AIFS as well as b_0, p and K. All the nodes of a particular priority have the same values for all these parameters. There are $n^{(1)}$ nodes of Class 1 and $n^{(0)}$ nodes of Class 0. Class 1 corresponds to a higher priority of service. The AIFS for Class 0 exceeds the AIFS of Class 1 by l slots. Thus, after every transmission activity in the channel, while Class 0 nodes wait to complete their AIFS, Class 1 nodes can attempt to transmit in those l slots. Also, if there is any transmission activity (by Class 1 nodes) during those l slots, then again the Class 0 nodes wait for another additional l slots compared to the Class 1 nodes, and so on.

As in [4] and [2], we need to model only the evolution of the backoff process of a node (i.e., the backoff slots after removing any channel activity such as transmissions or collisions) to

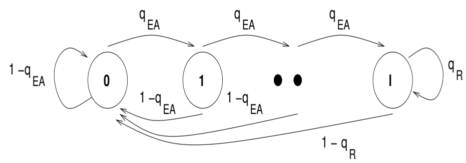


Fig. 4. AIFS differentiation mechanism: Markov model for remaining number of AIFS slots.

obtain the collision probabilities. For convenience, let us call the slots in which only Class 1 nodes can attempt as *excess AIFS* slots, which will correspond to the subscript EA in the notation. In the *remaining* slots (corresponding to the subscript R in the notation) nodes of either class can attempt. Let us view such groups of slots, where different sets of nodes contend for the channel, as different *contention periods*. Let us define

$\beta_i^{(1)}$:= the attempt probability of a Class 1 node for all $i, 1 \leq i \leq n^{(1)}$, in the slots in which a Class 1 node can attempt (i.e., all the slots)

$\beta_i^{(0)}$:= the attempt probability of a Class 0 node for all $i, 1 \leq i \leq n^{(0)}$, in the contention periods during which Class 0 nodes can attempt (i.e., slots that are not Excess AIFS slots)

Note that in making these definitions we are modeling the attempt probabilities for Class 1 as being constant over all slots, i.e., the Excess AIFS slots and the remaining slots. This simplification is just an extension of the Bianchi's approximation, and has been shown to yield results that match well with simulations (see [13]).

Now the collision probabilities experienced by nodes will depend on the contention period (*excess AIFS* or *remaining* slots) that the system is in. The approach is to model the evolution over contention periods as a Markov Chain over the states $(0, 1, 2, \cdots, l)$, where the state s, $0 \leq s \leq (l-1)$, denotes that an amount of time equal to s slots has elapsed since the end of the AIFS for Class 1. These states correspond to the *excess AIFS* period in which only Class 1 nodes can attempt. In the *remaining* slots, when the state is $s = l$, all nodes can attempt.

In order to obtain the transition probabilities for this Markov chain we need the probability that a slot is idle. Using the decoupling assumption, the idle probability in any slot during the *excess AIFS* period is obtained as,

$$q_{EA} = \prod_{i=1}^{n^{(1)}} (1 - \beta_i^{(1)}) \tag{6}$$

Similarly, the idle probability in any of the remaining slots is obtained as,

$$q_R = \prod_{i=1}^{n^{(1)}} (1 - \beta_i^{(1)}) \prod_{j=1}^{n^{(0)}} (1 - \beta_j^{(0)}) \tag{7}$$

The transition structure of the Markov chain is shown in Figure 4. As compared to [13], we have used a simplification that the maximum contention window is much larger than l. If this were not the case then some nodes would certainly attempt before reaching l. In practice, l is small (e.g., 1 slot or 5 slots; see [3]) compared to the maximum contention window.

Let $\pi(EA)$ be the stationary probability of the system being in the *excess AIFS* period; i.e., this is the probability that the above Markov chain is in states 0, or 1, or $\cdots$, or $(l-1)$. In addition, let $\pi(R)$ be the steady state probability of the system being in the remaining slots, i.e., state l of the Markov chain. Solving the balance equations for the steady state probabilities, we obtain,

$$\begin{aligned} \pi(EA) &= \frac{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1}}{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1} + \frac{q_{EA}^l}{1-q_R}} \\ \pi(R) &= \frac{\frac{q_{EA}^l}{1-q_R}}{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1} + \frac{q_{EA}^l}{1-q_R}} \end{aligned} \tag{8}$$

The average collision probability of a node is then obtained by averaging the collision probability experienced by a node over the different contention periods. The average collision probability for Class 1 nodes is given by, for all i, $1 \leq i \leq n^{(1)}$,

$$\begin{aligned} \gamma_i^{(1)} &= \pi(EA) \left(1 - \prod_{j=1, j\neq i}^{n^{(1)}} (1 - \beta_j^{(1)})\right) + \pi(R) \\ &\times \left(1 - \left(\prod_{j=1, j\neq i}^{n^{(1)}} (1 - \beta_j^{(1)}) \prod_{j=1}^{n^{(0)}} (1 - \beta_j^{(0)})\right)\right) \end{aligned} \tag{9}$$

Similarly, the average collision probability of a Class 0 node is given by, for all i, $1 \leq i \leq n^{(0)}$,

$$\gamma_i^{(0)} = 1 - \left(\prod_{j=1}^{n^{(1)}} (1 - \beta_j^{(1)}) \prod_{j=1, j\neq i}^{n^{(0)}} (1 - \beta_j^{(0)})\right) \tag{10}$$

Our analysis in the remaining section now generalises the analysis of [13] and also establishes uniqueness of the fixed point and the property that the fixed point is balanced over nodes in the same class. Define $G^{(1)}(\cdot)$ and $G^{(0)}(\cdot)$ as in (1) (except that the superscripts here denote the class dependent backoff parameters, with nodes within a class having the same parameters). Then the average collision probability obtained from the above equations can be used to obtain the attempt rates by using the relations

$$\beta_i^{(1)} = G^{(1)}(\gamma_i^{(1)}), \text{ and } \beta_j^{(0)} = G^{(0)}(\gamma_j^{(0)}) \tag{11}$$

for all $1 \leq i \leq n^{(1)}$, $1 \leq j \leq n^{(0)}$. We obtain fixed point equations for the collision probabilities by substituting the attempt probabilities from (11) into (9) and (10) (and also into (6) and (7)). We have a continuous mapping from $[0, 1]^{n^{(1)}+n^{(0)}}$ to $[0, 1]^{n^{(1)}+n^{(0)}}$. It follows from Brouwer's fixed point theorem that there exists a fixed point.

B. Uniqueness of the Fixed Point

Lemma 6.1: If $F^{(\cdot)}$ is one-to-one, then collision probabilities of all the nodes of the same class are identical; i.e., the fixed points are balanced within each class.

Proof: See Appendix. ■

Theorem 6.1: The set of equations (9), (10) and (11) (together with (8), (6) and (7)), representing the fixed point equations for the AIFS model, has a unique solution if the corresponding functions $G^{(1)}$ and $G^{(0)}$ are monotone decreasing and $F^{(1)}$ and $F^{(0)}$ are one-to-one.

Proof: See Appendix. ■

Remark: It follows from the earlier results in this paper (see, for example, Theorem 5.2) that if $G^{(0)}(\cdot)$ and $G^{(1)}(\cdot)$ are of the form in (5), and if $K^{(i)} \geq 1, p^{(i)} \geq 2$, and $b_0^{(i)} > 2p^{(i)}+1$, for $i = 0, 1$, then the fixed point will be unique.

C. Numerical Results (Fixed Point Analysis, CMP and ns-2 Simulation)

Although the numerical accuracy of the fixed point analysis has been reported before (see [4], [13]), for completeness, in Figures 5 and 6, we compare the collision probability obtained using the fixed point analysis with ns-2 simulation and the CMP simulator. Figure 5 plots the collision probabilities of AC_VO (access category for voice; the high priority, as in [3]) nodes and AC_BE (access category for best-effort traffic, e.g., TCP; the low priority) nodes, with the number of AC_BE nodes fixed to 4. Figure 6 plots the collision probabilities of AC_VI (access category for video; the high priority) nodes and AC_BE (the low priority) nodes with the number of AC_BE nodes fixed to 12. AC_VO, AC_VI and AC_BE correspond to the IEEE 802.11e EDCA access categories. As observed in the plots, the AIFS model works very well whenever $l \ll CW_{min}$ of the traffic classes (see Technical report [20] for additional plots comparing the fixed point analysis with the simulations).

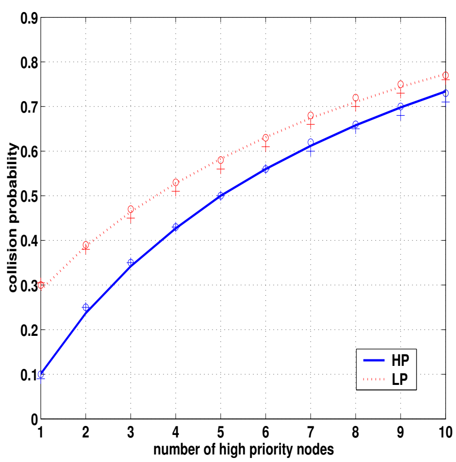


Fig. 5. Plots of collision probability of AC_VO (HP) nodes and AC_BE (LP) nodes with the number of AC_BE nodes fixed to 4. The lines correspond to the fixed point analysis, the "$+$" correspond to the ns-simulations and "o" correspond to the CMP simulator. The 95% confidence interval lies within 1% of the simulation estimate.

Remarks 6.1 (AIFS Differentiation and Multistability): It has been observed that (see [1]) as the number of nodes in the system increases, AIFS provides non-preemptive service to high priority nodes, starving the low priority nodes. This may lead to long periods of time when high priority nodes get serviced while the low priority nodes wait. We capture this behaviour using the Markov model in Figure 4. This

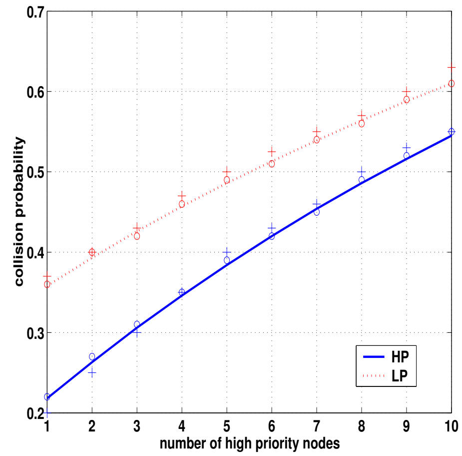


Fig. 6. Plots of collision probability of AC_VI (HP) nodes and AC_BE (LP) nodes with the number of AC_BE nodes fixed to 12. The lines correspond to the fixed point analysis, the "$+$" correspond to the ns-simulations and "o" correspond to the CMP simulator. The 95% confidence interval lies within 1% of the simulation estimate.

cannot be viewed as multistability (as seen in Section IV), because AIFS always gives preferential access to the high priority nodes, while starving the low priority nodes, and never the other way. Further, in our analysis on AIFS, the attempt probability $\beta^{(i)}$ of a class i corresponds to only those slots in which class i can attempt (rather than all slots). The variation in attempt rate and collision probability, due to AIFS, is captured using the Markov model shown in Figure 4.

VII. Multiple Access Categories per Node

In this section we further generalize our fixed point analysis to include the possibility of multiple access categories (or queues) per node. We consider n nodes and c_i access categories (ACs) per node i; the ACs can be of either AIFS class (for simplicity, we consider only two AIFS classes) and $c_i = c_i^{(1)} + c_i^{(0)}$ (the superscripts refering to the AIFS classes as before). The ACs in a node need not have the same $G(\cdot)$. Since there are multiple ACs per node, each with its own backoff process, it is possible that two or more ACs in a node complete their backoffs at the same slot. This is then called *Virtual Collision*, and is resolved in favour of the queue with the highest *Collision Priority* in the node. We label the ACs from 1 to c_i, with AC 1 corresponding to the highest collision priority in the node and AC c_i corresponding to the least collision priority. Unlike the single access category per node case where a collision is caused whenever any two nodes (equivalently, ACs) attempt in a slot, here, a AC sees a collision in a slot only when a AC of some other node or a higher priority AC of the same node attempts in that slot. A low priority AC of a node cannot cause collision to a higher priority AC in the same node. In Section VII-A we will study multiple access categories per node without AIFS (i.e., all the ACs wait only for DIFS) and consider AIFS later in Section VII-B.

We assume that, in a node (say i), the AIFS of Class 0 ACs (with $c_i^{(0)}$ ACs) exceeds the AIFS of the higher priority Class 1 ACs (with $c_i^{(1)}$ ACs) by l slots. Also we assume that the Class 1 ACs have a higher collision priority compared to Class 0 ACs in a node. This assumption conforms with the way access categories are defined in the IEEE 802.11e standard. Also, when collision priorities are interchanged with AIFS priorities, the actual performance of the system would be hard to characterise.

A. Without AIFS

Let $\gamma_{i,j}$ be the collision probability of AC j of node i and $\beta_{i,j}$ be the attempt probability of AC j of node i, when the AC can attempt. The fixed point equations for this system are, for all $i = 1, \cdots, n$ (and $j = 1, \cdots, c_i$),

$$\beta_{i,j} = G_{i,j}(\gamma_{i,j}) \tag{12}$$
$$\gamma_{i,j} = 1 - \prod_{m=1}^{j-1}(1-\beta_{i,m}) \prod_{\{k=1,k\neq i\}}^{n} \prod_{l=1}^{c_k}(1-\beta_{k,l}) \tag{13}$$

where $G_{i,j}(\cdot)$ depend on the backoff parameters of AC j of node i. The term $\prod_{m=1}^{j-1}(1-\beta_{i,m})$ in the above equation corresponds to the higher priority ACs in the same node. Observe that the $G_{i,j}(\cdot)$ definition allows the possibility of different backoff parameters (b_0, p, K) within a node.

Theorem 7.1: The fixed point equations in $\boldsymbol{\gamma}$, obtained by substituting (12) in (13) has a unique solution when $G_{i,j}$ is monotone decreasing and $F_{i,j}(\gamma) := (1-\gamma)(1-G_{i,j}(\gamma))$ is one-to-one for all $i = 1, \cdots, n$ and $j = 1, \cdots, c_i$.

Proof: See Technical Report [20]. ∎

B. With AIFS

In this section, we analyse the system where nodes have ACs of either AIFS class (the case where there are only Class 1 ACs can be modeled using the approach in Section VII-A). Define for $1 \le i \le n$, $1 \le j \le c_i$, $C_{i,j} \in \{0,1\}$ to be the AIFS class of AC j in node i. Writing the fixed point equations for i, j s.t. $C_{i,j} = 1$, we obtain,

$$\begin{aligned}\gamma_{i,j} = 1 - \Bigg(&\pi(EA)\prod_{m=1}^{j-1}(1-\beta_{i,m}) \\ &\times \prod_{\{k=1,k\neq i\}}^{n} \prod_{\{1\le l\le c_k: C_{k,l}=1\}} (1-\beta_{k,l}) \\ &+ \pi(R)\prod_{m=1}^{j-1}(1-\beta_{i,m}) \prod_{k=1,k\neq i}^{n}\prod_{l=1}^{c_k}(1-\beta_{k,l})\Bigg)\end{aligned} \tag{14}$$

and for i, j s.t. $C_{i,j} = 0$, we obtain,

$$\gamma_{i,j} = 1 - \prod_{m=1}^{j-1}(1-\beta_{i,m}) \prod_{\{k=1,k\neq i\}}^{n} \prod_{l=1}^{c_k}(1-\beta_{k,l}) \tag{15}$$

and $\beta_{i,j} = G_{i,j}(\gamma_{i,j})$. $\pi(EA)$ and $\pi(R)$ are defined as before (see (8)), with q_{EA} and q_R defined as

$$q_{EA} = \prod_{k=1}^{n} \prod_{\{1\le l\le c_k: C_{k,l}=1\}} (1-\beta_{k,l})$$
$$q_R = \prod_{k=1}^{n}\prod_{l=1}^{c_k}(1-\beta_{k,l}) \tag{16}$$

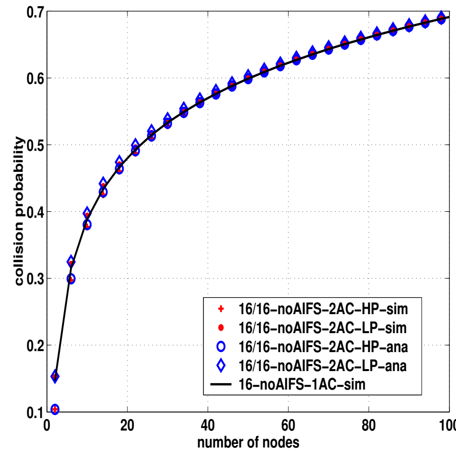


Fig. 7. Collision probability of high priority AC (HP) and low priority AC (LP) in a system of nodes with two ACs. Both simulation (sim) and analysis (ana) are plotted. The backoff parameters of both the ACs (in all the nodes) are identical with $b_0 = 16$ and AIFS = DIFS. Also plotted is the collision probability (obtained from simulation) for single AC per node case with same backoff parameters and twice the number of nodes. In all the cases $p = 2$ and $K = 7$. For the simulation results, the 95% confidence interval lies within 1% of the mean value.

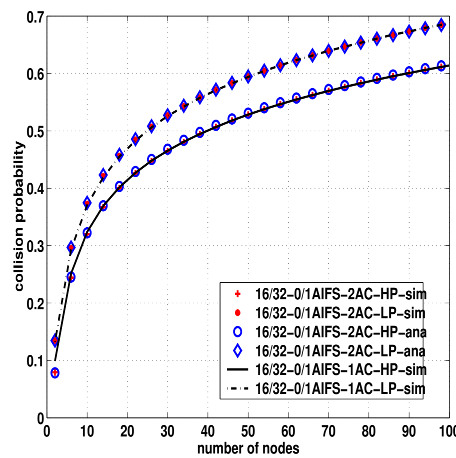


Fig. 8. Collision probability of high priority AC (HP) and low priority AC (LP) in a system of nodes with two ACs. Both simulation (sim) and analysis (ana) are plotted. For the high priority AC, $b_0 = 16$ and AIFS = DIFS, while for the low priority AC we have $b_0 = 32$ and AIFS = DIFS + 1 slot. Also plotted is the collision probability (from simulation) of two classes of nodes when the two ACs of a node are considered as independent ACs in separate nodes. In all the cases $p = 2$ and $K = 7$. For the simulation results, the 95% confidence interval lies within 1% of the mean value.

Theorem 7.2: The fixed point equations (14) and (15) have a unique solution when $G_{i,j}$ are monotone decreasing and $F_{i,j}(\cdot)$ are one-to-one for all $i = 1, \cdots, n$ and for each i, for all $j = 1, \cdots, c_i$.

Proof: See Appendix. ∎

Figures 7 and 8 plot performance results for the multiple ACs per node case. In Figure 7, we consider a set of homogeneous nodes each with two access categories. The backoff parameters for either AC are the same ($b_0 = 16, p = 2, K = 7$ and AIFS = DIFS). The figure plots the collision probability of the higher priority (HP) AC and the low priority (LP) AC obtained from CMP simulator as well as from analysis. Also plotted in comparison is the collision probability (from simulation) for the single AC per node case with twice the number of nodes. Notice that, except for small n, the performance of the high priority AC and the low priority AC are almost identical (the backoff parameters are identical), and close to the performance of the single AC per node case (see Remark 7.1 below).

In Figure 8, we again consider a set of nodes each with two access categories. The higher priority AC has $b_0 = 16$ and AIFS = DIFS, while the low priority AC has $b_0 = 32$ and AIFS = DIFS + 1 slot. $p = 2$ and $K = 7$ for either case. Figure 8 plots the collision probability of the high priority AC and the low priority AC from simulation as well as the analysis. Also plotted is the collision probability for the two classes of nodes (from simulation) obtained by modeling the two ACs in a node as independent ACs in separate nodes. Notice again that except for small n, the performance of the multiple queue per node case is close to the performance of the single queue case.

Remarks 7.1: The above observations from Figures 7 and 8 can be understood as follows. From the fixed point equations in Section VII, we see that for the high priority AC in any node, only one term corresponding to the low priority AC of the same node is missing (for the systems in Figures 7 and 8 with two ACs), in comparison to the case in which all the ACs are in $2n$ separate nodes. Hence, as n increases, the effect this single AC in the same node diminishes, and the performance of the multiple queue per node case coincides with the performance of the single queue per node case each with one of the original ACs. ■

VIII. Conclusions and Further Research Directions

In this paper we have studied a multidimensional fixed point equation arising from a model of the backoff process of the EDCA access mechanism in IEEE 802.11e Wireless LANs. Our first concern was the consequences of the nonuniqueness of the fixed point solution and conditions for uniqueness. We demonstrated via examples of homogeneous systems that even when the balanced fixed point is unique, the existence of unbalanced fixed points coexists with the observation of severe short term unfairness in simulations. Further, in such examples the balanced fixed point solution does not capture the long run average behaviour of the system. With these observations in mind, we concluded that it is desirable to have systems in which there is a unique fixed point, even for a nonhomogeneous system.

We have provided simple sufficient conditions on the node backoff parameters that guarantee that a unique fixed point exists. We have shown that the default IEEE 802.11 parameters satisfy these sufficient conditions. The IEEE 802.11e standard motivated us to consider the nonhomogeneous case, and in this case our results suggest certain *safe* ranges of parameters that guarantee the uniqueness of the fixed point while providing service differentiation.

Further, using the fixed point analysis, in [20], we were also able to obtain insights into how the different backoff parameters provide throughput differentiation between the nodes in a nonhomogeneous system. We observed that using initial backoff window, in general, a fixed throughput ratio can be achieved. On the other hand, using p and $AIFS$ the service can be significantly biased towards the high priority class, with the differentiation increasing in favour of the high priority class as the load in the system increases. We also observed that the effect of collision priority, where there are multiple access categories per node, decreases as the number of nodes increases.

The fixed point approach is simply a heuristic that is found to work well in some cases. Our work in this paper suggests where it might not work and where it might work. In a recent work [15], the authors have proved that for random backoff algorithms, when the number of sources grow large, the system is indeed decoupled, providing a theoretical justification of decoupling arguments used in the analysis.

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APPENDIX

A. Proof of Lemma 6.1

Rewriting (9), for all i, $1 \le i \le n^{(1)}$, we get,

$$\begin{aligned}(1-\gamma_i^{(1)}) &= \prod_{j=1,j\neq i}^{n^{(1)}}(1-\beta_j^{(1)})[\pi(EA) \\ &+ \pi(R)\prod_{k=1}^{n^{(0)}}(1-\beta_k^{(0)})]\end{aligned}$$

Multiplying by $(1-\beta_i^{(1)})$ and using the fact that $\beta_i^{(1)} = G^{(1)}(\gamma_i^{(1)})$, we have,

$$(1-\gamma_i^{(1)})(1-G^{(1)}(\gamma_i^{(1)})) = \pi(EA)q_{EA} + \pi(R)q_R \tag{17}$$

In (17), we see that the right hand side is independent of i. Hence, if the left hand side function, $F^{(1)}(\gamma) := (1-\gamma)(1-G^{(1)}(\gamma))$, is one to one, then $\gamma_i^{(1)} = \gamma_j^{(1)}$ for all $1 \le i,j, \le n^{(1)}$. Similarly, we can see from (10) that, for all i, $1 \le i \le n^{(0)}$,

$$(1-\gamma_i^{(0)})(1-G^{(0)}(\gamma_i^{(0)})) = q_R \tag{18}$$

Hence again, $\gamma_i^{(0)} = \gamma_j^{(0)}$ for all $1 \le i,j, \le n^{(0)}$, if $F^{(0)}$ is one to one.

B. Proof of Theorem 6.1

From Lemma 6.1, we already know that the fixed point is balanced within a class. Now, assume that there exist two vector fixed point solutions, $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$, with the first $n^{(1)}$ elements of $\boldsymbol{\gamma}$ being $\gamma^{(1)}$ and the remaining $n^{(0)}$ elements being $\gamma^{(0)}$. Similarly, the first $n^{(1)}$ elements of $\boldsymbol{\lambda}$ are $\lambda^{(1)}$ and the next $n^{(0)}$ elements are $\lambda^{(0)}$.

Let us, in this proof, denote the value of q_R (see (7)) for the fixed point $\boldsymbol{\gamma}$ as $q_R(\boldsymbol{\gamma})$ and for the fixed point $\boldsymbol{\lambda}$ as $q_R(\boldsymbol{\lambda})$; similarly, we do for q_{EA} and for other variables.

Lemma B.1: Let $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ be two fixed point solutions and let $F^{(0)}$ be one-to-one. If $\gamma^{(1)} < \lambda^{(1)}$, then $\gamma^{(0)} > \lambda^{(0)}$. Also, $\gamma^{(1)} = \lambda^{(1)}$ *iff* $\gamma^{(0)} = \lambda^{(0)}$.

Proof: Without loss of generality, let $\gamma^{(1)} < \lambda^{(1)}$. Then $G^{(1)}(\gamma^{(1)}) > G^{(1)}(\lambda^{(1)})$ (see Lemma 5.1). Hence,

$$(1-G^{(1)}(\gamma^{(1)}))^{n^{(1)}} < (1-G^{(1)}(\lambda^{(1)}))^{n^{(1)}}$$

If we assume $\gamma^{(0)} < \lambda^{(0)}$, then $q_R(\gamma^{(0)}) > q_R(\lambda^{(0)})$ (see (18)). Hence, we require

$$\begin{aligned}(1-G^{(1)}(\gamma^{(1)}))^{n(1)}(1-G^{(0)}(\gamma^{(0)}))^{n(0)} > \\ (1-G^{(1)}(\lambda^{(1)}))^{n(1)}(1-G^{(0)}(\lambda^{(0)}))^{n(0)}\end{aligned}$$

Or,

$$(1-G^{(0)}(\gamma^{(0)}))^{n(0)} > (1-G^{(0)}(\lambda^{(0)}))^{n(0)}$$

which implies $\gamma^{(0)} > \lambda^{(0)}$, which is a contradiction.

If $\gamma^{(0)} = \lambda^{(0)}$, then $q_R(\gamma^{(0)}) = q_R(\lambda^{(0)})$. Hence, $(1-G^{(1)}(\gamma^{(1)}))^{n^{(1)}} = (1-G^{(1)}(\lambda^{(1)}))^{n^{(1)}}$, Or, $\gamma^{(1)} = \lambda^{(1)}$. Hence, if $\gamma^{(1)} < \lambda^{(1)}$, then $\gamma^{(0)} > \lambda^{(0)}$. Let $\gamma^{(0)} \neq \lambda^{(0)}$, then $q_R(\gamma^{(0)}) \neq q_R(\lambda^{(0)})$. Hence, $(1-G^{(1)}(\gamma^{(1)}))^{n^{(1)}} \neq (1-G^{(1)}(\lambda^{(1)}))^{n^{(1)}}$, Or, $\gamma^{(1)} \neq \lambda^{(1)}$. ■

Now, using (8), write the right hand side of (17) as

$$J(q_{EA},q_R,l) := \frac{q_{EA}(1+q_{EA}+\cdots+q_{EA}^{l-1}) + q_R\frac{q_{EA}^l}{1-q_R}}{1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1}+\frac{q_{EA}^l}{1-q_R}} \tag{19}$$

Lemma B.2: If $\gamma^{(1)} < \lambda^{(1)}$, then $J(q_{EA}(\boldsymbol{\gamma}), q_R(\boldsymbol{\gamma}), l) < J(q_{EA}(\boldsymbol{\lambda}), q_R(\boldsymbol{\lambda}), l)$.

Proof:

Consider $J(q_{EA}, q_R, l)$ (see (19)).

$$J(q_{EA},q_R,l) = \frac{q_{EA}(1+q_{EA}+\cdots+q_{EA}^{l-1}) + q_R\frac{q_{EA}^l}{1-q_R}}{1+q_{EA}+\cdots+q_{EA}^{l-1}+\frac{q_{EA}^l}{1-q_R}}$$

Expanding and rewriting the above equation, we get,

$$= \frac{q_{EA}+q_{EA}(q_{EA}-q_R)+\cdots+q_{EA}^{l-1}(q_{EA}-q_R)}{q_{EA}+q_{EA}(q_{EA}-q_R)+\cdots+(1-q_R)}$$

which is of the form $\frac{f1}{f1+f2}$. When $\gamma^{(1)} < \lambda^{(1)}$, then $\gamma^{(0)} > \lambda^{(0)}$ (from the previous lemma). Hence,

$$\begin{aligned}q_{EA}(\boldsymbol{\gamma}) &- q_R(\boldsymbol{\gamma}) \\ &= \prod_{i=1}^{n^{(1)}}(1-G^{(1)}(\gamma^{(1)}))(1-\prod_{i=1}^{n^{(0)}}(1-G^{(0)}(\gamma^{(0)}))) \\ &< \prod_{i=1}^{n^{(1)}}(1-G^{(1)}(\lambda^{(1)}))(1-\prod_{i=1}^{n^{(0)}}(1-G^{(0)}(\lambda^{(0)}))) \\ &= q_{EA}(\boldsymbol{\lambda}) - q_R(\boldsymbol{\lambda})\end{aligned}$$

Also, we can see that,

$$q_{EA}(\boldsymbol{\gamma}) < q_{EA}(\boldsymbol{\lambda})$$
$$q_R(\boldsymbol{\gamma}) < q_R(\boldsymbol{\lambda})$$

Using the above three inequalities, we can see that,

$$J(q_{EA}(\boldsymbol{\gamma}), q_R(\boldsymbol{\gamma}), l) < J(q_{EA}(\boldsymbol{\lambda}), q_R(\boldsymbol{\lambda}), l)$$

■

If $\gamma^{(1)} < \lambda^{(1)}$, then $(1-\gamma^{(1)})(1-G^{(1)}(\gamma^{(1)})) > (1-\lambda^{(1)})(1-G^{(1)}(\lambda^{(1)}))$. However, from the above lemma and the right hand side of (17), we see that we have a contradiction.

C. Proof of Theorem 7.2

Consider c_i access categories per node i with $c_i^{(1)}$ ACs $(1,\cdots,c_i^{(1)})$ with $AIFS^{(1)}$, and the remaining $c_i^{(0)}$ ACs $(c_i^{(1)}+1,\cdots,c_i)$ with AIFS = $AIFS^{(1)}$ + l slots. The fixed point equations for the system are given in (14) and (15).

As before, by Brouwer's fixed point theorem, there exists a fixed point for the system of equations. Assume that there exist two fixed point solutions for the above system of equations, $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ with $\gamma_{i,j}$ and $\lambda_{i,j}$ as elements.

Let us, in this proof, denote the value of q_R (see (16)) for the fixed point $\boldsymbol{\gamma}$ as $q_R(\boldsymbol{\gamma})$ and for the fixed point $\boldsymbol{\lambda}$ as $q_R(\boldsymbol{\lambda})$; similarly, we do for q_{EA} and for other variables.

In a node i, consider two ACs of the same AIFS class, i.e., j and $j-1$ s.t. $C_{i,j} = C_{i,j-1}$. From (14) or (15), we see that,

$$(1-\gamma_{i,j}) \quad = \quad (1-\gamma_{i,j-1})(1-G_{i,j-1}(\gamma_{i,j-1}))$$

or,

$$(1-\gamma_{i,j}) = F_{i,j-1}(\gamma_{i,j-1})$$

Hence, using the one-to-one property of $F_{i,j}(\cdot)$ if $\gamma_{i,j} < \lambda_{i,j}$, then $\gamma_{i,k} < \lambda_{i,k}$ for all k such that $C_{i,j} = C_{i,k}$,

Now consider all those nodes with $C_{i,c_i} = 0$, i.e., the least collision priority AC in a node is of AIFS class 0. We then have, using (15) and (16),

$$\begin{aligned} (1-\gamma_{i,c_i})(1-G_{i,c_i}(\gamma_{i,c_i})) &= q_R(\boldsymbol{\gamma}) \\ (1-\lambda_{i,c_i})(1-G_{i,c_i}(\lambda_{i,c_i})) &= q_R(\boldsymbol{\lambda}) \end{aligned}$$

i.e., $F_{i,c_i}(\gamma_{i,c_i}) = q_R(\boldsymbol{\gamma})$ and $F_{i,c_i}(\lambda_{i,c_i}) = q_R(\boldsymbol{\lambda})$. If $q_R(\boldsymbol{\gamma}) > q_R(\boldsymbol{\lambda})$, then $\gamma_{i,c_i} < \lambda_{i,c_i}$ for all i s.t. $C_{i,c_i} = 0$. If $q_R(\boldsymbol{\gamma}) = q_R(\boldsymbol{\lambda})$, then $\gamma_{i,c_i} = \lambda_{i,c_i}$ for all i s.t. $C_{i,c_i} = 0$. Combining the above two results, we see that for all i,j s.t. $C_{i,j} = 0$, either $\gamma_{i,j} > \lambda_{i,j}$ or $\gamma_{i,j} = \lambda_{i,j}$ or $\gamma_{i,j} < \lambda_{i,j}$.

Without loss of generality, assume that the collision probability of Class 0 ACs is more in $\boldsymbol{\gamma}$ than in $\boldsymbol{\lambda}$ ($\boldsymbol{\gamma^{(0)}} > \boldsymbol{\lambda^{(0)}}$, $\boldsymbol{\gamma^{(0)}}$ and $\boldsymbol{\lambda^{(0)}}$ are the vector of collision probabilities corresponding to AIFS class 0 in the vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ respectively). Hence, $q_R(\boldsymbol{\gamma}) < q_R(\boldsymbol{\lambda})$. Also, $q_{EA}(\boldsymbol{\gamma}) < q_{EA}(\boldsymbol{\lambda})$ (the proof is similar to that provided for AIFS with single AC per node and is not provided), which implies $\boldsymbol{\gamma^{(1)}} < \boldsymbol{\lambda^{(1)}}$.

Now consider the expression $F(\cdot)$ for the least collision priority Class 1 AC, say j, of any node i (see (14)),

$$\begin{aligned} (1-\gamma_{i,j})(1-G_{i,j}(\gamma_{i,j})) &= \pi(EA,\boldsymbol{\gamma})q_{EA}(\boldsymbol{\gamma}) \\ &+ \pi(R,\boldsymbol{\gamma})q_{R_{(i,j)}}(\boldsymbol{\gamma}) \\ (1-\lambda_{i,j})(1-G_{i,j}(\lambda_{i,j})) &= \pi(EA,\boldsymbol{\gamma})q_{EA}(\boldsymbol{\lambda}) \\ &+ \pi(R,\boldsymbol{\gamma})q_{R_{(i,j)}}(\boldsymbol{\lambda}) \end{aligned}$$

where $q_{R_{(i,j)}} = \prod_{m=1}^{j}(1-\beta_{i,m})\prod_{\{1\le k\le n, k\ne i\}}\prod_{l=1}^{c_k}(1-\beta_{k,l})$. Notice that $q_R^{(i,j)}$ is similar to q_R except for terms corresponding to the Class 0 (with lower collision priority) ACs in node i. Hence, if $\boldsymbol{\gamma^{(0)}} > \boldsymbol{\lambda^{(0)}}$, then not only is $q_{EA}(\boldsymbol{\gamma}) < q_{EA}(\boldsymbol{\lambda})$ and $q_R(\boldsymbol{\gamma}) < q_R(\boldsymbol{\lambda})$, but also, $q_{R_{i,j}}(\boldsymbol{\gamma}) < q_{R_{i,j}}(\boldsymbol{\lambda})$. Expanding $(1-\cdot_{i,j})(1-G_{i,j}(\cdot_{i,j}))$, we get,

$$\begin{aligned} &(1-\cdot_{i,j})(1-G_{i,j}(\cdot_{i,j})) = \\ &\frac{(1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1})q_{EA}+\frac{q_{EA}^l}{1-q_R}q_{R_{(i,j)}}}{1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1}+\frac{q_{EA}^l}{1-q_R}} \\ = &\frac{q_{EA}+q_{EA}(q_{EA}-q_R)+\cdots+q_{EA}^l(q_{R_{(i,j)}}-q_R)}{q_{EA}+q_{EA}(q_{EA}-q_R)+\cdots+(1-q_R)} \end{aligned}$$

where $q_{EA}-q_R = q_{EA}(1-\prod_{k=1}^{N}\prod_{\{l=1,C_l^k=0\}}^{n_k}(1-\beta_{k,l}))$ and $q_R^{(i,j)} - q_R = q_R^{(i,j)}(1-\prod_{\{l=1,C_l^i=0\}}^{n_i}(1-\beta_{i,l}))$. Clearly, if $\boldsymbol{\gamma^{(0)}} > \boldsymbol{\lambda^{(0)}}$, then $q_{EA}(\boldsymbol{\gamma}) - q_R(\boldsymbol{\gamma}) < q_{EA}(\boldsymbol{\lambda}) - q_R(\boldsymbol{\lambda})$ and $q_{R_{i,j}}(\boldsymbol{\gamma}) - q_R(\boldsymbol{\gamma}) < q_{R_{i,j}}(\boldsymbol{\lambda}) - q_R(\boldsymbol{\lambda})$. Also, we know that $1-q_R(\boldsymbol{\gamma}) > 1-q_R(\boldsymbol{\lambda})$. From the above observations, we see that, $(1-\gamma_{i,j})(1-G_{i,j}(\gamma_{i,j})) < (1-\lambda_{i,j})(1-G_{i,j}(\lambda_{i,j}))$, which clearly implies that $\gamma_{i,j} > \lambda_{i,j}$. Hence we have $\boldsymbol{\gamma^{(1)}} > \boldsymbol{\lambda^{(1)}}$ which is a contradiction.

Also, we can see that $\boldsymbol{\gamma^{(1)}} = \boldsymbol{\lambda^{(1)}}$ *iff* $\boldsymbol{\gamma^{(0)}} = \boldsymbol{\lambda^{(0)}}$ (the proof is similar to that in Theorem 6.1 and is not provided here).

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