Delay Optimal Schedule for a 2-hop Vehicular Relay Network

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Abstract

We study a scheduling problem in a mobile network scenario where vehicles are used as relays. A fixed source node wants to transfer a file of a known size to a fixed destination node, located beyond its communication range. In the absence of any infrastructure connecting the two nodes, we consider the possibility of communication using vehicles passing by. Vehicles arrive at the source node at renewal instants and are known to travel towards the destination node with speed v from a given distribution. The source node communicates packets of the file to the destination node using these vehicles as relays. We assume that the vehicles communicate with the source node and the destination node only, and hence, every packet communication involves only two hops. In this setup, we study the source node's sequential decision problem of transferring packets of the file to vehicles as they pass by, with the objective of minimizing the time until the file is completely received at the destination node. We study both the finite file size case and the infinite file size case. In the finite file size case, using a Markov decision process (MDP) framework, we study the average delay minimization problem. In the infinite file size case, we study the optimal trade-off achievable between the average queueing delay experienced by the packets at the source node buffer and the average transit delay at the vehicles.

1 Introduction

We consider a scheduling problem in a mobile network scenario where vehicles are used as relays. A stationary source node wishes to send a file of a given size to a stationary destination node, located beyond its communication range. In the absence of a communication infrastructure connecting the source node and the destination node, we study the possibility of data transfer by relaying the packets using vehicles passing by. More precisely, we assume that the source and the destination of the file transfer are located by a road. At some random times, vehicles equipped with radio transceivers (and willing to serve as relays) pass by the source node towards the destination node, with speed v from a known distribution. The source node communicates packets of the file to the destination node, by using these vehicles as relays. In this work, we assume that the vehicles communicate with the source node and the destination node only, and they do not communicate among themselves. Thus, all packet communication between the source and the destination involves only two hops. Packet delay in this network scenario comprises of two components; queueing delay at the source node (before the packet is relayed) and transit delay at the vehicles (until the packet is delivered). The transit delay is assumed to be a function of the speed of the vehicle, and we assume that the source node has full knowledge of the vehicle speed at the time of relaying.

As the decision to relay a packet is taken only once (by the source node), there arises a natural tradeoff between the queueing delay of the packet and its transit delay. Small average queueing delay leads to large average transit delay and vice versa. Our objective is to study the source node's sequential decision problem of transferring packets of the file to vehicles as they pass by, with the objective of minimizing the time until the file is completely received at the destination. We study both the finite file size case and the infinite file size case. In the finite file size case, we are interested in minimizing the average delay to transfer the file (queueing delay plus the transit delay). In the infinite file size case, we study the asymptotic tradeoff achievable between the average queueing delay of the packets at the source node buffer and the average transit delay of the packets at the vehicles.

The above situation would arise in a wireless data network where there is limited access to backbone infrastructure and vehicles are used as a means of communication (as a relay). The fixed infrastructure of the wireless data network would consist of data posts (or info stations) located at various points along the roads. The data posts, in general, need not communicate directly with each other and we envision the possibility of using vehicles passing by to route packets among the data posts. Some of the data posts are connected to the backbone network, which in turn is connected to the Internet (we would call such data posts as access-data posts). Except for a few access-data posts, the network requires very little infrastructure, i.e., the data posts can be deployed anywhere along the roads, even without any direct connectivity to any other data posts or a fixed network. The data posts function both like routers as well as (WLAN) access points in hot-spots. End users communicate via the data posts, and the packets are routed appropriately to their destination using the vehicles. The main difference between the proposed architecture and the traditional wireless setup is that, we maintain network connectivity among the data posts using the vehicles and hence, there is no interference or the need of network design.

Figure 1 shows an example scenario studied in this work. A "source car" wishes to communicate a file to a "destination car", beyond its communication range. The first phase of data communication involves transferring the file from the source car to the stationary node S. The file is then communicated to node D using a "relay car", which passes by nodes S and D. The file is finally transferred from the stationary node D to the destination car, when the destination car passes by node D. In this work, we restrict ourselves to a simple one hop communication problem between two such data posts, a source node (like S) and a destination node (like D). We assume that the source node has a file with z

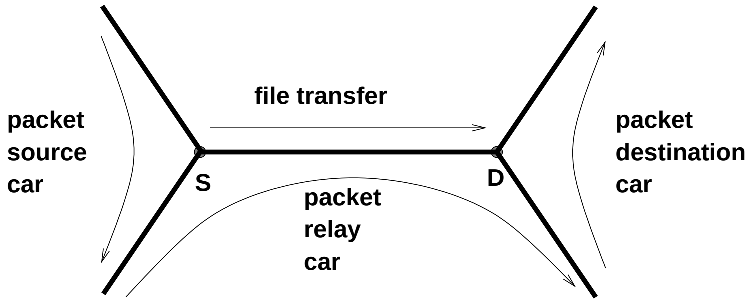

Figure 1: Vehicular Relay Network Scenario. “Source car” has z data packets to communicate to the “destination car”. The packets are first transferred from the source car to the stationary node S. Node S communicates the packets to node D using a “relay car” that passes by the nodes S and D. Finally, the packets are transferred from the stationary node D to the “destination car”, when it passes by.

packets to communicate to the destination node and study the sequential decision problem of relaying packets of the file to passing by vehicles.

Outline of the paper :

In Section 2, we describe the network model, and in Section 3, the optimization problem. We study the average delay minimization problem for the finite file case in Section 4. In Section 5, we study the asymptotic optimal tradeoff achievable between the queueing delay and the transit delay of the packets, for the infinite file size case. Section 6 concludes the paper and discusses future work.

2 The Network Model

A stationary source node intends to send a file of a fixed size (z packets) to a stationary destination node, located at a distance s metres away (i.e., a road of length s connects the two). At random times, vehicles that drive in the direction of the destination node enter the communication range of the source node. We assume that the interarrival times of the vehicles are i.i.d. and have a c.d.f. $I(\cdot)$ known to the source node. The ith vehicle that enters the communication range of the source node travels at an average speed v_i towards the destination node, where v_i is assumed to be from an i.i.d. sequence with known c.d.f. $V(\cdot)$. We assume that the ith vehicle (with speed v_i) takes $\frac{s}{v_i}$ seconds to cover the distance between the source and the destination. Further, we assume that the interarrival times of the vehicles are distributed independent of the speeds of the vehicles. As the ith vehicle enters the communication range of the source node, the source node gets the information about this event as well as the speed of the vehicle, v_i.

In this work, we assume that the source node can relay at most a single packet (of size L bytes) to the relay vehicles, independent of the vehicle speeds. We acknowledge that the time spent by the vehicles in the communication range of the source/destination will depend on the speed of the vechicles. This may permit the relay vehicles to carry different amounts of data depending on their speed. However, when the data posts (such as S and D in Figure 1) are located near a junction, the vehicle speeds in the coverage region will have little correlation with the average speed achieved over the distance s to the destination. Correlating the average speed of the vehicle to the actual speed in the coverage radius (over a very short duration of time) may not be appropriate as well. Hence, we restrict to single packet per relay vehicle model. Packet relay to vehicles (with different speeds) is achieved either by coding packets at different data rates or by restricting the set of vehicles to which communication is permitted.

Due to randomness in the interarrival distribution and the vehicular speeds, it is possible that the different packets of a file arrive at the destination in a random order. We assume that the destination has the capability to reassemble these packets. Also, it is possible that more than one vehicle is in the coverage region of the source or the destination node at any given point of time. We assume that both the source and the destination nodes are capable of simultaneous communication with vehicles, possibly in different, non-interfering bands.

3 The Optimization Problem

Packet delay in the network comprises queueing delay at the source node, transmission delay (depends on the bit rate at which data is transmitted to/from the relay vehicle) and transit delay at the relay vehicle. In this work, we assume that the transmission delay is very small as compared to the transit delay and study only the queueing delay and the transit delay of the packets. In other words, we assume that the packets are relayed instantaneously between the source/destination node and the relay vehicles.

There is a natural tradeoff between the queueing delay and the transit delay in the network. We can minimize the queueing delay by choosing every vehicle as a relay, thereby increasing the transit delay of the packets. Similarly, by relaying only to high speed vehicles, we can decrease the transit delay of the packets while increasing the queueing delay at the source buffer. Note that when minimizing the expected sojourn time of a packet (queueing delay plus the transit delay), if a slow vehicle arrives, it might be optimal to ignore it and to wait until the next vehicle arrives in the hope that it would be faster. In fact we would wish it to be sufficiently faster so as to compensate for the extra waiting time. In this work, we study this tradeoff between the queueing delay and the transit delay and find the delay minimizing policy. We study the following two versions of the scheduling problem in this paper.

1. **Finite File Size:** We study the average delay minimization problem for a single file with z packets ($1 < z < \infty$) at the source node. As we can relay only one packet per vehicle, this requires that the source node use z vehicles to relay the packets to

the destination. The goal is to minimize $\tau(v) = \mathsf{E}[\mathcal{T}(v)]$ for all v, where $\mathcal{T}(v)$ is the random time from the moment a vehicle with speed v arrives in the communication range of the source, until the file is completely transferred to the destination. We note here that $\mathcal{T}(v)$ is the maximum of the sojourn times of the z packets in the network, and not just the sojourn time of the last transmitted packet.

2. **Infinite Fize Size:** We assume that the source node has infinite packets to communicate to the destination node. More precisely, there is a packet arrival process at the source node, independent of the vehicle arrival process. Packets are queued at the source buffer until transmission, and our aim is to study the impact of the scheduling policy on the queueing delay and the transit delay of the packets. We first obtain the maximum throughput sustainable in the network for a given transit delay constraint. Using the throughput-transit delay curve, we then study the asymptotic optimal tradeoff achievable between the queueing delay and the transit delay of the packets.

4 Finite File Size

In this section, we will study the case where the source node has a file with z packets ($1 < z < \infty$) to communicate to the destination node. We are interested in minimizing the average delay in transfering the file completely to the destination. Every time a vehicle enters the communication range of the source node, the source node has to make a decision on using the current vehicle (with speed v) as a relay. As discussed in Section 2, we assume that only one packet is relayed through a vehicle, and hence, the vehicular communication requires z relay vehicles to complete the data transfer. Starting with the first vehicle to arrive, the decision problem evolves over vehicle arrival instants $\{T_0 = 0, T_1, T_2, \cdots\}$ with vehicle speeds $\{V_0 = v, V_1, V_2, \cdots\}$. As discussed in Section 2, $\{T_k, k \geq 0\}$ are renewal instants with i.i.d. interarrival times $\{I_{k+1} = T_{k+1} - T_k, k \geq 0\}$ and the vehicle speeds $\{V_k, k \geq 0\}$ are i.i.d. with c.d.f. $V(\cdot)$. Let X_k denote the residual number of packets of the file, at the source node, at time instant T_k. The system permits two possible actions: "to relay" and "not to relay". Define Y_k as the action chosen at time instant T_k, i.e., the number of packets relayed to the k^{th} vehicle. In our context, we have, $Y_k \in \{0, 1\}$. When $X_k = 0$, $Y_k = 0$ is the only permissible action. Define $\{D_k, k \geq 0\}$ as the instant (from T_k) when the last of any previously relayed data will reach the destination. At time instant T_k, $T_k + D_k$ lower bounds the minimum time required to complete the file transfer. $\{(X_k, V_k, D_k), k \geq 0\}$, denotes the system state, and the system state dynamics is now given by,

$$\begin{aligned} X_{k+1} &= X_k - Y_k \\ D_{k+1} &= \left(\max\left\{D_k, I_{\{Y_k>0\}}\frac{s}{V_k}\right\} - I_{k+1}\right)^+ \end{aligned}$$

Define $R_k(x_k, v_k, d_k, y_k)$ as,

$$R_k(x_k, v_k, d_k, y_k) = \begin{cases} 0 & x_k = 0 \\ I_{k+1} & x_k > 1 \\ I_{k+1} & x_k = 1 \text{ and } y_k = 0 \\ \max\left\{d_k, \frac{s}{v_k}\right\} & x_k = 1 \text{ and } y_k = 1 \end{cases} \tag{1}$$

$R_k(x_k, v_k, d_k, y_k)$ can be viewed as the single stage cost associated with the system state (x_k, v_k, d_k) and action y_k. For a sequence $\{(X_k, V_k, D_k, Y_k), k \geq 0\}$, satisfying the above dynamics, define the total cost function as

$$\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, Y_k)$$

Notice that the above total cost expression is the random time of delivery of the file to the destination from the moment $T_0 = 0$. Our aim is to minimize the average value of the above total cost function.

Given that $X_0 = z, V_0 = v$ and $D_0 = 0$, we wish to choose the policy $\{\pi_k, k \geq 0\}$, so as to minimise the expected delay in delivery of the file, i.e., we wish to solve the following problem,

$$\inf_{\{\pi_k, k\geq 0\}} \mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, Y_k)\middle| X_0 = z, V_0 = v, D_0 = 0\right] \tag{2}$$

where the infimum is over the set of all feasible policies. The problem has been formulated in a Markov decision process framework with a total cost criterion. Suppose that the speed of the vehicle is continuously distributed within a bounded set, i.e., $V_k \in [v_{min}, v_{max}]$. Further, assume that the interarrival time distribution is continuously distributed. Then, $D_k \in [0, \infty)$, and the state space of the problem (X_k, V_k, D_k) is a Borel set. The action space Y_k is finite in our case. When $\mathsf{E}[I]$, the mean interarrival time, is finite, the single stage cost function R, and its expectation is bounded as well. Then, from [1], we see that there exists a stationary deterministic Markov policy that achieves the minimum for (2), i.e., there exists an optimal policy π^* such that $\pi_k^* \equiv \pi^*(x_k, v_k, d_k) \in \{0, 1\}$. Also, the stationary optimal policy $\pi^*(x, v, d)$ can be obtained by solving the Bellman's equation.

Theorem 4.1 *Let $\tau^*(x, v, d)$ be the optimal value of (2). Then, $\tau^*(x, v, d)$ satisfies the following dynamic programming (DP) equation.*

$$\tau^*(x, v, d) := \begin{cases} 0 & x = 0 \\ \min_{y \in \{0,1\}} \left\{ \mathsf{E}[I] + \mathsf{E}_{I,V}[\tau^*(x-y, V, (\max(d, I_{\{y>0\}} \frac{s}{v}) - I)^+)] \right\} & x > 1 \\ \min \left\{ \mathsf{E}[I] + \mathsf{E}_{I,V}[\tau^*(x, V, (d-I)^+)], \max\left(\frac{s}{v}, d\right) \right\} & x = 1 \end{cases} \quad (3)$$

The optimal policy $\pi^*(x, v, d)$ is the stationary policy that chooses the y that minimizes the right hand side expresssion of (3).

4.1 One Shot Problem

Consider the special case, $z = 1$. The Bellman equation (3) simplifies to

$$\tau^*(1, v) = \min \left\{ \mathsf{E}[I] + \int_{u=v_{min}}^{v_{max}} \tau^*(1, u) \mathrm{d}V(u), \frac{s}{v} \right\} \quad (4)$$

and $\tau^*(0, v) = 0$. From (4), we see that it is optimal to transmit (i.e., $y = 1$) when, $\frac{s}{v}$, the total cost of relaying using the current vehicle is less than the expected future cost, i.e., when

$$\frac{s}{v} \leq \mathsf{E}[I] + \int_{u=v_{min}}^{v_{max}} \tau^*(1, u) \mathrm{d}V(u) \quad (5)$$

Define v^* as the v that satisfies the above expression with equality, i.e.,

$$\begin{aligned} \frac{s}{v^*} &= \mathsf{E}[I] + \int_{u=v_{min}}^{v_{max}} \tau^*(1, u) \mathrm{d}V(u) \\ &= \mathsf{E}[I] + \int_{u=v_{min}}^{v^*} \tau^*(1, u) \mathrm{d}V(u) + \int_{v^*}^{v_{max}} \tau^*(1, u) \mathrm{d}V(u) \end{aligned} \quad (6)$$

For any $v \geq v^*$, the cost-to-go value is $\frac{s}{v}$ (follows from (4)). And, for all $v < v^*$, the cost-to-go value is the same. Hence, for all $v < v^*$, $\tau^*(1, v) = \tau^*(1, v^*) = \frac{s}{v^*}$. Substituting in (6), we have,

$$\begin{aligned} \frac{s}{v^*} &= \mathsf{E}[I] + \int_{v^*}^{v_{max}} \left(\frac{s}{u}\right) \mathrm{d}V(u) + \left(\frac{s}{v^*}\right) \left(1 - \int_{v^*}^{v_{max}} \mathrm{d}V(u)\right) \\ &= \mathsf{E}[I] + \int_{v^*}^{v_{max}} \left(\frac{s}{u} - \frac{s}{v^*}\right) \mathrm{d}V(u) + \frac{s}{v^*} \end{aligned}$$

Rearranging terms, we have,

$$\int_{v^*}^{v_{max}} \left(\frac{s}{v^*} - \frac{s}{u}\right) \mathrm{d}V(u) = \mathsf{E}[I] \quad (7)$$

Notice that the left hand side of (7) is a convex decreasing function of v^*, and hence, there exists a unique solution for v^*.

Remarks 4.1 *The optimal policy has a simple interpretation. Rearranging (7), we have,*

$$\frac{1}{\int_{v^*}^{v_{max}} \mathrm{d}V(u)} \left(\int_{v^*}^{v_{max}} \frac{s}{u} \mathrm{d}V(u) + \mathsf{E}[I] \right) = \frac{s}{v^*}$$

A vehicle of speed v is used as a relay only when the time required for the vehicle to reach the destination is less than the mean time that we need to wait for another vehicle which can be used as a relay and the subsequent time to travel to the destination.

4.2 Piecewise Transmission Problem

Now, consider the general case, $z > 1$. From the dynamic program (3), we conclude the following.

Theorem 4.2

1. *$\tau^*(x, v, d)$ is a non-decreasing function of x and d, and a non-increasing function of v.*
2. *The optimal policy $\pi^*(x, v, d)$ is a threshold policy for every x, with the threshold v_x^* a non-increasing function of x and d.*

Proof: See Appendix A. □

4.2.1 A sub-optimal policy

We will now obtain a sub-optimal scheduling policy for the average delay minimization problem, such that the average delay is within a constant bound from τ^*. Clearly, using every vehicle as a relay will yield a sub-optimal policy with a constant bound of $\frac{s}{v_{min}}$. Also, treating the file as z independent packets and using the scheduling policy from the one-shot model will yield an average delay within $z\tau^*$. Alternatively, here, we will obtain a much tighter bound by considering a different optimization problem. For a given scheduling policy π, define $\mathcal{T}_1^\pi, \mathcal{T}_2^\pi, \cdots, \mathcal{T}_z^\pi$ as the random time (from $T_0 = 0$) in which packets $1, 2, \cdots, z$ reach the destination node. Our original objective is to solve the following optimization problem.

$$\inf_\pi \{\mathsf{E}_{I,V}[\max(\mathcal{T}_1^\pi, \mathcal{T}_2^\pi, \cdots, \mathcal{T}_z^\pi)]\} \tag{8}$$

Alternatively, we will solve (9) and show that the optimal scheduling policy for (9) provides a close approximation to the original problem (8). Consider the following modified optimization problem,

$$\inf_\pi \{\max(\mathsf{E}_{I,V}[\mathcal{T}_1^\pi], \mathsf{E}_{I,V}[\mathcal{T}_2^\pi], \cdots, \mathsf{E}_{I,V}[\mathcal{T}_z^\pi])\} \tag{9}$$

Let $\tilde{\tau}$ and $\tilde{\pi}$ be the optimal value and the optimal scheduling policy of the alternate objective function, i.e.,

$$\tilde{\tau} := \inf_\pi \{\max(\mathsf{E}_{I,V}[\mathcal{T}_1^\pi], \mathsf{E}_{I,V}[\mathcal{T}_2^\pi], \cdots, \mathsf{E}_{I,V}[\mathcal{T}_z^\pi])\}$$

$$\tilde{\pi} = (\tilde{v}_1, \cdots, \tilde{v}_z)$$

The new objective function is straightforward to solve and the optimization problem involves obtaining the z threshold values $\tilde{v}_i$ for all $1 \le i \le z$, where,

$$\mathsf{E}_{I,V}[\mathcal{T}_i^{\tilde{\pi}}] = \sum_{j=1}^{i-1} \frac{\mathsf{E}[I]}{\int_{\tilde{v}_j}^{v_{max}} dV(u)} + \int_{\tilde{v}_i}^{v_{max}} \frac{s}{u} dV(u) \tag{10}$$

We know that, $\tilde{\tau} = \max(\mathsf{E}_{I,V}[\mathcal{T}_1^{\tilde{\pi}}], \cdots, \mathsf{E}_{I,V}[\mathcal{T}_z^{\tilde{\pi}}]) \ge \mathsf{E}_{I,V}[\mathcal{T}_z^{\tilde{\pi}}]$. Also, from Jensen's inequality, we have,

$$\begin{aligned} \tau^* &= \inf_{\pi} \{\mathsf{E}_{I,V}[\max(\mathcal{T}_1^{\pi}, \mathcal{T}_2^{\pi}, \cdots, \mathcal{T}_z^{\pi})]\} \\ &\ge \inf_{\pi} \{\max(\mathsf{E}_{I,V}[\mathcal{T}_1^{\pi}], \mathsf{E}_{I,V}[\mathcal{T}_2^{\pi}], \cdots, \mathsf{E}_{I,V}[\mathcal{T}_z^{\pi}])\} \\ &= \tilde{\tau} \end{aligned}$$

Hence, $\mathsf{E}_{I,V}[\mathcal{T}_z^{\tilde{\pi}}] \le \tilde{\tau} \le \tau^*$. Now the actual delay of the policy $\tilde{\pi}$ is given by $\mathsf{E}_{I,V}[\max(\mathcal{T}_1^{\tilde{\pi}}, \cdots, \mathcal{T}_z^{\tilde{\pi}})]$. We know that

$$\mathcal{T}_i^{\tilde{\pi}} \le \mathcal{T}_z^{\tilde{\pi}} + \frac{s}{\tilde{v}_1}$$

holds true for all $1 \le i \le z$ and for every sample space. This is because, $\tilde{v}_1 \le \tilde{v}_2 \le \cdots \le \tilde{v}_z$ (follows from the equations (10)), and $\frac{s}{\tilde{v}_1}$ is the maximum transit delay incurred by any packet. Hence,

$$\max(\mathcal{T}_1^{\tilde{\pi}}, \cdots, \mathcal{T}_z^{\tilde{\pi}}) \le \mathcal{T}_z^{\tilde{\pi}} + \frac{s}{\tilde{v}_1}$$

Taking expectation on both the sides, we get,

$$\mathsf{E}_{I,V}[\max(\mathcal{T}_1^{\tilde{\pi}}, \cdots, \mathcal{T}_z^{\tilde{\pi}})] \le \mathsf{E}_{I,V}[\mathcal{T}_z^{\tilde{\pi}}] + \frac{s}{\tilde{v}_1}$$

But we know that $\mathsf{E}_{I,V}[\mathcal{T}_z^{\tilde{\pi}}] \le \tau^*$. Hence,

$$\mathsf{E}_{I,V}[\max(\mathcal{T}_1^{\tilde{\pi}}, \cdots, \mathcal{T}_z^{\tilde{\pi}})] \le \tau^* + \frac{s}{\tilde{v}_1}$$

which shows that the average delay of $\tilde{\pi}$ is bounded within $\frac{s}{\tilde{v}_1}$ of the optimal value τ^*, which is our desired result.

5 Infinite File Size

In this section, we study the case where the source node has infinite packets to communicate to the destination node. Packets arrive at the source node according to some point process independent of the vehicle speeds and the vehicle interarrival times. The packets are queued until transmission in an infinite buffer at the source node. We are interested in studying the average packet delay in this network scenario, at the queue in the source node and in transit at the relay vehicles. In particular, we are interested in the tradeoff achievable between the queueing delay and the transit delay of the packets.

Every time a vehicle enters the communication range of the source node, the source node has to make a decision on using the current vehicle (with speed v) as a relay. Here again, we assume that only one packet is relayed using a vehicle. Starting with the first vehicle to arrive, the decision problem evolves over vehicle arrival instants $\{T_0 = 0, T_1, T_2, \cdots\}$ with vehicle speeds $\{V_0, V_1, V_2, \cdots\}$. Let X_k denote the number of packets in the source buffer at time instant T_k and let Y_k denote the number of packets relayed through the kth vehicle. As before, $Y_k \in \{0, 1\}$. The system state dynamics for the problem is now given by,

$$X_{k+1} = (X_k - Y_k)^+ + A_k$$

where A_k is the number of packets arriving at the source buffer during the time interval $I_{k+1} = T_{k+1} - T_k$. We assume that $\{A_k, k \geq 0\}$ is an i.i.d. sequence with mean $\mathsf{E}[A]$. Also, A_k is assumed to be independent of X_k and Y_k, but may depend on the interarrival time I_{k+1}. Define λ as $\lambda := \frac{\mathsf{E}[A]}{\mathsf{E}[I]}$, the mean number of packets arriving at the source node per second.

Following the definitions of X_k, V_k, Y_k and T_k for $k \geq 0$, we define the throughput, the transit delay and the queueing delay of the system as follows.

Definition 5.1 *The long term expected time average throughput of the system is defined as*

$$\lim_{k\to\infty} \frac{\sum_{i=0}^{k} \mathsf{E}[Y_i]}{\sum_{i=0}^{k} \mathsf{E}[T_{i+1} - T_i]} = \frac{1}{\mathsf{E}[I]} \lim_{k\to\infty} \frac{1}{k} \sum_{i=0}^{k} \mathsf{E}[Y_i]$$

Definition 5.2 *The long term expected time average transit delay of the packets is defined as*

$$s \lim_{k\to\infty} \frac{\frac{1}{k}\sum_{i=0}^{k} \mathsf{E}\left[\frac{Y_i}{V_i}\right]}{\frac{1}{k}\sum_{i=0}^{k} \mathsf{E}[Y_i]}$$

Definition 5.3 *The long term expected time average queueing delay of the system is defined as*

$$\lim_{k\to\infty} \frac{\sum_{i=0}^{k} \mathsf{E}[X_i(T_{i+1} - T_i)]}{\sum_{i=0}^{k} \mathsf{E}[T_{i+1} - T_i]} = \lim_{k\to\infty} \frac{1}{k} \sum_{i=0}^{k} \mathsf{E}[X_i]$$

The definitions follows from renewal reward theorem (RRT) and the fact that X_k is independent of $(T_{k+1} - T_k)$. Define $R_k(x_k, v_k, y_k)$ as

$$R_k(x_k, v_k, y_k) = \frac{1}{\lambda} x_k + r(v_k, y_k)$$

$R_k(x_k, v_k, y_k)$ is the single stage cost associated with the system state (x_k, v_k) and action y_k. The cost expression comprises of two components : the first term $\frac{x_k}{\lambda}$ corresponds to the queue length (or the queueing delay) and the second term $r(v_k, y_k)$ corresponds with the decision of using a vehicle with speed v_k as a relay. We are interested in an average cost minimization problem and our objective function is to find a policy $\pi \equiv \{\pi_k, k \geq 0\}$ that minimizes the following cost function.

$$\limsup_{k\to\infty} \frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\left(\frac{X_k}{\lambda} + r(V_k, Y_k)\right)\right]$$

$\frac{1}{\lambda}\limsup_{k\to\infty}\frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k} X_k\right]$ corresponds with the average queueing delay of the system. Now, let $r(v_k, y_k)$, the cost associated with using a vehicle of speed v_k be given by, $r(v_k, y_k) := \frac{1}{\lambda\mathsf{E}[I]}\frac{y_k}{v_k}$. Then, $r(v_k, y_k)$ is the transit delay incurred by the packet with the vehicle of speed v_k. Now, the second term of the objective function becomes

$$\limsup_{k\to\infty} \frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\frac{1}{\lambda\mathsf{E}[I]}\frac{Y_k}{V_k}\right] = \frac{1}{\lambda\mathsf{E}[I]}\limsup_{k\to\infty}\frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\frac{Y_k}{V_k}\right]$$

which is the expected average transit delay (since $\limsup_{k\to\infty}\frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k} Y_i\right] = \lambda\mathsf{E}[I]$). The average delay minimization policy can now be obtained by minimizing the following optimization problem.

$$\limsup_{k\to\infty} \frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\left(\frac{X_k}{\lambda} + \frac{1}{\lambda\mathsf{E}[I]}\frac{Y_k}{V_k}\right)\right]$$

As we are interested in studying the tradeoff achievable between the queueing delay and the transit delay, we will introduce an additional parameter β, $\beta > 0$, and study the following modified cost minimization problem.

$$\limsup_{k\to\infty} \frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\left(\frac{X_k}{\lambda} + \beta\frac{1}{\lambda\mathsf{E}[I]}\frac{Y_k}{V_k}\right)\right]$$

In [2], Berry and Gallager have studied a similar problem for a single downlink wireless channel. [2] considers a slotted wireless fading channel, where the data rate achievable over a slot is a function of the channel fade gain h during that slot and the power allocated P. Packets arrive into an infinite buffer independent of the queue length and the channel evolution process, and the objective was to study the tradeoff achiveable between the queueing delay at the buffer and the average power required to support the arrival process. The single stage cost function in [2] was $X_i + \beta P_i$, where X_i is the queue size at the ith slot, P_i is the power allocated during that slot and β is a Lagrangian multiplier. The optimization problem in [2] was to find a policy π that minimizes the following objective function.

$$\limsup_{k\to\infty} \frac{1}{k}\mathsf{E}\left[\sum_{i=0}^{k}\frac{X_i}{\lambda}+\beta P_i\right]$$

Observe that, our problem formulation is very similar to the problem formulation in [2] and hence, the results from [2] can be directly extended to our framework by suitably interpreting the average transit delay in our framework with the average power in [2]. More formally, we will first define a concave throughput - transit delay function $C(d)$ equivalent to the the concave throughput - power function $C(P)$ used in the Berry and Gallager model. The tradeoff then follows along the proof in [2].

Queueing delay is a function of throughput (the rate at which packets are served by the source node), and throughput itself is a function of the transit delay incurred by the packets. Accomodating large transit delay leads to large throughput and small queueing delay for the packets. The following definition provides the transit delay constrained throughput for an infinitely backlogged queue.

Definition 5.4 *Let $C(d)$ be the maximum throughput sustainable for an infinitely backlogged system, for a given average transit delay constraint d. Then, $C(d)$ is defined as,*

$$\begin{aligned} C(d) \quad &:= \quad \max\frac{1}{\mathsf{E}[I]}\int \pi(u)\mathrm{d}V(u) \qquad (11)\\ \textit{s.t.} \quad & \qquad \frac{s}{\int \pi(u)dV(u)}\int\left(\frac{\pi(u)}{u}\right)dV(u)\le d \end{aligned}$$

where $\pi(u)$ is the fraction of vehicles with speed v which are used as a relay by the source node, and, $\pi(u)\in[0,1]$.

$C(d)$ is a concave non-decreasing function in d. The throughput maximizing policy π^* is a threshold based policy, i.e., there exists a v^* such that for all $v>v^*$, $\pi^*(v)=1$.

Let λ, the average arrival rate of packets into the system, be given. Define d_λ as the minimum average transit delay incurred to support the arrival rate λ, i.e., $C(d_\lambda)=\lambda$. Trivially, we know that for any scheduling policy with an average transit delay $d\le d_\lambda$, the queueing delay in the system will be infinite. Now, consider a fixed schedule π_d obtained from the optimization problem (11) such that the average transit delay is d, where $d>d_\lambda$.

$$\frac{s}{\int \pi_d(u)dV(u)}\int\left(\frac{\pi_d(u)}{u}\right)dV(u)=d$$

For $d=d_\lambda+O(\delta)$, $C(d)=C(d_\lambda)+O(\delta)=\lambda+O(\delta)$. Suppose that we use the fixed schedule π_d to relay packets to vehicles, independent of the queue length at the source node

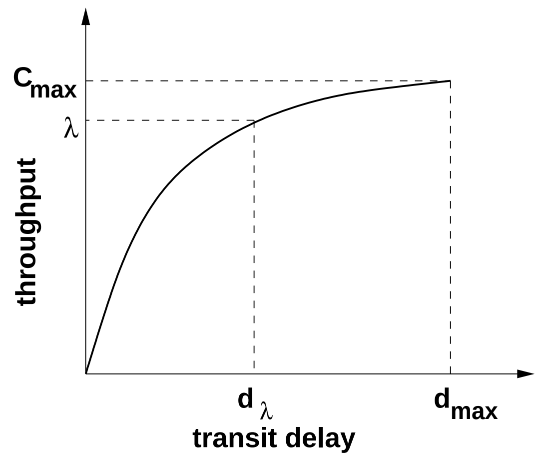

Figure 2: Throughput vs Transit delay for the vehicular network scenario (from (11)). The maximum transit delay, d_{max} and the maximum throughput achievable, C_{max} are given by $d_{max} = s \int \frac{1}{u} dV(u)$ and $C_{max} = \frac{1}{\mathsf{E}[I]}$. Given λ, the arrival rate of the packets into the system, d_λ is the minimum transit delay incurred in supporting the arrival process, and $C(d_\lambda) = \lambda$.

buffer. Then, we can achieve an average queue length (or delay) of $O\left(\frac{1}{\delta}\right)$ (an upper bound for the average queue length of a G/G/1 queue with load $\frac{\lambda}{\lambda + O(\delta)}$ is $O\left(\frac{1}{\delta}\right)$). Instead, in [2], Berry and Gallager developed a threshold based algorithm, where, two schedules $\pi_{d_\lambda - \delta}$ and $\pi_{d_\lambda + \delta}$ were used depending on the queue size being below and above a predetermined threshold. This threshold based policy was shown to reduce the average queueing delay to $O\left(\frac{1}{\sqrt{\delta}}\right)$. The following theorem summarizes the above discussion, whose proof follows directly from the tradeoff studied in [2].

Theorem 5.5 *Let λ, the average arrival rate of packets into the source node buffer, be given. And, let d_λ be the minimum average transit delay incurred to support the throughput λ, i.e., $C(d_\lambda) = \lambda$. Suppose that $C'(d_\lambda) > 0$ and let the arrival process A_k be a compact subset of R^+. Then, for an excess average transit delay of δ (i.e., for an average transit delay of $d_{opt} + \delta$), the average queueing delay in the system scales as $O\left(\frac{1}{\sqrt{\delta}}\right)$.*

6 Conclusion

In this paper, we have considered a scheduling problem in a mobile network scenario, where vehicles are used as relays. A stationary source node has z packets to communicate to a stationary destination node, and passing by vehicles are used as relays to transfer the file to the destination. All packet communication involves only two hops, and we are interested in minimizing the average queueing delay and the average transit delay of the

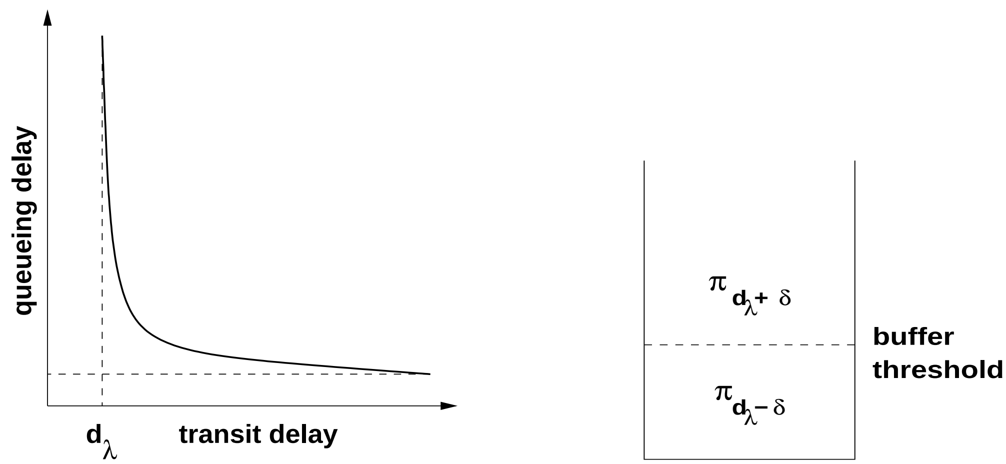

Figure 3: Figure 3(a) plots the optimal tradeoff achievable between the average queueing delay and the average transit delay. Observe that the queueing delay approaches infinity as the transit delay approaches d_λ. Figure 3(b) shows the buffer threshold based scheduling policy proposed in [2]. When the queue in the buffer is less than the threshold, a policy $\pi_{d_\lambda-\delta}$ is used and when the queue exceeds the threshold, policy $\pi_{d_\lambda+\delta}$ is used to schedule packets.

packets in the network. We studied both the finite file size case and the infinite file size case. In the finite file size case, we obtained the average delay minimizing schedule using a Markov decision process framework. We also obtained a simple sub-optimal scheduling policy whose average delay is within a constant from the optimal value (obtained from the MDP formulation). In the infinite file size case, we studied the asymptotic tradeoff achievable between the queueing delay and the transit delay of the packets. By defining the maximum throughput sustainable for a given transit delay constraint, we showed that the average queueing delay of the system scales as $O\left(\frac{1}{\sqrt{\delta}}\right)$ for an excess average transit delay of $O(\delta)$.

References

[1] Ralph E Strauch, Negative Dynamic Programming, The Annals of Mathematical Statistics, Vol. 37, No. 4, August, 1966.

[2] Randall A. Berry and Robert G. Gallager, Communication over Fading Channels with Delay Constraints, IEEE Transactions on Information Theory, Vol. 48, No. 5, 2002.

A Proof of Theorem 4.2

Theorem A.1 *$\tau^*(x, v, d)$ is a non-decreasing function of x and d, and a non-increasing function of v.*

Proof: Let $\pi \equiv \{\pi_k, k \geq 0\}$ be any stationary Markov policy. For the single stage cost function $R_k(x_k, v_k, d_k, y_k)$ given by (1), the random total cost function for the scheduling policy π is given by,

$$\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi(X_k, V_k, D_k))$$

and the expected total cost (or the average delay in delivery) for a file with z packets is now given by

$$\mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi(X_k, V_k, D_k))\middle| X_0 = z, V_0 = v, D_0 = d\right]$$

Consider two file sizes z and $z+1$. We need to prove that $\tau^*(z, v, d) \leq \tau^*(z+1, v, d)$. For π, a stationary Markov policy, define π_{-1}, another stationary Markov policy, as

$$\pi_{-1}(x, v, d) = \pi(x+1, v, d)$$

and $\pi_{-1}(0, v, d) = 0$. Now, it is straightforward to see that

$$\left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi_{-1}(X_k, V_k, D_k))\middle| X_0 = z, V_0 = v, D_0 = d\right]$$
$$\leq \left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi(X_k, V_k, D_k))\middle| X_0 = z+1, V_0 = v, D_0 = d\right]$$

for every sample space of the interarrival distribution and the vehicular speed distribution. In other words, for any policy π, there exists a scheduling policy π_{-1} such that the random total delay in delivering a file with $z+1$ packets (and with policy π) is greater than or equal to the random total delay in delivering a file with z packets (and with a policy π_{-1}). Taking expectation over the sample spaces, we have,

$$\mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi_{-1}(X_k, V_k, D_k))\middle| X_0 = z, V_0 = v, D_0 = d\right]$$
$$\leq \mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k, V_k, D_k, \pi(X_k, V_k, D_k))\middle| X_0 = z+1, V_0 = v, D_0 = d\right]$$

But we know that

$$\tau^*(z{+}1,v,d) = \inf_{\pi} \mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k,V_k,D_k,\pi(X_k,V_k,D_k))\middle| X_0=z+1, V_0=v, D_0=d\right]$$

Hence,

$$\begin{aligned}
\tau^*(z+1,v,d) &\geq \inf_{\pi_{-1}} \mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k,V_k,D_k,\pi_{-1}(X_k,V_k,D_k))\middle| X_0=z, V_0=v, D_0=0\right] \\
&\geq \inf_{\pi} \mathsf{E}_{I,V}\left[\sum_{k=0}^{\infty} R_k(X_k,V_k,D_k,\pi_(X_k,V_k,D_k))\middle| X_0=z, V_0=v, D_0=0\right] \\
&= \tau^*(z,v,d)
\end{aligned}$$

The proof for the ordering of τ with respect to v and d follows similar arguments and is omitted here.

□

Theorem A.2 *The optimal policy $\pi^*(x,v,d)$ is a threshold policy for every x, with the threshold v_x^* a non-increasing function of x and d.*

Proof: We know that $\tau^*(x,v,d)$ satisfies the following DP, given in (3),

$$\tau^*(x,v,d) := \begin{cases} 0 & x=0 \\ \min_{y\in\{0,1\}}\left\{\mathsf{E}[I] + \mathsf{E}_{I,V}[\tau^*(x-y,V,(\max(d,I_{\{y>0\}}\frac{s}{v})-I)^+)]\right\} & x>1 \\ \min\left\{\mathsf{E}[I]+\mathsf{E}_{I,V}[\tau^*(x,V,(d-I)^+)], \max\left(\frac{s}{v},d\right)\right\} & x=1 \end{cases}$$

and $\pi^*(x,v,d)$ is the stationary policy that chooses the y that minimizes the right hand side expression of the above DP.

Consider the case $x=1$. The optimal policy chooses the minimum of

$$\left\{\mathsf{E}[I]+\mathsf{E}_{I,V}[\tau^*(x,V,(d-I)^+)], \max\left(\frac{s}{v},d\right)\right\}$$

Note that the first term is independent of v. Since $\max\left(\frac{s}{v},d\right)$ is a monotone decreasing function with v, we see that the optimal policy is a threshold based policy, i.e., $\pi(1,v,d) = 1$, for all v such that

$$\mathsf{E}[I]+\mathsf{E}_{I,V}[\tau^*(x,V,(d-I)^+)] \geq \max\left(\frac{s}{v},d\right)$$

Now, consider the case $x>1$. The optimal policy chooses the minimum of the following term.

$$\left\{\mathsf{E}[I]+\mathsf{E}_{I,V}[\tau^*(x,V,(d-I)^+), \mathsf{E}[I]+\mathsf{E}_{I,V}\left[\tau^*\left(x-1,V,\left(\max\left(d,\frac{s}{v}\right)-I\right)^+\right)\right]\right\}$$

As before, the first term is independent of v. And, the second term is a monotone decreasing function with v (since $\tau^*(x, v, d)$ is a non-increasing function with d). Hence, we see that the optimal policy is a threshold based policy for $x > 1$ as well.
Proof to be completed. $\square$