Cooperative Profit Sharing in Coalition Based Resource Allocation in Wireless Networks

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***Abstract*—We consider a network in which several service providers offer wireless access to their respective subscribed customers through potentially multi-hop routes. If providers cooperate by jointly deploying and pooling their resources, such as spectrum and infrastructure (e.g., base stations), and agree to serve each others' customers, their aggregate payoffs, and individual shares, may substantially increase through opportunistic utilization of resources. The potential of such cooperation can, however, be realized only if each provider intelligently determines who it would cooperate with, when it would cooperate, and how it would deploy and share its resources during such cooperation. Also, when the providers share their aggregate payoffs, developing a rational basis for such sharing is imperative for the stability of the coalitions. We model such cooperation using the theory of *transferable payoff coalitional games.* We show that the optimum cooperation strategy, which involves the acquisition, deployment and allocation of the channels and base stations (to customers), can be computed as the solution of a concave or an integer optimization. We next show that the grand coalition is stable in many different settings, i.e., if all providers cooperate, there is always an operating point that maximizes the providers' aggregate payoff, while offering each a share that removes any incentive to split from the coalition. Such stabilizing payoff shares can be computed by solving the duals of the above optimizations. The optimal cooperation strategy and the stabilizing payoff shares can be obtained in polynomial time using distributed computations and limited exchange of confidential information among the providers. Our numerical evaluations reveal that cooperation substantially enhances individual provider's payoffs under the optimal cooperation strategy and several different payoff sharing rules.**

I. Introduction

A. Motivation

We have witnessed a significant growth in commercial wireless services in the past few years, and the trend is likely to continue in the foreseeable future. Satisfaction of this increasing demand is contingent upon efficient utilization of the transmission resources, which are either under-utilized (e.g., spectrum - utilization of licensed spectrum is at times only 15% [1]), or costly (e.g. infrastructure). Cooperation among wireless providers, whereby different providers may form a coalition and pool their resources, such as spectrum and infrastructure like base stations (or access points) and relay nodes, and serve each others' customers, has the potential to substantially improve the utilization of the available resources, We now elucidate the benefits of such cooperation using a sequence of examples.

We first demonstrate how cooperation may substantially enhance throughput through efficient opportunistic utilization of resources and lower overall energy consumption of the customers through multi-hop relaying; both the above result in higher customer satisfaction and payoffs for the providers. Transmission qualities of available channels randomly fluctuate with time and space, owing to customer mobility and propagation conditions. Also, in secondary access networks, the providers may be secondary users who do not license channels but communicate when the license holders (primary users) do not use the channels. Such access opportunities may only arise sporadically. Since all customers of all providers do not need to be served simultaneously, and the channels of different providers may not be unavailable or have poor qualities simultaneously, spectrum pooling can enhance throughput by mitigating service fluctuations resulting from occasional variations in channel qualities and availabilities, and instantaneous traffic overloads. In multi-hop wireless networks (e.g., mesh networks), cooperation increases the number of available relays (mesh points). This in turn increases the number of multi-hop routes to each customer, thereby decreasing the total power usage and increasing the total throughput of the customers. Also, the customers may be induced to serve as relays, perhaps, in lieu of service discounts. Then the enhancement in throughput and energy consumption owing to cooperation magnifies as the coalitions have a larger set of customers, and therefore a larger number of multi-hop routes.

Cooperation also reduces the costs incurred by the providers and thereby increases their net payoffs. A provider can acquire a channel by paying a fixed licensing cost or usage based charges, or a combination of the two. The first case arises when the providers are primary users who license the channels from government agencies, and the other option arises when they are secondary users who use the channels licensed by the primaries. When the providers do not cooperate, they may need to operate as secondary users and opt primarily for usage based charges, as the volume of their individual traffic may not justify other options. Since cooperation allows the providers to pool the customers, the resulting higher aggregate traffic may allow them to license channels, share the licensing fees and thereby reduce the individual costs. Next, deploying and maintaining base stations constitutes one of the major costs in expanding the networks. Cooperation may reduce the expansion costs by allowing the providers to deliver desired

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coverage and throughput guarantees while deploying fewer base stations. For example, for a provider whose customer base is concentrated in a particular region, traffic demand is low but non-zero (owing to customer mobility) in other regions. The provider must deploy base stations even in the regions of low traffic intensity so as to provide universal coverage (otherwise the customers would desert). If instead, the provider cooperates with another provider whose traffic demand is concentrated in a different region, both may satisfy coverage requirements by deploying base stations only in the regions where their individual demands are concentrated, and thereby reduce individual operational expenses.

B. *Research Challenges and Contributions*

Several research challenges must, however, be addressed before large scale cooperation can be realized. First, commercial service providers are selfish entities who seek to maximize their individual payoffs. Therefore, they will cooperate only when cooperation increases their individual incomes. Even so, a provider may refuse to join a coalition if it perceives that its share of the aggregate payoff is not commensurate to the amount it invested and the wealth it generated. The former depends on the transmission rates in the channels it has acquired and the locations and the number of base stations it has deployed, while the latter depends on its customer base. So, developing a rational basis for determining the individual shares of the aggregate payoff is imperative. Note that the aggregate payoff and the individual shares depend on the providers' cooperation strategies. Specifically, each provider needs to decide which providers it would cooperate with, which channels would use, the locations of its base stations, and when it should serve the customers of other providers. The sharing mechanism and the optimal cooperation strategies for providers depend on each other and must be obtained jointly.

We present a framework to determine the optimal decisions of the providers using tools from *transferable payoff coalitional game theory* [2]. The framework also provides a rational basis for sharing the aggregate payoff. The first network setup we consider is an access network where providers pool their spectrum, base stations and customers (Section IV). We assume that the locations of base stations and the set of channels they have access to are determined a priori, but the providers decide how they would allocate the base stations and the channels of the coalition, to the customers. We then obtain optimal decision rules for the providers and a strategy for sharing the resulting aggregate payoff as solutions of concave optimization problems. This sharing strategy ensures that it is optimal for all providers to cooperate. Specifically, if any subset of providers split from the grand coalition (the coalition of all providers), irrespective of how they cooperate and the way they share their aggregate payoff, at least one provider in this subset receives less payoff than what it received in the grand coalition. In coalitional game terminology, such a sharing scheme is said to belong to the *core* of the game. This result is of interest in itself as many cooperative games have empty cores, and the specific games we consider do not satisfy some standard sufficiency conditions for non-emptiness of the core (e.g., convexity of the game).

In the subsequent sections, we extend the formulation and results. We first consider the cases where the providers also need to determine the locations of their base stations or the set of channels each base station has access to (Section V). The optimal cooperation strategy can now be obtained by solving an integer optimization with nonzero duality gap unlike in concave optimizations used before. We obtain the optimal decision rules and the payoff sharing mechanism using unimodularity arguments. Subsequently, we extend the results in Section IV to multi-hop wireless networks (Section VI). We consider other profit sharing mechanisms, namely, the nucleolus and the Shapley value, and investigate whether they stabilize the grand coalition of providers (Section VII). We examine the impact of providers' cooperation on the customers and propose a framework for optimal (and selective) acceptance of service level agreements (SLAs) by the providers (Section VIII). In the context of the resource pooling game we numerically evaluate and compare the providers' payoff increases resulting from cooperation under different sharing mechanisms and different payoff functions as a function of the number of customers and base stations (Section IX).

II. Related Work

Interactions among different entities in wireless networks have primarily been investigated from the following extreme perspectives. In the first, each entity is assumed to select its actions so as to maximize its own incentive without coordinating with others, e.g., [3]. This scenario, which has been investigated using noncooperative game theory, in general suffers from inefficient utilization of resources [4]. The other perspective has been to assume that entities selflessly choose their actions so as to optimize a global utility function even when such actions may deteriorate individual incentives of some entities e.g., [5]. We investigate interactions among providers assuming that each provider would be willing to cooperate and coordinate its actions with others when such cooperation enhances its individual incentives.

We obtain optimal cooperation schemes using the framework of cooperative game theory. This choice of tools allows us to combine the desirable features of the extreme approaches studied in the existing literature, that of allowing entities to choose their actions guided by selfish objectives, and of maximizing global utility functions. Surprisingly, cooperative game theory has seen only limited use in wireless context so far. Nash bargaining solutions have been proposed for power control and spectrum sharing among multiple users [6]. Coalitional games have been used recently for modeling cooperation among nodes in the physical layer [7], [8], collaborative sensing by secondary users in cognitive radio networks [9], rate allocation in multiple access channels (MAC) [10], rate allocation among mobiles and admission control in heterogeneous wireless access environments [11], and studying cooperation among single antenna receivers and transmitters in an interference channel [12]. Our problem formulation, solution techniques, and results significantly differ from the above owing to the difference in contexts - our focus is on cooperative resource allocation and subsequent payoff sharing among providers at the network and MAC layers. To our knowledge, our work is the first to investigate cooperation among wireless providers.

Coalitional game theory has been used for studying cooperation in other communication networks as well (see [13] for a survey). For instance, Shapley value based profit sharing has been proposed and investigated for incentivizing cooperation among peers [14] and among internet service providers [15]. Our framework can be used to study core allocations for coalitional games among internet service providers (Section VI).

III. System Model

A. Communication Model

Consider a network with a set of providers $\mathcal{N}$. Each provider i deploys a set of base stations (or access points) in order to serve its set of customers $\mathcal{M}_i$. Each base station has access to a certain set of channels (e.g., FDM carriers in GSM and OFDMA systems),[1] and each base station-channel pair is referred to as a service unit. Thus, a provider's resources are its service units. Let $\mathcal{B}_i$ be the set of service units of provider i, $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$ and $\mathcal{M}_i \cap \mathcal{M}_j = \emptyset$ for $i \neq j$. For a $\mathcal{S} \subseteq \mathcal{N}$, let $\mathcal{B}_\mathcal{S}$ and $\mathcal{M}_\mathcal{S}$ denote the set of service units and customers associated with providers in $\mathcal{S}$. Thus $\mathcal{B}_\mathcal{N}$ and $\mathcal{M}_\mathcal{N}$ are the sets of all service units and all customers, respectively.

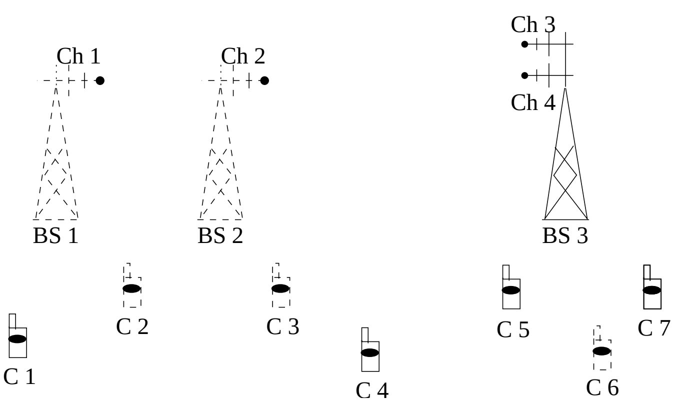

Fig. 1: The dashed (solid, resp.) base stations, channels and customers) belong to provider 1 (2, resp.). Provider 1 owns 2 base stations each of which has access to one channel, and thus corresponds to 1 service unit each. Provider 2 owns 1 base station which has access to two channels, and thus corresponds to 2 service units. Thus $\mathcal{B}_1 = \{1, 2\}$ and $\mathcal{B}_2 = \{3, 4\}$. Also, $\mathcal{M}_1 = \{2, 3, 6\}$ and $\mathcal{M}_2 = \{1, 4, 5, 7\}$.

Assumption III.1. *We assume that the achievable rates of a customer-service unit pair do not depend on communications of other customers and service units.*

Each customer or a service unit may be involved in at most one communication at a given time (*time sharing*). We assume, unless mentioned otherwise, that a) the locations of the base stations and the channels they have access to are predetermined, and b) the service units and the customers communicate through single-hop links. We show how these assumptions can be relaxed in Sections V and VI, respectively. Each customer j negotiates a *service level agreement* (SLA) in form of a minimum rate guarantee m_j with its provider.

For ease of exposition, we consider only downlink communications in our model (the results easily extend to the case where communications involve both uplinks and downlinks). We assume that when customer j is served by service unit k, j receives at a rate r_{jk}, a random variable which is a function of the location of customer j and the state of channel k both of which can vary randomly. Let ω represent a network state (customer location, channel qualities resulting from fading and channel access of primary users[2]), Ω be the collection of all ωs and $\mathbb{P}(\omega)$ be the probability that the network state is ω. The rates r_{jk} are functions of ω and are denoted as $r_{jk}(\omega)$. We assume that $|\Omega|$ is finite, since (i) feasible service rates in any practical communication system belong to a finite set, and (ii) we can partition the geographical region where the network is deployed in such a way that the service rates are identical in each partition.

B. A Coalition Game Model

We now propose a coalitional game theory framework that models the interactions of the providers.

Definition III.1. *A coalition $\mathcal{S} \subseteq \mathcal{N}$ is a subset of providers who cooperate. We refer to $\mathcal{N}$ as the grand coalition.*

Definition III.2. *A coalitional game with transferable payoff $< \mathcal{N}, v >$ consists of a finite set $\mathcal{N}$ (set of providers) and a characteristic function $v(\cdot)$ that associates with every nonempty subset $\mathcal{S}$ of $\mathcal{N}$, a real number $v(\mathcal{S})$, which is the maximum aggregate payoff (or profit) available for division in any arbitrary way among the members of $\mathcal{S}$.*

A service unit can serve a customer only when either both are associated with the same provider, or the providers associated with them are in a coalition. Consider a network state ω. Let $\alpha_{jk}(\omega) \in [0, 1]$ be the fraction of time service unit k serves customer j. When the provider associated with customer j is in coalition $\mathcal{S}$, the rate received by j is $y_j(\omega) = \sum_{k \in \mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) r_{jk}(\omega)$. Note that $r_{jk}(\omega)$ does not depend on $\{\alpha_{lm}(\omega), l \in \mathcal{M}_\mathcal{N}, m \in \mathcal{B}_\mathcal{N}\}$ due to assumption III.1. When customers associated with provider i receive rates $\mathbf{y}_i(\omega) = \{y_j(\omega), j \in \mathcal{M}_i\}$, i gains a benefit (e.g., revenue from the customers) of $U_i(\mathbf{y}_i(\omega))$, where $U_i(\cdot)$ is a concave function. Next, owing to the tariffs imposed by spectrum regulators or by the license holders of the channels, provider i incurs a cost of $V_i(\mathbf{z}_i(\omega))$, where $\mathbf{z}_i(\omega) = \{z_k(\omega), k \in \mathcal{B}_i\}$, $z_k(\omega) = \sum_{j \in \mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega)$ is the total fraction of time service unit k

[1] We assume that each base station has a separate radio available for every channel. Most of our formulations and all our results go through even when some base stations have fewer radios than channels - wherever applicable we mention the necessary changes in the formulations in this case.

[2] In case a provider is a secondary user of a licensed channel, the available rate in the channel depends on the usage patterns of the primary user, as the secondary can use the channel only when it is not being used by the primary.

is used and $V_i(\cdot)$ is a convex function.[3] Then the profit (or payoff) of a coalition $\mathcal{S}$ is the sum of the U_is for $i \in S$ minus the sum of the V_is for $i \in S$. We assume that the benefit and cost functions $U_i(\cdot), V_i(\cdot)$ are decided apriori (based on spectrum regulation, customer charging policies etc.), and do not investigate the optimal selections of these functions.

Providers in a coalition $\mathcal{S}$ have to decide how to schedule service units to customers, i.e., select the variables $\alpha_{jk}(\omega)$s, for each $\omega \in \Omega$, based on the benefit and cost functions $U_i(\cdot), V_i(\cdot)$, and the service unit to customer rates $r_{jk}(\omega)$s so as to attain the maximum possible payoff $v(\mathcal{S})$ subject to possible service level agreements.

C. How the Framework Relates to Existing Wireless Networks

We now illustrate via examples how our framework can be used to model specific communication systems. Consider elastic data transfers in the downlink of a CDMA cellular system (e.g., used for internet access of cellular subscribers) [16, Chapter 5] with provider set $\mathcal{N}$. Owing to simplicity of physical layer implementations, a base station k always transmits at a pre-determined fixed power P_k. This happens even when no mobile associated with it requires downlink transmission [17]. Each base station has access to only one band and thus the service units are same as the base stations. Customers in a cell are served on a *time-sharing* basis, i.e., a base station transmits to at most one customer at a given time. Also, at any given time, a customer receives transmissions from at most one base station. Then, $\{\alpha_{jk}(\omega)\}$ represent the fractions of time customers are served by different base stations. When base station k transmits to customer j and the network realization is ω, the achievable rate $r_{jk}(\omega)$ from k to j is a function of the downlink SINR $\text{SINR}_{jk}(\omega)$ [16, Chapter 5], where

$$\text{SINR}_{jk}(\omega) = \frac{h_{jk}(\omega)P_k}{\sum_{i' \in \mathcal{B}_{\mathcal{N}} \setminus \{k\}} h_{ji'}(\omega)P_{i'} + N_0 W},$$

$h_{jk}(\omega)$ are the channel gains between customer-base station pairs, N_0 is the power spectral density of the additive noise and W is the spectrum bandwidth.[4] Thus, $\text{SINR}_{jk}(\omega)$ and hence $r_{jk}(\omega)$, is independent of which customers are being served by other base stations. Thus, assumption III.1 holds.

Next, consider downlink communications in a multi-cell OFDMA system [16, Chapter 6]. Different providers acquire non-overlapping bands and the bandwidth acquired by a provider is divided into several channels (sub-carriers in OFDM terminology) (For small-scale providers, some of these channels can be secondary access channels or spectrum white-spaces acquired from primary users). Each provider partitions its set of sub-carriers into reuse groups, assigning one such group of sub-carriers to each base station in such a way to ensure that inter-cell interference to simultaneous transmissions in other base station sub-carrier pairs is negligible. At any given time, a base station assigns a sub-carrier to only one customer, but more than one sub-carrier can be assigned to a customer (*multiple allocation*). Thus, the intra-cell interference is negligible as well. Also, each base station, in each state ω, assigns a fixed transmit power to each of its carriers. Thus, the rate that a customer gets from a service unit (which denotes a base station and sub-carrier pair) to which it is assigned depends only on the channel gain from the corresponding base station sub-carrier pair to itself, channel usage of primary users as applicable, and not on the assignments of other customers and service units. Hence assumption III.1 holds.

IV. Resource Pooling Game

A. Optimal Allocation of Customers to Service units

The characteristic function $v(\mathcal{S})$ for a coalition $\mathcal{S} \subseteq \mathcal{N}$, is the maximum aggregate payoff of providers in $\mathcal{S}$ and is given by the following concave optimization problem.

$\text{P}(\mathcal{S}) : \max \sum_{\substack{i \in \mathcal{S} \\ \omega \in \Omega}} \mathbb{P}(\omega) \Big(U_i(\mathbf{y}_i(\omega)) - V_i(\mathbf{z}_i(\omega)) \Big)$

subject to:

1) $y_j(\omega) = \sum_{k \in \mathcal{B}_{\mathcal{S}}} \alpha_{jk}(\omega) r_{jk}(\omega), \quad j \in \mathcal{M}_{\mathcal{S}}, \omega \in \Omega$
2) $z_k(\omega) = \sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jk}(\omega), \quad k \in \mathcal{B}_{\mathcal{S}}, \omega \in \Omega$
3) $\sum_{k \in \mathcal{B}_{\mathcal{S}}} \alpha_{jk}(\omega) \le 1, \quad j \in \mathcal{M}_{\mathcal{S}}, \omega \in \Omega$
4) $\sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jk}(\omega) \le 1, \quad k \in \mathcal{B}_{\mathcal{S}}, \omega \in \Omega$
5) $\sum_{\omega \in \Omega} \mathbb{P}(\omega) y_j(\omega) \ge m_j, \quad j \in \mathcal{M}_{\mathcal{S}}$
6) $\alpha_{jk}(\omega) \ge 0, \quad j \in \mathcal{M}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}, \omega \in \Omega$

Constraints (3) ensure that for all $j \in \mathcal{M}_{\mathcal{S}}$, the fraction of time customer j is served is at most 1. Constraints (4) ensure that the fraction of time each service unit $k \in \mathcal{B}_{\mathcal{S}}$ serves is at most 1.[5] Constraints (5) provide the minimum service guarantees. Incidentally, constraints (3), (4) arise from the time-sharing model,[6] but for the multiple allocation model (see

[3]We say $\mathbf{a} \ge \mathbf{b}$ if the inequality is satisfied for each component. Then, a function $f(\cdot)$ is increasing if $f(\mathbf{a}) \ge f(\mathbf{b})$ for any $\mathbf{a} \ge \mathbf{b}$. Natural revenue and cost connotations would imply that $U_i(\cdot), V_i(\cdot)$ are increasing and 0 at the origin - though our formulations and analytical results do not rely on these assumptions. Again, usually, $U_i(\mathbf{y}) = \sum_{j \in \mathcal{M}_i} g_{ij}(y_j)$, where $g_{ij}(\cdot)$ is an increasing concave (either strict or linear) revenue function chosen by provider i for customer j. We therefore allow a provider to choose different revenue functions for different customers. The revenue functions are assumed to be concave since customers would pay in accordance with their satisfactions, which are usually concave functions of rates (increase sub-linearly in practice).

[4]This SINR expression assumes that all base stations use the same band. This facilitates smooth hand-overs but provides poor SINR to the mobiles at cell boundaries owing to high interference from neighboring base stations. Note that CDMA technology can provide acceptable rates even in presence of low SINRs. Nevertheless, in some implementations, neighboring base stations are allocated different bands. In that case, we sum over all co-channel base stations to obtain the aggregate interference in the denominator.

[5]When a base station has access to multiple channels with only 1 radio, constraint (4) must be modified to bound the sum of $\alpha_{jk}(\omega)$ over customers $j \in \mathcal{M}_{\mathcal{S}}$, and service units k corresponding to the base station by 1. For example, if the base station has access to c channels, the fractional associations to the corresponding c service units, $k_1, \ldots, k_c$, satisfy the constraint $\sum_{l=1}^{c} \sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jk_l}(\omega) \le 1, \quad \omega \in \Omega$. It can be shown that all the subsequent results extend to this scenario.

[6]The system can be represented by a complete bipartite graph where the customers and the service units represent the nodes and there exists a link between every customer-service unit node pair. Under the time-sharing model, any customer-service unit assignment corresponds to a matching in the above graph. Note that for each ω, $\{\alpha_{jk}(\omega)\}$ comprise a feasible allocation of service units to customers if and only if there exists a corresponding collection of matchings $L_1, L_2, \ldots$ and a collection of non-negative real numbers $\gamma_1, \gamma_2, ..$ such that (i) $\sum_i \gamma_i = 1$, $\gamma_i \ge 0$ and (ii) if the service unit - customer allocation follows matching L_i for γ_i fraction of time for each i, then service unit k transmits to customer j for $\alpha_{jk}(\omega)$ fraction of time for all j, k. Constraints (3), (4) provide the necessary and sufficient condition for feasibility of $\{\alpha_{jk}(\omega)\}$ for each ω [19].

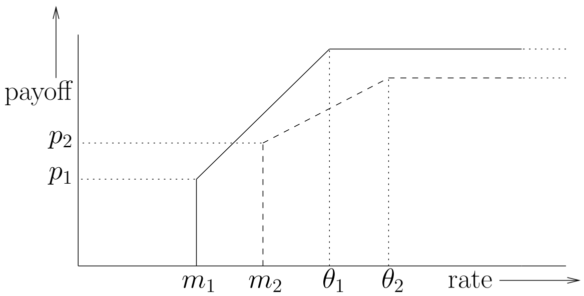

Fig. 2: Examples of revenue functions. The customers pay fixed costs p_js for being guaranteed minimum average rates m_js, but do not pay additional costs for rates beyond θ_js.

the last paragraph of Section III-C), only (4) suffice - all results presented below extend even in absence of (3).

Assumption IV.1. *$P(\{i\})$ is feasible for each $i \in \mathcal{N}$, i.e., each provider can support the minimum rates of its customers even when it does not cooperate with other providers.*

Then P$(\mathcal{S})$ is feasible for each $\mathcal{S} \subseteq \mathcal{N}$. Also, the optimization problem P$(\mathcal{S})$ provides the maximum aggregate payoff of the providers in a coalition $\mathcal{S}$ and also the optimal service unit-customer allocations that attain this maximum.

Finally, we examine whether the above resource allocation framework captures the intricacies of existing wireless traffic. We focus on data as it is fast emerging as the predominant component of wireless traffic. Many emerging applications, such as streaming video, require certain minimum rate, and the quality of service is critically sensitive to the service rate. Thus, minimum rate constraints are likely to be integral components of service agreements in near future, and providers are likely to charge (i) fixed fees that are increasing functions of the minimum rates agreed upon, and (ii) additionally for service rates they can provide over and above the required minimum value. A customer may however be willing to pay additionally for rates only up to a certain maximum rate value determined by his QoS requirements.[7] The following simple pricing strategy captures the above features. If the average rate a customer of provider i receives is r, and he has negotiated a minimum rate guarantee of m, then he pays $d_i \max(\min(r, \theta) - m, 0) + e'_i m$, where θ is the maximum rate the customer needs (Fig. 2 with $p_j = e'_j m_j$). Owing to the minimum rate constraints (5) in P$(\mathcal{S})$, each customer's average rate is at least m_j. Thus,

$$U_i(\mathbf{y}_i) = d_i \sum_{j \in \mathcal{M}_i} \max(y_j, \gamma_j) + e_i m_j \text{ with } e_i = e'_i + d_i$$

captures the above pricing strategy. Note that $U_i(\cdot)$ is a concave function for each i. Finally, constraints (5) in P$(\mathcal{S})$ apply to the average service rates; more stringent QoS demands may require constraints on service rates in each ω, i.e., given certain desired minimum rates $m_j(\omega)$ for different $\omega \in \Omega$, $y_j(\omega) \geq m_j(\omega)$ for each $\omega \in \Omega$. The modified optimization P$(\mathcal{S})$ continues to be a concave maximization with linear constraints, and all subsequent results apply. Alternatively, "soft" minimum rate guarantees may be ensured in each ω by choosing strict concave revenue functions. Specifically, higher the degree of concavity of the revenue functions (that is lower the second derivatives), a provider incurs higher additional revenue in any ω by enhancing the service rate of a customer who is receiving a low rate at that ω as opposed to enhancing that of a customer who is receiving a high rate at that ω. Thus, providers are more likely to equalize the service rates of all customers at each ω, and thereby ensure certain minimum rates to each customer at every ω.

[7]For instance, for layered video streaming [20], all customers need a minimum rate for an acceptable quality video, but they do not need more than the rate required to decode the finest layer.

B. Sharing Aggregate Payoffs

A rational basis for sharing the maximum aggregate payoff is imperative to motivate the providers to join the grand (or any other) coalition. We use a solution concept from coalitional games known as the *core* to provide such a basis.

Definition IV.1. *For any real valued vector $\mathbf{x} = (x_i, i \in \mathcal{N})$ and any coalition $\mathcal{S}$, we let $x(\mathcal{S}) = \sum_{i \in S} x_i$. Such a vector is said to be an imputation if $x(\mathcal{N}) = v(\mathcal{N})$ and $x_i \geq v(\{i\})$ for all $i \in \mathcal{N}$. The core of the coalitional game with transferable payoff $\langle N, v \rangle$ is the set of all imputations $\mathbf{x}$ for which $x(\mathcal{S}) \geq v(\mathcal{S})$ for all $\mathcal{S} \subset \mathcal{N}$. In other words,*

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^{\mathcal{N}} : x(\mathcal{N}) = v(\mathcal{N}), x(\mathcal{S}) \geq v(\mathcal{S}), \forall \mathcal{S} \subset \mathcal{N}\}. \tag{1}$$

An imputation provides the payoff shares of providers in a grand coalition such that no provider's payoff is below what it earns in absence of cooperation. The core is a collection of imputations that provide stronger guarantees: no coalition has any incentive to split from the grand coalition if the providers share the aggregate payoff $v(\mathcal{N})$ as per an imputation $\mathbf{x}$ in the core. To see this, suppose a set of providers $\mathcal{S} \subset \mathcal{N}$ split from the grand coalition, form a separate coalition, and share their aggregate payoff $v(\mathcal{S})$ as per $\mathbf{w}$. A provider $i \in \mathcal{S}$, however, would agree to split only if $w_i > x_i$. Thus, $v(\mathcal{S}) = \sum_{i \in \mathcal{S}} w_i > \sum_{i \in \mathcal{S}} x_i$ which contradicts the fact that $\mathbf{x} \in \mathcal{C}$. Therefore, every imputation in the core renders the grand coalition stable.

We now elucidate $v(\cdot)$ and $\mathcal{C}$ using a simple example.

Example IV.1. *Let $\mathcal{N} = \{1, 2\}$, $\mathcal{B}_i = \{i\}, i = 1, 2$, and $\mathcal{M}_i = \{2i-1, 2i\}, i = 1, 2$. Let $r_{jk} = P$ for $j \in \mathcal{M}_1$, and $r_{jk} = Q$ for $j \in \mathcal{M}_2$, for all $k \in \mathcal{B}_{\mathcal{N}}$, $P < Q$ and $m_j = 0, \forall j \in \mathcal{M}_{\mathcal{N}}$. Let $U_i(\mathbf{x}) = \sum_{j \in \mathcal{M}_i} x_i$ and $V_i(\cdot) = 0$ for each $i \in \mathcal{N}$. Then $v(\{1\}) = P$, $v(\{2\}) = Q$, and $v(\{1, 2\}) = 2Q$ (when the providers cooperate, the aggregate benefit is maximized when only 2's customers are served and this maximum is $2Q$). Then, $\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^2 : x_1 + x_2 = 2Q, x_1 \geq P, x_2 \geq Q\}$. For instance, $(\frac{Q+P}{2}, \frac{3Q-P}{2})$ is an imputation in the core. When $1, 2$ cooperate, the benefit (revenue) earned from 1's (2's, resp.) customers is 0 ($2Q$, resp.), and therefore less (more, resp.) than its payoff under the above imputation. Provider 1's payoff is positive since its service unit fetches part of the coalition revenue by* *serving 2's customers. Also, this imputation increases each provider's payoff by $\frac{Q-P}{2}$ as compared to that in absence of cooperation (i.e., $x_i - v(\{i\}) = \frac{Q-P}{2}$).*

In several coalitional games the core is empty, i.e., no allocation can stabilize the grand coalition [2, Example 260.3], and in general it is NP-hard to determine whether the core of a coalitional game is nonempty [21]. A sufficient condition for the core to be nonempty is the convexity of the coalitional game, i.e., $v(\mathcal{S}) + v(\mathcal{T}) \le v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T})$ for all $\mathcal{S}, \mathcal{T} \subseteq \mathcal{N}$ [2, pp. 260]. But, as the following example illustrates, the game we are considering need not be convex.

Example IV.2. *Let $\mathcal{N} = \{1,2,3\}$, $\mathcal{B}_i = \{i\}, i = 1,2,3$, $\mathcal{M}_i = \{i\}, i = 1,2,3$. Let $r_{1k} = R, k \in \mathcal{B}_\mathcal{N}$, $r_{j1} = P, j \in \{2,3\}$ and $r_{jk} = Q, j \in \{2,3\}, k \in \{2,3\}$ and $P > Q$. Let $m_j = 0$ for all $j \in \mathcal{M}_\mathcal{N}$. Let $U_i(\mathbf{x}) = \sum_{j\in\mathcal{M}_i} x_i$ and $V_i(\cdot) = 0$ for each $i \in \mathcal{N}$. Thus $v(\{1\}) = R, v(\{1,2\}) = v(\{1,3\}) = R + P$, and $v(\{1,2,3\}) = R + P + Q$. Let $\mathcal{S} = \{1,2\}$ and $\mathcal{T} = \{1,3\}$. Then $v(\mathcal{S}) + v(\mathcal{T}) = 2R + 2P$ and $v(\mathcal{S}\cup\mathcal{T}) + v(\mathcal{S}\cap\mathcal{T}) = 2R+P+Q$. Thus $v(\mathcal{S})+v(\mathcal{T}) > v(\mathcal{S}\cup\mathcal{T}) + v(\mathcal{S}\cap\mathcal{T})$. Hence, this game is not convex.*

Nevertheless, we next show that the game $< \mathcal{N}, v >$ always has a nonempty core. Our proof technique is similar to ones presented in [22]–[25]. The proof is constructive in that it provides an imputation in $\mathcal{C}$ as well.

We obtain the dual of the optimization problem $\mathrm{P}(S)$ following dual formulation techniques in [26, Chapter 5].[8] Let $\lambda, \beta \in \mathbb{R}^{\mathcal{M}_\mathcal{S}\times\Omega}$, $\nu, \gamma \in \mathbb{R}^{\mathcal{B}_\mathcal{S}\times\Omega}$, and $\rho \in \mathbb{R}^{\mathcal{M}_\mathcal{S}}$. Let $g_{i\omega}(\lambda,\rho) = \max_{\mathbf{y}_i(\omega)\ge 0}\big(\mathbb{P}(\omega)U_i(\mathbf{y}_i(\omega)) + \sum_{j\in\mathcal{M}_i} y_j(\omega)(\lambda_j(\omega) + \rho_j\mathbb{P}(\omega))\big)$ and $h_{i\omega}(\nu) = \max_{\mathbf{z}_i(\omega)\ge 0}\big(- \mathbb{P}(\omega)V_i(\mathbf{z}_i(\omega)) + \sum_{k\in\mathcal{B}_i} z_k(\omega)\nu_k(\omega)\big)$. Then we have the following as the dual of $\mathrm{P}(S)$:

$$\mathrm{D}(\mathcal{S}) \ : \ \min \sum_{i\in\mathcal{S}} \Big(\sum_{\omega\in\Omega} \big(g_{i\omega} + h_{i\omega} + \sum_{k\in\mathcal{B}_i}\gamma_k(\omega) + \sum_{j\in\mathcal{M}_i}\beta_j(\omega)\big) - \sum_{j\in M_i} m_j\rho_j\Big)$$

subject to:

I) $\lambda_j(\omega)r_{jk}(\omega) + \nu_k(\omega) + \beta_j(\omega) + \gamma_k(\omega) \ge 0, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$

II) $\beta_j(\omega), \gamma_k(\omega), \rho_j \ge 0, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$

Clearly, $\mathrm{D}(\mathcal{S})$ is feasible for each $\mathcal{S} \subseteq \mathcal{N}$. Formulate $\mathrm{D}(\mathcal{N})$ by appropriately defining vectors $\lambda, \beta, \gamma, \nu, \rho$ and let $\mathcal{D}^*$ be the set of optimal solutions of $\mathrm{D}(\mathcal{N})$. Then, $\mathcal{D}^* \ne \emptyset$. Let

$$\mathcal{I} = \Big\{\mathbf{x}^* \in \mathbb{R}^\mathcal{N} : x_i^* = \sum_{\omega\in\Omega}\Big(g_{i\omega}(\lambda^*,\rho^*) + h_{i\omega}(\nu^*) + \sum_{k\in\mathcal{B}_i}\gamma_k^*(\omega) + \sum_{j\in\mathcal{M}_i}\beta_j^*(\omega)\Big) - \sum_{j\in\mathcal{M}_i} m_j\rho_j^* \text{ for some } (\lambda^*,\nu^*,\beta^*,\gamma^*,\rho^*) \in \mathcal{D}^*\Big\}$$

Here is the main result:

Theorem IV.1. *$\mathcal{I} \ne \emptyset$ and $\mathcal{I} \subseteq \mathcal{C}$.*

Proof: Since $\mathcal{D}^* \ne \emptyset$, $\mathcal{I} \ne \emptyset$. We show that for an arbitrary $\mathbf{x}^* \in \mathcal{I}$, $\mathbf{x}^* \in \mathcal{C}$. Note that since $U_i(\cdot)$s and $V_i(\cdot)$s are concave and convex functions respectively, the objective function of $\mathrm{P}(\mathcal{S})$ is concave. Also, the constraints of $\mathrm{P}(\mathcal{S})$ are all linear. Therefore, $\mathrm{P}(\mathcal{S})$ is maximizing a concave function over a convex set. Thus, strong duality holds.

Now, consider an arbitrary $\mathbf{x}^* \in \mathcal{I}$, corresponding to one $(\lambda^*,\nu^*,\beta^*,\gamma^*,\rho^*) \in \mathcal{D}^*$. Clearly $x^*(\mathcal{N}) = \sum_{i\in\mathcal{N}} x_i^*$ is the optimal value of $\mathrm{D}(\mathcal{N})$. Since $\mathrm{D}(\mathcal{S})$ is the dual of $\mathrm{P}(\mathcal{S})$ for each $\mathcal{S} \subseteq \mathcal{N}$, by strong duality $x^*(\mathcal{N}) = v(\mathcal{N})$. Now we only need to show that $x^*(\mathcal{S}) = \sum_{i\in\mathcal{S}} x_i^* \ge v(\mathcal{S})$ for any $\mathcal{S} \subset \mathcal{N}$. By strong duality, $v(\mathcal{S})$ equals the optimum value of $\mathrm{D}(\mathcal{S})$. Consider the sub-vectors $\lambda^*_\mathcal{S}, \nu^*_\mathcal{S}, \beta^*_\mathcal{S}, \gamma^*_\mathcal{S}, \rho^*_\mathcal{S}$ consisting of the components of $\lambda^*, \nu^*, \beta^*, \gamma^*, \rho^*$ in $\mathcal{S}$. Clearly these sub-vectors constitute a feasible solution of $\mathrm{D}(\mathcal{S})$, and $x^*(\mathcal{S})$ is the value of the objective function of $\mathrm{D}(\mathcal{S})$ for the above feasible solution. Therefore, the optimal value of $\mathrm{D}(\mathcal{S})$ is a lower bound for $x^*(\mathcal{S})$, i.e., $x^*(\mathcal{S}) \ge v(\mathcal{S})$. ∎

Thus, any imputation in $\mathcal{I}$ stabilizes the grand coalition. It also ensures that the payoffs of the providers are commensurate with the resource they invest and the wealth they generate. For ease of exposition, let there be no minimum rate requirements and let the benefit and cost functions be linear. Then, $g_{i\omega}(\lambda^*,\rho^*) = h_{i\omega}(\nu^*) = 0$, and provider i's payoff x_i^* equals the sum of the Lagrange multipliers corresponding to the constraints (3), (4) for its customers and service units ($\beta_j^*(\omega), \gamma_k^*(\omega)$, respectively). Lagrange multiplier $\gamma_k^*(\omega)$ ($\beta_j^*(\omega)$, resp.) is high only when service unit k (customer j, resp.) is fully utilized, i.e., serves customers (is served, resp.) all the time, and provides high transmission rates, $r_{jk}(\omega)$ and cost less (pay more, resp.) per unit time (bandwidth, resp.). Thus, i's Lagrange multipliers and hence i's payoff is high when it invests more resource and/or generates more wealth.

C. Computation Complexity and Distributed Computation

Note that $\mathrm{P}(\mathcal{S})$, $\mathrm{D}(\mathcal{S})$ are concave optimizations with linear constraints, and $\mathrm{P}(\mathcal{S})$ ($\mathrm{D}(\mathcal{S})$, resp.) has $O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_N||\Omega|)$ variables and constraints ($O(\max(|\mathcal{M}_\mathcal{N}| + |\mathcal{B}_N|)|\Omega|)$ variables, $O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_N||\Omega|)$ constraints resp.).[9] Under certain technical conditions, which involve existence of ϑ-concordant barrier functions [27, Chapter 2, Definition 2.1, Chapter 3.1], iterative interior point algorithms compute $\epsilon-$solutions for such optimizations using computation time that grows polynomially in the number of variables and constraints given the desired accuracy parameter ϵ, ϑ, and the distance between the optimal solution and the starting point of the iterations; the latter is bounded if the feasible set is bounded [27, Chapter 4].[10] The technical conditions hold for a large class of objective functions, including linear, logarithmic etc. [27,

[8]The notations can be explained considering $|\Omega| = 1$, $\mathcal{M}_1 = \{4,5,6\}$ and $\mathcal{M}_2 = \{7,8,9\}$. A vector $x \in \mathbb{R}^{\mathcal{M}_1\times\Omega}$ will have components x_4, x_5 and x_6 corresponding to customers $4, 5$ and 6 respectively. Similarly, a vector $x \in \mathbb{R}^{\mathcal{M}_2\times\Omega}$ will have components x_7, x_8 and x_9 corresponding to customers $7, 8$ and 9 respectively.

[9]Note that we have fewer dual variables as compared to primal constraints as the dual variables corresponding to some primal non-negativity constraints can be omitted without any imprecision.

[10]An ϵ-solution is one that (i) attains an objective value that is at most ϵ less than the maximum value and (ii) satisfies the feasibility constraints within an error margin of ϵ.

Chapter 10], and interior point algorithms have been known to perform well even in their absence [26, Chapter 11.5.1]. It can be shown that the optimization problem
$\max \sum_{i=1}^{Q} \phi_i(x_i)$
subject to: $\mathbf{x} \in \mathcal{G}$
where $\phi_i : \mathbb{R} \rightarrow \mathbb{R}$ are concave functions, $|\phi^{''}(x)| \leq B$ and $\mathcal{G} \subset \mathbb{R}^Q$ is specified by C linear constraints involving W variables ($W \geq Q$) and $A = \max_{i=1}^{Q} (\max_{\mathbf{x}\in\mathcal{G}} x_i - \min_{\mathbf{x}\in\mathcal{G}} x_i)$, may be solved within error margin ϵ in $O(W^2 \left(C + \frac{Q^2 A^2 B}{\epsilon}\right)^{3/2})$ time [18, Appendix A]. Note that P$(\mathcal{S})$ satisfies the above conditions with $A = \max_{j\in\mathcal{M}_\mathcal{N}, k\in\mathcal{B}_\mathcal{N}, \omega\in\Omega} r_{jk}(\omega)$, $W \sim O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_\mathcal{N}||\Omega|), Q \sim O\left((|\mathcal{M}_\mathcal{N}| + |\mathcal{B}_\mathcal{N}|)|\Omega|\right)$ and $C \sim O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_\mathcal{N}||\Omega|)$, if the revenue and cost functions are additive, i.e., $U_i(\mathbf{y}) = \sum_{j\in\mathcal{M}_i} g_{ij}(y_j)$ and $V_i(\mathbf{z}) = \sum_{k\in\mathcal{B}_i} h_{ik}(z_k)$ (which is likely to be the case in practice) and $-B \leq g''_{ij}(x) \leq 0$ and $0 \leq h''_{ik}(x) \leq B$ for all $x \in [\min_{i=1}^{Q} \min_{\mathbf{x}\in\mathcal{G}} x_i, \max_{i=1}^{Q} \max_{\mathbf{x}\in\mathcal{G}} x_i]$. A large class of revenue and benefit functions have bounded second derivatives, e.g., $g_{ij}(y_j) = \log(\gamma_{ij} + y_j)$ or $g_{ij}(y_j) = \frac{(\gamma_{ij}+y_j)^{1-\alpha}}{1-\alpha}$ for arbitrary positive γ_{ij}s. The dual D$(\mathcal{S})$ also satisfies the above conditions provided additionally (i) the minimum rate constraints do not exist, (ii) $|g''_{ij}(x)| \geq \delta, |h''_{ij}(x)| \geq \delta$ for some $\delta > 0$and for all $x \in [\min_{i=1}^{Q} \min_{\mathbf{x}\in\mathcal{G}} x_i, \max_{i=1}^{Q} \max_{\mathbf{x}\in\mathcal{G}} x_i]$ and (iii) the first and third derivatives of these functions are upper bounded. Then, B is a function of the bounds in (ii) and (iii). For D$(\mathcal{S})$, $W \sim O(|\mathcal{M}_\mathcal{N}| + |\mathcal{B}_\mathcal{N}|)$, A is the absolute value of the maximum of the above first derivatives and C, Q are as for P$(\mathcal{S})$.

The computation times can be large since $|\Omega|$, typically, is large. This may not however pose a major challenge as the computations are done off-line using large work-stations and at a slower time-scale (only when the network state statistics change or the coalitions are assessed). Also, whenever customers do not have minimum rate constraints (see Constraints (5)), we can solve both P$(\mathcal{S})$, D$(\mathcal{S})$ by solving separate convex optimizations, one for each $\omega \in \Omega$ - the number of variables and constraints for each such optimization depends only on $|\mathcal{M}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|$.[11] This separability allowed us to solve the above optimizations for reasonably large systems using Monte Carlo simulations (Section IX).[12]

Concave optimizations with linear constraints can be solved in distributed manner using the theory of subgradients, as described in [28] for example. The advantage of this distributed computation is that each provider i needs to know only its benefit and cost functions $U_i(\cdot), V_i(\cdot)$ (and not those of the others), the link rates r_{jk} only when either j is its customer or k is its service unit. The need for limited access to global information ensures confidentiality of operations.

For brevity we describe the distributed computations only for P$(\mathcal{S})$ - an imputation in the core may be similarly computed via solving D$(\mathcal{S})$. We consider the case that $U_i(\cdot), V_i(\cdot)$ have bounded partial derivatives, and the customers do not have minimum rate requirements; therefore, owing to the separability described above, we focus on the optimization for only one ω. Based on message exchanges with other providers, each provider iteratively updates (i) the downlink allocations $\alpha_{jk}^{(n)}$ from its service units to all customers, (ii) the rates of its customers $y_j^{(n)}$ and (iii) the total time allocation for its service units $z_k^{(n)}$ and the iterations provably converge to the optimum (the superscript n indicates the iteration index). At the end of each iteration, each provider i communicates (i) the $\{\alpha_{jk}^{(n)}\}$ iterates for all its service units (i.e., $k \in \mathcal{B}_i$), and (ii) indicators indicating the status of the satisfaction of the constraints (1), (3) for its customers (i.e., $j \in \mathcal{M}_i$), to the providers whose service units can serve its customers (i.e., those with positive r_{jk} to its customers). These indicators are used by other providers in the updates for the next iterations.

We describe the indicators and the update process next. Let $l_{1j}^{(n)}$ be 1 if for customer j at the end of the nth iteration the LHS exceeds the RHS of constraint (1), -1 if RHS exceeds the LHS, and 0 otherwise. Next, $l_{2k}^{(n)}$ is defined similarly for constraint (2) (for service unit k). Now, $l_{3j}^{(n)}$ is 1 if for customer j the LHS exceeds the RHS of constraint (3) and 0 otherwise. Next, $l_{4k}^{(n)}$ is defined similarly for constraint (4) (for service unit k). We now describe the update for each provider i, using constants δ_n, K that would be described later. In the $n+1$th iteration, provider i (i) for each of its customers j, obtains $y_j^{(n+1)}$ by adding $y_j^{(n)}$ and $\delta_n \left(\frac{\partial}{\partial y_j} U_i(\mathbf{y}_i^{(n)}) - K l_{1j}^{(n)}\right)$, (ii) for each of its service units k, obtains $z_k^{(n+1)}$ by decrementing $z_k^{(n)}$ by $\delta_n \left(\frac{\partial}{\partial z_k} V_i(\mathbf{z}_i^{(n)}) + K l_{2k}^{(n)}\right)$, and (iii) for each customer j and its service unit k (not necessarily its customer though) such that $r_{jk} > 0$, obtains $\alpha_{jk}^{(n+1)}$ by adding $\alpha_{jk}^{(n)}$ and $\delta_n K \left(r_{jk} l_{1j}^{(n)} + l_{2k}^{(n)} - l_{3j}^{(n)} - l_{4k}^{(n)}\right)$.

The updates of the optimization variables depend on the derivatives of the objective functions and also on whether the constraints are satisfied - intuitively, the iterates successively move closer to the optimum value of the objective function subject to the constraints. Formally, similar to the proof of [28, Theorem 5], it can be shown that for any (a) K exceeding the maximum value of the partial derivatives of the $U(\cdot), V(\cdot)$ functions and (b) $\{\delta_n\}$ such that $\sum_n \delta_n = \infty$, $\lim_{n\to\infty} \delta_n = 0$, $\{\alpha_{jk}^{(n)}\}$ converge to the optimum allocations [18].

D. Insights From the Framework

Now we discuss how this framework can provide useful insights about the relation between a provider's payoff share, the resources it contributes, and the wealth it generates. Among the demands and assets in possession of a provider, one could be more constrained than the others. For instance, a provider might have a lot of customers, but few service units. Then, increasing the number of service units could boost the

[11] This separability speeds up the computations as the computation times for the optimizations are super-linear in the number of variables and constraints.

[12] In each run of the Monte Carlo simulation, we generate a network state ω, using the distribution on the service unit-customer rates, and solve the optimizations $P(\mathcal{S})$ for the coalitions $\mathcal{S}$ for the given ω. Subsequently, we computed the average of the payoffs of the providers over a large number of runs. Using ergodicity it can be analytically shown that as the number of runs tend to infinity, the averages converge to the optimum solution.

payoff generated by the provider, while adding to the number of customers might not change it. An intuitive observation then is that the provider that offers more of the demand or asset that is sought most by the majority in the coalition, is likely to receive a larger share of the aggregate payoff. The following example will further elucidate this.

Example IV.3. *Consider the setting in Example IV.1 except that $\mathcal{N} = \{1,2,3\}$, $|\mathcal{M}_1| = 5$, $|\mathcal{M}_2| = |\mathcal{M}_3| = 2$, $|\mathcal{B}_1| = 2$, $|\mathcal{B}_2| = 3$, and $|\mathcal{B}_3| = 4$, $r_{jk} = P$ for all $j \in \mathcal{M}_{\mathcal{N}}$ and $k \in \mathcal{B}_{\mathcal{N}}$. Then, $v(\{i\}) = 2P$ for $i \in \mathcal{N}$, $v(\{1,2\}) = 5P$, $v(\{1,3\}) = 6P$, $v(\{2,3\}) = 4P$, and $v(\{1,2,3\}) = 9P$. An example allocation in the core $(\frac{7P}{2}, \frac{5P}{2}, 3P)$ fetches payoff gains of $\frac{3P}{2}, \frac{P}{2}$ and P to the three providers as compared to the case when they do not cooperate. Also, somewhat contrary to intuition, provider 1, who has the least number of service units, attains the highest payoff. This is because the other providers, i.e., $2,3$ have fewer customers than service units, and these excess service units are utilized only when 1 joins the coalition along with its customers. Thus, 1 is adding the most value to the coalition by bringing in the demand that is sought out by others: note that $v(\{2,3\}) = v(\{2\}) + v(\{3\})$ but $v(\{1,2,3\}) > v(\{1\}) + v(\{2\}) + v(\{3\})$. Also, the providers' shares of the aggregate payoff are usually largely determined by parameters other than their decision variables, e.g., the number of customers, service units, etc.*

In Example IV.3, if provider 2 can somehow expand its customer base, e.g., by extensive advertising, its share increases, although the aggregate payoff remains the same. Thus, a provider can accordingly decide how to upgrade its resources.

V. Resource Deployment Games

We now consider a service unit deployment game which allows the providers to maximize the aggregate payoffs and also enhance individual payoffs by deciding which bands to lease from spectrum regulators or primary users and also where to deploy base stations in addition to deciding their allocations to customers. Redefine $\mathcal{B}_i$ to be the set of service units *available* to provider i; $\mathcal{B}_i$, $i \in \mathcal{N}$ are assumed to be disjoint. A provider i can use a service unit k available to it once it "opens" it by paying a fixed fee f_k; it subsequently pays usage based charge for using it (the $V(\cdot)$ functions in the previous sections) which depends only on the amount of usage and is 0 if k is not used. Let $b_k = 1$ if provider i opens $k \in \mathcal{B}_i$ and 0 otherwise. A provider need not open all service units available to it, and thus the b_ks constitute its decision variables (in addition to the $\{\alpha_{jk}\}$s). A service unit $k \in \mathcal{B}_i$ can then serve customer j if $a)$ $b_k = 1$ and $b)$ service unit k and customer j are associated with the same provider or with different providers who are in a coalition.

We now describe how (i) spectrum acquisition game and (ii) base station location game can be captured in the setting of a service unit deployment game. In the former, providers decide the channels each base station has access to. Each available service unit corresponds to a base station-available channel pair, and thus if a provider decides to allow a base station access to a specific channel k, it opens the corresponding service unit, by paying the spectrum regulator (a government agency or a license holder) the fixed fee (membership charge), f_k, and pays usage-based charges for using it subsequently. Depending on the spectrum pricing model, either the membership charge or the usage-based charge may be zero, or both may be positive.[13] In a base station deployment game, a provider decides the locations of its base stations. We initially assume that each base station has access to only one band, and thus, the service units are the same as the base stations.The set of candidate locations of base stations of provider i constitutes its set of available service units, and the band available to a candidate location is decided apriori (based on interference conditions). A provider can construct a base station at a candidate location k by paying the fixed establishment (and maintenance) cost f_k. Usually, it will not pay any usage based costs subsequently and the $V(\cdot)$ functions are 0.

A. Characteristic Function Formulation

We now formulate the characteristic functions of the service unit deployment game. We assume $m_j = 0$ for all $j \in \mathcal{M}_{\mathcal{N}}$, $U_i(\mathbf{x}) = \sum_{j \in \mathcal{M}_i} a_j x_j$, $V_i(\mathbf{x}) = \sum_{k \in \mathcal{B}_i} s_k x_k$ for all $i \in \mathcal{N}$, and also that customers are static and the quality of channels do not vary with time, i.e. $|\Omega| = 1$.

For a coalition $\mathcal{S} \subseteq \mathcal{N}$, the payoff $v(\mathcal{S})$ is then obtained by solving the following optimization problem.

$\mathrm{P_c}(\mathcal{S}) : \max \sum_{\substack{j \in \mathcal{M}_{\mathcal{S}} \\ k \in \mathcal{B}_{\mathcal{S}}}} \alpha_{jk}(r_{jk} a_j - s_k) - \sum_{k \in \mathcal{B}_{\mathcal{S}}} f_k b_k$

subject to:

1) $\sum_{k \in \mathcal{B}_{\mathcal{S}}} \alpha_{jk} \leq 1, \quad j \in \mathcal{M}_{\mathcal{S}}$
2) $\sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jk} \leq b_k, \quad k \in \mathcal{B}_{\mathcal{S}}$
3) $\alpha_{jk} \geq 0, \quad j \in \mathcal{M}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}$
4) $b_k \in \{0,1\}, \quad k \in \mathcal{B}_{\mathcal{S}}$

Constraints (1) ensure that the total fraction of time customer j is being served, is upper bounded by 1. A service unit k can serve at most 1 fraction of time if it is open and can not serve otherwise, by constraints (2). The following example illustrates how cooperation may change providers' decisions regarding the opening of service units.

Example V.1. *Let $\mathcal{N} = \{1,2\}$, $\mathcal{B}_1 = \mathcal{M}_1 = \{1\}$ and $\mathcal{B}_2 = \mathcal{M}_2 = \{2,3\}$, where $f_1 = 0$, and $f_2 = f_3 = f$. Let $r_{11} = r_{12} = r_{32} = Q$, $r_{21} = r_{22} = r_{33} = P$, and $r_{jk} = 0$ otherwise. Suppose $f < P$, and $s_1 < Q < P$. Let $a_j = 1$ for each j and $s_k = 0$ for $k > 1$. Now $v(\{1\}) = Q - s_1$. Also $v(\{2\}) = 2P - 2f$ and $v(\{1,2\}) = \max[2P - f - s_1, 2P + Q - 2f - s_1]$, where the former payoff is the result of opening just service unit 3, while the latter arises in the event of opening both $2,3$. If $Q < f < P$, opening both service units is optimal when not in coalition, while opening only 3 is optimal otherwise. Thus, a provider may need to open fewer service units while in coalition, which is beneficial for large opening fees.*

[13] In the settings where the providers have decided apriori which channels the base stations have access to and only need to determine the service unit-customer allocations, as in Sections III, IV, the fixed fee is paid for each channel irrespective of the allocation decisions - thus these do not alter the optimum allocations and hence were not explicitly considered.

B. Nonemptyness of the Core

We proceed to prove that the core of the coalitional game $< \mathcal{N}, v >$, with characteristic function $v(\cdot)$ given by $\mathrm{P_c}(\mathcal{S})$, is nonempty. Note that the aggregate payoff of a coalition now is given by an integer (rather than concave) optimization problem. As a result, the strong duality used in Section IV does not hold in general. Our proof relies on unimodularity arguments instead and proceeds in two steps.

Step (i): Consider the coalitional game $< \mathcal{N}, \hat{v} >$, where $\mathcal{N}$ is the same set of providers and the characteristic function $\hat{v}(\cdot)$ is given by the LP, $\mathrm{P_{relaxed}}(\mathcal{S})$. $\mathrm{P_{relaxed}}(\mathcal{S})$ is the linear relaxation of $\mathrm{P_c}(S)$, where the constraints $b_k \in \{0,1\}$ are now replaced by $b_k \in [0,1]$. We show that the core of the coalitional game $< \mathcal{N}, \hat{v} >$, $\hat{\mathcal{C}}$, is nonempty.

Using $\lambda \in \mathbb{R}^{\mathcal{M}_\mathcal{S}}$, and $\nu, \gamma \in \mathbb{R}^{\mathcal{B}_\mathcal{S}}$, we construct the following LP as the dual of $\mathrm{P_{relaxed}}(\mathcal{S})$
$\mathrm{D_{relaxed}}(\mathcal{S}) : \min \sum_{j \in \mathcal{M}_\mathcal{S}} \lambda_j + \sum_{k \in \mathcal{B}_\mathcal{S}} \gamma_k$
subject to:

1) $\lambda_j + \nu_k \geq r_{jk} a_j - s_k, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}$
2) $\nu_k - \gamma_k \leq f_k, \quad k \in \mathcal{B}_\mathcal{S}$
3) $\lambda_j, \nu_k, \gamma_k \geq 0, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}$

Let $\mathcal{D}^*_{\text{relaxed}}$ constitute the set of optimal solutions of $\mathrm{D_{relaxed}}(\mathcal{N})$. Define: $\mathcal{I}_\mathrm{c} := \{\mathbf{x}^* \in \mathbb{R}^\mathcal{N} : x^*_i = \sum_{j \in \mathcal{M}_i} \lambda_j + \sum_{k \in \mathcal{B}_i} \gamma_k \quad \text{for some} \quad (\lambda^*, \nu^*, \beta^*, \gamma^*) \in \mathcal{D}^*_{\text{relaxed}}\}$.

Theorem V.1. *$\mathcal{I}_c \neq \emptyset$, and $\mathcal{I}_c \subseteq \hat{\mathcal{C}}$*

Proof: The proof is identical to that of Theorem IV.1. ∎

Step (ii): Next, we prove that, for any coalition $\mathcal{S} \subseteq \mathcal{N}$,$\mathrm{P_{relaxed}}(\mathcal{S})$ has an integral optimum solution, which therefore constitutes an optimum solution of $\mathrm{P_c}(\mathcal{S})$. We use the fact that, the constraints of $\mathrm{P_{relaxed}}(\mathcal{S})$ can be represented as a totally unimodular matrix:

Definition V.1. *A matrix A is totally unimodular if every square submatrix of A has determinant either 0, 1 or -1.*

We have the following sufficient conditions for the matrix A to be totally unimodular [29].

Theorem V.2. *Suppose A can be partitioned into two disjoint sets B and C, with the following properties:*

1) *Every column of A contains at most two non-zero entries;*
2) *Every entry in A is 0, $+1$, or -1;*
3) *If two non-zero entries in a column of A have the same sign, then the row of one is in B, and the other in C;*
4) *If two non-zero entries in a column of A have opposite signs, then the rows of both are in B or both in C.*

Then A is totally unimodular.

Now, consider the following linear program
P: $\max c^T x$
subject to: $Ax \leq b, \quad x \geq 0$
We have the following theorem [30].

Theorem V.3 ([30]). *The linear program*
P: $\max c^T x$ *subject to:* $Ax \leq b, \quad x \geq 0$
has an optimal integral solution if $(1) A$ is totally unimodular, and $(2) b$ contains only integers.

For any coalition $\mathcal{S} \subseteq \mathcal{N}$, $\mathrm{P_{relaxed}}(\mathcal{S})$ satisfies the sufficiency conditions in the above theorem, and therefore has an integral optimum solution. Thus:

Theorem V.4. *For any coalition $\mathcal{S} \subseteq \mathcal{N}$, the integer program* $\mathrm{P_c}(\mathcal{S})$ $v(\mathcal{S}) = \hat{v}(\mathcal{S})$ *for all* $\mathcal{S} \subseteq \mathcal{N}$.

Theorem V.4 implies that $\mathcal{C} = \hat{\mathcal{C}}$. Thus, from Theorem V.1,

Theorem V.5. *$\mathcal{I}_c \neq \emptyset$, and $\mathcal{I}_c \subseteq \mathcal{C}$.*

It follows directly from this theorem that the optimal service unit opening decisions and the service unit allocations and an imputation in the core can be obtained by solving the linear programs $\mathrm{P_{relaxed}}(\mathcal{N})$, $\mathrm{D_{relaxed}}(\mathcal{N})$, which can be done in polynomial time. Specifically, $\mathrm{P_{relaxed}}(\mathcal{N})$ has $W \sim O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_\mathcal{N}|)$ variables and $C \sim O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_\mathcal{N}|)$ constraints, $\mathrm{D_{relaxed}}(\mathcal{N})$ has $W \sim O(|\mathcal{M}_\mathcal{N}| + |\mathcal{B}_\mathcal{N}|)$ variables and $C \sim O(|\mathcal{M}_\mathcal{N}||\mathcal{B}_\mathcal{N}|)$ constraints. Thus each can be solved using Karmarkar's interior point algorithm [31] in $O(C^{\frac{3}{2}} W^2 L)$ time where the obtained solution and the optimal solution match in L most significant digits. [14] Also, the linear programs $\mathrm{P_{relaxed}}(\mathcal{N})$, $\mathrm{D_{relaxed}}(\mathcal{N})$ can be solved by the providers in a distributed manner and without revealing their confidential information such as the revenue and costs $\{a_j, s_k\}$ to each other, using the sub-gradient technique as described in Section IV. Finally, the resulting imputation, which belong to the core, distributes the aggregate grand-coalition payoff among providers in accordance with the Lagrange-multipliers of $\mathrm{P_{relaxed}}(\mathcal{N})$, which as explained in Section IV, are commensurate with the resource investments and wealth generated by the providers.

C. Generalizations

Finally, we discuss how we can relax our earlier simplifying assumptions. First, when the customers' locations and channels' qualities are random, i.e., $|\Omega| > 1$, then we can prove using an extended duality technique that the core is nonempty and obtain an imputation in the core in polynomial time under an additional assumption: $\alpha_{jk}(\omega) \leq 1/|\mathcal{B}_\mathcal{N}|$ for each j, k, ω [18]. Similar results can be shown for a joint spectrum acquisition and base station location game where providers have to decide both the locations of the base stations and the set of channels each base station has access to (allowing each base station access to multiple channels) [18]. This additional assumption does not cause any loss of generality for the network states ω in which there are several customers with identical transmission rates from the service units (such network states arise frequently when the number of customers is large). In such cases, the aggregate payoff may be maximized if each service unit time-shares among the customers that have identical transmission conditions - thus, even the optimizations that do not impose this condition explicitly will choose small $\alpha_{jk}(\omega)$s.

[14] Thus, L is the number of accuracy digits of the generated solution. Often, the computation time results are stated in units of L, e.g., $O(C^{3/2} V^2)$ per accuracy digit in the algorithm output.

VI. Cooperation In Multi-hop Networks

Cooperation in multi-hop networks allows the cooperating providers to redirect their traffic through possibly better multi-hop routes. Consider a network in which customers can communicate with service units via potentially multi-hop routes, that is, via other customers which act as relays. If now a set of providers agree to cooperate by pooling their service units and customers, not only they benefit from sharing others' service units (as in single-hop networks), but they also have access to more relay nodes. This, in turn, can increase the capacity of the network, as well as its power efficiency,thereby enhancing the payoffs of the providers.[15] For instance in Figure 3, in absence of cooperation, provider 1 can send data to customer 6, only through $C3$, but when the providers cooperate, it can also send through $C4, C5$. We formulate the interactions among the providers in a multi-hop network using a coalitional game model, prove that the core of this game is non-empty, obtain polynomial-time computable (i) optimal strategies that maximize the aggregate payoffs and (ii) payoff shares for individual providers that render the grand coalition stable (i.e., an imputation in the core) and are also commensurate with the resource investment and wealth generated by the providers.

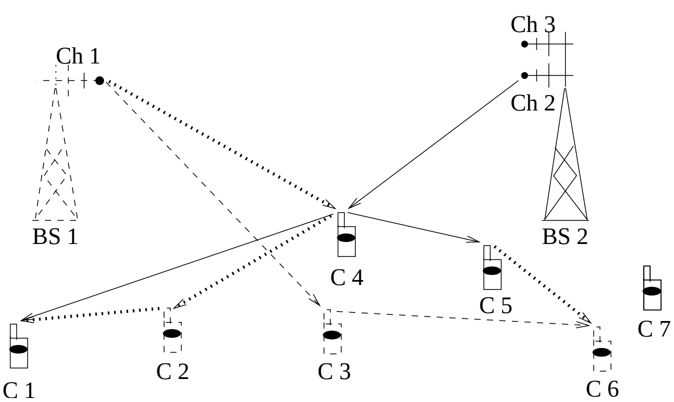

Fig. 3: A multihop network with two providers: the dashed objects (base stations, channels,customers and links) belong to provider 1 and the solid objects belong to provider 2. Here, $\mathcal{B}_1 = \{1\}, \mathcal{B}_2 = \{2, 3\}, \mathcal{M}_1 = \{2, 3, 6\}$ and $\mathcal{M}_2 = \{1, 4, 5, 7\}$. Providers 1 and 2 want to send data to customers 6 and 1 respectively, through multi-hop routes. The thick dotted links are those resulting from providers' cooperation.

As in single-hop networks, providers determine the allocations of the service units. But, an interesting question is: who determines the communication routes - providers or customers who relay the traffic? When a customer relays others' packets, it essentially provides a service that enhances the providers' payoffs and consumes its time and energy, and must therefore be compensated via discounts from the providers. Such discounts must depend on how much traffic each customer relays, and how much time and energy it invests in relaying. Thus, in its relaying role, a customer is like any other "resource" (like spectrum for example) whose utilization fetches benefits and also incurs costs. Thus, the providers determine the multi-hop routes so as to best utilize the customers' service potentials and to regulate the costs they incur. Note that a customer can regulate its participation in relaying through a *maximum relaying agreement* with its provider that limits the amount of time it can be used for relaying others' traffic.

We now describe how the resource pooling game formulated in Sections III, IV may be generalized to allow multi-hop transmissions. Consistent with the downlink communication assumption, we assume that service units transmit to customers (who are either sinks or relays), but do not receive from them. We assume that a pair of customers can communicate with each other (to relay packets) without interfering with the communications of other customer-customer or customer-service unit pairs (owing to appropriate channel allocations for example). Similar transmission models have extensively been assumed in related contexts, e.g., [5]. The wireless link to a customer j from a service unit or another customer k can transmit packets at a rate r_{jk}, a random variable which is a function of the location of customer j and the state of channel k. A customer and a service unit, or two customers, can communicate only when both are associated with the same provider or the providers associated with them are in a coalition. For instance in Figure 3, the links $C5 - C6$ and $C2 - C1$ arise when the two providers cooperate.

The service rate of a customer j is defined as the total rate at which traffic intended for j reaches j. Let τ_j be the maximum fraction of time customer j spends as a relay. Let $\beta^{j_1}_{j_2 k}(\omega) \in [0, 1]$ be the fraction of time, customer j_2 receives the packets destined for customer j_1, from customer or service unit k when the network state is ω; $\beta^{j_2}_{j_1 j_2}(\omega) = 0$ for all j_1, j_2 and ω. The providers determine the routes through the choice of the allocations $\{\beta^{j_1}_{j_2 k}(\omega)\}$.

Consider a coalition $\mathcal{S}$ and a network realization ω. When the provider associated with customer j is in $\mathcal{S}$, j receives a service rate $y_j(\omega) = \sum_{k \in \mathcal{B}_\mathcal{S} \cup \mathcal{M}_\mathcal{S}} \beta^j_{jk}(\omega) r_{jk}(\omega)$. Let $\mathbf{y}_i(\omega) = \{y_j(\omega), j \in \mathcal{M}_i\}$. Then, provider i gains a benefit (e.g., revenue from the customers) of $U_i(\mathbf{y}_i(\omega))$. Next, customer j relays the traffic for $t_j(\omega)$ fraction of time, where $t_j(\omega) = \sum_{j_1, j_2 \in \mathcal{M}_\mathcal{S} \setminus j, k \in \mathcal{B}_\mathcal{S}} (\beta^{j_1}_{j_2 j}(\omega) + \beta^{j_1}_{j j_2}(\omega) + \beta^{j_1}_{jk}(\omega))$. Let $p_{jk}(\omega)$ represent the power usage of customer j when it transmits to (and thereby relays others' traffic to) customer k. Then a customer j in a coalition $\mathcal{S}$, has a total power usage of $z_j(\omega) = \sum_{j_1 \in \mathcal{M}_\mathcal{S} \setminus \{j\}, k \in \mathcal{M}_\mathcal{S}} \beta^{j_1}_{kj}(\omega) p_{jk}(\omega)$ in relaying packets. Let $\mathbf{z}_i(\omega) = \{z_j(\omega), j \in \mathcal{M}_i\}$ and $\mathbf{t}_i(\omega) = \{t_j(\omega), j \in \mathcal{M}_i\}$. Then, provider i incurs a cost of $V_i(\mathbf{z}_i(\omega), \mathbf{t}_i(\omega))$ owing to the compensations (i.e., service discounts) required by its customers for spending time and energy in relaying packets. Functions $U_i(\cdot)$ ($V_i(\cdot)$, resp.) are concave (convex, resp.) and are decided apriori, possibly through prior negotiations with the customers. We assume that the locations of service units and the set of channels they have access to are determined a priori. Thus, we do not consider fixed service unit deployment costs or channel licensing costs.

[15] Note that for certain customers, the increase in the power usage may not be proportional to that in their service rates, but cooperation increases the power efficiency of the network as a whole.

The aggregate payoff available to providers in a coalition is the difference between their benefits and costs. Therefore, in order to maximize their aggregate payoff, providers in a coalition must optimally decide the allocations $\{\beta^{j_1}_{j_2k}\}$, based on the network state, and benefit and cost functions, subject to the minimum rate and maximum relaying constraints. Let $v(\mathcal{S})$ denote the maximum aggregate payoff achievable by a coalition $\mathcal{S}$. Then, $v(\mathcal{S})$ is the maximum value of the objective function of the following concave optimization:

$\mathrm{P_m}(\mathcal{S}) : \max \sum_{\substack{i\in\mathcal{S}\\ \omega\in\Omega}} \mathbb{P}(\omega)\Big(U_i(\mathbf{y}_i(\omega)) - V_i(\mathbf{z}_i(\omega), \mathbf{t}_i(\omega))\Big)$

subject to:

1) $y_j(\omega) = \sum_{k\in\mathcal{B}_\mathcal{S}\cup\mathcal{M}_\mathcal{S}} \beta^j_{jk}(\omega) r_{jk}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
2) $t_j(\omega) = \sum_{\substack{j_1,j_2\in\mathcal{M}_\mathcal{S}\setminus j\\ k\in\mathcal{B}_\mathcal{S}}} \Big(\beta^{j_1}_{j_2j}(\omega) + \beta^{j_1}_{jj_2}(\omega) + \beta^{j_1}_{jk}(\omega)\Big), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
3) $z_j(\omega) = \sum_{\substack{j_1\in\mathcal{M}_\mathcal{S}\setminus j\\ k\in\mathcal{M}_\mathcal{S}}} \beta^{j_1}_{kj}(\omega) p_{jk}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
4) $\sum_{k\in\mathcal{M}_\mathcal{S}\cup\mathcal{B}_\mathcal{S}} \beta^{j_1}_{j_2k}(\omega) r_{j_2k}(\omega) = \sum_{j\in\mathcal{M}_\mathcal{S}} \beta^{j_1}_{jj_2}(\omega) r_{jj_2}(\omega), \quad j_1 \neq j_2 \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
5) $t_j(\omega) + \sum_{k\in\mathcal{B}_\mathcal{S}\cup\mathcal{M}_\mathcal{S}} \beta^j_{jk}(\omega) \le \theta, \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
6) $\sum_{j_1,j_2\in\mathcal{M}_\mathcal{S}} \beta^{j_1}_{j_2k}(\omega) \le \theta, \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
7) $\sum_{\omega\in\Omega} \mathbb{P}(\omega) y_j(\omega) \ge m_j, \quad j \in \mathcal{M}_\mathcal{S}.$
8) $\sum_{\omega\in\Omega} \mathbb{P}(\omega) t_j(\omega) \le \tau_j, \quad j \in \mathcal{M}_\mathcal{S}.$
9) $\beta^{j_1}_{j_2k}(\omega) \ge 0, \quad j_1, j_2 \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S} \cup \mathcal{M}_\mathcal{S}, \omega \in \Omega$

Constraints (4) ensure that the set of $\beta^{j_1}_{j_2l}$s satisfy the flow feasibility constraints, while constraints (5) and (6) guarantee that they constitute a feasible allocation for $\theta \le 2/3$.[16] Constraints (7) and (8) impose minimum rate and maximum relaying guarantees, respectively. Similar to Assumption IV.1, we assume that $\mathrm{P_m}(\{i\})$ is feasible for each $i \in \mathcal{N}$, and thus, $\mathrm{P_m}(\mathcal{S})$ is feasible for each $\mathcal{S} \subseteq \mathcal{N}$.

Similar to the proof of Theorem IV.1, one can formulate the dual problem of the concave maximization $\mathrm{P_m}(\mathcal{N})$ (which is always feasible) and subsequently, define the set $\mathcal{I}$ appropriately. The same proof technique then shows that $\mathcal{I}$ belongs to the core. Hence, nonemptiness of the core follows.

Finally, similar formulations may be used to model cooperation among internet service providers (ISPs) in the same tier. Specifically, peer ISPs may form coalitions where the providers in the same coalition route traffic to the customers (i.e., end users or the ISPs in lower tiers) through each other's routers (analogous to our service units). The characteristic function $v(\mathcal{S})$ now represents the total profit of the ISPs in a coalition $\mathcal{S}$, and can be obtained by solving a concave maximization with linear constraints, similar to $\mathrm{P_m}(\mathcal{S})$ - the differences in this optimization are that (i) there is only one ω as the link qualities will not vary randomly in wireline networks, (ii) cost functions $V_j(\cdot)$ are zero as the routers belong to the ISPs (iii) constraint (5), (6) on the fraction of time each service unit and relay is used must be replaced by link capacity constraints. The duality gap continues to be zero. Hence, it can be shown similar to the proof of Theorem IV.1 that the core is non-empty and an allocation in the core can be obtained in polynomial time.

[16] At each ω, the system can be represented by a graph where the customers and the service units represent the nodes and there exists a link between any two nodes (only one of which can be a service unit) j, k such that $r_{jk}(\omega) > 0$. Any customer-service unit and customer-customer assignment corresponds to a matching in the above graph. Note that $\{\beta^l_{jk}(\omega)\}$ comprise a feasible allocation of service units to customers if and only if there exists a corresponding collection of matchings $L_1, L_2, \ldots$ and a collection of non-negative real numbers $\gamma_1, \gamma_2, ..$ such that (i) $\sum_i \gamma_i = 1$, $\gamma_i \ge 0$ and (ii) if the allocations follow matching L_i for γ_i fraction of time for each i, then customer j receives from customer or service unit k for $\sum_{j_1\in\mathcal{M}_\mathcal{S}} \beta^{j_1}_{jk}(\omega)$ fraction of time for all j, k. A sufficient condition for feasibility is that the fraction of time each service unit or customer communicates is below θ, where θ is a constant in $[0, 1]$ and depends on the network topology. For bipartite networks, for instance, $\theta = 1$, which is also a necessary condition [19]. It has been shown that in general, $\theta = \frac{2}{3}$ is a sufficient but not a necessary condition [19]. Nevertheless, utilization would usually be less than $2/3$ so as to avoid inordinate queuing delays. Thus, constraints (5), (6) provide the necessary and sufficient conditions for feasibility of $\{\beta^l_{jk}(\omega)\}$ for each ω and $\theta \le 2/3$ - the θ value is chosen based on delay constraints.

VII. Other Solution Concepts: Nucleolus And Shapley Value

We now investigate aggregate payoff sharing among providers using two other well known solution concepts in coalitional games, namely the nucleolus and the Shapley value, and examine whether these payoff shares stabilize the grand coalition (i.e., belong to the core).

A. Nucleolus

Definition VII.1. *The excess of a non-empty coalition $\mathcal{S} \subset \mathcal{N}$ under an imputation $\mathbf{x}$ is $e_\mathcal{S}(\mathbf{x}) = v(\mathcal{S}) - x(\mathcal{S})$. Let $e^{(1)}(\mathbf{x}) = \max_{\mathcal{S}:\phi\subset\mathcal{S}\subset\mathcal{N}} = e_\mathcal{S}(\mathbf{x})$, i.e., $e^{(1)}(\mathbf{x})$ is the maximum among the excesses of the non-empty and proper subsets of the grand coalition, $e^{(2)}(\mathbf{x})$ is the second maximum etc. The nucleolus is the imputation $\mathbf{x}$ that lexicographically minimizes the excesses, i.e., has the minimum value of $e^{(1)}(\mathbf{x})$ among all the imputations, subject to minimizing $e^{(1)}(\mathbf{x})$ minimizes $e^{(2)}(\mathbf{x})$, and so on.*

Recall that $v(\mathcal{S})$ ($\mathbf{x}(\mathcal{S})$, resp.) are the maximum aggregate payoff and (aggregate payoff under $\mathbf{x}$, resp.) of coalition $\mathcal{S}$. Thus, one can think of $e_\mathcal{S}(\mathbf{x})$ as a measure of dissatisfaction of $\mathcal{S}$ under $\mathbf{x}$. Then, the nucleolus is the payoff share (of the aggregate grand coalition payoff) that equalizes the dissatisfactions of the coalitions as far as possible.

The nucleolus of any transferable payoff coalitional game is a singleton [2, pp. 288]. Whenever the core of a coalitional game is nonempty, its nucleolus belongs to the core.

When there are only two providers, the excesses of the coalitions $\{1\}, \{2\}$ for an imputation $\mathbf{x} = (x_1, x_2)$ are $(v(\{1\}) - x_1, v(\{2\}) - x_2)$ - these are the negatives of the payoff gains brought about by cooperation. Since $\mathbf{x}$ is an imputation, $x_1 + x_2 = v(\{1, 2\})$ is a constant, and thus the sum of the two excesses is also constant. Also, since the nucleolus minimizes the maximum excess, it equalizes the two excesses. Thus, the nucleolus is the payoff vector $((v(\{1, 2\} + v(\{1\}) - v(\{2\}))/2, (v(\{1, 2\} + v(\{2\}) - v(\{1\}))/2)$. Thus, in Example IV.1, the nucleolus payoff allocations are $\frac{Q+P}{2}, \frac{3Q-P}{2}$

respectively.[17] The imputations obtained earlier by solving the dual of the aggregate payoff maximization problems do not necessarily equalize the payoff gains, but rather distributes the payoffs in accordance with the resource investments and the wealth generated by the providers (Section IX).

B. Shapley Value

Definition VII.2. *For any i, and $\mathcal{S} \subset \mathcal{N}$ such that $i \notin \mathcal{S}$, let $\Delta_i(\mathcal{S}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S})$. The Shapley value is the imputation $\mathbf{x}$ for which*

$$x_i = \frac{1}{n!} \sum_{U \in \mathcal{U}} \Delta_i(\mathcal{S}_i(U)), \quad (2)$$

where $\mathcal{U}$ is the set of all orderings of the set of players, and $\mathcal{S}_i(U)$ is the set of players preceding i in ordering U.

In Example IV.1, $\Delta_i(\varnothing) = v(\{i\}), \Delta_1(\{2\}) = Q, \Delta_2(\{1\}) = 2Q - P$, and the Shapley value is $\big((Q+P)/2, (3Q-P)/2\big)$.

Shapley value is the unique imputation that attains certain desirable game-theoretic properties like *symmetry*, *dummy player allocation* and *additivity* [2, pp. 292]. For two player transferable payoff coalitional games the Shapley value is the imputation $(v(\{1,2\} + v(\{1\}) - v(\{2\}))/2, (v(\{1,2\} + v(\{2\}) - v(\{1\}))/2$; it is therefore identical to the nucleolus and belongs in the core. But, in case of three or more players, the Shapley value need not be in the core, and therefore need not stabilize the grand coalition:

Example VII.1. *Let $\mathcal{N} = \{1,2,3\}$, $\mathcal{B}_i = \{i\}$ and $\mathcal{M}_i$ be nonempty for each provider i. Let $r_{j2} = 1, j \in \mathcal{M}_1 \cup \mathcal{M}_3, r_{j1} = r_{j3} = 1, j \in \mathcal{M}_2$ and $r_{jk} = 0$ otherwise. Also, let $m_j = 0$, $\forall j \in \mathcal{M}_{\mathcal{N}}$, $U_i(\mathbf{x}) = \sum_{j \in \mathcal{M}_i} x_j$ and $V_i(\cdot) = 0$, $i \in \mathcal{N}$. Clearly, $v(\{i\}) = 0 \ \forall\ i$, $v(\{1,2\}) = v(\{2,3\}) = v(\{1,2,3\}) = 2, v(\{1,3\}) = 0$. From (2) and Table I, the Shapley value of the providers is $\mathbf{x} = (\frac{2}{6}, \frac{8}{6}, \frac{2}{6})$. Note that $x_1 + x_2 = \frac{10}{6} < v(\{1,2\})$. Hence $\mathbf{x} \notin \mathcal{C}$.*

TABLE I: All possible orderings and marginal contributions of the players.

U	$\Delta_1(U)$	$\Delta_2(U)$	$\Delta_3(U)$
123	0	2	0
132	0	2	0
213	2	0	0
231	0	0	2
312	0	2	0
321	0	2	0

Computation complexity: Since the number of coalitions increases exponentially with increase in the number of providers, naive strategies for evaluating $e^{(1)}(\mathbf{x})$ for a given imputation $\mathbf{x}$ require exponential computation time. Thus, naive strategies for evaluating the nucleolus which minimizes the above among all imputations also require exponential computation time. Computation of the Shapley value through (2) also requires exponential time as the number of possible orderings of the providers increases exponentially with increase in the number of providers. On the other hand, the imputations obtained by solving the duals of the aggregate payoff maximization problems are polynomial time computable and also stabilize the grand coalition.

VIII. Impact of Cooperation on Customers

Cooperation enhances providers' aggregate payoffs which are increasing functions of the customers' service rates. Thus, intuitively, the rates of most of the customers increase when the providers cooperate. Cooperation may however decrease the rates of some of the customers, and therefore induce unfairness. In Example IV.1 when the providers do not cooperate, all customers may receive non-zero rates; but the customers of provider 1 receive no service when the providers cooperate.

The unfairness is however mitigated when the providers' benefit functions are strictly concave. For example, if the benefit function in Example IV.1 is logarithmic (instead of linear), i.e., $U_i(\mathbf{y}_i) = \sum_{j \in \mathcal{M}_i} \log(1+y_j)$, then it can be shown that each customer of provider 1 is served $[1-(1/P-1/Q)]/2$ fraction of time (assuming $1/P - 1/Q < 1$ which for example happens if $P > 1$) [18]. Note that when $P >> 1$ (since $Q > P$, then $Q >> 1$ as well), then each customer of provider 1 (and of provider 2 as well) is served approximately 50% of time irrespective of whether the providers cooperate. Thus, cooperation does not induce any unfairness in this case.[18] The benefit functions may be chosen during negotiations between providers and customers and may also be controlled by regulatory bodies (e.g., FCC in USA).

Our coalitional game framework also allows the customers to mitigate this unfairness (even in presence of linear benefit functions) by imposing minimum rate constraints through SLAs (Example IV.1 had no SLAs), e.g., all the customers in Example IV.1 may ask for a minimum rate $\frac{P}{2}$. Then, $v(\{1\}) = P, v(\{2\}) = Q, v(\{1,2\}) = P + Q$, and each customer receives the same rate irrespective of cooperation. But, then, the core has the unique imputation of (P, Q) which provides no payoff gain to any provider as compared to when they do not cooperate. The question then is whether provider 1 should accept the above SLA? More generally, should providers accept any SLA? The following example suggests that the providers ought to accept SLAs, but selectively.

Example VIII.1. *Again consider Example IV.1, with the difference that each customer of provider 1 requests an SLA equal to $\frac{P}{2}$. Moreover, customers in $\mathcal{M}_1$ do not require service rates above $\frac{3P}{4}$, and as a result will not pay for any extra service. Let the grand coalition payoff be shared*

[17] The aggregate payoff of the coalition is maximized by only serving $2'$s customers. But, if 1's customers leave, $v(\{1\}) = 0, v(\{2\}) = Q$, and $v(\{1,2\}) = 2Q$ and the nucleolus is $(\frac{Q}{2}, \frac{3Q}{2})$. Thus, although 1's customers do not receive any service from the coalition, and therefore do not generate any revenue, their mere presence enhances 1's payoff (from $Q/2$ to $\frac{Q+P}{2}$).

[18] Under logarithmic benefit functions, cooperation does not enhance the providers payoffs in this case either. This happens since each customer has the same rate from all the service units. However, when customers have rate-diversity, i.e., have potentially different rates from different service units, cooperation substantially enhances the payoffs of individual providers for logarithmic and several other strictly concave benefit functions (Section IX).

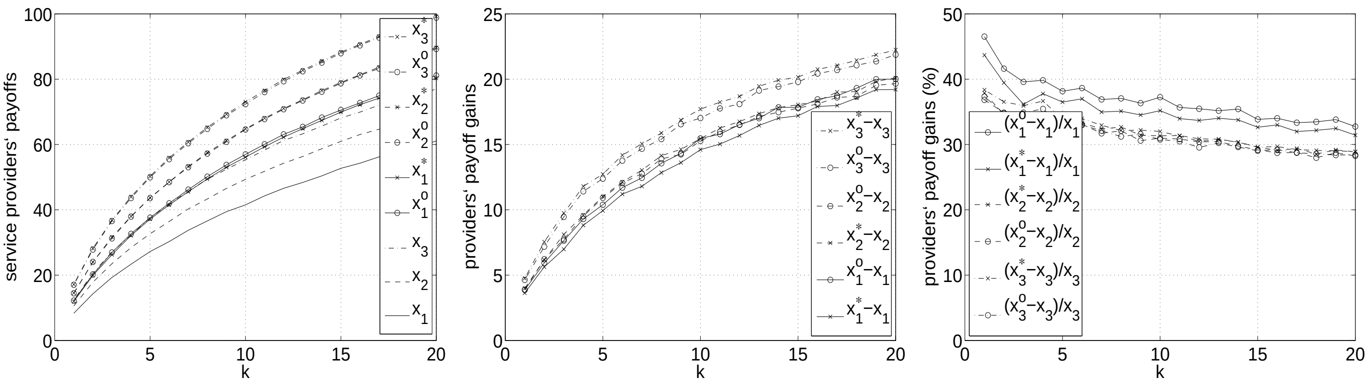

Fig. 4: The left, middle and right sub-plots respectively show providers' payoffs, payoff gains and percentage payoff-gains as functions of the number of customers: the three providers have $3k, 4k$ and $5k$ customers, respectively.

among the providers as per the nucleolus. If provider 1 *rejects both SLAs, customers in* $\mathcal{M}_1$ *leave and we have:* $v(\{1\}) = 0, v(\{2\}) = Q$*, and* $v(\{1,2\}) = 2Q$*. Consequently, providers' payoffs will be* $(x_1, x_2) = (\frac{Q}{2}, \frac{3Q}{2})$*. Instead, if provider* 1 *accepts one of the SLAs and rejects the other, we have:* $v(\{1\}) = \frac{3P}{4}, v(\{2\}) = Q$*, and* $v(\{1,2\}) = \frac{P}{2} + \frac{3Q}{2}$*, which lead to payoffs* $(x_1, x_2) = (\frac{5P+2Q}{8}, \frac{10Q-P}{8})$*. Finally, if provider* 1 *accepts both SLAs, we have:* $v(\{1\}) = P, v(\{2\}) = Q$*, and* $v(\{1,2\}) = P + Q$*, and therefore,* $(x_1, x_2) = (P, Q)$*. If* $Q > 5P/2$ *(*$Q < 3P/2$*, resp.), then it is optimal for provider* 1 *to reject (accept, resp.) both SLAs. If* $\frac{3P}{2} < Q < 5P/2$*, then it is optimal for provider* 1 *to accept only one of the SLAs.*

We now introduce a framework that allows the providers in a coalition to jointly decide which SLAs to accept. Clearly, the optimal cooperation strategy of a coalition $\mathcal{S}$ then involves selecting a set of SLAs that maximize the aggregate payoff - let $\hat{v}(\mathcal{S})$ be this maximum aggregate payoff. Let s_j be a decision variable indicating whether customer j's SLA is accepted: $s_j = 1$ if so and $s_j = 0$ otherwise. Then, $\hat{v}(\mathcal{S})$ is given by the maximum value of the objective function of P($\mathcal{S}$) in Section IV, with constraints (3), (5) being $\sum_{k \in \mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) \leq s_j, \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega$, $\sum_{\omega \in \Omega} \mathbb{P}(\omega) y_j(\omega) \geq s_j m_j, \quad j \in \mathcal{M}_\mathcal{S}$ respectively. Note that for any customer j, the minimum rate constraint (modified constraint (5)) is nontrivial, only if $s_j = 1$. Also, for a customer j with $s_j = 0$, $\alpha_{jk}(\omega) = 0$, for each $k \in \mathcal{B}_\mathcal{S}$ and at each ω because of constraint (6) and modified constraint (3). These two conditions ensure that in any optimal solution of the above optimization problem, only customers with accepted SLAs are served. Thus, the solution of this integer optimization provides the optimum set of acceptable SLAs. Establishing the non-emptiness of the core of this coalitional game remains open.

IX. Quantitative Evaluations

In the context of the resource pooling game (Section IV), we evaluate the benefits of cooperation and compare different payoff sharing schemes such as the dual-based payoff shares (Section IV) and the nucleolus (Section VII) for a range of benefit functions.

We first consider a logarithmic revenue (benefit) function $U_i(\mathbf{y}_i) = \sum_{j \in \mathcal{M}_i} \log(1 + y_j)$ and zero cost function $V_i(\mathbf{z}_i) = 0$ for each provider $i \in \mathcal{N}$. Thus, $U_i(\mathbf{y}_i)$ is a strictly concave function and assumes positive values except when $\mathbf{y}_i$ is the zero vector and in this case the revenue is 0. Note that logarithmic functions have been widely used as satisfaction functions of customers and therefore constitute good candidates for the revenues they pay (and hence for the benefits the providers incur). The cost functions are zero when the providers acquire the resources (spectrum, base stations) apriori by paying fixed (licensing or deployment) fees and do not incur subsequent usage based costs.[19] Also, we assume that the customers do not have SLAs as is typically the case for elastic transfers from the Internet (e.g., file transfers). We allow the service unit-customer rates r_{jk} to be uniformly distributed over the set $\{0, 100, 200\}$Kbps, and to be independent across service unit-customer pairs (j, k). The characteristic functions $v(\mathcal{S})$ for different coalitions $\mathcal{S}$ and the dual based imputation in the core can now be obtained by solving the concave optimization P($\mathcal{S}$), D($\mathcal{S}$) (Section IV). The nucleolus can subsequently be computed using Definition VII.1. We denote the payoff of a provider i (i) in absence of cooperation as x_i (i.e., $x_i = v(\{i\})$), (ii) in the grand coalition as x_i^o (nucleolus) or x_i^* (via solving dual optimization). Owing to large state spaces we use *Monte Carlo simulations* in our evaluations.

We first consider 3 providers, $B_1 = B_2 = B_3 = 1$, and $M_1 = 3k, M_2 = 4k, M_3 = 5k$ where k ranges from 1 to 20 (Figure 4). The plots show that cooperation leads to substantial payoff improvements for all providers, and the payoff-gains increase as the number of customers increase. As expected (from Definition VII.1), *the nucleolus distributes the payoff gains more equitably than the dual based profit-share* which allocates payoff gains in increasing order of the number of customers (wealth generated), reserving the highest payoff gain for the provider with the highest number of customers.

[19]Recall that the fixed service unit deployment and acquisition fees need to be considered explicitly only when the deployment and acquisition of service units constitute optimization decision variables as in the resource deployment game in Section V, and not when these are decided apriori as in the resource pooling game of Section IV.

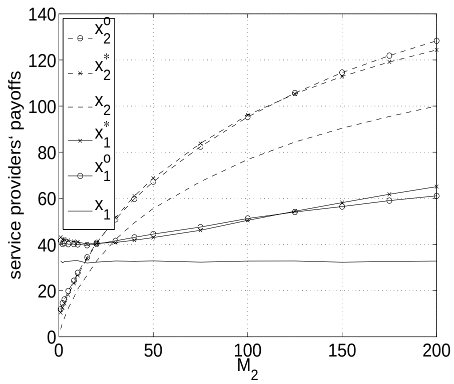

Fig. 5: Providers' payoffs as functions of number of customers: the first provider has 20 customers while the number of customers of the second, M_2, is varied.

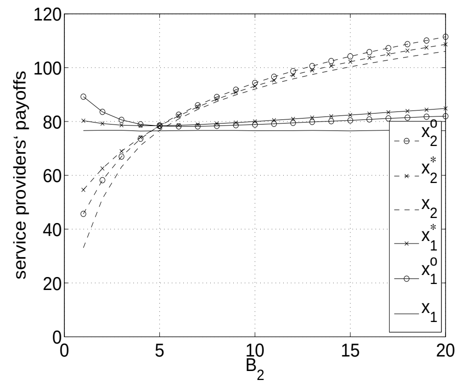

Fig. 6: Providers' payoffs as functions of number of base stations: the first provider owns 5 service units while the number of service units, B_2, of the second is varied.

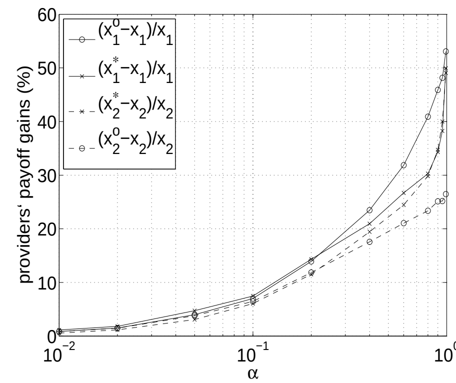

Fig. 7: The percentage payoff gains of the providers are plotted as functions of α.

Nevertheless, the payoffs of each provider are similar under both payoff sharings, and also to those under the Shapley value (see [18]). *The percentage gains in payoffs due to cooperation are quite significant (*$30\% - 40\%$*) for each provider*.

Henceforth, for simplicity, we focus on 2 providers. Note that the Shapley value is the same as the nucleolus in this case (Section VII - paragraph before Example VII.1). We investigate the impact of varying the (i) demand (number of customers) and (ii) asset (number of service units) of only one provider while keeping the other's demand and asset fixed. First, let $N = 2$, $B_1 = B_2 = 1$, $M_1 = 20$ and vary the number of customers M_2 of provider 2 (Figure 5). Next, we let $N = 2$, $M_1 = M_2 = 20$, $B_1 = 5$, and vary the number of service units B_2 of provider 2 (Figure 6). As the demand (or asset) of the second provider is increased, the payoff of the second provider increases under both the nucleolus and dual-based payoff sharing rules, but that of the first may either increase (Figure 5) or decrease (Figure 6), depending on how its importance in the cooperation changes due to the increase in the demand (or asset) of the second. Mathematically, $x_1^o = v(\{1,2\})+v(\{1\})-v(\{2\})/2$, and as the demand (asset) of the second increases, $v(\{1,2\}), v(\{2\})$ increase but $v(\{1\})$ does not change. Thus, the difference $v(\{1,2\})-v(\{2\})$ may either increase, or decrease. Nevertheless, the payoff of the first always exceeds that it attains without cooperation. Also, in both cases the provider with the larger demand or asset obtains higher payoffs under both sharing rules.

We now investigate how the choice of the revenue function affects providers' payoff gains. In particular, we consider the generalized $\alpha-$fair revenue function [32]: $U_i(\mathbf{y}_i) = \sum_{j\in\mathcal{M}_i} \frac{(y_j)^{1-\alpha}}{1-\alpha}$, where $0 < \alpha < 1$. Note that for each j $\frac{\partial^2}{\partial^2 y_j} U_i(\mathbf{y}_i) = -\alpha(y_j)^{-1-\alpha}$ and thus intuitively the "concavity" of the revenue function increases with increase in α (the function is linear if $\alpha = 0$). We plot the providers' percentage payoff gains as a function of α, for $N = 2$, $B_1 = B_2 = 1$ and $M_1 = 10$ and $M_2 = 20$ (Figure 7). Nucleolus and the dual based sharing rules provide similar payoff gains. More importantly, *the percentage payoff gains for both providers increase significantly with increase in α - thus, higher the concavity, the more beneficial cooperation is.* This can be explained as follows. For small α (i.e., nearly linear benefit functions), at any ω, the aggregate revenue is maximized by allocating each service unit to one customer. Next, given that the number of customers (10 or 20) significantly exceeds the number of service units (1) of each provider, usually (i.e., for most ω) each provider's service unit has excellent transmission conditions to at least one of its own customers. Thus, cooperation can not enhance the aggregate customers' rates, nor the providers' aggregate and hence individual payoffs. As α increases, the aggregate payoff increases when more equitable rates are provided to the customers at each ω. When not in coalition, in order to roughly equalize the rates of the customers, each provider's service unit must therefore serve customers with poor transmission quality r_{jk} for considerable fractions of time. When the providers cooperate, usually, most of the customers have high transmission rates from at least one service unit - thus equitable rates can also be provided by allowing each service unit to time-share among the customers (not necessarily of the same provider) that have good transmission quality from it. Thus, equity is attained through good match between customers and service units and without compromising the overall customer rates and providers' revenues. Thus, cooperation substantially enhances aggregate, and therefore individual, payoffs.

Finally, we illustrate the benefits of cooperation and compare the dual and nucleolus based payoff shares in presence of SLAs. We consider 3 providers each with 3 service units and 10 customers. Now, $r_{jk} = 100$ Kbps (200Kbps, resp.) with probability 0.8 (0.2, resp.). Each provider has 3 *premium* and 7 *best effort* customers: the former have negotiated SLAs which guarantee a minimum average rate m. We consider linear revenue functions:

$$U_i(\mathbf{y}_i) = \sum_{j=1}^{3} (\beta m + \alpha(y_j - m)) + \sum_{j=4}^{10} \alpha y_j$$

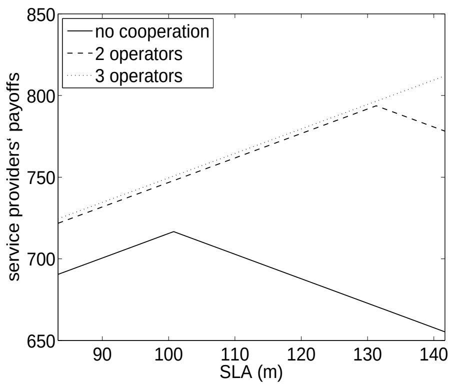

Fig. 8: Providers' payoffs as a function of the guaranteed rate to the premium customers

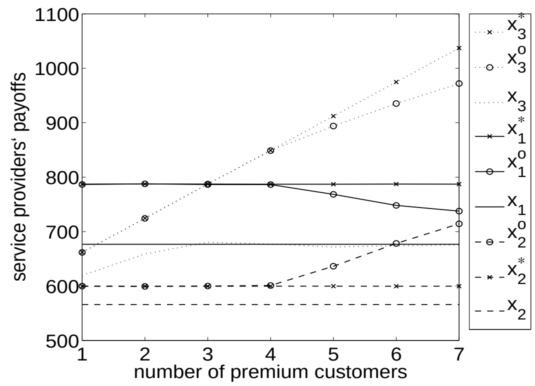

Fig. 9: Providers' payoffs as a function of the number of the premium customers of the third provider. x_1^*, x_2^*, x_3^* and x_1^o, x_2^o, x_3^o are payoffs corresponding to the dual allocation and the nucleolus respectively. x_1, x_2, x_3 are the providers' payoffs if they do not cooperate.

where $\beta > \alpha$ captures the higher payoff per Kbps for the service guarantees to the premium customers. We choose $\alpha = 1$ and $\beta = 1.5$. The revenue $\alpha \sum_{j=1}^{10} y_j$ is denoted as "usage based revenue" and the rest $(\beta - \alpha)3m$ is the fixed fee associated with SLAs. Due to symmetry, providers receive equal payoffs under both dual and nucleolus based shares. As Figure 8 reveals, cooperation enhances each provider's revenue: the increase is significant when the size of each coalition increases from 1 to 2, and somewhat less when the size increases to 3. For small m, a provider does not need to compromise on the efficient usage of resources (i.e., it preferentially serves the customers with high transmission rates). Each provider's payoff increases linearly with m in this region due to the increase of the fixed fees associated with m. However, beyond a certain threshold, each provider needs to schedule a few lower rate links to the premium customers (instead of the higher rate links to the best effort customers) to satisfy the SLAs. This lowers the aggregate service rates, and each provider's payoff decreases linearly with increase in m. Cooperation increases this threshold and also the aggregate rate of all the customers by allowing the scheduling of higher rate links more often.

Next, we consider an asymmetric scenario where each provider has 10 customers as before, but they respectively have $3, 0, k$ premium customers; k is varied from 1 to 7. All the premium customers demand a minimum guaranteed rate of 125Kbps. It turns out that a provider alone cannot guarantee 125Kbps to more than 3 customers. Similarly, any two providers can support at most 8 premium customers together. Thus, P$(\{3\})$ is not feasible for $k > 3$, and assumption IV.1 no longer holds. For $k > 3$, we define $v(\{3\})$ as the objective function of P$(\{3\})$ with 3 premium customers, for $k > 5$, $v(\{1, 3\})$ is the objective function of P$(\{1, 3\})$ with 5 premium customers, and for $k > 8$, $v(\{2, 3\})$ is the objective function of P$(\{2, 3\})$ with 8 premium customers. It turns out that the dual and nucleolus payoff shares are in the core, and hence the core is non-empty. Figure 9 plots the providers' payoffs as functions of the number of premium customers of the third provider under both allocations. The dual based allocation equally divides the total usage based payoffs among all providers, and allocates the fixed fees of each provider's customers to the provider. Thus, the payoffs of providers $1, 2$ do not change with increase in k, but that of provider 3 increases linearly with increase in k. The nucleolus however transfers a part of the fixed fees provider 3 earns to other providers - intuitively such transfer is warranted as provider 3 can not support all its premium customers by itself for $k > 3$. Thus, payoff shares of all providers change with increase in k, and evidently, the nucleolus based payoff gains are more equitable than the dual based ones. In all the allocations, a provider with larger number of premium customers gets a larger payoff share, and each provider's payoff increases substantially due to cooperation.

X. Conclusion and Future Work

We studied cooperation among providers in wireless networks. If providers cooperate, they can jointly decide how to deploy their service units, pool their service units and allocate them to the joint pool of customers in an optimal fashion. We formulated the problem as a transferable utility coalitional game. We showed nonemptyness of cores in various scenarios (see Theorems IV.1, V.5 etc.) implying that cooperation is not only globally optimal, but also makes each of the providers better off. Our proof technique is constructive and yields an optimal resource allocation and corresponding profit shares. Our numerical evaluations reveal that cooperation substantially enhances individual provider's payoffs.

We now outline some open problems. The computation time for an allocation in the core may be high since it depends polynomially on the number of possible channel state and mobile location realizations ($|\Omega|$), which is large. Obtaining near-optimal solutions with low computation time remains open. Next, in practice, coalition formation can incur overheads, e.g., from increased computing requirements. Investigating the stability of the grand coalition considering the coalition formation overhead constitutes an open problem. Finally, we considered a system where the customer subscriptions and the providers' revenue function have already been determined. Investigating cooperation among the providers when the customers dynamically decide their subscription based on the revenue functions, and how providers can dynamically and optimally select the revenue functions so as to enhance their individual share of the overall profit remain open.

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