Optimal Forwarding in Delay Tolerant Networks with Multiple Destinations

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***Abstract*—We study the trade-off between delivery delay and energy consumption in a delay tolerant network in which a message (or a file) has to be delivered to each of several destinations by epidemic relaying. In addition to the destinations, there are several other nodes in the network that can assist in relaying the message. We first assume that, at every instant, all the nodes know the number of relays carrying the packet and the number of destinations that have received the packet. We formulate the problem as a controlled continuous time Markov chain and derive the optimal closed loop control (i.e., forwarding policy). However, in practice, the intermittent connectivity in the network implies that the nodes may not have the required perfect knowledge of the system state. To address this issue, we obtain an ODE (i.e., fluid) approximation for the optimally controlled Markov chain. This fluid approximation also yields an asymptotically optimal open loop policy. Finally, we evaluate the performance of the deterministic policy over finite networks. Numerical results show that this policy performs close to the optimal closed loop policy.**

I. Introduction

Delay tolerant networks (DTNs) [1] are sparse wireless ad hoc networks with highly mobile nodes. In these networks, the link between any two nodes is up when these are within each other's transmission range, and is down otherwise. In particular, at any given time, it is unlikely that there is a complete route between a source and its destination.

We consider a DTN in which a short message (also referred to as a *packet*) needs to be delivered to multiple (say, M) destinations. There are also N potential relays that do not themselves "want" the message but can assist in relaying it to the nodes that do. At time $t = 0$, N_0 of the relays have copies of the packet. All nodes are assumed to be mobile. In such a network, a common technique to improve packet delivery delay is *epidemic* relaying [2]. We consider a controlled relaying scheme that works as follows. Whenever a node (relay or destination) carrying the packet meets a relay that does not have a copy of the packet, then the former has the option of either copying or not copying. When a node that has the packet meets a destination that does not, the packet can be delivered.

The authors' work on this paper was supported by the Indo-French Center for the Promotion of Advanced Research (IFCPAR), Project 4000-IT-1, and by DAWN (an Associates program of INRIA, France).

We want to minimize the duration to copy the packet to a significant (say α) fraction of the destinations receive the packet; we refer to this duration as *delivery delay*. On the one hand, copying the packet to a relay incurs a transmission cost. On the other hand, this copying increases the number of carriers of the packet and thereby potentially reduces the delivery delay. We focus on the problem of the control of forwarding.

Related work: Analysis and control of DTNs with single-source and single-destination has been widely studied. Groenevelt et al. [3] modeled epidemic relaying and two-hop relaying using Markov chains, and derived the average delay and number of copies generated until the time of delivery. Zhang et al. [4] developed a unified framework based on ordinary differential equations to study epidemic routing and its variants.

Neglia and Zhang [5] were the first to study the optimal control of relaying in DTNs with a single destination and multiple relays. They assumed that all the nodes have perfect knowledge of the number of nodes carrying the packet. Their optimal closed loop control is a threshold policy - when a relay that does not have a copy of the packet is met, the packet is copied if and only if the number of relays carrying the packet is below a threshold. Due to the assumption of complete knowledge, the performance reported is a lower bound for the cost in a real system.

Altman et al. [6] addressed the optimal relaying problem for a class of *monotone relay strategies* which includes epidemic relaying and two-hop relaying. In particular, they derived *static* and *dynamic* relaying policies. Altman et al. [7] considered optimal discrete-time two-hop relaying. They also employed stochastic approximation to facilitate online estimation of network parameters. In another paper, Altman et al. [8] considered a scenario where active nodes in the network continuously spend energy while *beaconing*. Their paper studied the joint problem of node activation and transmission power control. These works ([6], [7], [8]) heuristically obtain fluid approximations for DTNs and study open loop controls. Li et al. [9] considered several families of open loop controls and obtain optimal controls within each family.

Deterministic fluid models expressed as ordinary differential equations have been used to approximate large Markovian systems. Kurtz [10] obtained sufficient conditions for the convergence of Markov chains to such fluid limits. Darling [11] and subsequently, Darling and Norris [12] generalized Kurtz's results. Darling [11] considers the scenario when the Markovian system satisfies the conditions in [10] only over a given set. He shows that the scaled processes, until they exit from this set, converge to a fluid limit. Darling and Norris [12] generalize the conditions for convergence, e.g., uniform convergence of the mean drifts of Markov chains and Lipschitz continuity of the limiting drift function, prescribed in [10]. Gast and Gaujal [13] use differential inclusions to address the scenario where the limiting drift functions are not continuous, and hence the differential equations are not well defined. Gast et al. [14] study an optimization problem on a large Markovian system. They show that solving the limiting deterministic problem yields an asymptotically optimal policy for the original problem.

Our Contributions: We formulate the problem as a controlled continuous time Markov chain (CTMC) [15], and obtain the optimal policy (Section III). The optimal policy relies on complete information of the network state, but availability of such information is constrained by the same connectivity problem that limits packet delivery. In the incomplete information setting, the decisions of the nodes would have to depend upon their beliefs about the network state. The nodes would need to update their beliefs continuously with time, and also after each meeting with another node. Such belief updates would involve maintaining a complex information structure and are often impractical for nodes with limited memory and computation capability. Moreover, designing closed loop controls based on beliefs is a difficult task [16], more so in our context with multiple decision makers and all of them equipped with distinct partial information.

In view of the above difficulties, we adopt the following approach. We show that when the number of nodes is large, the optimally controlled network evolution is well approximated by a deterministic dynamical system (Section IV). Towards this, we extend the existing differential equation approximation results to Markovian systems [10], [11] for which the mean drift rates are discontinuous and do not converge uniformly. The limiting deterministic dynamics then suggests a deterministic control (for the finite network) that is asymptotically optimal. Our notion of asymptotically optimality is identical to the one proposed in [14]. Our numerical results illustrate that the deterministic policy performs close to the complete information optimal closed loop policy for a wide range of parameter values (Section V). We omit some of the proofs for lack of space. All the proofs can be found in our technical report [17].

II. The System Model

We consider a set of $K := M + N$ mobile nodes. These include M destinations and N relays. At $t = 0$, a packet is generated and immediately copied to N_0 relays (e.g., via a broadcast from a cellular network). Alternatively, these N_0 nodes can be thought of as source nodes.

1) Mobility Model: We model the point process of the *meeting instants* between pairs of nodes as independent Poisson point processes, each with rate λ. Groenevelt et al. [3] validate this model for a number of common mobility models (random walker, random direction, random waypoint). In particular, they establish its accuracy under the assumptions of small communication range and sufficiently high speed of nodes.

2) Communication Model: Two nodes may communicate only when they come within transmission range of each other, i.e., at the so called *meeting instants*. The transmissions are assumed to be instantaneous. We assume that that each transmission of the packet incurs unit energy expenditure at the transmitter.

3) Relaying Model: We assume that a controlled epidemic relay protocol is employed.

Throughout, we use the terminology relating to the spread of infectious diseases. A node with a copy of the packet is said to be *infected*. A node is said to be *susceptible* until it receives a copy of the packet from another infected node. Thus at $t = 0$, N_0 nodes are infected while $M + N - N_0$ are susceptible.

A. The Forwarding Problem

The packet has to be disseminated to all the M destinations. However, the goal is to minimize the duration until a fraction α ($\alpha < 1$) of the destinations receive the packet.

At each meeting epoch with a susceptible relay, an infected node (relay or destination) has to decide whether to copy the packet to the susceptible relay or not. Copying the packet incurs unit cost, but promotes the early delivery of the packet to the destinations. We wish to find the trade-off between these costs by minimizing

$$\mathbb{E}\{\mathcal{T}_d + \gamma \mathcal{E}_c\} \tag{1}$$

where $\mathcal{T}_d$ is the time until which at least $M_\alpha := \lceil \alpha M \rceil$ destinations receive the packet, $\mathcal{E}_c$ is the total energy consumption due to transmissions of the packet and γ is the parameter that relates energy consumption cost to delay cost. Varying γ helps studying the trade-off between the delay and the energy costs.

III. Optimal Forwarding

We derive the optimal forwarding policy under the assumption that, at any instant of time, all the nodes have full information about the number of relays carrying the packet and the number of destinations that have received the packet.

A. The MDP Formulation

Let $t_k, k = 1, 2, \ldots$ denote the meeting epochs of the infected nodes (relays or destinations) with the susceptible nodes. Let $t_0 := 0$ and define $\delta_k := t_k - t_{k-1}$ for $k \geq 1$.

Let $m(t)$ and $n(t)$ be the numbers of infected destinations and relays, respectively, at time t. In particular, $m(0) = 0$ and $n(0) = N_0$, and the forwarding process stops at time t if $m(t) = M$. We use m_k and n_k to mean $M(t_k-)$ and $N(t_k-)$ respectively. Let e_k describe the type of the susceptible node that an infected node meets at t_k; $e_k \in \mathcal{E} := \{d, r\}$ where d

and r stand for destination and relay, respectively. The state of the system at a meeting epoch t_k is given by the tuple

$$s_k := (m_k, n_k, e_k).$$

Since the forwarding process stops at time t if $m(t) = M$, the state space is $[M-1] \times [N_0 : N] \times \mathcal{E}$.[1]

Let u_k be the action of the infected node at meeting epoch $t_k, k = 1, 2, \ldots$. The control space is $\mathcal{U} \in \{0, 1\}$, where 1 is for *copy* and 0 is for *do not copy*. The embedding convention described above is shown in Figure 1.

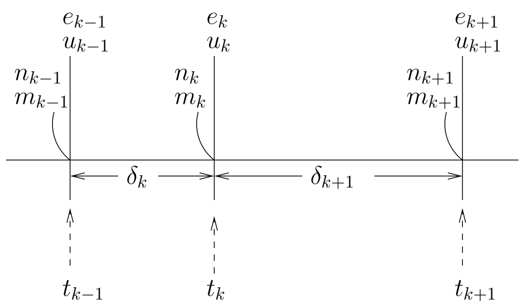

Fig. 1. Evolution of the controlled Markov chain $\{s_k\}$. Note that (m_k, n_k) is embedded at t_k-.

We treat the tuple (δ_{k+1}, e_{k+1}) as the random disturbance at epoch t_k. Note that for $k = 1, 2, \ldots$, the time between successive decision epochs, δ_k, is independent and exponentially distributed with parameter $(m_k + n_k)(M + N - m_k - n_k)\lambda$. Furthermore, with "w.p." standing for "with probability", we have

$$e_k = \begin{cases} d & \text{w.p. } p_{m_k,n_k}(d) := \frac{M-m_k}{M+N-m_k-n_k}, \\ r & \text{w.p. } p_{m_k,n_k}(r) := \frac{N-n_k}{M+N-m_k-n_k}. \end{cases}$$

1) Transition Structure: From the description of the system model, the state at time t_{k+1} is given by $s_{k+1} = (m_k + u_k, n_k, e_{k+1})$ if $e_k = d$, and $s_{k+1} = (m_k, n_k + u_k, e_{k+1})$ if $e_k = r$. Recall that e_{k+1} is a component in the random disturbance. Thus the next state is a function of the current state, the current action and the current disturbance as required for an MDP.

2) Cost Structure: For a state action pair (s_k, u_k) the expected single stage cost is given by

$$g(s_k, u_k) = \gamma u_k + \mathbb{E}\left\{\delta_{k+1} 1_{\{m_{k+1} < M_\alpha\}}\right\}.$$

Furthermore, it can be observed that

$$g(s_k, u_k) = \begin{cases} \gamma u_k \text{ if } s_k \text{ is such that } m_k \geq M_\alpha \\ \gamma \text{ if } s_k = (M_\alpha - 1, n, d) \text{ and } u_k = 1 \\ \gamma u_k + C_d(s_k, u_k) \text{ otherwise,} \end{cases}$$

where

$$C_d(s_k, u_k) = \frac{1}{(m_k + n_k + u_k)(M + N - m_k - n_k - u_k)\lambda}$$

[1]We use notation $[a] = \{0, 1, \ldots, a\}$ and $[a : b] = \{a, a+1, \ldots, b\}$ for $b \geq a + 1$ and $a, b \in \mathbb{Z}_+$.

is the mean time until the next decision epoch. The quantity γ is expended whenever $u_k = 1$, i.e., the action is to copy.

3) Policies: A policy π is a sequence of mappings $\{u_k^\pi, k = 1, 2, \ldots\}$, where $u_k^\pi : [M-1] \times [N_0 : N] \times \mathcal{E} \to \mathcal{U}$. The cost of an admissible policy π starting at a meeting epoch and state $s = (m, n, e)$ is

$$J_\pi(s) = \sum_{k=0}^{\infty} \mathbb{E}\Big\{g(s_k, u_k^\pi(s_k)) \big| s_0 = s\Big\}.$$

Let Π be the set of all admissible policies. Then the optimal cost function is defined as

$$J(s) = \min_{\pi \in \Pi} J_\pi(s).$$

A policy π is called stationary if u_k^π are identical, say u, for all k. For brevity we refer to such a policy as the stationary policy u. A stationary policy $u^* \equiv \{u^*, u^*, \ldots\}$ is optimal if $J_{u^*}(s) = J(s)$ for all states s.

4) Total Cost: We can translate the optimal cost to go from the first meeting instant into optimal total cost. Recall that at the first decision instant t_1, the state s_1 is $(0, N_0, r)$ or $(0, N_0, d)$ depending on whether the susceptible node met is a relay or a destination. The objective function (1) can then be restated as

$$\mathbb{E}_\pi\{\mathcal{T}_d + \gamma \mathcal{E}_c\} = \frac{1}{\lambda N_0 (M + N - N_0)} + \bigg(\frac{N - N_0}{M + N - N_0} J_\pi(0, N_0, r) + \frac{M}{M + N - N_0} J_\pi(0, N_0, d)\bigg), \quad (2)$$

where the subscript π shows dependence on the underlying policy. In the right hand side, $\frac{1}{\lambda N_0 (M+N-N_0)}$ is the average delay until the first decision instant which has to be borne under any policy.

B. Optimal Policy

Since the single stage cost $g(\cdot)$ takes nonnegative values for all possible values of its arguments, Proposition 1.1 in [15, Chapter 3] implies that the optimal cost function will satisfy the following Bellman equation. For $s = (m, n, e)$,

$$J(s) = \min_{u \in \{0,1\}} A(s, u)$$
$$\text{where } A(s, u) = g(s, u) + \mathbb{E}\left(J(s')|s, u\right).$$

Here s' denotes the next state which depends on s, u and the random disturbance in accordance with the transition structure described above. The expectation is taken with respect to the random disturbance. Furthermore, since the action space is finite, there exists a stationary optimal policy u^* such that, for all s, $u^*(s)$ attains minimum in the above Bellman equation (see [15, Chapter 3]). In the following we characterize this stationary optimal policy.

First, observe that it is always optimal to copy to a destination, that is, the optimal policy satisfies $u^*(m, n, d) = 1$ for all $(m, n) \in [M-1] \times [N_0 : N]$. Moreover, once the α fraction of destinations have gotten the packet, no further delay cost is incurred, and so relays are of no further help: $u^*(m,n,r) = 0$ for all $(m,n) \in [M_\alpha : M-1] \times [N_0 : N]$.

Next, focus on a reduced state space $[M_\alpha - 1] \times [N_0 : N] \times \{r\}$. Consider the following *one step look ahead policy* [15, Section 3.4]. At each meeting with a susceptible relay, compare the following two action sequences.

1) $0s$: do not copy and stop where "stop" means that no copying is done to susceptible relays in future as well,
2) $1s$: copy to this relay and then stop.

The infected node chooses action 0 or 1 depending on whether $0s$ or $1s$ has lesser cost. Formally, consider a network state (m,n,r). The costs to go corresponding to the action sequences $0s$ and $1s$ are, respectively,

$$J_{0s}(m,n,r) = (M-m)\gamma + \sum_{j=m}^{M_\alpha - 1} \frac{1}{\lambda(n+j)(M-j)} \text{ and}$$

$$J_{1s}(m,n,r) = (M-m+1)\gamma + \sum_{j=m}^{M_\alpha - 1} \frac{1}{\lambda(n+j+1)(M-j)}.$$

The differences of costs to go are given by

$$\begin{aligned}\Phi(m,n) &:= J_{0s}(m,n,r) - J_{1s}(m,n,r) \\ &= \sum_{j=m}^{M_\alpha - 1} \frac{1}{\lambda(n+j)(n+j+1)(M-j)} - \gamma \end{aligned} \quad (3)$$

for all $(m,n) \in [M_\alpha - 1] \times [N_0 : N]$. The one step look ahead policy $u^o : [M_\alpha - 1] \times [N_0 : N] \times \{r\} \to \mathcal{U}$ is

$$u^o(m,n,r) = \begin{cases} 1 \text{ if } \Phi(m,n) > 0, \\ 0 \text{ if } \Phi(m,n) \leq 0. \end{cases}$$

One step look ahead policies have been shown to be optimal for stopping problems under certain conditions (see [18, Section 4.4] and [15, Section 3.4]). However, our problem is not a stopping problem. More precisely, action 0 in our problem is not equivalent to *stop* as the resulting state is not a *terminal state*; a susceptible relay that is met in future may be copied even if the one met at present is not. Nonetheless, we prove that the above one step look ahead policy is the optimal policy for our forwarding problem.

Furthermore, we can extend the definition of $\Phi(m,n)$ to all $(m,n) \in [M-1] \times [N_0 : N]$. We use the standard convention that a sum over an empty index set is 0. Thus, for $m \geq M_\alpha$, $\Phi(m,n) = -\gamma$, and so $u^o(m,n,r) = 0$ which is consistent with the optimal policy. Hence, we get the following theorem.

Theorem 3.1: The optimal policy $u^* : [M-1] \times [N_0 : N] \times \mathcal{E} \to \mathcal{U}$ satisfies

$$u^*(m,n,e) = \begin{cases} 1 \text{ if } e = d, \\ 1 \text{ if } e = r \text{ and } \Phi(m,n) > 0, \\ 0 \text{ if } e = r \text{ and } \Phi(m,n) \leq 0. \end{cases}$$

Proof: See Appendix A. ■

Remarks 3.1: Observe that $\Phi(m,n)$ is decreasing in m for a given n and also decreasing in n for a given m. Thus the optimal policy has the following properties.

1) If $u^*(m,n,r) = 0$, then $u^*(i,n,r) = 0$ for all $m < i < M$.
2) If $u^*(m,n,r) = 0$, then $u^*(m,j,r) = 0$ for all $n < j < N$.

Thus the optimal solution can be given a "stopping" interpretation. More precisely, if the packet is not copied at a meeting with a susceptible relay, it is not copied to relays in future meetings. A priori however, we did not know if such a "stopping" was optimal.

We illustrate the optimal policy using an example. Let $M = 15, N = 50, N_0 = 10, \alpha = 0.8, \lambda = 0.001$ and $\gamma = 1$. The shaded region in Figure 2 corresponds to the states in which the optimal action (at meeting with a relay) is to copy. Thus, for example, if 5 destinations have the packet, then relays are copied if and only if there are 24 or less infected relays. If 7 destinations already have the packet and there are 20 infected relays, then no further copying to relays is done.

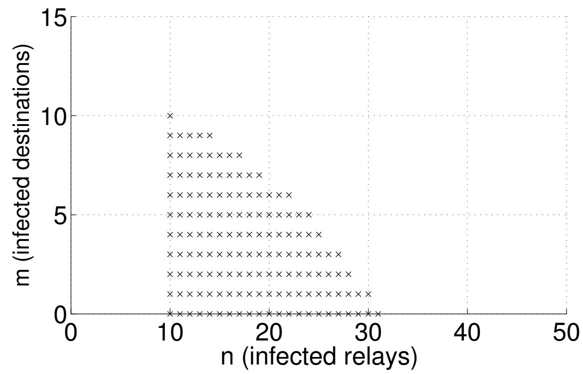

Fig. 2. An illustration of the optimal policy. The symbols 'X' mark the states in which the optimal action (at meeting with a relay) is to copy

IV. Asymptotically Optimal Forwarding

In states $[M_\alpha - 1] \times [N_0 : N] \times \{r\}$, the optimal action, which is governed by the function $\Phi(m,n)$, requires perfect knowledge of the network state (i.e., m and n). However this may not be available to the decision maker due to intermittent connectivity. In this section, we derive an asymptotically optimal policy that does not require knowledge of network's state but depends only on the time elapsed since the generation of the packet. Such a policy is implementable if the packet is time-stamped on generation and nodes' clocks are synchronized.

A. Asymptotic Deterministic Dynamics

Our analysis closely follows Darling [11]. It is straightforward to show that following are the conditional expected drift rates of the optimally controlled CTMC. For $(m(t), n(t)) \in [M-1] \times [N_0 : N]$,

$$\frac{\mathrm{d}\mathbb{E}(m(t)|(m(t),n(t)))}{\mathrm{d}t} = \lambda(m(t)+n(t))(M-m(t)),$$

$$\frac{\mathrm{d}\mathbb{E}(n(t)|(m(t),n(t)))}{\mathrm{d}t} = \lambda(m(t)+n(t))(N-n(t)) 1_{\{\Phi(m(t),n(t))>0\}}.$$

Recalling that $K = M + N$, we study large K asymptotics. Towards this, we normalize the system variables and parameters as follows.

$$\left.\begin{aligned} X = \frac{M(K)}{K},\ Y = \frac{N(K)}{K}, \\ X_\alpha = \frac{\alpha M(K)}{K},\ Y_0 = \frac{N_0(K)}{K}, \\ \lambda(K) = \frac{\Lambda}{K}, \gamma(K) = \frac{\Gamma}{K}, \\ x^K(t) = \frac{m(t)}{K} \text{ and } y^K(t) = \frac{n(t)}{K}. \end{aligned}\right\} \tag{4}$$

Remarks 4.1: The pairwise meeting rate and the copying cost both must scale down as K increases. Otherwise, the delivery delay will be negligible and the total transmission cost will be enormous for any policy, and no meaningful analysis is possible.

Now, for $(x^K(t), y^K(t)) \in [0, X - 1/K] \times [Y_0, Y]$, the drift rates can be rewritten as follows.[2]

$$\begin{aligned} \frac{\mathrm{d}\mathbb{E}(x^K(t)|(x^K(t), y^K(t)))}{\mathrm{d}t} &= f_1^K(x^K(t), y^K(t)) \\ &:= \Lambda(x^K(t) + y^K(t))(X - x^K(t)), \\ \frac{\mathrm{d}\mathbb{E}(y^K(t)|(x^K(t), y^K(t)))}{\mathrm{d}t} &= f_2^K(x^K(t), y^K(t)) \\ := \Lambda(x^K(t) + y^K(t))&(Y - y^K(t))1_{\{\phi^K(x^K(t), y^K(t))>0\}}, \end{aligned}$$

where

$$\phi^K(x, y) = \sum_{j=Kx}^{\lceil KX_\alpha \rceil - 1} \frac{1}{K\Lambda(y + \frac{j}{K})(y + \frac{j+1}{K})(X - \frac{j}{K})} - \Gamma.$$

We also define $x(t), y(t) \in [0, X] \times [Y_0, Y]$ as functions satisfying the following ODEs: $x(0) = 0, y(0) = Y_0$, and for $t \geq 0$,

$$\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= f_1(x(t), y(t)) := \Lambda(x(t) + y(t))(X - x(t)), \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= f_2(x(t), y(t)) := \Lambda(x(t) + y(t))(Y - y(t)) \\ &\qquad 1_{\{\phi(x(t), y(t))>0\}} \end{aligned}$$

where[3]

$$\phi(x, y) = \int_{z=x}^{X_\alpha} \frac{dz}{\Lambda(y + z)^2(X - z)} - \Gamma.$$

Finally, we define

$$\tau^K = \inf\{t \geq 0 : x^K(t) \geq X_\alpha\}, \tag{5}$$

$$\tau = \inf\{t \geq 0 : x(t) \geq X_\alpha\}. \tag{6}$$

Note that τ^K is a stopping time for the random process $(x^K(t), y^K(t))$. Since $f_1^K(x, y)$ is bounded away from zero, $\tau^K < \infty$ with probability 1. Similarly, $\tau < \infty$, and is also a deterministic time instant.

We now prove the following result which is similar to [11, Theorem 2.8].

Theorem 4.1: Assume that $\alpha < 1$ and $Y_0 > 0$. Then, for every $\epsilon, \delta > 0$,

$$\lim_{K\to\infty} \mathbb{P}\left(\sup_{0\leq t\leq\tau} \|(x^K(t), y^K(t) - (x(t), y(t))\| > \epsilon\right) = 0,$$
$$\lim_{K\to\infty} \mathbb{P}\left(|\tau^K - \tau| > \delta\right) = 0.$$

Proof: We give only an outline of the proof; for details see [17]. Observe that $f_2^K(x, y)$ does not converge uniformly to $f_2(x, y)$, and $f_2(x, y)$ is not Lipschitz over $[0, X_\alpha] \times [Y_0, 1]$. Hence, the results of Darling [11] do not directly apply in our context. However, we use the facts that

(a) $\phi^K(x, y)$ converges uniformly to $\phi(x, y)$,
(b) the drift rates, $f_1(x, y)$ and $f_2(x, y)$, are bounded from below and above,
(c) $f_1(x, y)$ is Lipschitz and $f_2(x, y)$ is locally Lipschitz, and
(d) for all small enough $\nu \in \mathbb{R}$, and all (x, y) on the graph of '$\phi(x, y) = \nu$', the direction in which the ODE progresses, $(f_1(x, y), f_2(x, y))$, is not tangent to the graph.

As a consequence of fact (a) we can obtain a "tube" around the curve '$\phi(x, y) = 0$' such that, for large enough K, the curve '$\phi^K(x, y) = 0$' is inside the tube. Outside the tube the dynamics of the ODE is Lipschitz. Hence, from Darling [11], the ODE is a good approximation of the controlled Markov chain until one of them hits the tube. Facts (b) and (d), along with the dynamics of the controlled Markov chain, imply that both exit the tube within a short time, and therefore their separation is controlled when they enter the other side of the tube. From then on the result in Darling [11] applies. ∎

We illustrate Theorem 4.1 using an example. Let $X = 0.2, Y = 0.8, \alpha = 0.8, Y_0 = 0.2, \Lambda = 0.05$ and $\Gamma = 50$. In Figure 3, we plot $(x(t), y(t))$ and sample trajectories of $(x^K(t), y^K(t))$ for $K = 100, 200$ and 500. We indicate the states at which the optimal policy stops copying to relays, i.e., $\phi^K(x^K(t), y^K(t))$ goes below 0 (see Theorem 3.1) and the states at which the fraction of infected destinations crosses X_α. We also show the corresponding states in the fluid model. The plots show that for large K, the fluid model captures very well the random dynamics of the network.

B. Asymptotically Optimal Policy

Observe that $\phi(x, y)$ is decreasing in x and y both of which increase with t. Consequently $\phi(x(t), y(t))$ decreases with t. We define

$$\tau^* := \inf\{t \geq 0 : \phi(x(t), y(t)) \leq 0\}. \tag{7}$$

The limiting deterministic dynamics suggests the following policy u^∞ for the original forwarding problem.[4]

[2] More precisely, $(x^K(t), y^K(t))$ lies on a scaled two-dimensional integer lattice of the from $(i/K, j/K)$ for some $i, j \in \mathbb{Z}_+$.

[3] We use the convention that an integral assumes 0 value if its lower limit exceeds the upper limit. So, $\phi(x, y) = -\Gamma$ if $x \geq X_\alpha$.

[4] Observe that the policy u^∞ does not require knowledge of m and n. The infected node readily knows the type of the susceptible node (d or r) at the decision epoch.

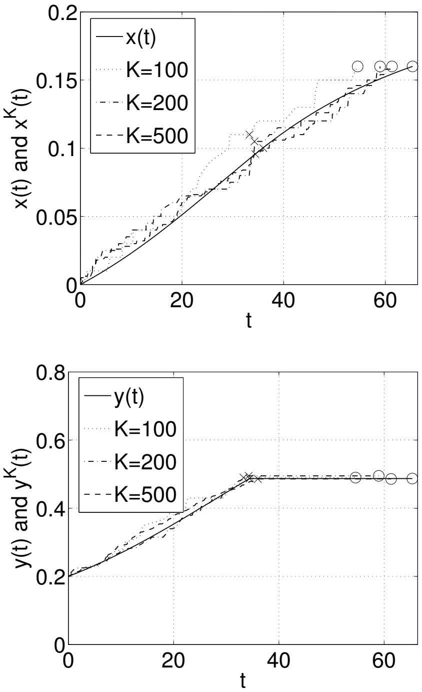

Fig. 3. Simulation results: The top and bottom sub-plots respectively show the fractions of infected destinations and relays as a function of time. $(x^K(t), y^K(t))$ are obtained from a simulation of the controlled CTMC, and $(x(t), y(t))$ from the ODEs. The marker 'X' indicates the states at which copying to relays is stopped whereas 'O' indicates the states at which α fraction of destinations have been copied.

$$u^{\infty}(m,n,e) = \begin{cases} 1 \text{ if } e = d, \\ 1 \text{ if } e = r \text{ and } t \le \tau^*, \\ 0 \text{ if } e = r \text{ and } t > \tau^*. \end{cases}$$

We show that the policy u^{∞} is asymptotically optimal in the sense that its expected cost approaches the expected cost of the optimal policy u^* as the network grows. Gast et al. [14] have also defined a similar notion of asymptotic optimality. Let us restate (2) as

$$\mathbb{E}_{\pi}^{K}\{\mathcal{T}_d + \gamma \mathcal{E}_c\} = \frac{1}{K\Lambda Y_0(1-Y_0)} + \left(\frac{Y - Y_0}{1 - Y_0} J_{\pi}(0, Y_0, r) + \frac{X}{1 - Y_0} J_{\pi}(0, Y_0, d)\right).$$

We have used superscript K to show the dependence of cost on the network size. Then, we establish the following result.

Theorem 4.2:

$$\lim_{K\to\infty} \left(\mathbb{E}_{u^{\infty}}^{K}\{\mathcal{T}_d + \gamma \mathcal{E}_c\} - \mathbb{E}_{u^*}^{K}\{\mathcal{T}_d + \gamma \mathcal{E}_c\}\right) = 0.$$

Proof: See [17]. ■

Distributed Implementation: We now describe how the asymptotically optimal policy is implemented in distributed fashion. Assume that all the nodes are time synchronized.[5] Suppose that the packet is generated at the source at time t_0 (we assumed $t_0 = 0$ for the purpose of analysis). Given the system parameters $M, N, \alpha, N_0, \lambda$ and γ, the source first extracts $X, Y, X_\alpha, Y_0, \Lambda$ and Γ as in (4). Then, it calculates τ^* (see (7)), and stores $t_0 + \tau^*$ as a header in the packet.

The packet is immediately copied to N_0 relays. When an infected node meets a susceptible relay, it compares $t_0 + \tau^*$ with the current time. The susceptible relay is not copied to if the current time exceeds $t_0 + \tau^*$. However, all the infected nodes continue to carry the packet, and to copy to susceptible destinations as and when they meet.

[5]In practice, due to variations in the clock frequency, the clocks at different nodes will drift from each other. But the time differences are negligible compared to the delays caused by intermittent connectivity in the network. Moreover, when an infected node meets a susceptible node, clock synchronization can be performed before the packet is copied.

V. Numerical Results

We now show some numerical results to demonstrate the performance of the deterministic control. Let $X = 0.2, Y = 0.8, \alpha = 0.8, Y_0 = 0.2$ and $\gamma = 0.5$. We vary λ from 0.00005 to 0.05 and use $K = 50, 100$ and 200. In Figure 4, we plot the total number of copies to relays and the delivery delays corresponding to both the optimal and the asymptotically optimal deterministic policies. Evidently, the deterministic policy performs close to the optimal policy on both the fronts. We observe that, for a fixed K, both the mean delivery delay and the mean number of copies to relays decrease as λ increases. We also observe that, for a fixed λ, the mean delivery delay decreases as the network size grows. Finally, for smaller values of λ, the mean number of copies to relays increases with the network size, and for larger values of λ, vice-versa happens.

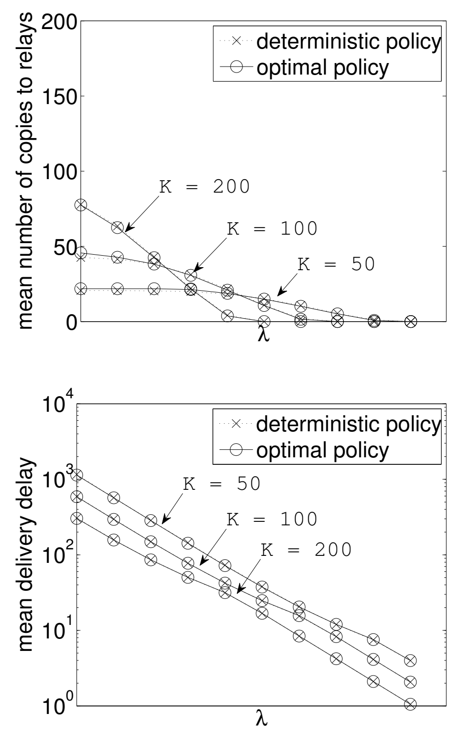

Fig. 4. The top and bottom sub-plots, respectively, show the total number of copies to relays and the delivery delays corresponding to both the optimal and the deterministic policies.

VI. Conclusion

We studied the control of forwarding in DTNs employing epidemic relaying, and obtained the optimal policy (Theorem 3.1). We obtained an asymptotically optimal policy that does not require any information on the dynamic network state, and hence is feasible (Theorem 4.2). In order to do so, we also extended the existing differential equation approximation results for Markov chains to controlled Markov chains (Theorem 4.1).

In our future work we want to study the scenario where packets come with a life-time and the goal is to maximize the fraction of destinations that receive the packet subject to the energy constraint. We also want to study the adaptive controls for the case when the network parameters (M, N, λ etc.) are not known to the source.

Appendix A
Proof of Theorem 3.1

We first recall the following monotonicity properties of the one step look ahead policy u^o (see (3) and Remarks 3.1:

1) If $u^o(m,n,r)=0$, then $u^o(i,n,r)=0$ for all $m<i<M$.
2) If $u^o(m,n,r)=0$, then $u^o(m,j,r)=0$ for all $n<j<N$.

We show the optimality of the policy u^o for states $(m,n,s)\in[M_\alpha-1]\times[N_0:N-1]\times\{r\}$ proceeding in raster scan order starting from the top right (i.e., the state $(M_\alpha-1,N-1,r)$). To do this, we divide the set of states into the following types:

(a) *T1:* $\{(m,n,r):\ u^o(m,n+1,r)=u^o(m+1,n,r)=0\}$
(b) *T2:* $\{(m,n,r):\ u^o(m,n{+}1,r)=1,\ u^o(m{+}1,n,r)=0\}$
(c) *T3:* $\{(m,n,r):\ u^o(m+1,n,r)=1\}$

(a) *Optimality for type T1 states:* We show this iteratively. Consider the state $(M_\alpha-1,N-1,r)$. If the infected node does not copy to the susceptible relay, it will not copy in future either because at all potential future decision epochs (meetings with susceptible relays) the system state remains the same, $(M_\alpha-1,N-1,r)$. On the other hand, if the infected node copies, there will be no more susceptible relays and hence no more decision epochs. Hence to obtain the optimal action it suffices to consider only two policies $0s$ and $1s$. Thus $u^*(M_\alpha-1,N-1,r)=u^o(M_\alpha-1,N-1,r)$. Now, consider a type T1 state (m,n,r) such that $u^*(i,j,r)=u^o(i,j,r)=0$ for all $i\geq m, j\geq n, (i,j)\neq(m,n)$. If it is optimal to copy at (m,n,r), the state moves to $(m,n+1,r)$ and the optimal policy from there onwards is not to copy. So $1s$ is optimal. If it is optimal not to copy at (m,n,r), then the state at the next decision epoch is either (m,n,r) or (m',n,r) with $m'>m$, and the optimal decision remains not to copy any further. Thus, in this case, $0s$ is optimal. Consequently, we have $A((m,n,r),0)=J_{0s}(m,n,r)$ and $A((m,n,r),1)=J_{1s}(m,n,r)$. Clearly then, to obtain the optimal action, it suffices to consider only two action sequences $0s$ and $1s$, and thus, $u^*(m,n,r)=u^o(m,n,r)$.

(b) *Optimality for type T2 states:* For a type T2 state (m,n,r), $u^o(m,n{+}1,r)=1$ and $u^o(m{+}1,n,r)=0$. Thus, the monotonicity properties of the policy u^o imply that $u^o(m,n,r)=1$ and $u^o(j,n,r)=0$ for all $m<j\leq M-1$. Also, for all $m<j\leq M-1$, (j,n,r) are type T1 states. Hence, from part (a), $u^*(j,n,r)=u^o(j,n,r)=0$ for all these states. We prove that $u^*(m,n,r)=1$ via contradiction. Suppose $u^*(m,n,r)=0$. Then, the next state to be visited will be $(m{+}1,n,r)$. Moreover, no susceptible relays will be copied in future as well because $u^*(j,n,r)=0$ for all $m<j\leq M-1$. In particular, $J(m,n,r)=A(m,n,r),0)=J_{0s}(m,n,r)$. But $J_{0s}(m,n,r)>J_{1s}(m,n,r)$. Hence our supposition is false and $u^*(m,n,r)=1$.

(c) *Optimality for type T3 states:* Consider a type T3 state (m,n,r). By definition of type T3 states, there exists a $j\geq m+1$ such that $u^o(j,n,r)=1$. Let m_n^* be the maximum j such that $u^o(j,n,r)=1$. We show the optimality of policy u^o inductively. Note that, for each n, (m_n^*,n,r) is either a type T1 or type T2 state. Thus, from parts (a) and (b), $u^*(m_n^*,n,r)=u^o(m_n^*,n,r)=1$. Let us assume that $u^*(j,n,r)=u^o(j,n,r)=1$ for all $m_n^*\geq j\geq m+1$. Then, in the following, we show that $u^*(m,n,r)=u^o(m,n,r)=1$. This completes the induction step.

We define

$$\begin{aligned}\psi(m,n) &:= J_{0s}(m,n,r)-J(m,n,r),\\ \theta_0(m,n) &:= J_{0s}(m,n,r)-A((m,n,r),0),\\ \text{and } \theta_1(m,n) &:= J_{1s}(m,n,r)-A((m,n,r),1).\end{aligned}$$

The action sequences that give rise to the two cost terms in the definition of $\theta_0(m,n)$, both do not copy to the susceptible relay met at present. Let j be the number of infected destinations at the next decision epoch when a susceptible relay is met; j can be $m,m+1,...,M$. All interim decision epochs were meetings with susceptible destinations, and both policies copy at these meetings. Hence, both policies incur the same cost until this epoch, and differ by $\psi(j,n)$ in the costs to go (from this epoch onwards). Averaging the difference over j, and noting that $\psi(j,n)=0$ for $j>M_\alpha-1$, we get

$$\theta_0(m,n)=\sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1}p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n).^6 \quad (8)$$

Since $A((m,n,r),0)\geq J(m,n,r)$, and so $\theta_0(m,n)\leq\psi(m,n)$, we get

$$\begin{aligned}&\sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1}p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n)\leq\psi(m,n)\\ \text{or, } & p_{m,n}(r)\psi(m,n)+p_{m,n}(d)\\ &\times\sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1}p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n)\leq\psi(m,n)\\ \text{or, } & \sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1}p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n)\leq\psi(m,n) \quad (9)\end{aligned}$$

which is obtained by transposing the first term on the left in the previous expression to the right.

[6] We use the standard convention that a product over an empty index set is 1, which happens when $j=m$.

Next, we establish the following lemma.

Lemma A.1: $\theta_1(m,n) \geq \theta_1(m+1,n)$.

Proof: Note that both the action sequences that lead to the two cost terms in the definition of $\theta_1(m,n)$, copy at state (m,n,r). Subsequently, both incur equal costs until a decision epoch when an infected node meets a susceptible relay. Also, at any such state $(j,n+1,r)$, $j \geq m$, the costs to go differ by $\psi(j,n+1)$. Hence,

$$\theta_1(m,n) = \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n+1}(d)\right) p_{j,n+1}(r)\psi(j,n+1)$$
$$= p_{m,n+1}(r)\psi(m,n+1) + p_{m,n+1}(d)\theta_1(m+1,n)$$

where

$$\theta_1(m+1,n) = \sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1} p_{l,n+1}(d)\right) p_{j,n+1}(r)\psi(j,n+1).$$

Thus it suffices to show that

$$\psi(m,n+1) \geq \theta_1(m+1,n).$$

which is same as (9) with n replaced by $n+1$. ■

Next, observe that, for all $m \leq j \leq m_n^*$,

$$\begin{aligned}\psi(j,n) &= J_{0s}(j,n,r) - \min\{A((j,n,r),0), A((j,n,r),1)\}\\ &= \max\{\theta_0(j,n), \Phi(j,n) + \theta_1(j,n)\}. \qquad (10)\end{aligned}$$

Moreover, from the induction hypothesis, the optimal policy copies at states (j,n,r) for all $m+1 \leq j \leq m_n^*$. Hence, for $m+1 \leq j \leq m_n^*$,

$$\psi(j,n) = \Phi(j,n) + \theta_1(j,n).$$

Finally, $\psi(j,n) = 0$ for all $m_n^* < j \leq M_\alpha - 1$ as the optimal policy does not copy in these states. Hence, from (8),

$$\begin{aligned}&\theta_0(m,n)\\ &= p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\} + p_{m,n}(d)\\ &\quad\times \sum_{j=m+1}^{m_n^*}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right) p_{j,n}(r)\big(\Phi(j,n)+\theta_1(j,n)\big)\\ &< p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\} + p_{m,n}(d)\\ &\quad\times \big(\Phi(m,n)+\theta_1(m,n)\big)\sum_{j=m+1}^{m_n^*}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right) p_{j,n}(r)\\ &\leq p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\}\\ &\quad + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big)\\ &= \max\{p_{m,n}(r)\theta_0(m,n) + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big),\\ &\qquad \Phi(m,n)+\theta_1(m,n)\}, \qquad (11)\end{aligned}$$

where the first inequality holds because $\Phi(m,n)$ is strictly decreasing and $\theta_1(m,n)$ is also decreasing in m for fixed n (from (3) and Lemma A.1, respectively). The second inequality follows because the summation term is a probability which is less than 1. Now suppose that $\theta_0(m,n) \geq \Phi(m,n) + \theta_1(m,n)$. Then

$$\begin{aligned}&\max\{p_{m,n}(r)\theta_0(m,n) + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big),\\ &\qquad \Phi(m,n)+\theta_1(m,n)\}\\ &= p_{m,n}(r)\theta_0(m,n) + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big)\\ &\leq \theta_0(m,n)\end{aligned}$$

which contradicts (11). Thus, we conclude that

$$\theta_0(m,n) < \Phi(m,n) + \theta_1(m,n).$$

This further implies that $\psi(m,n) = \Phi(m,n) + \theta_1(m,n)$ (see (10)), and so that $u^*(m,n,r) = u^o(m,n,r) = 1$.

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