Design of a TDD Multisector TDM MAC for the WiFiRe Proposal for Rural Broadband Access

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***Abstract*— The WiFiRe (WiFi Rural Extension) proposal for rural broadband access is being developed under the aegis of CEWIT. The system leverages the widely available, and highly cost-reduced, WiFi chipsets. However, only the physical layer from these chipsets is retained. A single base station carries several WiFi transceivers, each serving one sector of the cell, and all operating on the same WiFi channel in a time division duplex (TDD) manner. We replace the contention based WiFi MAC with a single-channel TDD multisector TDM MAC similar to the WiMax MAC. In this paper we discuss in detail the issues in designing such a MAC for the purpose of carrying packet voice telephony and for Internet access. The problem of determining the optimal spatial reuse is formulated and the optimal spatial reuse and the corresponding cell size is derived. Then the voice and data scheduler is designed. It is shown how throughput fairness can be implemented in the data scheduler. A capacity assessment of the system is also provided.**

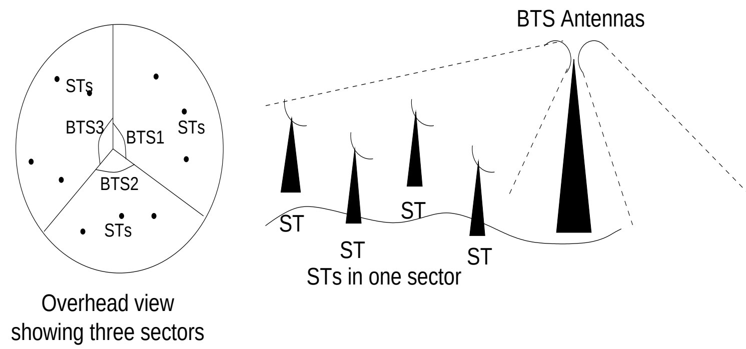

Fig. 1. WiFiRe network configuration. The figure on the left shows a deployment with three sectors, and the figure to the right shows a tall tower carrying several BSs, with sector antennas, and several STs in a sector, with lower height directional antennas.

I. INTRODUCTION

The WiFiRe standard for rural Internet access (see [1], and [7]) is being developed under the aegis of CEWIT, IIT Madras, as a technology for providing wireless broadband voice and data access for rural areas. The following are the key features of the current version of this standard.

- In order to leverage the price advantage of using existing mass produced integrated circuits, the physical layer has been taken to be the same as that of IEEE 802.11, the popular standard for wireless local area networks (WLANs).
- One access point (AP) (or base station controller (BSC)), using a single IEEE 802.11 channel, will serve a “cell” with about 80-120 villages spread over a 15 Km to 20 Km radius.
- The cell will be sectored (typically $60°$), with each sector containing a directional base station (BS) antenna. There will be one fixed subscriber terminal (ST) in each village, which could be connected to voice and data terminals in the village by a local area network. The ST antennas will also be directional, thus permitting reliable communication between the BS antenna in a sector and all STs in that sector. However, because of antenna side-lobes, transmitters in each sector will interfere with receivers in other sectors. Depending on the attenuation levels, a scheduled transmission in one sector may exclude the simultaneous scheduling of certain transmitter-receiver pairs in other sectors. Further, simultaneous transmissions will interfere, necessitating a limit on the number of simultaneous transmissions possible. A typical configuration of a WiFiRE system is shown in Figure 1.
- There will be one MAC controller for all the sectors in a cell. The multiple access mechanism will be time division duplexed multisector TDM (TDD-MSTDM) scheduling of slots. Time is divided into frames, which contain traffic slots. The set of slots in a frame is partitioned into contiguous uplink and downlink segments. During the downlink segment, in each slot, one or zero transmissions can take place in each sector; and similarly in the uplink segment. Because of site and installation dependent path loss patterns, and because of time varying traffic requirements, the schedule will need to be computed on-line.

The objective of this work is to abstract out the basic scheduling problem, to develop a mathematical formulation for the problem, to provide some scheduling algorithms, and to provide a capacity assessment of the MAC architecture.

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A. Related Literature

Bhagwat et al. [5] have discussed issues related to using 802.11 family of wireless technologies for long distance transmission in rural environment, such as the quality of 802.11 PHY performance outdoors, range extension, spectral vs. cost efficiency. The authors provide details of the 802.11-based mesh network deployed in the Digital Gangetic Plains Project providing voice and data services to villages. Raman and Chebrolu [9] discuss the issues in using CSMA/CA in networks including long distance links. CSMA/CA is designed to resolve contention in the indoor environment, but is inefficient in long distance point to point links. The authors provide a new MAC for mesh networks synTX/synRX, which in the context of our problem translates to saying that the antennas at the base station should either all be in transmit mode or all in receive mode and the transmissions should satisfy some power relations. These ideas have been made use in the spatial reuse model that we discuss.

Shetiya [6] considers the joint routing and scheduling problem in WiMax mesh networks. A dynamic programming problem is formulated to maximize the throughput, and is found to be computationally complex to solve. Heuristics are used to attain a near optimal solution by considering the routing and scheduling problems separately.

B. Preview of Contributions

We begin by developing a model for antenna coverage and spatial reuse in a single channel multi-sector operation. It is seen that in multi-sector operation, depending on the path loss, receive sensitivity and the antenna directivity, the number of simultaneous transmissions can be 3 or 4. We then set up an abstraction for the TDD, single channel multi-sector scheduling problem. We begin by developing capacity bounds for fair rate allocation, sum rate and sum of log rate.This analysis also shows how the sectors should be angularly oriented. We then develop a scheduling methodology for voice and data traffic.

C. Overview of the Paper

Section II sets up the notations used through the rest of the report and also explains the voice and TCP traffic models used. Section III explains the model used to characterize the interference in the network by disallowing transmission in some regions and by limiting the total number of simultaneous transmissions. Section IV provides bounds on the capacity of the system. The optimum antenna positioning can be obtained based on these bounds. Section V gives the scheduling problem in hand. A dynamic programming problem formulation of the problem is given in Section C. In this section, we also propose a greedy heuristic scheduler for uplink and downlink. Section G gives a scheduling algorithm to improve the fairness among users.

II. Problem Setup

A. Some Typical System Parameters

Typically, there will be 80 to 120 subscriber terminals (STs) in a 15 to 20 km radius covered by a 6 sector system. Each station will be associated with a base transceiver station (BTS). The TDD-MSTDM scheduling is done over a frame. A typical frame time is 10ms with slot time of 32μs, giving rise to 312 slots per frame. The frame is divided into downlink and uplink segments in a ratio which is a design parameter. During downlink transmissions, a significant amount of power from the transmitting BTS reaches other BTSs, the distance separating them being very small. So, when downlink transmissions are scheduled in any one of the sectors, other BTSs cannot be in the receive mode. Therefore, downlink and uplink transmissions must alternate over the entire cell. It follows that the ratio of number of slots in uplink to that in downlink must be the same in all sectors. This ratio is kept constant. A beacon marks the beginning of the frame and also carries the scheduling MAP. A total of 24 slots are needed for the beacon in every frame.

All links are at 11 Mbps. A slot is of 32μs. At 11 Mbps, this is 44 bytes. A VOIP packet is 40 bytes long. Thus, assuming that the MAC overhead is 4 bytes per packet, the transmission of a VOIP packet can be done in a slot. Each transport block (TB) has a 96μs PHY overhead, i.e., 3 slots. Hence, the minimum size of a transport block is 4 slots. A TB should fit into an integral number of slots. An uplink TB is always for one ST, but downlink TBs can be for multiple STs. There is a maximum size of TB (T_{max}) which indicates PHY limitations or may correspond to higher layer limitations.

Implications for Scheduling: Since each TB involves a 3 slot overhead, it is advantageous to use long TBs. However, this would result in starving some STs while favoring others. Note also that, because of T_{max}, there is a minimum overhead of $\frac{3}{T_{max}}$ slots per slot.

B. Directional Antennas and Intersector Interference

The radiation pattern of a typical antenna used in the deployment is shown in Figure 2. Based on the antenna pattern, we can divide the region into an *association region*, a *taboo region* and a *limited interference region* with respect to each BTS.

The radial zone over which the directional gain of the antenna is above -3dB is called the association region. In our analysis, we take the directional gain to be constant over this region. Any ST which falls in this region of a BTS antenna j is associated to the BTS j.

The region on either side of the association region where the directional gain is between -3dB and -15dB is called the taboo region. Any ST in this region of BTS j causes significant interference to the transmissions occurring in Sector j. When a transmission is occurring in Sector j, no transmission is allowed in this region.

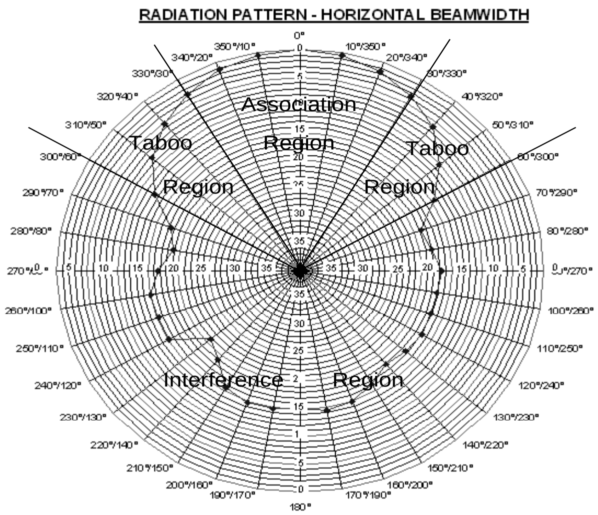

Fig. 2. Radiation pattern of a typical BS antenna that could be used in the deployment. The association region is a $60°$ sector centered at the $0°$ mark, the taboo region is $30°$ on either side of this association region, and the limited interference region covers the remaining $240°$.

In the limited interference region the directional gain of the BTS antenna is below -15dB. A single transmission in this region of BTS j may not cause sufficient interference to the transmission in Sector j. But a number of such transmissions may add up causing the SINR of a transmission in Sector j to fall below the threshold required for error free transmission. We take care of this by limiting the total number of simultaneous transmissions in the system as explained in Section III.

As an example, for the antenna pattern shown in Figure 2, the association region is a $60°$ sector centered at the $0°$ mark, the taboo region is $30°$ on either side of this association region, and the limited interference region covers the remaining $240°$.

C. Notation and Terminology

- n — the number of BTSs (e.g., 6); BTSs are indexed clockwise; for Sector j the interference region in the previous counter-clockwise sector will be denoted by $j-$ and the next clockwise sector will be indexed by $j+$.
- n_0 — the number of sectors that can have simultaneous transmissions; see Section III.
- m — the number of STs (e.g., 40 or 120); the number of STs in Sector j will be denoted by m_j and the number of taboo STs in the previous counter-clockwise previous sector by m_{j-} ; similarly we define m_{j+}.
- N — the number of slots in a scheduling frame. Each slot is used either for downlink communication or for uplink communication. For example, $N = 312$ slots, as per the numerical values provided earlier.
- $\mathbf{A}$ — the association matrix; an $m \times n$ matrix, where each row corresponds to an ST and each column corresponds to a BTS. The (i, j)th element of the matrix is a 1 if the ith ST is associated with BTS j. Otherwise it is 0. We will sometimes refer to each nonzero element of $\mathbf{A}$ as a *link*.
- $B(i)$ — the BTS with which ST i, $1 \leq i \leq m$ is associated. i.e., $\mathbf{A}_{i,B(i)}$ = 1.
- $\mathbf{I}$ — the exclusion matrix; an $m \times n$ matrix, where each row corresponds to an ST and each column corresponds to a BTS. The (i, j)th element of the matrix is 1 if ST i is taboo for BTS j or i is associated with j. Otherwise it is 0. Note that the matrices $\mathbf{A}$ and $\mathbf{I}$ together define the scheduling constraints.
- $\mathbf{u}$: — *Activation vector:* a $1 \times n$ matrix, where the ith element denotes which ST in Sector i is transmitting. If we decide to transmit between ST j and its BTS, say BTS i, the ith element of $\mathbf{u}$ is j. Evidently an activation vector should satisfy $|\{j : u_j > 0\}| \leq n_0$. Also, $\mathbf{u}$ must satisfy the exclusion constraints given by $\mathbf{I}$.
- $\mathbf{U}$: — *Maximal activation vector:* If no more links in an activation vector can be activated without causing interference to some other transmissions scheduled in the same vector then this activation vector is maximal.
- $\mathcal{U}$: — *Activation set:* the set of all maximal activation vectors.
- $\mathbf{S}$: — *A schedule:* A schedule is an $N \times (n + 1)$ matrix, with rows corresponding to slots and columns (except the last column) corresponding to sectors, where the (i, j)th entry corresponds to the link in the jth sector that is scheduled to transmit in the ith slot. If no ST in the jth sector can transmit in the ith slot of the frame (because this will interfere with other scheduled transmissions in the frame) the corresponding entry is 0. The last column indicates the number of consecutive slots for which the activation vector is used.
- $\mathcal{I}(\mathbf{s})$: — This is the set of links that can interfere with any of the links in $\mathbf{s}$.

D. Traffic Models, QoS Objectives

A possible network architecture for a WiFiRe deployment is shown in Figure 3. There are a number of telephones and PCs connected to the WiFiRe BTS through STs. Several BTSs are controlled by a single base station controller (BSC). The BSC is connected to the Internet and the PSTN through switches. All telephony traffic is carried as VoIP over the WiFiRe access network. For this purpose, notice that there is a *gateway* between the PSTN and the WiFiRe network.

For packet voice telephony, we assume that the voice coder emits a frame every 20ms. Assuming a frame time of 10ms

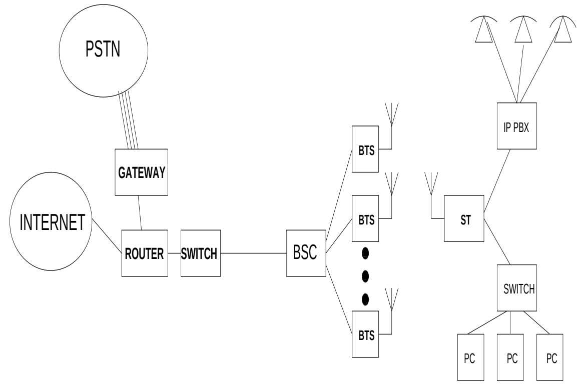

Fig. 3. A possible network access architecture for a WiFiRE deployment.

we need one voice packet to be transmitted in each direction (uplink and downlink), every 2 frames. This also implies that if a voice packet is transmitted in the frame following the one in which it arrives, then its delay is bounded by 20ms. We propose to admit only so many VoIP calls, so that the probability of a voice packet not getting transmitted in the slot after the one in which it arrives is small, say 0.02. We note that the slot utilization can be optimized by performing silence suppression before the periodically arriving voice packets enter the system. This will give rise to an on-off packet arrival process for each VoIP call in each direction. The on-off process will be random (typically modeled by a Markov process). For calls between the BTS and ST i, it suffices to allocate $C_i \leq m_{v,i}$ ($m_{v,i}$ is the number of voice calls for ST i) slots per frame in the uplink and downlink such that the desired probability of packet dropping is achieved [2, Chapter 5].

Assuming a mean call holding time of 3 minutes ($\mu^{-1} =$ 3 minutes) and a call arrival rate of 3 calls per hour ($\lambda = 3/60$ per minute), which are typical values for a home telephone, $\rho = \lambda/\mu = 0.15$ erlangs. The frame time in WiFiRE is 10ms. Given that the vocoder emits one packet every 20ms, CBR traffic requires one uplink slot per call in every 2 frames, and VBR traffic requires almost one uplink slot per call in every 4 frames. So, with 2 slots reserved for voice calls per ST per frame, the number of calls that can be supported is $N_v = 4$ for CBR traffic, and $N_v = 8$ for VBR traffic. With $\rho = 0.15$ and $N_v = 4$, we can have 7 subscribers with a probability of blocking as low as 0.02. With $\rho = 0.15$ and $N_v = 8$, we can have 24 subscribers at probability of blocking 0.02. With 4 slots reserved per ST these numbers are 24 for CBR calls and 65 for VBR traffic. Given the village economics we expect that just 2-4 slots per ST may be all that are required.

One VOIP packet is 40 bytes. The MAC header has been taken to be 4 bytes, so that transmission of a lone voice packet can be completed in 4 slots (3 slots of PHY overhead + 1 slot voice packet). It is possible to send more voice packets in a single transport block without additional PHY overhead. Thus, for a single call from an ST, we need 4 slots each in uplink and downlink every 2 frames; for two calls from an ST, we need 5 slots each in the uplink and downlink every 2 frames and so on.

For TCP controlled data traffic (the predominant traffic over the Internet), we assume that this wireless access system is the bottleneck along the path. As a first model, we assume that TCP packets are backlogged in each direction (i.e., at the BTS and the STs) and the scheduling objective is to pack as many TCP packets as possible into the schedule, after ensuring that voice QoS is met. We will also consider the problem of ensuring some form of fairness between the TCP users.

III. Spatial Reuse Model

In [3], the authors prove that maximizing the cardinality of independent sets used in a schedule need not necessarily increase the throughput, since as the cardinality of the set increases, the prevailing SINR drops, thereby resulting in an increase in the probability of error, decreasing the throughput. Hence it is necessary to limit the cardinality of the independent set used so as to satisfy the SINR requirements. i.e., there is a limit to the number of simultaneous transmissions possible.

In this section we consider the problem of finding the maximum number of simultaneous transmissions possible in different sectors in the uplink and the downlink. We assume that there is no power control in the downlink. The BTS transmits to all the STs at the same power. We can have static power control in the uplink. Each ST transmits to the BTS at a fixed power, such that the average power received from different STs at the BTS is the same. The STs near the BTS transmit at a lower power and the ones farther away transmit at a higher power.

A. Uplink

In the uplink, we assume that there is static power control. All STs transmit at a power such that the average power received at the BTS is P times noise power. Let the maximum power that can be transmitted by an ST be P_t times noise power. Let R_0 be the distance such that when P_t is transmitted by an ST at distance R_0, the average power received at the BTS is P_0 times noise power, where P_0 is the minimum SNR required to decode a frame with a given probability of error. Also, let R be such that when P_t is transmitted from an ST at distance R, power received at the BTS is P times noise power, i.e., $\frac{P}{P_0} = \left(\frac{R}{R_0}\right)^{-\eta}$

In the presence of interferers, the power required at the receiver will be greater than P_0 times noise. Let P be the power required, so that the receiver decodes the frame with a given probability of error, in the presence of interferers. The directional gain of the BTS antenna is -15dB in other

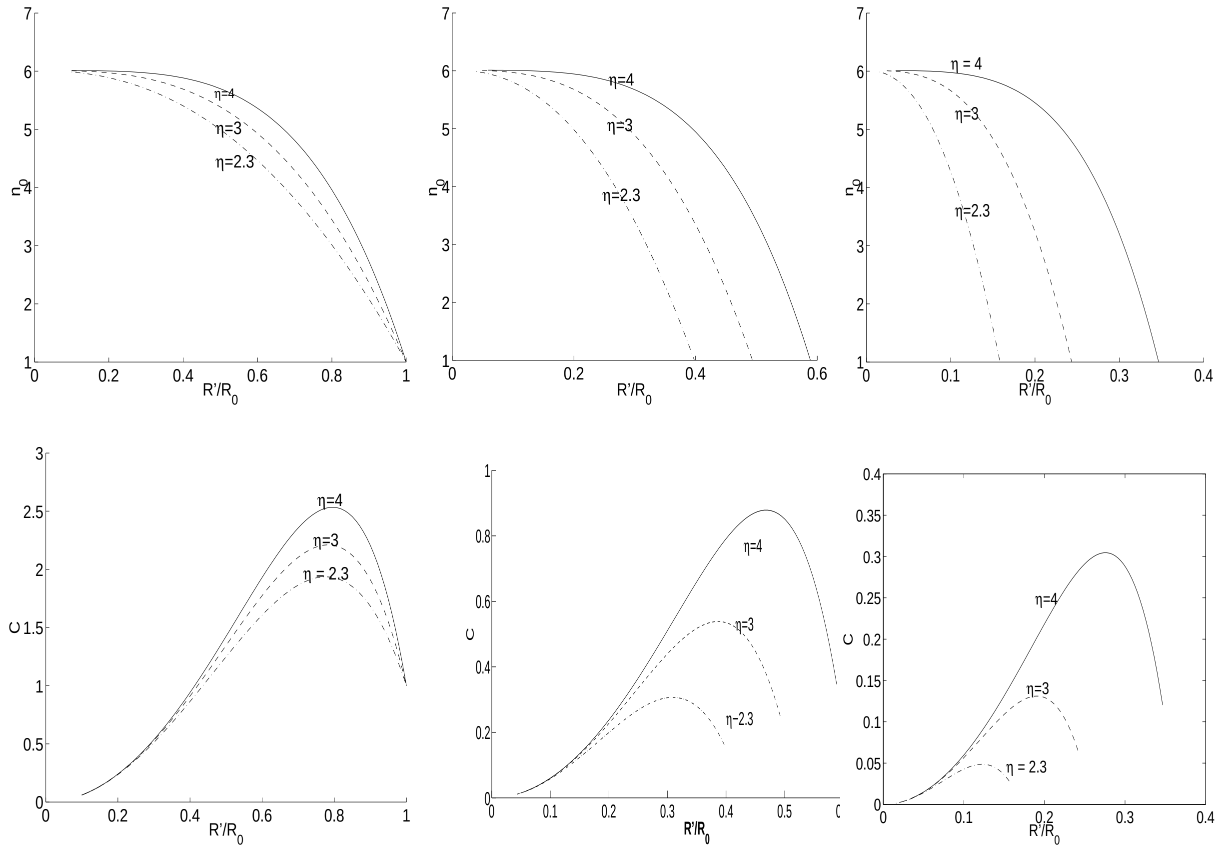

Fig. 4. Variation of the number of simultaneous transmissions possible (n_0) and system capacity (C) with coverage relative to a reference distance R_0 for $\eta = 2.3, 3, 4$ and $\sigma = 0, 4, 8$. Plots for $\sigma = 0, 4, 8$ are shown left to right.

sectors. Hence, the interference power from a transmission in any other sector would be $10^{-\frac{3}{2}}P$. For decoding a frame with less than a given probability of error, we need a SNR of P_0 at the receiver. If there are $n_0 - 1$ simultaneous transmissions, the path loss factor being η,we need R such that

$$\begin{aligned} \Psi_{rcv} &= \frac{P_0(\frac{R}{R_0})^{-\eta}}{1 + (n_0 - 1)10^{-\frac{3}{2}}P_0(\frac{R}{R_0})^{-\eta}} \geq P_0 \\ n_0 &\leq 1 + \frac{(\frac{R}{R_0})^{-\eta} - 1}{10^{-\frac{3}{2}}P_0(\frac{R}{R_0})^{-\eta}} \end{aligned}$$

To provide a margin for fading, we consider a reduced range R' such that $10\log\left(\frac{R'}{R}\right)^{-\eta} \geq 2.3\sigma$ where σ is the shadowing standard deviation. Thus 99% of the STs in a circle of radius R' around the BTS can have their transmit power set so that the average power P is received at the BTS in the uplink.

Notice that, to allow spatial reuse, the coverage of the system needs to be reduced to $R' < R_0$. There is thus a tradeoff between spatial reuse and coverage, which is captured by the spatial capacity measure $C = n_0 R'^2$, which has units slots $\times$ km^2. We note that this measure has the same motivation as the *bit metres per second* measure introduced in [8].

The variation of the number of transmissions and system capacity with coverage is as shown in Figure 4. We can see that, for each η, that there is an optimal n_0 and R' such that the coverage is maximum.

The coverage for which the capacity is maximum can be obtained from $\frac{dC}{dr'} = 0$ where $r' = \frac{R'}{R_0}$ Thus we get the optimum value of r' and n_0 as

$$r' = \left(10^{-\frac{2.3\sigma}{10}} \frac{1 + 10^{-\frac{3}{2}}P_0}{1 + \frac{\eta}{2}}\right)^{\frac{1}{\eta}}$$

$$n_0 = \frac{(1 + 10^{-\frac{3}{2}}P_0)\eta}{10^{-\frac{3}{2}}P_0(\eta + 2)}$$

The results are shown in Table 1.

B. *Downlink*

In the downlink, the transmit power is kept constant. In downlink, assuming that the BTS antennas transmit at a power P_t times noise, and repeating the calculations as for uplink, we find that n_0 for downlink gives the same expression as for uplink. The plots and tables for uplink applies for downlink also.

σ / η	0	4	8
2.3	0.77	0.31	0.12
3	0.78	0.39	0.20
4	0.80	0.47	0.28

η	2.3	3	4
n_0	3	3	4

TABLE 1

THE OPTIMUM COVERAGE NORMALIZED TO R_0 AND THE OPTIMUM NUMBER OF SIMULTANEOUS TRANSMISSIONS IN A MULTI SECTOR SYSTEM FOR DIFFERENT VALUES OF η AND σ.

C. *Number of Sectors*

Once we get the maximum number of simultaneous transmissions possible, n_0, we get some idea about the number of sectors required in the system. In an n_0 sector system, when a transmission occurs in the taboo region between Sector j and Sector $j+1$, no more transmissions can occur in Sectors j and $j+1$. So, the number of simultaneous transmissions can be at most $n_0 - 1$, one in Sector j and $j+1$ and at most one each in each of the other sectors. Thus maximum system capacity cannot be attained with $n_0 - 1$ sectors. With $n_0 + 1$ sectors, we can choose maximal independent sets such that the sets are of cardinality n_0. So, we need at least $n_0 + 1$ sectors in the system. From the spatial reuse model we see that we can have up to 4 simultaneous transmissions in the system, so we need at least 5 sectors in the system.

IV. CHARACTERISING THE AVERAGE RATE REGION

There are m STs. Suppose a scheduling policy assigns $k_j(t)$ slots, out of t slots, to ST j, such that $\lim_{t\to\infty} \frac{k_j(t)}{t}$ exists and is denoted by r_j. Let $\mathbf{r} = (r_1, r_2, \ldots r_m)$ be the rate vector so obtained. Denote by $\mathcal{R}(n)$ the set of achievable rates when the maximum number of simultaneous transmissions permitted is n. Notice that for $n_1 > n_2$, $\mathcal{R}_1 \supset \mathcal{R}_2$. This is evident because any sequence of scheduled slots with $n = n_2$ is also schedulable with $n = n_1$. In the previous section, we have determined the maximum value of n, i.e., n_0. Denote $\mathcal{R}_0 = \mathcal{R}(n_0)$. A scheduling policy will achieve an $r \in \mathcal{R}_0$. In this section, we provide some understanding of $\mathcal{R}_0$ via bounds.

A. *An Upper Bound on Capacity*

Suppose each ST has to be assigned the same rate r. In this subsection an upper bound on r is determined. In general, the rate vector $(r, r, \ldots r) \notin \mathcal{R}_0$. The upper bound is obtained via simple linear inequalities. Consider the case $n \geq 3$. Suppose one wishes to assign an equal number of slots k to each ST in the up link. There are N_U uplink slots in a frame. Consider Sector j, which contains m_j STs. Thus $k \cdot m_j$ slots need to be allocated to uplink transmission in Sector j. When STs in the interference region $j-$ or $j+$ transmit, then no ST in Sector j can transmit. Suppose $k_{j\pm}$ slots are occupied by such interference transmission. Now it is clear that

$$k \cdot m_j + k_{j\pm} = N_U$$

because whenever there is no transmission from the interference region for sector j there can be a transmission from sector j. Let m_{j-} and m_{j+} denote the number of STs in the interference regions adjacent to Sector j. Since the nodes in $j-$ and $j+$ can transmit together, we observe that

$$k_{j\pm} \geq \max(k \cdot m_{j-}, k \cdot m_{j+})$$

with equality if transmission in $j-$ and $j+$ overlap wherever possible. Hence one can conclude that for any feasible scheduler that assigns k slots to each ST in the uplink

$$k \cdot m_j + \max(k \cdot m_{j-}, k \cdot m_{j+}) \leq N_U$$

For large frame time N, divide the above inequality by N and denote the rate of allocation of slots by r. Thus if out of t slots, each ST is allocated k slots, then $r = \lim_{t\to\infty} \frac{k}{t} \leq 1$

$$r \cdot m_j + r \cdot \max(m_{j-}, m_{j+}) \leq \phi_u$$

where ϕ_u is the fraction of frame time allocated to the uplink or

$$r \leq \frac{\phi_u}{m_j + \max(m_{j-}, m_{j+})}$$

This is true for each j. So,

$$r \leq \frac{\phi_u}{\max_{1\leq j\leq n}(m_j + \max(m_{j-}, m_{j+}))}$$

For the case $n = 2$ for $j \in \{1, 2\}$ denote the interfering nodes in the other sector by m_j. One easily sees that

$$r \leq \frac{\phi_u}{\max(m_1 + m_1', m_2 + m_2')}$$

B. *An Inner Bound for the Rate Region*

In this section a rate set $\mathcal{R}_L$ is obtained such that $\mathcal{R}_L \subset \mathcal{R}_0$. i.e., $\mathcal{R}_L$ is an inner bound to the achievable rate set.

Reuse constraint graph: The vertices of this graph represent the links in the WiFiRe cell. In any slot we consider only uplinks or only downlinks. Two vertices in the graph are connected if a transmission in one link can cause interference to a transmission in the other link. The reuse constraint graph is represented as $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$ is the set of edges.

Clique: A fully connected subgraph of the reuse constraint graph. A transmission occurring from an ST in a clique can interfere with all other STs in the clique. At most one transmission can occur in a clique at a time.

Maximal clique: A maximal clique is a clique which is not a proper subgraph of another clique.

Clique incidence matrix: Let κ be the number of maximal cliques in $(\mathcal{V}, \mathcal{E})$. Consider the $\kappa \times m$ matrix $\mathcal{Q}$ with

$$\mathcal{Q}_{i,j} = \begin{cases} 1 & \text{if link } j \text{ is in clique } i \\ 0 & \text{o.w.} \end{cases}$$

By the definition of $\mathbf{r}$ and $\mathcal{Q}$, a necessary condition for $\mathbf{r}$ to be feasible is (denoting by $\mathbf{1}$, the column vector of all 1s.)

$$\mathcal{Q} \cdot \mathbf{r} \leq \mathbf{1}$$

since at most one link from a clique can be activated. In general, $\mathcal{Q} \cdot \mathbf{r} \leq \mathbf{1}$ is not sufficient to guarantee the feasibility of $\mathbf{r}$. This condition is sufficient if the graph is linear. A linear graph is one in which the nodes can be indexed in such a way that if nodes $i, j, i < j$, are in the clique then each node k with $i \leq k \leq j$ is also in the clique; i.e., the nodes of each maximal clique are contiguous in the indexing. Such a graph will have a clique incidence matrix of the form

$$\mathcal{Q} = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \dots \\ \vdots & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \end{bmatrix}$$

The reuse constraint graph in the multisector scheduling problem being considered has a ring structure. In any case, the set of rate vectors satisfying $\mathcal{Q} \cdot \mathbf{r} \leq 1$ provides an outer bound to the rate set. A linear subgraph can be extracted from the reuse constraint graph by deleting one sector, or equivalently setting all rates in one sector to 0. Let us index the STs in such a way that we can write a rate vector $\mathbf{r}$ as:

$$\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_m)$$

where, for $1 \leq k \leq m$, $\mathbf{r}_k$ is the rate vector for the STs in Sector k. Since deleting one sector yields a linear graph, if $\mathbf{r}$ is such that $\mathbf{r}_k = \mathbf{0}$, and $\mathcal{Q} \cdot \mathbf{r} \leq \mathbf{1}$ then $\mathbf{r}$ is a feasible rate vector (which assigns 0 rates to all STs in Sector k). Linear combinations of feasible rate vectors are also feasible (since time sharing can be done over the schedules that achieve these vectors). Define

$$\hat{\mathcal{R}} := \{\mathbf{r} : \mathcal{Q} \cdot \mathbf{r} \leq \mathbf{1}, \text{for some} k, 1 \leq k \leq m, \mathbf{r}_k = \mathbf{0}\}$$

Further, let $\mathcal{R}_L$ be the convex hull of $\hat{\mathcal{R}}$. By the above discussion, it follows that $\mathcal{R}_L \subset \mathcal{R}_0$. Thus we have an inner rate region. We will use this inner bound in the next section.

C. Optimum Angular Positioning of the Antennas

As can be seen from the previous section, the feasible rates set, $\mathcal{R}_0$, of the system depends on the spatial distribution of the STs around the BTS. Thus the $\mathcal{R}_0$ varies as the sector orientation is changed. A system where the antennas are oriented in such a way that most STs fall in the association region of the BTSs rather than in the taboo region will have more capacity than one in which more STs are in the taboo regions.

One sector boundary is viewed as a reference. Let $\mathcal{R}_0(\theta)$ denote the feasible rate set, when this boundary is at an angle θ with respect to a reference direction. Then, for each $0 \leq \theta \leq \frac{360°}{n}$, we have a rate region $\mathcal{R}_0(\theta)$, where n is the number of sectors. Since $\mathcal{R}_0(\theta)$ is not known, the inner bound $\mathcal{R}_L(\theta)$ (obtained earlier) is used in the following analysis. If each vector $\mathbf{r}$ is assigned a utility function $U(\mathbf{r})$, then one could seek to solve the problem

$$\max_{0 \leq \theta \leq \frac{360°}{n}} \max_{\mathbf{r} \in \mathcal{R}_0(\theta)} U(\mathbf{r})$$

and then orient the sectors corresponding to the optimum value of θ.

We can examine various forms for the utility function. The optimization can be done so as to maximize the average rate allocated to each ST, with the constraint that each ST gets the same average rate. This is called *max-min fairness*. Trying to optimize the rates such that the rate to each ST is maximized will adversely affect the sum capacity of the system. So, take $U(\mathbf{r}) = \sum_{j=1}^{m}(r_j)$. This maximises the sum capacity of the system, giving preference to STs that are in a favourable position, causing less interference to other STs. This has an impact on the fairness of the system. To improve the fairness, we can take the log utility function, $U(\mathbf{r}) = \sum_{i=1}^{m} \log(r(i))$; this is known to lead to what is called *proportional fairness*.

Evaluating the upper and lower bounds derived above, we find that the bounds are close to each other. So, we only report the results from the lower bound. Hence, we have computed $\max_{\mathbf{r} \in \mathcal{R}_L(\theta)} U(\mathbf{r})$ for various values of θ. For each θ we obtain a vector $\mathbf{r}$ of average rates. We evaluate the fairness of this vector by using the the fairness index is given by $\gamma(\mathbf{r}) = \frac{(\sum_{1=1}^{m} r_i)^2}{m \sum_{1=1}^{m} r_i^2}$. If the rates to different STs are equal, then fairness index is 1, and the index decreases if there is rate variability between the STs.

In Figure 5 we plot, as a function of θ, the total rate (left panel) over all the STs for each of the three utility functions, and also the fairness index (right panel) (the lower bounds are plotted here). It can be seen that maximizing the sum rate gives high overall capacity, but poor fairness. On the other hand, maximizing the average rate to each ST gives good fairness, but low sum capacity. Maximizing $\sum_{i=1}^{m} \log r_i$ gives a good trade off between maximizing the system capacity and providing fairness. It is interesting to note that in maximum $\sum_{i=1}^{m} \log r_i$ case, the sum capacity is higher when fairness is lower and vice versa. For example, at $\theta = 10$, we can see that the sum rate is close to 4. The fairness index is also close to 1. So, we may choose this orientation to operate the network.

Note that the above computation can be done off-line once the ST locations are known. Then the sector orientations can be obtained from this analysis.

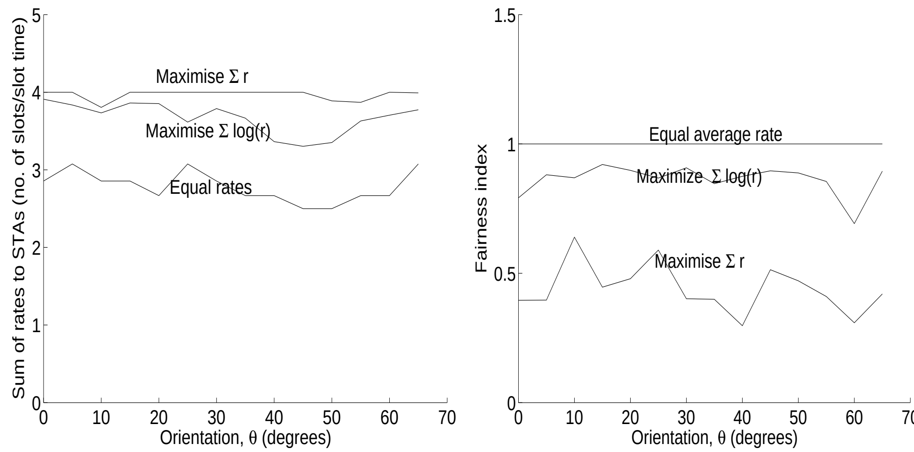

Fig. 5. Variation of sum rate and fairness index with antenna orientation for different utility functions.

V. Scheduling: Problem Formulation and Scheduler Design

Based on the discussion up to this point, the scheduling problem we are faced with is the following.

First partition the frame of size N slots into a contiguous part with N_D downlink slots and an uplink part with N_U uplink slots, such that $N_D + N_U = N - N_B$, where N_B is the number of slots required for the periodic beacon. Typically we will have $N_D \gg N_U$. This is because data transfer traffic is highly asymmetric, as users download a lot more than they upload. During downloads, long TCP packets (up to 1500 bytes) are received in the downlink and one 40 byte TCP ACK is sent in the uplink for every alternate TCP data packet received.

Now, when $m_{v,i}, 1 \leq i \leq m$, VoIP calls are admitted for ST i, we need to determine the number of slots C_i to be reserved in the uplink and downlink subframes for ST i, such that the QoS targets are met for all the voice calls. For doing this, evidently the set of vectors $\mathbf{C} = (C_1, \ldots, C_m)$ that are feasible (i.e., can be scheduled) needs to be known. For each $\mathbf{A}$ and $\mathbf{I}$, there will be an optimal set of such vectors $\mathcal{C}_{opt}(\mathbf{A}, \mathbf{I})$, and for any practical scheduler, there will be an achievable set of admissible vectors $\mathcal{C} \in \mathcal{C}_{opt}(\mathbf{A}, \mathbf{I})$.

Once the required vector of voice payload slots has been scheduled, we need to schedule as many additional payload slots, so as to maximize the traffic carrying capacity for TCP while ensuring some fairness between the flows.

A. Obtaining the Activation Set

Consider a graph with the STs and BTSs as nodes and the communication links between the STs and BTSs as edges. An activation vector is a matching on this graph. [10] proposes randomized algorithms that can be used for finding near-maximal matchings, with complexity O(Number of nodes). But, the inherent graph in the problem we consider being bipartite in nature, and the scheduling being centralized, the maximal matchings can be found without randomized algorithms.

The algorithm for enumerating a maximal activation vector is as follows.

Algorithm 5.1:

1) Choose a link to be included in the activation vector. This might be based on criteria such as (i) the link with the longest queue length, (ii) or the link that received the lowest average rate over previous window of frames.
2) Eliminate all the links that can cause interference to transmission on the set of links chosen until this point.
3) Choose the best link (according to the above criterion) from the remaining set of links.
4) Repeat Steps 2 and 3 until there are no more links that can be activated, or the set contains n_0 links (the maximum that can be activated at a time).

Remarks 5.1:

Suppose that we consider only TCP traffic. If we try to maximize the number of useful slots without any regard to fairness, then the schedule will be to use the vector $\mathbf{U}$ with maximum cardinality ($\mathbf{U} \in \mathcal{U}$) for all slots. Then, a bound on the total number of slots available for transmission would be

$$|\mathbf{U}| \left(\frac{T_{max} - 3}{T_{max}} \right) N$$

if the frame size N is an integral multiple of T_{max}. Note that the maximal activation vector depends on the antenna used, the distribution of STs, etc. If there are several activation vectors with with the maximum cardinality, we can achieve some fairness by cycling between these vectors every T_{max} slots. Still, this throughput maximising approach may result in some STs getting starved. ■

B. The Optimal Scheduling Problem for Uplink

As an illustration, we focus here on the uplink scheduling problem. The uplink scheduling problem avoids the slight complexity in the downlink problem, that, in the downlink, we can combine transmissions from a BTS to multiple STs in the same TB.

An instance of a scheduling problem is defined by an association matrix $\mathbf{A}$, an interference matrix $\mathbf{I}$, and a vector $\mathbf{C} \in \mathcal{C}_{opt}(A, I)$ of required uplink voice capacities for the STs. We formulate the uplink scheduling problem as a *constrained dynamic program* over a finite horizon N_U, i.e., over the indices $k \in \{1, 2, \ldots, N_U\}$, where N_U is the number of uplink slots in a frame. The state of the system at the beginning of slot k is denoted by $(\mathbf{x}_k, \mathbf{q}_k)$, where

$\mathbf{x}_k$: a $1 \times m$ vector with x_{ki} denoting the number of consecutive slots for which ST i has been transmitting; clearly, $\mathbf{x}_0 = (0, 0, \ldots, 0)$

$\mathbf{q}_k$: a $1 \times m$ vector, with q_{ki} being the number of required voice slots yet to be scheduled; clearly $\mathbf{q}_0 = \mathbf{C} = [C_1, C_2, \ldots, C_m]$

At the beginning of slot k, $k \in \{0, 1, \ldots, N_U - 1\}$ an activation vector $\mathbf{u}_k \in \mathcal{U}$ is selected. Then in the slot, all links appearing in $\mathcal{U}$ are allowed to transmit, with the voice queues being depleted first. The state evolves as follows.

$$\begin{aligned} x_{k+1,i} &= f_1(x_{ki}, u_{ki}) \\ &= \begin{cases} (x_{k,i}+1) I_{\{u_{k,B(i)}=i\}} & x_{k,i} + I_{\{u_{k,B(i)}=i\}} < T_{max} \\ 0 & x_{k,i} + I_{\{u_{k,B(i)}=i\}} = T_{max} \end{cases} \end{aligned}$$

i.e., if ST i is scheduled in slot k, $(\mathbf{u}_{k,B(i)} = 1)$ and the maximum TB length has not been reached, then we increment the burst length from ST i by 1, else the burst length is reset to 0. Further, $\mathbf{q}_k$, $k \geq 0$, evolves as follows.

$$q_{k+1,i} = f_2(q_{ki}, x_{ki}, u_{ki}) = (q_{k,i} - I_{\{u_{k,B(i)}=1\}} I_{\{x_{k,i}>3\}})^+$$

i.e., the number of required voice slots reduces by 1 provided the overhead part of the current TB has elapsed. Note that $x^+ = \max(0, x)$.

We now define the reward structure. We wish to satisfy the need for voice slots and having done that, we wish to maximize the number of slots remaining for TCP data. Define the reward in slot k, $0 \leq k \leq N-1$ by

$$g_k(\mathbf{x}_k, \mathbf{u}_k) = \sum_{i=1}^{m} I_{\{u_{k,B(i)}=i\}} I_{\{x_{ki}>3\}}$$

i.e., this is the total number of payload slots scheduled in slot k. Clearly, $g_k(\mathbf{x}_k, \mathbf{u}_k) \leq n$, since there can be at most n transmissions at a time.

Then, we set a terminal cost

$$g_N = \begin{cases} 0 & \mathbf{q}_N = \mathbf{0} \\ -\infty & \text{if } q_{Ni} > 0 \text{ for some } i \end{cases}$$

i.e., we incur an infinite cost if we are unable to schedule all the required voice slots.

A scheduling policy maps the state at slot k to a vector $\mathbf{u} \in \mathcal{U}$. Let π denote a generic policy. Then there is a sequence of functions $\pi_k : (\mathbf{x}, \mathbf{q}) \to \mathcal{U}$ that define the policy. Define

$$J_N^\pi(0, \mathcal{C}) = \sum_{k=0}^{N-1} g_k^\pi(\mathbf{X}_k, \mathbf{u}_k) + g_N^\pi$$

where $\mathbf{X}_k$ evolves as explained as above under the policy π. We wish to solve

$$\max_\pi J_N^\pi(0, \mathcal{C})$$

and obtain the optimal policy. Let $J_N^*(0, \mathcal{C})$ denote the optimal value and π^* be an optimal policy. Since the number of policies are finite for each $C \in \mathcal{C}_{opt}(A, I)$, $J_N^*(0, \mathcal{C})$ is finite and there exists a π^*.

C. On Solving the DP formulation

In the dynamic programming formulation of the scheduling problem, the state of the system can be written as, $\mathbf{X}_k = \begin{pmatrix} \mathbf{x}_k \\ \mathbf{q}_k \end{pmatrix}$. The system evolves as

$$\mathbf{X}_{k+1} = f(\mathbf{X}_k, \mathbf{u}_k) = \begin{pmatrix} f_1(\mathbf{x}_k, \mathbf{u}_k) \\ f_2(\mathbf{q}_k, \mathbf{x}_k, \mathbf{u}_k) \end{pmatrix}$$

The single stage reward in using the control $\mathbf{u}_k$, when the system is in state $\mathbf{X}_k$ is given by

$$g(\mathbf{X}_k, \mathbf{u}_k) = \sum_{i=1}^{m} I_{\{u_{k,B(i)=i}\}} . I_{\{x_{ki}>3\}}$$

Terminal cost is

$$J_N(\mathbf{X}_N) = \begin{cases} 0 & \mathbf{q}_N = \mathbf{0} \\ -\infty & o.w. \end{cases}$$

The control $\mathbf{u}_k$, i.e., the activation vector to be used in slot k, is the one that attains the maximum in the recursion

$$J_k(\mathbf{X}_K) = \max_{\mathbf{u}_k} \{ g(\mathbf{X}_k, \mathbf{u}_k) + J_{k+1}(f(\mathbf{X}_k, \mathbf{u}_k), \mathbf{u}_k) \}$$

We can do a backward recursion, with all possible $\mathbf{X}_N$, and proceed to find the controls that maximize the reward at each stage. The set of controls, $\{\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{N-1}\}$ that maximizes $J_0(\mathbf{0}, \mathbf{q}_0)$ as obtained by the above recursion is the optimal schedule.

But, this approach is feasible [4, Chapter 1] only when the number of stations is small. The size of space occupied by $\mathbf{x}_k$ is almost $(\frac{mT_{max}}{n})^n$, since one station could be transmitting in each sector, and x_{ki} may be any integer between 0 to T_{max}. So also, the number of controls that can be applied increases as $O(N^6)$, since we can choose one station from each sector, such that it obey the constraints. The exponential increase in the size of state space and control space with the number of stations make this approach infeasible.

D. A Greedy Heuristic Scheduler for Voice in Uplink

At each slot k, we heuristically build an activation vector $\mathbf{u}_k \in \mathcal{U}$ starting from an ST in $\{i : q_{k,i} = \max_j q_{k,j}\}$. Then we follow the approach in Algorithm 5.2 each time we choose an ST with max residual $q_{k,j}$

Algorithm 5.2:

1) Modify the voice queue lengths to include the overhead slots required. i.e., if an ST has a voice queue of 2 packets, add 3 slots of PHY overhead to make the queue length 5.
2) Initially, slot index $k = 0$. Let ST i be such that
$$q_{ki} = \max_{l=1\ldots m} \{q_{kl}\}$$
i.e., The ST with longest voice queue at the beginning of slot k is i. Form activation vector $\mathbf{u}$ with link i activated. i.e., $\mathbf{u} = \{i\}$
3) Let ST j be such that
$$q_{kj} = \max_{l} \{q_{kl} : l \notin \mathcal{I}(\mathbf{u})\}$$
j is such that it is the noninterfering ST with maximum queue length. Augment $\mathbf{u}$ with link j. Now, find $\mathcal{I}(\mathbf{u})$ corresponding to the new $\mathbf{u}$.
4) Repeat Step 3 until the activation vector that we get is a maximal activation vector.
5) Let
$$n = \{q_{kl} : \min_{l=1,\ldots,m} (q_{kl}, l \in \mathbf{u})\}$$
i.e., n is the minimum number of slots required for the first ST in $\mathbf{u}$ to complete its transmission. Use $\mathbf{u}$ in the schedule from kth to $(k+n)$th slot.
$$q_{k+n,i} = \begin{cases} q_{k,i} - n & \text{for} \quad i \in \mathbf{u}' \\ q_{k,i} & \text{for} \quad i \notin \mathbf{u}' \end{cases}$$
and $k = k + n$ i.e., slot index advances by n, and the queue length for the STs at the beginning of $k + n$th slot is n less
6) At the end of the $k + n$th slot,
$$\mathbf{u} = \mathbf{u} - \{l : q_{kl} = min(q_{kl}, l \in \mathbf{u})\}$$
i.e., remove from the activation vector, those STs that have completed their voice slot requirement.
7) Go back to Step 3 and form maximal activation vector including $\mathbf{u}$. Continue the above procedure until $\mathbf{q} = \mathbf{0}$ or $n = N_U$ In this step, we form a new activation vector with the remaining STs in the activation vector (which need more slots to complete their requirement).
8) Once the voice packets are transmitted, we serve the TCP packets in the same way, except that if in forming a maximal activation set, it is found that the only schedulable ST has only TCP packets to send, then TCP packets are scheduled.

If $\mathbf{q} > \mathbf{0}$ when $n = N_U$, the allocation is infeasible.

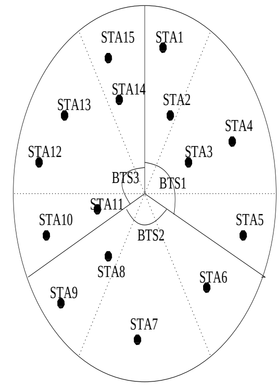

Fig. 6. A typical deployment of a system with 3 sectors and 15 STs.

E. A Greedy Heuristic Scheduler for Voice in Downlink

The difference of the downlink scheduling problem from the uplink scheduling problem is that in the downlink, a transport block can contain packets to multiple STs. By combining the voice packets to different STs to a single TB, we save considerable PHY overhead. For transmitting a single voice packet needs 4 slots, where 3 slots are for the PHY header. Transmitting 2 voice packets need only 5 slots. So, it is always advantageous to have transmissions in longer blocks. This can be done by grouping together the STs to those which are heard only by ith BTS, those heard by ith and $(i-1)$th BTS, but associated to the ith BTS and those heard by ith and $(i-1)$th BTS, but associated to the $(i-1)$th BTS, for all values of i.

In Figure 6 we show a simple deployment, with 3 BTSs. In each sector the taboo regions are also shown. STs 3 and 4 are associated with BTS 1 and are not in either of the taboo regions. So, any ST in the interference set of 3 will also be in the interference set of 4. Any transmission to ST 3 can equivalently be replaced by a transmission to 4. Thus, they form a group for the down link schedule. Similarly, STs 8 and 9 are associated with BTS 2 and interfere with BTS 3. They are associated to the same BTS and cause interference to the same STs. So, ST 8 and 9 also form a group.

The STs are grouped together based on the above criterion. The queue length of each group would be the sum of queue lengths of the STs forming the group. The greedy heuristic scheduler for the uplink scheduling problem can then be used over these groups.

F. Round Robin Scheduling

A low complexity scheduler can be designed as follows. The uplink and downlink parts of the frame may further be divided into two contiguous parts. Alternate sectors are served in these two parts. For example, with 6 sectors, Sectors 1, 3, 5 are served in the first part, and Sectors 2, 4, 6 can be served in the second part of the frame. Interference between adjacent sectors can be eliminated in this way. Within the

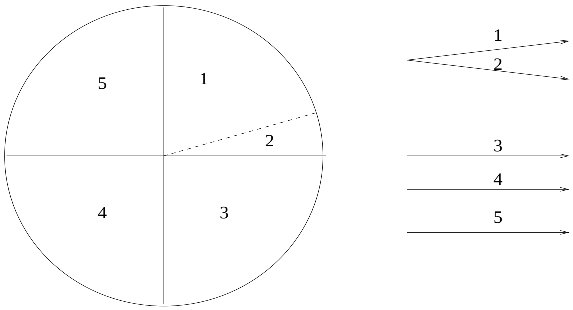

Fig. 7. Deployment of a system with 4 sectors and 5 STs

round robin scheduler, the STs can be sceduled based on queue lengths. With the number of sectors equal to $2n_0$, the performance of this scheduler would be equivalent to that of the scheduler discussed in Section D, since we can have n_0 transmissions going on in each slot, with this scheduler. But, with $n_0 = 4$, this would require 8 sectors in the system. With the number of sectors less than $2n_0$, the number of simultaneous transmissions would be less than n_0 with the round robin algorithm, whereas we can have up to n_0 transmissions with the greedy algorithm.

The round robin scheduler can achieve maximum throughput when the distribution of villages and traffic is uniform. But under admissible traffic it might lead to instability and unfairness. This can be demonstrated by a simple example. Consider the deployment of 5 STs in four sectors, as shown in Figure 7. STs 1 and 2 are in the same sector. The scheduling constraints are that $n_0 = 3$ and Links 2 and 3 cannot transmit together. The arrival rate vector is denoted by a vector $\mathbf{a}$, where a_i is the arrival rate at the BTS for ST i ST i. An arrival rate $(1/2, 1/2, 1/2, 1/2, 1)$(slots/slot time) is admissible, but, not schedulable by a round robin scheduler.This is clear from the examples in Tables 2 and 3. Table 2 shows the a schedule that schedules maximal independent sets. Table 3 shows the way the round robin scheduler schedules the STs, where STs 1 and 2 are scheduled only in alternate bursts, so that the two STs have to share the slots, such that they are scheduled only in half the slots. We see that the service rates applied to ST 1 and ST 2 are $\frac{1}{4}$ and $\frac{1}{4}$ and to ST 5 is $\frac{1}{2}$.

1	2	1	2	1	2	1
3		3		3		3
	4		4		4	
5	5	5	5	5	5	5

TABLE 2
SCHEDULE FOR MAXIMAL INDEPENDENT SET SCHEDULER.

Another observation is that with the increase in variability of the distribution of STs in sectors, the round robin scheduler tends to become unfair.

1		2		1		
	3		3		3	
4		4		4		4
	5		5		5	

TABLE 3
SCHEDULE FOR ROUND ROBIN SCHEDULER.

G. Fair Scheduling for Data

To provide fairness to users, we keep track of the average rates allocated to STs over time. The STs with low average rate are given the chance to transmit first. Maximal independent sets are formed starting from the ST with the lowest average rate. Once the slots for voice transmission are scheduled, we attempt to include TCP transmissions in blocks of size T_{max}, so that the PHY overhead per slot is minimized.

Let $\mathbf{R}_k$ be the vector of average rates allocated to STs until the kth slot and $\mathbf{r}_k$ be the vector of rates allocated to the STs in the kth slot.

$$\mathbf{R}_{k+1} = \alpha\mathbf{R}_k + (1-\alpha)\mathbf{r}_k$$

1) Given a rate vector $\mathbf{R}$, obtain a maximal independent set as follows
   a) $\mathbf{u}_1 = \{i_1\}$
      $i_1 = \arg\min_{1\leq j\leq n} \mathbf{R}_j$
      $\mathcal{I}(\mathbf{u}_1)$ is the set of links interfering with the links in $\mathbf{u}_1$. In this step, we select the ST with the smallest average rate R_k for transmission.
   b) Choose $i_2 \in \arg\min_{1\leq j\leq n, i_2\notin\mathcal{I}(\mathbf{u_1})} \mathbf{R}_j$
      $\mathbf{u}_1 = \{i_1, i_2\}$. In this step, we select one of the non interfering STs with minimum average rate for transmission.
   c) Repeat the above until a maximal independent set is obtained. Now, we have a set with STs which have received low average rates in the previous slots. So, once all STs transmit their voice packets, we schedule these STs for data packets.
2) Let l_1 denote the number of nodes in $\mathbf{u_1}$ at the end of step 1. Repeat the above for the remaining $n - l_1$ nodes. Now we have a maximal independent set from the remaining $N_u - l_1$ nodes. If any one of the l_1 nodes can be activated along with the maximal independent set formed from the $N_u - l_1$ nodes, add that till one get a maximal independent set. This yields $\mathbf{u}_1, \mathbf{u}_2 \ldots \mathbf{u}_k$ such that each node is included at least once. Each node is included at least once since a given number of slots is to be reserved for each ST in every frame.
3) Now, we need to schedule $\mathbf{u}_1$ for t_1, $\mathbf{u}_2$ for t_2, etc. To maximize throughput, we take $t_j = T_{max}$ or number of voice slots required. The vectors in the initial part of the schedule had low average rate over frames. So, they get priority to send data packets. So, starting from j=1, i.e., from the first activation vector, if the sum of

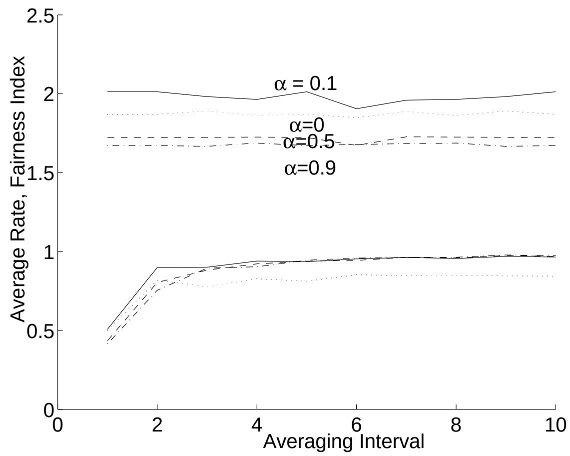

Fig. 8. Variation of total rate and fairness index with averaging interval for different values of α. The upper set of plots are of the total rate, and bottom set are for the fairness index.

n_0 θ		Number of voice calls per station		
		1	2	3
3, 10°	min d/l rate	164	148	134
	max d/l rate	178	182	167
	sum d/l rate	13749	12852	11690
3, 10°	min d/l rate	163	151	136
	max d/l rate	179	173	177
	sum d/l rate	13545	12798	11799
3, 10°	min d/l rate	167	153	137
	max d/l rate	180	173	161
	sum d/l rate	13883	13000	11750
3, 10°	min d/l rate	224	204	190
	max d/l rate	294	278	258
	sum d/l rate	19807	18377	17007
3, 10°	min d/l rate	204	194	177
	max d/l rate	283	255	274
	sum d/l rate	19312	17919	16430
3, 10°	min d/l rate	172	165	140
	max d/l rate	212	208	190
	sum d/l rate	15573	14078	12499

TABLE 4

SIMULATION RESULTS FOR DOWNLINK DATA RATES WITH 80 STS IN 6 SECTORS, AVERAGED OVER 30 RANDOM DEPLOYMENTS. THE DATA THROUGPUTS ARE GIVEN IN KILO BITS PER SECOND.

number of slots allocated to STs in the frame is less than N_u, $t_j = T_{max}$. Else, $t_j =$ number of voice slots required.Therefore transmission takes place in blocks of length equal to T_{max} as long as it is possible.

4) Update the rate vector as

$$\mathbf{R}_{k+1} = \alpha \mathbf{R}_k + (1 - \alpha)\mathbf{r}_k$$

We simulated the algorithm and obtained the fairness index of the rates allocated, and the total rate achieved for various values of α. These are plotted vs. the rate averaging interval in Figure 8. The averaging interval on the x axis is the number of frames over which the average throughput or fairness index is calculated. The fairness index is found to be close to one unless the averaging interval is very small. This occurs partly because of the small number of STs considered. A larger α in the rate averaging algorithm yields a smaller average throughput.

VI. VOICE AND DATA CAPACITY: SIMULATION RESULTS

The scheduling algorithm discussed in Section D was implemented in a MATLAB simulation. The PHY rate is 11 Mbps. We consider a random distribution of 80 STs in 6 sectors. The spatial reuse n_0 of 3 or 4 has been considered, and the taboo regions in each sector, on either side of the sector, are $\theta = 10°, 20°, 30°$. Simulation is done with all STs having the same number of ongoing voice calls: 1, 2 or 3. One VoIP call requires one slot every alternate frame. A voice packet that arrives in the system is scheduled within the next two frames. If the scheduling constraints do not allow the voice packet to be transmitted within two frame times of arrival, the packet is dropped. In the simulation, we have assumed synchronous arrival of voice packets, i.e., if two voice calls are going on from an ST, packets for both calls arrive synchronously, in the same frame. The data traffic model is that all the STs have packets to be transmitted throughout.

The results are shown in Table [4] and Table [5]. Here, *min d/l rate* is the average of the minimum rate over STs in the downlink, averaged over 30 random deployments; *max d/l rate* is the average of the maximum rate over STs in the downlink, and *sum d/l rate* is the average of the sum of downlink rates to the STs. The same measures are also given for the uplink. The *packet drop u/l* is the fraction of voice packets dropped in the uplink, this being the bottleneck direction. All the rates indicated are in terms of the MAC payload. The PHY overhead has already been accounted for in the calculations.

Each voice call requires a payload of 44 Bytes every 20 ms, and hence 1.41 Mbps are utilised per voice call, in the uplink and downlink, for 80 STs. With a PHY rate of 11 Mbps, with $n_0 = 3$ we have an aggregate nominal rate of 22 Mbps in the downlink and 11 Mbps in the uplink (assuming that 2/3 of the frame time is allocated to the downlink). From the table, it can be seen that with 80 STs in 6 sectors, and 1 voice call, with a taboo region of $10°$ on either side of each sector, and $n_0 = 3$, each ST gets an average minimum data throughput of 164 Kbps, and the average total rate is 13.749 Mbps. Adding to this 1.41 Mbps, we obtain about 15.16 Mbps, for a nominal downlink bandwidth of 22 Mbps. The difference is because of PHY overheads, and the inability to fill up all slots in a frame. We notice that a second simultaneous call at each ST reduces the data throughput by less than 1 Mbps; this is because the packing can become more efficient. For this same case, with one voice call, the average minimum uplink data throughput is 17 kbps, and the average total downlink data throughput is 3.57 Mbps. Adding

n_0 θ		Number of voice calls per station		
		1	2	3
3, 10°	min u/l rate	17.1	8.1	0
	max u/l rate	85	59	34
	sum u/l rate	3570	2286	1229
	packet drop u/l	0	0.0029	0.0229
3, 20°	min u/l rate	13	5	0
	max u/l rate	88	57	31
	sum u/l rate	3510	2285	1110
	packet drop u/l	0	0.0033	0.0312
3, 30°	min u/l rate	16	5	0
	max u/l rate	83	62	43
	sum u/l rate	3463	2114	1176
	packet drop u/l	0	0.0042	0.0346
4, 10°	min u/l rate	38	18	0
	max u/l rate	106	92	78
	sum u/l rate	5161	3776	2906
	packet drop u/l	0	0.0029	0.0283
4, 20°	min u/l rate	25	9	0
	max u/l rate	157	168	160
	sum u/l rate	4833	3699	2771
	packet drop u/l	0	0.0025	0.0304
4, 30°	min u/l rate	15	7	0
	max u/l rate	92	70	53
	sum u/l rate	3468	2400	1359
	packet drop u/l	0	0.0029	0.0354

TABLE 5

SIMULATION RESULTS FOR UPLINK DATA RATES AND PACKET DROP WITH 80 STS IN 6 SECTORS, AVERAGED OVER 30 RANDOM DEPLOYMENTS. THE DATA THROUGPUTS ARE GIVEN IN KILO BITS PER SECOND.

to this 1.41 Mbps for voice, we obtain a total uplink utilisation of 5.18 Mbps over a nominal bandwidth of 11 Mbps allocated to the uplink. Because of being smaller, the uplink frame is more inefficiently packed.

If $n_0 = 3$ and a taboo region of width $\theta = 10°$, the fraction of voice packets dropped is 0.29% when we support 2 calls per ST and 2.29% when we support 3 calls per ST. With 3 voice calls per station, we can see that the packet drop is high, and the uplink capacities to some STs are 0. With $n_0 = 3$, the width of the taboo region does not have an effect on the system capacity, since we are always able to schedule in 3 sectors. With $n_0 = 4$, the system capacity reduces as θ increases. With $\theta = 30°$, we can usually schedule transmissions in just 3 sectors in a slot, even though the SINR constraints allows 4 transmissions in a slot.

VII. CONCLUSION

We consider the problem of finding the amount of spatial reuse possible with a single channel multi sector WIFiRE system with 802.11 MAC. It has been found that there is an optimum value for the number of simultaneous transmissions possible, so as to maximize the total system capacity. The number of simultaneous transmissions is found to depend on the path loss factor and the radiation pattern of the antenna. For the antenna pattern considered, and for path loss factor 2.3, as is applicable for rural environments, the number of simultaneous transmissions possible is found to be 3. This can be improved by the user of antennas with lesser back lobe radiation.

Also, for a given deployment, we find bounds to the system capacity, assuming full channel reuse. Based on this we can find the optimum positioning of the antenna, such that the log utility function of rates obtained by different STs is maximized. The bounds obtained here are weak since we do not consider the constraint imposed by the maximum number of simultaneous transmissions.

A constrained dynamic programming problem is found to give the optimum schedule for the system. But, the problem is intractable due to the explosion of state and action space. We employ a maximal weight algorithm were the weights are the queue lengths of the voice queue, such that each ST transmits in contiguous slots, so as to minimize the PHY overhead. For scheduling data in the system, we follow the maximal weight algorithm, where the weights are reciprocal of the average rate obtained by each ST in previous slots. The average considered is an exponential weighted average of rates. A simple round robin scheduler have also been considered. Scheduling examples are given for the greedy heuristic scheduler.

The different schedulers considered were implemented in MATLAB and the data throughput in each case is obtained as the average data throughput over deployments, in terms of payload slots. Deployments with different number of sectors and STs, width of taboo region have been considered for different voice loads, and the data throughput has been obtained in each case.

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