OTFS 2.0 (Zak-OTFS): A Waveform for Communication and Radar Sensing in 6G and Beyond

ECE Faculty Colloquium

IISc, Bangalore

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Joint work with S. K. Mohammed, R. Hadani, R. Calderbank

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Outline I

6G and beyond

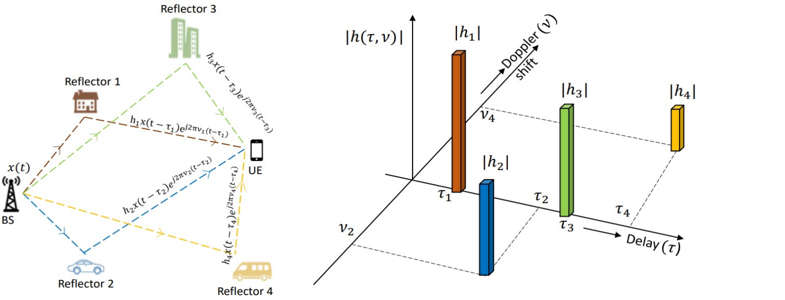
1. Operational space
    - 3D communication
    - confluence of terrestrial, UAV/drones/aeroplanes, LEO satellites
    - high relative velocities
2. Spectrum space
    - 28 GHz limited success so far
    - sub-6 GHz will continue to be important
    - mmWave frequencies (30 to 300 GHz) yet to make a mark
    - THz research is opening up
3. Application space
    - popular use of AR/VR/XR
    - holographic communication - widely anticipated XR use case
4. Physical layer space
    - waveforms
        - robust to high-mobility/high-Doppler
        - for integrated communication and radar sensing
    - intelligent surfaces for beamforming and modulation

High-Dopplers in 6G and beyond

- Dopplers in several KHz range
- Traditional multicarrier modulation schemes fail to deliver robust performance at such high Dopplers
  - reason: inter-carrier interference (ICI) due to Doppler

Delay-Doppler (DD) Domain

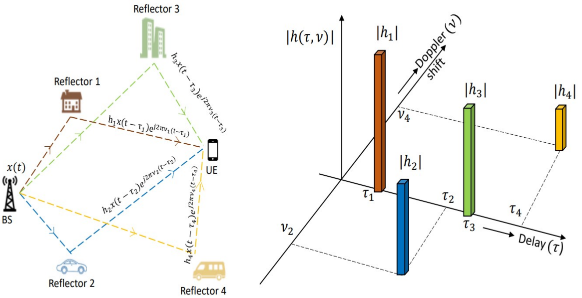
- Wireless channels are doubly-spread



- Delay-Doppler spreading function: $h(\tau,\nu) = \sum_{i=1}^{4} h_i \delta(\tau - \tau_i)\delta(\nu - \nu_i)$
- Received signal: $y(t) = \iint h(\tau,\nu) x(t-\tau) e^{j2\pi\nu(t-\tau)} \, d\tau d\nu = \sum_{i=1}^{4} h_i x(t-\tau_i) \, e^{j2\pi\nu_i(t-\tau_i)}$
- Signal received along i-th path: $h_i \underbrace{x(t-\tau_i)}_{\text{delay: } \tau_i} \underbrace{e^{j2\pi\nu_i(t-\tau_i)}}_{\text{Doppler shift: } \nu_i}$

Orthogonal time frequency space (OTFS 1.0) modulation*

- A promising modulation scheme for doubly-spread channels
- Information is multiplexed in the delay-Doppler (DD) domain
  - Map information from DD domain to time domain and transmit
    - DD domain → TF domain → time domain
- Channel is viewed/represented in DD domain



(*) R. Hadani, S. Rakib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch, and R. Calderbank, "Orthogonal time frequency space modulation," in *Proc. IEEE WCNC*, San Francisco, CA, USA, Mar. 2017.

OTFS - Signaling in DD domain

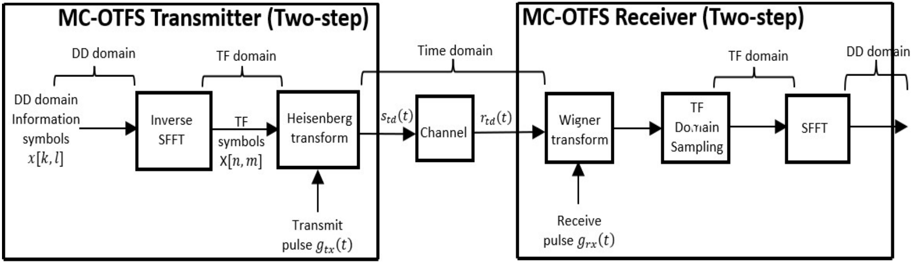


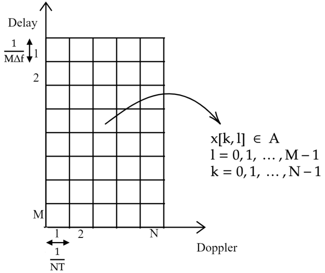
Figure: Multicarrier OTFS (OTFS 1.0)

- Tx
  - DD domain → TF domain: Inverse SFFT
  - TF domain → time domain: Heisenberg transform
- Rx
  - Time domain → TF domain: Wigner transform
  - TF domain → DD domain: SFFT

* Best Readings in Orthogonal Time Frequency Space (OTFS) and Delay Doppler Signal Processing.
https://www.comsoc.org/publications/best-readings/orthogonal-time-frequency-space-otfs-and-delay-doppler-signal-processing

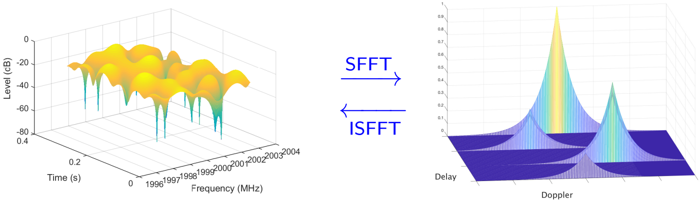
Delay-Doppler grid

- Signaling in DD grid



- NxM delay-Doppler grid
- NM information symbols
- Time duration of NT
- Bandwidth of $M\Delta f$

- Channel viewed in DD grid
- $h(\tau, \nu) = \sum_{i=1}^{P} h_i \delta(\tau - \tau_i)\delta(\nu - \nu_i)$



Input-output relation

- Received signal in DD domain[1]
  - for $\tau_i \triangleq \frac{\alpha_i}{M\Delta f}$ and $\nu_i \triangleq \frac{\beta_i}{NT}$, α_i and β_i are integers

$$y[k,l] = \sum_{i=1}^{P} h'_i \, x[(k-\beta_i)_N, (l-\alpha_i)_M] + v[k,l]$$

where $h'_i = h_i e^{-j2\pi\nu_i\tau_i}$, $h_i \sim \mathcal{CN}(0, 1/P)$

- Input-output relation can be vectorized as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v},$$

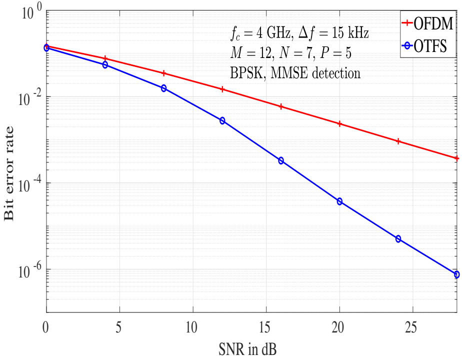
where $\mathbf{x}, \mathbf{y}, \mathbf{v} \in \mathbb{C}^{MN\times 1}$, $\mathbf{H} \in \mathbb{C}^{MN\times MN}$, $x_{k+Nl} = x[k,l]$, $y_{k+Nl} = y[k,l]$

- This vectorized system model has enabled development of several
  - signal detection (e.g., message passing detection) and DD channel estimation algorithms for OTFS

[1] P. Raviteja, K. T. Phan, and E. Viterbo, "Interference cancellation and iterative detection for orthogonal time frequency space modulation," *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 6501-6515, Oct. 2018.

Why OTFS?

- OTFS vs OFDM performance



Parameter	Value
Carrier frequency (GHz)	4
Subcarrier spacing (kHz)	15
Frame size (M, N)	$(12, 7)$
Number of paths (P)	5
Delay profile	Exponential
Maximum speed (km/h)	500
Maximum Doppler (Hz)	1875
Modulation scheme	BPSK

* Smallest resource block used in LTE: $M = 12$, $N = 7$

- MMSE detection

- OFDM performs poor due to Doppler induced ICI
- OTFS performs significantly better than OFDM

* G. D. Surabhi, R. M. Augustine, and A. Chockalingam, "On the diversity of uncoded OTFS modulation in doubly-dispersive channels," *IEEE Trans. Wireless Commun.*, vol. 18, no. 6, pp. 3049-3063, Jun. 2019.

OTFS 2.0 (Zak-OTFS): What and why?

- What?
  - Transmitter
    - DD domain-to-time domain conversion in one step (inverse Zak transform)
  - Receiver
    - Time domain-to-DD domain conversion in one step (Zak transform)

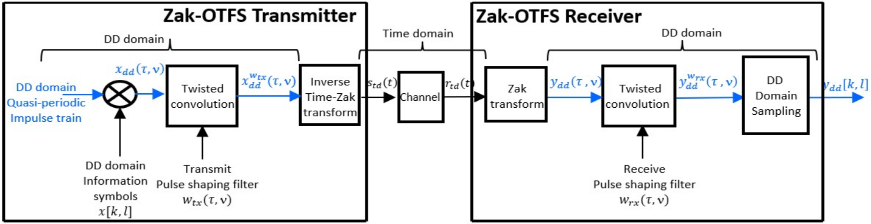


Figure: Signal processing in Zak-OTFS (OTFS 2.0)

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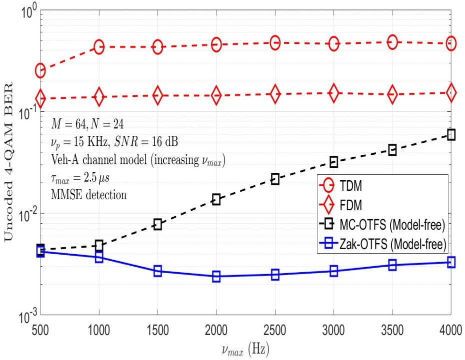
[1] S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, "OTFS – A mathematical foundation for communication and radar sensing in the delay-Doppler domain," to appear in IEEE BITS the Information Theory Magazine. Available in IEEE Xplore Early Access. Also available at https://arxiv.org/abs/2302.08696

[2] S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, "OTFS - Predictability in the delay-Doppler domain and its value to communication and radar sensing," under review. Available at https://arxiv.org/pdf/2302.08705.pdf

- Why?
  - Formal mathematical framework (Zak theory)
  - More robust to large channel spreads compared to OTFS 1.0
  - A good radar waveform

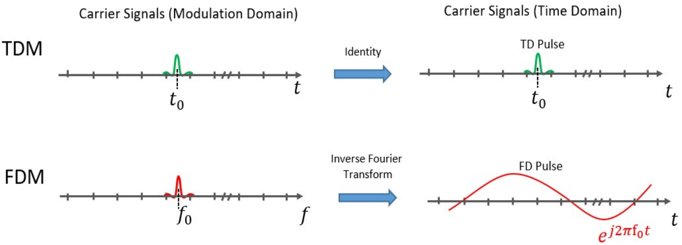


[2] S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, "OTFS - Predictability in the delay-Doppler domain and its value to communication and radar sensing," under review. Available at https://arxiv.org/pdf/2302.08705.pdf

Key phrases in OTFS 2.0 (Zak-OTFS)

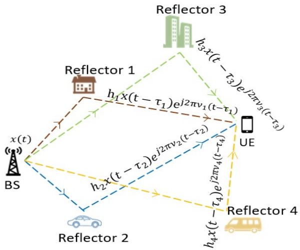
- Waveform
  - DD domain pulse
  - Pulsone: pulse train modulated by a tone
  - Quasi-periodic function
  - TD and FD pulses are special cases of pulsone
- Transforms
  - Zak and Inverse Zak transforms
- Operation
  - Twisted convolution
  - Cascade of twisted convolutions
- Important phenomenon
  - DD domain aliasing
- Preferred operating regime
  - Crystalline regime
  - Regime where crystallization condition holds
- Favorable attributes
  - Predictability of input-output relation
  - Non-fading
- Radar ambiguity function

TDM, FDM carrier waveforms

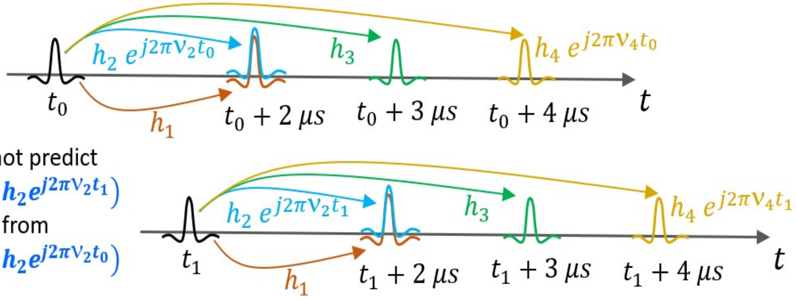


- TDM carrier waveform
  - A pulse in TD - localized in TD, not in FD (good for delay-only channels)
- FDM carrier waveform
  - A pulse in FD (sinusoid in TD) - localized in FD, not in TD (good for Doppler-only channel)
- Implication
  - In doubly-spread channels, TDM/FDM input-output (I/O) relation witnesses
    - fading (leading to BER degradation)
    - non-predictability (leading to frequent acquisition of channel)
- Predictability
  - Channel response to an impulse at any arbitrary location can be estimated/predicted from the response to a particular impulse (pilot impulse)

TDM: Non-predictable and fading channel interaction

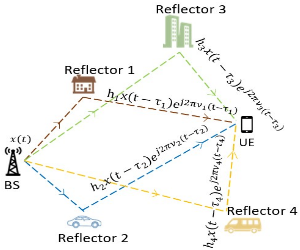


- Path delay (μs) and Doppler (Hz) shifts:
  $\tau_1 = \tau_2 = 2,\ \tau_3 = 3,\ \tau_4 = 4$
  $\nu_1 = \nu_3 = 0,\ \nu_2 = -950,\ \nu_4 = 750$
- Can we predict the response to $\delta(t - t_1)$ from the response to $\delta(t - t_0)$? No !!
- Fading interaction:
  $|h_1 + h_2 e^{j2\pi\nu_2 t_0}| \neq |h_1 + h_2 e^{j2\pi\nu_2 t_1}|$

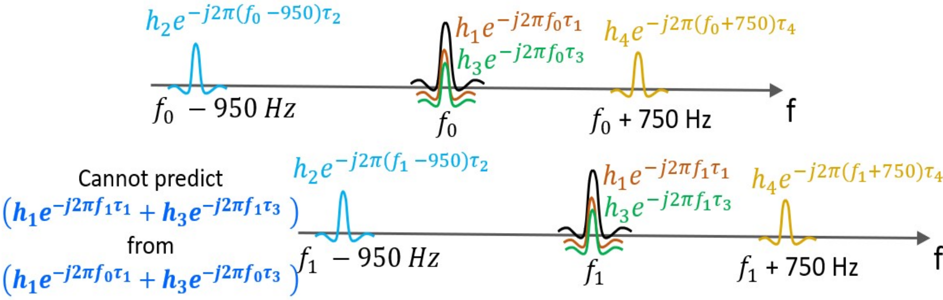


Cannot predict
$(h_1 + h_2 e^{j2\pi\nu_2 t_1})$
from
$(h_1 + h_2 e^{j2\pi\nu_2 t_0})$

FDM: Non-predictable and fading channel interaction



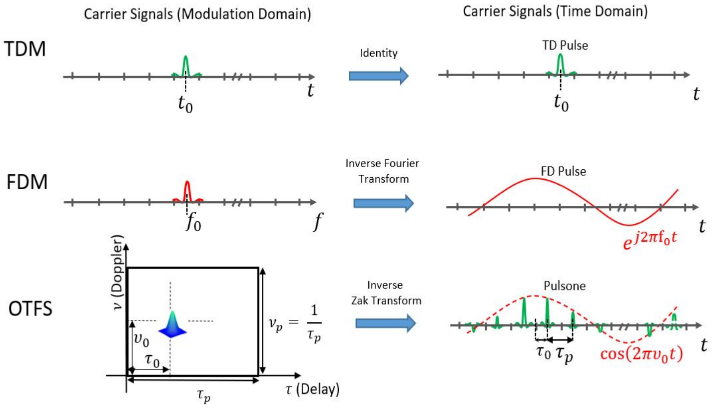
- Path delay (μs) and Doppler (Hz) shifts:
  $\tau_1 = \tau_2 = 2,\ \tau_3 = 3,\ \tau_4 = 4$
  $\nu_1 = \nu_3 = 0,\ \nu_2 = -950,\ \nu_4 = 750$
- Can we predict the response to $\delta(f - f_1)$ from the response to $\delta(f - t_0)$? No !!
- Fading interaction: $|h_1 e^{-j2\pi f_1 \tau_1} + h_3 e^{-j2\pi f_1 \tau_3}| \neq |h_1 e^{-j2\pi f_0 \tau_1} + h_3 e^{-j2\pi f_0 \tau_3}|$



Predictability and fading

- Predictability and fading
  - Attributes of channel interaction with the carrier waveform
- Predictability in doubly-spread channels
  - TDM interaction is NOT predictable as TD pulses are spread in frequency
  - FDM interaction is NOT predictable as FD pulses are spread in time
  - Pulses need to be localized in both TD and FD
  - NOT possible: Heisenberg's uncertainty principle
  - Can the obstruction of simultaneous TD/FD localization be eliminated?
  - Yes: Quasi-periodic pulses in the delay-Doppler (DD) domain

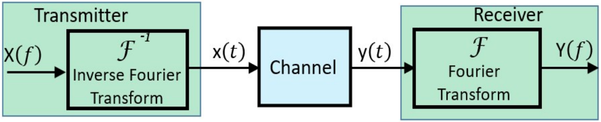
Information carrier in TDM/FDM/Zak-OTFS



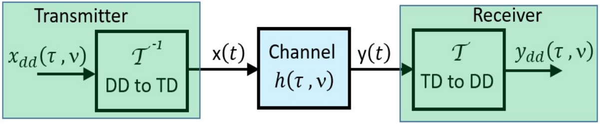
- Zak-OTFS carrier: Quasi-periodic pulse in DD domain. TD realization is a pulse train modulated by a sinusoid (Pulsone).

Modulation domain to time domain

- FD to TD (FDM)



- DD to TD



- Questions:
  - what is the transform $\mathcal{T}$?
  - what is the channel action, i.e., what operation between $h(\tau, \nu)$ and $x_{\text{dd}}(\tau, \nu)$, would give $y_{\text{dd}}(\tau, \nu)$?

Why Zak transform?

- In TDM and FDM
  - cascade of two channels: effective channel impulse response is linear convolution of the impulse responses of the two channels
  - channel action on carrier waveform is also described by linear convolution
- Cascade of two doubly-spread channels $h_1(\tau,\nu)$ and $h_2(\tau,\nu)$
  - effective channel response is twisted convolution of $h_1(\tau,\nu)$ and $h_2(\tau,\nu)$

$$
\begin{aligned}
h(\tau,\nu) &= h_2(\tau,\nu) *_\sigma h_1(\tau,\nu) \\
&= \iint h_2(\tau',\nu')\, h_1(\tau-\tau',\nu-\nu')\, e^{j2\pi\nu'(\tau-\tau')}\, d\tau'\, d\nu'
\end{aligned}
$$

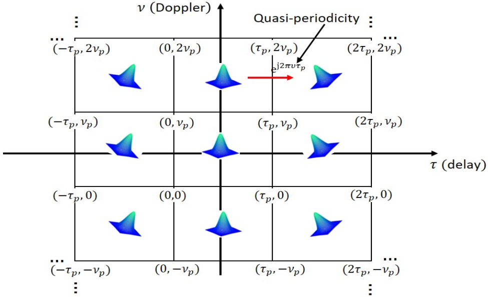
- What is the transform for which that the channel action is twisted convolution, i.e., what is the $\mathcal{T}$ for which

$$
y_{\text{dd}}(\tau,\nu) = h(\tau,\nu) *_\sigma \mathcal{T}\Big(x(t)\Big)
$$

  - Ans: $\mathcal{T}$ is Zak transform, denoted by $\mathcal{Z}_t$ parameterized by (τ_p,ν_p), $\nu_p = 1/\tau_p$

Why Zak transform?

- Zak transform: $x(t) \xrightarrow{\mathcal{Z}_t} x_{\text{dd}}(\tau, \nu) = \sqrt{\tau_p} \sum_{k=-\infty}^{\infty} x(\tau + k\tau_p)\, e^{-j2\pi k \nu \tau_p}$
- For any $n, m \in \mathbb{Z}$, $x_{\text{dd}}(\tau, \nu)$ satisfies $x_{\text{dd}}(\tau + n\tau_p, \nu + m\nu_p) = e^{j2\pi n \nu \tau_p}\, x_{\text{dd}}(\tau, \nu)$



- Any DD domain signal (which is the Zak-transform of some TD signal) is quasi-periodic in the DD domain with delay and Doppler periods τ_p and ν_p
- TD realization of a DD function exists only if it is quasi-periodic

* J. Zak, "Finite translations in solid state physics," *Phy. Rev. Lett.*, 19, pp. 1385-1387, 1967.
* A. J. E. M. Janssen, "The Zak transform: a signal transform for sampled time-continuous signals," *Philips J. Res.*, 43, pp. 23-69, 1988.

Zak Transform: From TD to DD

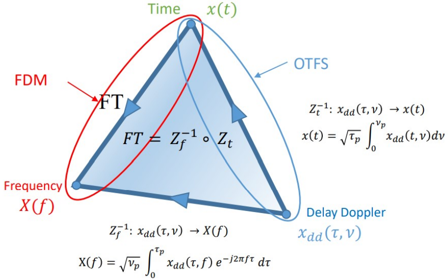
- A DD domain pulse is localized in the fundamental DD period

$$\mathcal{D}_0 \triangleq \left\{ (\tau, \nu) \,\middle|\, 0 \leq \tau < \tau_p,\ 0 \leq \nu < \nu_p \right\}$$

- Inverse time-Zak transform (DD $\rightarrow$ TD):

$$x_{\mathrm{dd}}(\tau, \nu) \xrightarrow{\mathcal{Z}_t^{-1}} x(t) = \mathcal{Z}_t^{-1}\left(x_{\mathrm{dd}}(\tau, \nu) \right) \triangleq \sqrt{\tau_p} \int_0^{\nu_p} x_{\mathrm{dd}}(t, \nu)\, d\nu$$

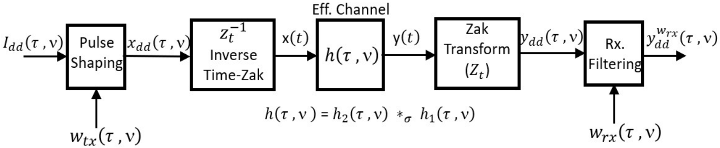
- Signal realizations and transforms



TDM, FDM, DD domain modulation

Modulation	Channel	Domain	Transform	Channel action	Information carrier
TDM	Delay-only	TD	Identity	Linear convolution	TD pulse
FDM	Doppler-only	FD	Fourier	Linear convolution	FD pulse
DD domain Modulation	Doubly-spread	DD	Zak	Twisted convolution	DD pulse

Tx/Rx signal processing



- $y_{\text{dd}}(\tau,\nu) = h(\tau,\nu) *_\sigma x_{\text{dd}}(\tau,\nu)$
- Pulse shaping at Tx: $x_{\text{dd}}(\tau,\nu) = w_{tx}(\tau,\nu) *_\sigma \overbrace{I_{\text{dd}}(\tau,\nu)}^{\text{information}}$
- $*_\sigma$ preserves quasi-periodicity
- Filtering at Rx: $y_{\text{dd}}^{w_{rx}}(\tau,\nu) = w_{rx}(\tau,\nu) *_\sigma y_{\text{dd}}(\tau,\nu)$
- I/O relation: $y_{\text{dd}}^{w_{rx}}(\tau,\nu) = w_{rx}(\tau,\nu) *_\sigma \Big(h(\tau,\nu) *_\sigma \big(w_{tx}(\tau,\nu) *_\sigma I_{\text{dd}}(\tau,\nu)\big)\Big)$
- $*_\sigma$ is associative: $\underbrace{y_{\text{dd}}^{w_{rx}}(\tau,\nu)}_{\text{output}} = \underbrace{(w_{rx}(\tau,\nu) *_\sigma h(\tau,\nu) *_\sigma w_{tx}(\tau,\nu))}_{\text{Eff. DD response } h_{dd}(\tau,\nu)} *_\sigma I_{\text{dd}}(\tau,\nu)$
- Twisted convolution → predictable I/O relation

Channel interaction of DD pulse

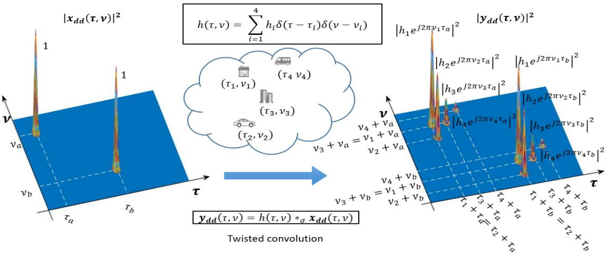


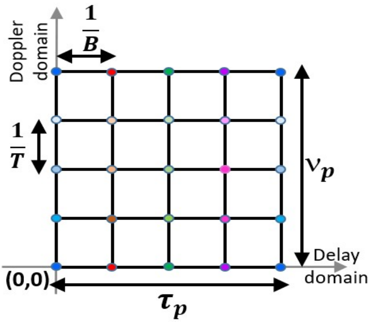
Figure: Channel response for two impulses at (τ_a, ν_a) and (τ_b, ν_b)

- **Question:** Can b-response be predicted from a-response?
- **Ans:** Yes. Provided the following condition (crystallization condition) holds

$$\tau_p > \underbrace{\left(\max_i \tau_i - \min_i \tau_i\right)}_{\text{max. delay spread}}, \quad \nu_p > \underbrace{\left(\max_i \nu_i - \min_i \nu_i\right)}_{\text{max. Doppler spread}}$$

- Prediction of the (n, m)-th term in b-response
  - Shift the (n, m)-th term in a-response by $(\tau_b - \tau_a, \nu_b - \nu_a)$
  - i-th path channel gain:
    $h_i e^{j2\pi\nu_i\tau_b}\, e^{j2\pi n(\nu_b+\nu_i)\tau_p} = h_i e^{j2\pi\nu_i\tau_a}\, e^{j2\pi n(\nu_a+\nu_i)\tau_p} \times e^{j2\pi\nu_i(\tau_b-\tau_a)}\, e^{j2\pi n(\nu_b-\nu_a)\tau_p}$
- Non-fading interaction: $|h_i e^{j2\pi\nu_i\tau_b}\, e^{j2\pi n(\nu_b+\nu_i)\tau_p}| = |h_i e^{j2\pi\nu_i\tau_a}\, e^{j2\pi n(\nu_a+\nu_i)\tau_p}| = |h_i|$

Zak-OTFS modulation



- Information symbols transmitted on DD pulses located on the Information Grid
- Information Grid: In $\mathcal{D}_0$,

  $M = \frac{\tau_p}{(1/B)} = B\tau_p$ points along delay domain and

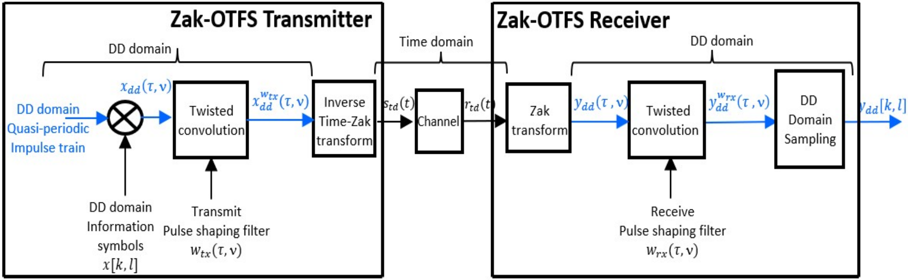
  $N = \frac{\nu_p}{1/T} = T\nu_p$ points along Doppler domain
- No. of information symbols in an OTFS frame

  $=$ No. of grid points in $\mathcal{D}_0$

  $= M \times N = B\tau_p \times T\nu_p$

  $= BT$ (time-bandwidth product)

Zak-OTFS: Transceiver signal processing

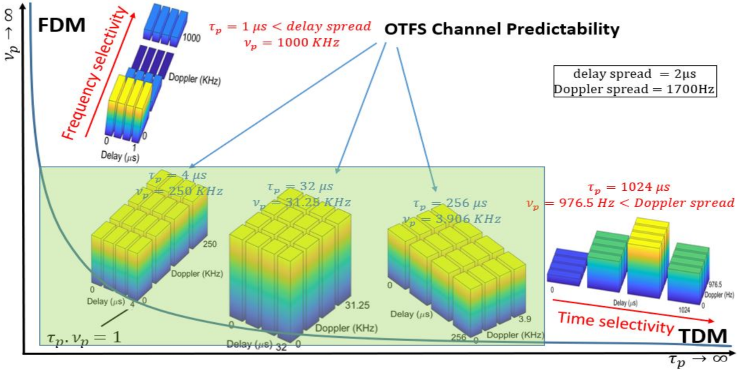


- I/O relation $y_{\text{dd}}^{w_{rx}}(\tau,\nu) = \underbrace{\left(w_{rx}(\tau,\nu) *_\sigma h(\tau,\nu) *_\sigma w_{tx}(\tau,\nu)\right)}_{h_{\text{dd}}(\tau,\nu)} *_\sigma x_{\text{dd}}(\tau,\nu)$
- Output $y_{\text{dd}}[k,l]$ is given by discrete twisted convolution of the input $x_{\text{dd}}[k,l]$ with the effective DD channel filter $h_{\text{dd}}[k,l]$

$$\begin{aligned} y_{\text{dd}}[k,l] &= \sum_{k',l' \in \mathbb{Z}} h_{\text{dd}}[k',l']\, x_{\text{dd}}[k-k',l-l']\, e^{j2\pi \frac{(k-k')}{M}\frac{l'}{N}} \\ &= h_{\text{dd}}[k,l] *_\sigma x_{\text{dd}}[k,l]. \end{aligned}$$

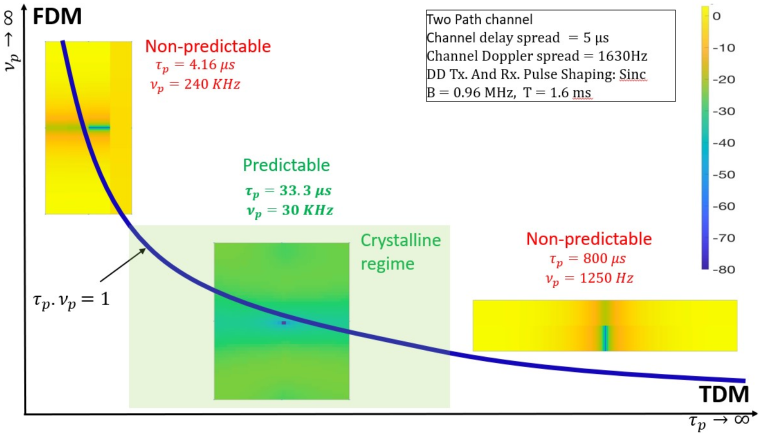
where $h_{\text{dd}}[k,l] \triangleq h_{\text{dd}}(\tau,\nu)\Big|_{\left(\tau=\frac{k\tau_p}{M},\ \nu=\frac{l\nu_p}{N}\right)}$

Non-fading I/O relation in crystalline regime



- Crystalline regime → No DD domain aliasing → Non-fading
- Average received power profile is flat
- $\tau_p \to \infty \Rightarrow \nu_p \to 0 \Rightarrow$ Doppler domain aliasing (Zak-OTFS → TDM)
- $\nu_p \to \infty \Rightarrow \tau_p \to 0 \Rightarrow$ Delay domain aliasing (Zak-OTFS → FDM)

Error in prediction of I/O relation



- PE is small in the crystalline regime (predictable I/O relation)
- PE is high in the non-crystalline regime (non-predictable I/O relation)

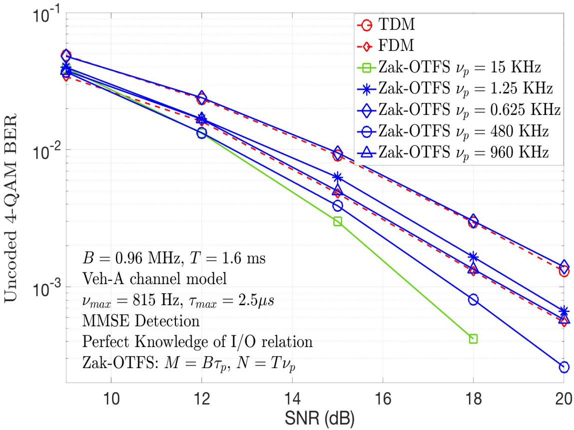
BER performance

- Simulation parameters
  - ITU Veh-A channel model

Path no. i	1	2	3	4	5	6
Rel. Delay τ_i (μs)	0	0.31	0.71	1.09	1.73	2.51
Rel. Power $\frac{\mathbb{E}[\|h_i\|^2]}{\mathbb{E}[\|h_1\|^2]}$ (dB)	0	-1	-9	-10	-15	-20

  - Path Doppler shift: $\nu_i = \nu_{max} \cos(\theta_i)$, $\nu_{max} = 815$ Hz, $\theta_i \sim$ i.i.d. Unif ([0 , 2π)])
  - Path channel gain: Rayleigh faded, $\sum_{i=1}^{6} \mathbb{E}[|h_i|^2] = 1$
  - Pulse shaping at Tx/Rx: Sinc pulses, $B = 0.96$ MHz, $T = 1.6$ ms
- BER performance
  - Uncoded 4-QAM symbols
  - DD domain LMMSE equalizer

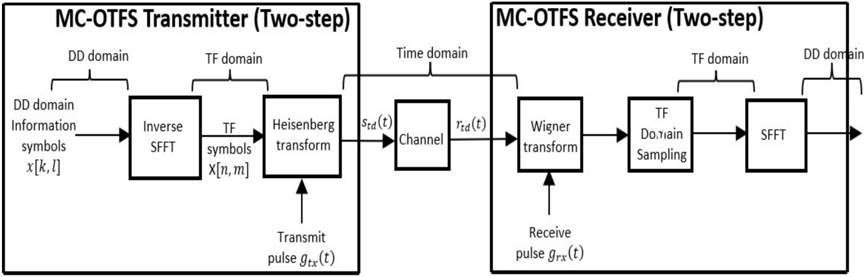
BER performance



- Crystalline regime: Zak-OTFS achieves BER better than TDM and FDM
- Crystalline regime: Non-fading Zak-OTFS I/O relation
- As $\nu_p \to \infty$, Zak-OTFS BER $\to$ FDM BER (Fading I/O relation)
- As $\nu_p \to 0$, Zak-OTFS BER $\to$ TDM BER (Fading I/O relation)

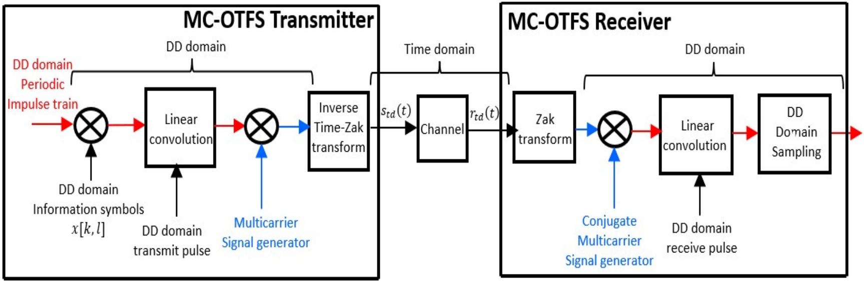
MC-OTFS (2-step OTFS) vs Zak-OTFS (1-step OTFS)

- MC-OTFS



- Most existing work on OTFS presume MC-OTFS
- MC-OTFS different from Zak-OTFS
- MC-OTFS: driven by compatibility with existing 4G/5G modems

MC-OTFS viewed under Zak framework



- Periodic DD signal (Not Quasi-periodic): $x(\tau,\nu) = \sum_{k,l\in\mathbb{Z}} x[k,l]\,\delta\left(\tau - \frac{k\tau_p}{M}\right)\,\delta\left(\nu - \frac{l\nu_p}{N}\right)$
- Pulse shaping: Linear convolution, not twisted convolution
- Pulse shaping waveform: SFFT of TF window (whose support is the time and bandwidth support of OTFS frame)
- Multicarrier generator $G_{\text{dd}}(\tau,\nu)$: Zak-transform of MC-OTFS Tx. pulse $g_{tx}(t)$. Needed to satisfy Quasi-periodicity

I/O relation in MC-OTFS and Zak-OTFS

- MC-OTFS vs. Zak-OTFS I/O relation

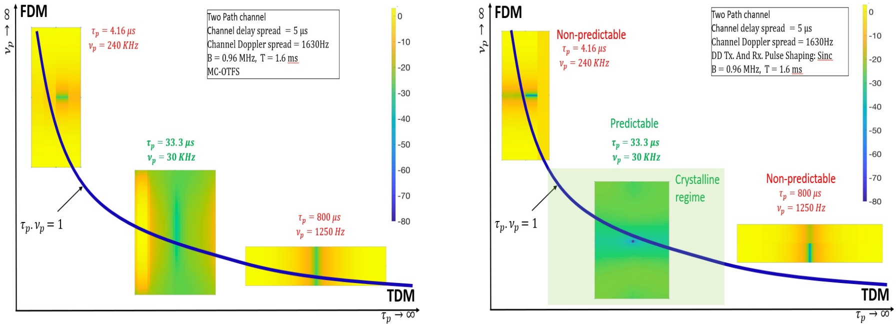
MC-OTFS I/O relation
$y^{w_{rx}}(\tau,\nu) = w_{rx}(\tau,\nu) \star \left[G^{*}_{\mathrm{dd}}(\tau,\nu) \cdot \left(h(\tau,\nu) *_{\sigma} \left\{ G_{\mathrm{dd}}(\tau,\nu) \cdot [w_{tx}(\tau,\nu) \star x(\tau,\nu)] \right\} \right) \right]$

Zak-OTFS I/O relation
$y^{w_{rx}}_{\mathrm{dd}}(\tau,\nu) = w_{rx}(\tau,\nu) *_{\sigma} h(\tau,\nu) *_{\sigma} w_{tx}(\tau,\nu) *_{\sigma} x_{\mathrm{dd}}(\tau,\nu) = h_{\mathrm{dd}}(\tau,\nu) *_{\sigma} x_{\mathrm{dd}}(\tau,\nu)$

- MC-OTFS I/O relation
  - Mix of linear convolution, multiplication and twisted convolution
  - Cannot be expressed as a simple action with some effective filter
  - Clearly not same as that of Zak-OTFS
    - Example: In delay-only channels, as $\nu_p \to 0$, Zak-OTFS $\to$ TDM (predictable I/O relation). MC-OTFS does not converge to TDM
  - Inefficient acquisition of I/O relation as simple prediction is difficult

I/O relation prediction error: MC-OTFS vs Zak-OTFS

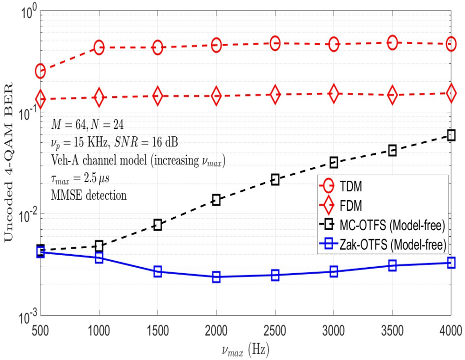
- MC-OTFS Zak-OTFS



- Non-crystalline regime: Both have similar PE
- Crystalline regime: PE of Zak-OTFS is better
- Zak-OTFS I/O relation is more predictable than MC-OTFS I/O relation

BER performance: MC-OTFS vs Zak-OTFS

- Zak OTFS more robust to large channel spreads compared to MC-OTFS



[2] S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, "OTFS - Predictability in the delay-Doppler domain and its value to communication and radar sensing," under review. Available at https://arxiv.org/pdf/2302.08705.pdf

Radar sensing

- Radar scene with single target, no reflector
- Tx. radar waveform: $s_{\text{td}}(t)$
- Received echo:

$$r_{\text{td}}(t) = h\, s_{\text{td}}(t-\tau)\, e^{j2\pi\nu(t-\tau)} + n_{\text{td}}(t)$$

- ML estimate of delay and Doppler

$$(\widehat{\tau}, \widehat{\nu}) = \arg\max_{\tau,\nu} |A_{r,s}(\tau,\nu)|$$

$$A_{r,s}(\tau,\nu) \triangleq \int r_{\text{td}}(t)\, s^*_{\text{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt \qquad \text{(Cross-ambiguity)}$$

- Detection of multiple targets and reflector: Peaks of cross-ambiguity
- Cross-ambiguity for general radar scene:

$$A_{r,s}(\tau,\nu) = h(\tau,\nu) *_{\sigma} A_{s,s}(\tau,\nu) + \int n_{\text{td}}(t)\, s^*_{\text{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt$$

- Ambiguity function of $s_{\text{td}}(t)$:

$$A_{s,s}(\tau,\nu) \triangleq \int s_{\text{td}}(t)\, s^*_{\text{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt$$

Ambiguity of TDM carrier waveform

- TDM pulse $s(t) = s_{\text{td}}(t) = \sqrt{B}\, sinc(Bt)$
  - Ambiguity function

$$A_{s,s}^{\text{tdm}}(\tau,\nu) = \begin{cases} \left(1 - \frac{|\nu|}{B}\right) e^{j\pi\nu\tau}\, sinc((B-|\nu|)\tau) & , |\nu| < B \\ 0 & , |\nu| \geq B \end{cases}$$

  - Peak $A_{s,s}^{\text{tdm}}(\tau,\nu)$ at $(\tau,\nu) = (0,0)$
  - For $\nu = 0$, the spread along delay domain is $\propto \frac{1}{B}$
  - Spread along Doppler domain is $2B$
  - Can resolve targets along delay domain <u>but not along Doppler</u>
  - Because TD pulses are localized in time and not in frequency

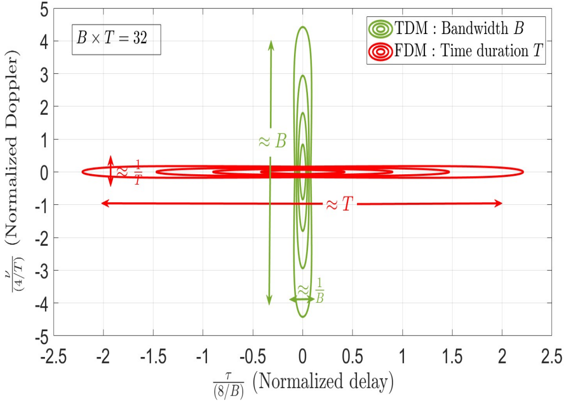
Ambiguity of FDM carrier waveform

- FDM pulse $s(f) = s_{\text{fd}}(f) = \sqrt{T}\, sinc(fT)$.
  - Ambiguity function

$$A_{s,s}^{\text{fdm}}(\tau,\nu) = \begin{cases} \left(1 - \frac{|\tau|}{T}\right) e^{j\pi\nu\tau}\, sinc((T - |\tau|)\nu) & , |\tau| < T \\ 0 & , |\tau| \geq T \end{cases}$$

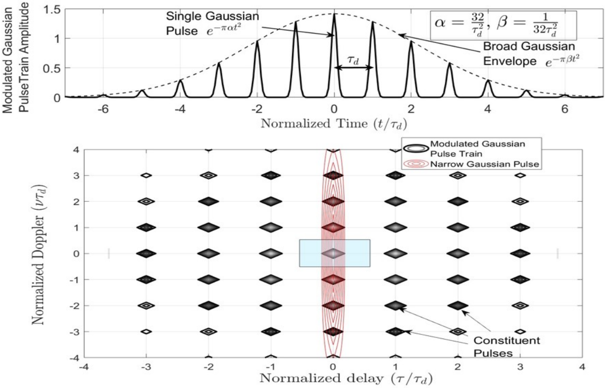
  - Peak $A_{s,s}^{\text{fdm}}(\tau,\nu)$ at $(\tau,\nu) = (0,0)$
  - For $\tau = 0$, the spread along Doppler domain is $\propto \frac{1}{T}$
  - Spread along delay domain is $2T$
  - Can resolve targets along Doppler domain <u>but not along delay</u>
  - Because, FD pulses are localized in frequency and not in time

Ambiguity of TD and FD pulses



- TD/FD pulses cannot resolve targets simultaneously along delay and Doppler
- A good radar waveform re-distributes "ambiguity" such that simultaneous delay-Doppler resolvability is achieved

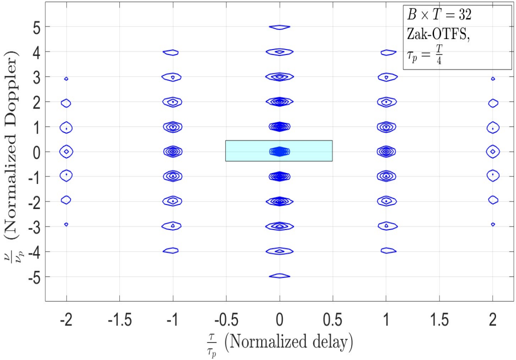
A good radar waveform



- Re-distributing ambiguity: P. M. Woodward (70 years back)*
- Woodward's trick: Modulate a train of narrow TD Gaussian pulses with a broad Gaussian envelope
- Woodward's waveform's resembalance to Zak-OTFS TD pulsone

*P. M. Woodward, *Probability and Information Theory with Applications to Radar*, Pergamon Press, 1953.

Ambiguity function of Zak-OTFS TD pulsone



- No ambiguity when crystallization condition is satisfied
- Delay and Doppler domain resolution are $\propto 1/B$ and $1/T$ respectively
- Ambiguity function can be expressed analytically in terms of the tx. pulse $w_{tx}(\tau, \nu)$
- Design of good radar waveforms therefore reduces to pulse design in the DD domain
- Zak theory provides a mathematical framework for design of good radar waveforms

Concluding remarks

- 6G presents an opportunity to reflect on wireless communication fundamentals with a focus on waveform design
- Information signaling and signal processing in the DD domain is attractive
- Zak-OTFS (OTFS 2.0)
  - information carrier: DD pulse (pulsone in TD)
  - DD pulse is localized in the fundamental DD period
  - channel action: twisted convolution
  - leads to predictability of I/O relation and non-fading in the crystalline regime
  - more robust to larger channel spreads compared to MC-OTFS (OTFS 1.0)
  - good radar waveform with well-localized ambiguity function
- Research on Zak-OTFS is wide open

Thank you