On the Capacity and Performance of Generalized Spatial Modulation

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***Abstract*—Generalized spatial modulation (GSM) uses N antenna elements but fewer radio frequency (RF) chains (R) at the transmitter. In GSM, apart from conveying information bits through R modulation symbols, information bits are also conveyed through the indices of the R active transmit antennas. In this letter, we derive lower and upper bounds on the the capacity of a (N, M, R)-GSM MIMO system, where M is the number of receive antennas. Further, we propose a computationally efficient GSM encoding method and a message passing-based low-complexity detection algorithm suited for large-scale GSM-MIMO systems.**

***Index Terms*—GSM-MIMO capacity, GSM encoding, combinadics, low-complexity detection, message passing.**

I. Introduction

SPATIAL modulation (SM) is emerging as a promising multi-antenna modulation scheme (see [1] and the references therein). SM uses multiple antenna elements but only one radio frequency (RF) chain at the transmitter. In SM, only one antenna element is activated in a given channel use and a QAM/PSK symbol is sent on the activated antenna; the remaining antenna elements remain silent. The index of the active antenna element also conveys information bits. It has been shown that SM can achieve better performance than spatial multiplexing (SMP) under certain conditions [1].

Generalized spatial modulation (GSM) is a generalization of SM, where the transmitter uses multiple (N) antenna elements and more than one (R) RF chain [2]. R among the N available antenna elements are activated in a given channel use, and R QAM/PSK symbols are sent simultaneously on the active antennas. The indices of the R active antennas also convey information bits. Both SM and SMP can be seen as special cases of GSM with $R = 1$ and $R = N$, respectively. It has been shown that for the same spectral efficiency, GSM can perform better than both SM and SMP [3]. While most studies on SM/GSM in the literature so far have focused mainly on performance analysis and receiver algorithms, capacity of SM/GSM systems remains to be studied. The need for capacity analysis of SM has also been highlighted in [1]. In [4], the authors have obtained the capacity of SM for MISO systems through simulation. However, an analytical characterization of the capacity of SM/GSM has not been explored. Our contribution in this letter addresses this gap. In particular, we derive lower and upper bounds on the capacity of GSM, which have not been reported before. Another contribution is the proposal of low complexity encoding and detection methods suited for GSM-MIMO systems with large number of antennas. The proposed encoding makes use of combinadic representations in combinatorial number system, and the detection makes use of layered message passing.

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II. GSM-MIMO System Model

Consider a (N, M, R)-GSM MIMO system with N antennas and R RF chains at the transmitter ($1 \leq R \leq N$), and M antennas at the receiver. An $R \times N$ switch connects the R RF chains to the N transmit antennas. In each channel use, R out of N transmit antennas are chosen and activated. The remaining $N - R$ antennas remain silent. The selection of the R antennas to activate in a channel use is done based solely on $\lfloor \log_2 \binom{N}{R} \rfloor$ information bits (not based on CSIT). Therefore, the indices of the active antennas convey $\lfloor \log_2 \binom{N}{R} \rfloor$ information bits per channel use. On the active antennas, R modulation symbols (one on each active antenna) from a modulation alphabet $\mathbb{A}$ are transmitted. This conveys $R \lfloor \log_2 |\mathbb{A}| \rfloor$ additional information bits. Therefore, the total number of bits conveyed in a channel use in GSM is given by

$$\eta_{gsm} = \underbrace{R \left\lfloor \log_2 \mathbb{A} \right\rfloor}_{\text{Modulation symbol bits}} + \underbrace{\left\lfloor \log_2 \tbinom{N}{R} \right\rfloor}_{\text{Antenna index bits}} \quad \text{bpcu.} \tag{1}$$

A. GSM Signal Set

Let $\mathbb{G}$ denote the GSM signal set, which is the set of all possible GSM signal vectors that can be transmitted. Therefore, if $\mathbf{x}$ is the $N \times 1$ signal vector transmitted by the GSM transmitter, then $\mathbf{x} \in \mathbb{G}$.

Let $\mathbf{s}$ denote the vector of modulation symbols transmitted in a channel use over the R chosen antennas, i.e., $\mathbf{s} \in \mathbb{A}^R$.

Definition: Define a matrix $\mathbf{A}$ of size $N \times R$ as the *antenna activation pattern matrix*. The matrix $\mathbf{A}$ represents a particular choice of R antennas from the available N antennas, such that the $N \times 1$ GSM signal vector $\mathbf{x} = \mathbf{A}\mathbf{s} \in \mathbb{G}$. The matrix $\mathbf{A}$ is a sparse matrix consisting only of 1's and 0's, with exactly one non-zero entry in every column and one non-zero entry in R rows. If $I_1, I_2, \ldots, I_r, \ldots, I_R$ are the indices of the chosen antennas, then $\mathbf{A}$ is constructed as

$$A_{ij} = \begin{cases} 1 & \text{if } j = r \text{ and } i = I_r \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where A_{ij} denotes the element in the ith row and jth column of $\mathbf{A}$. For example, in a system with $N = 8$ and $R = 4$, to activate antennas 1, 3, 6 and 8, the matrix $\mathbf{A}$ is given by[1]

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T . \tag{3}$$

Note that the indices of the non-zero rows in matrix $\mathbf{A}$ give the support of the GSM signal vector $\mathbf{x}$. Out of the $\binom{N}{R}$ possible

[1] Note that, since the active antennas are chosen based only on information bits and not based on CSIT (i.e., GSM does not require CSIT), the $\mathbf{A}$ matrix does not depend on the channel matrix.

antenna activation choices, only $2^{\lfloor \log_2 \binom{N}{R} \rfloor}$ are needed for signaling. Let $\mathcal{A}$ denote this set of all allowed antenna activation pattern matrices, where $|\mathcal{A}| = 2^{\lfloor \log_2 \binom{N}{R} \rfloor}$ and $\mathbf{A} \in \mathcal{A}$. Let $L \triangleq |\mathcal{A}|$. Now, $\mathbb{G}$ is given by

$$\mathbb{G} = \left\{ \mathbf{x} : \mathbf{x} = \mathbf{A}\mathbf{s}, \text{ for some } \mathbf{A} \in \mathcal{A}, \mathbf{s} \in \mathbb{A}^R \right\}. \tag{4}$$

Note that any GSM signal vector $\mathbf{x}_j \in \mathbb{G}$, $j = 1, \ldots, |\mathbb{G}|$, can be represented as $\mathbf{x}_j = \mathbf{A}_i \mathbf{s}_k$ with some $\mathbf{A}_i \in \mathcal{A}$, $i = 1, \ldots, |\mathcal{A}|$ and $\mathbf{s}_k \in \mathbb{A}^R$, $k = 1, \ldots, |\mathbb{A}|^R$, and $|\mathbb{G}| = |\mathcal{A}||\mathbb{A}|^R$. Conversely, given any $\mathbf{A}_i \in \mathcal{A}$ and $\mathbf{s}_k \in \mathbb{A}^R$, there exists a GSM signal vector $\mathbf{x}_j \in \mathbb{G}$ such that $\mathbf{x}_j = \mathbf{A}_i \mathbf{s}_k$. Since $\mathbf{A}_i$ and $\mathbf{s}_k$ are chosen by two independent information bit sequences, $\mathbf{A}_i$ and $\mathbf{s}_k$ are independent. That is, $p(\mathbf{x}) = p(\mathbf{A}\mathbf{s}) = p(\mathbf{A})p(\mathbf{s})$. The $M \times 1$ received signal vector $\mathbf{y} = [y_1 \; y_2 \cdots \; y_M]^T$ at the receiver can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \; = \; \mathbf{H}\mathbf{A}\mathbf{s} + \mathbf{w}, \tag{5}$$

where $\mathbf{x} \in \mathbb{G}$ is the transmit vector, $\mathbf{H} \in \mathbb{C}^{M \times N}$ is the channel gain matrix whose (i, j)th entry $H_{i,j} \sim \mathcal{CN}(0, 1)$ denotes the complex channel gain from the jth transmit antenna to the ith receive antenna, and $\mathbf{w} = [w_1 \; w_2 \cdots \; w_M]^T$ is the noise vector whose entries are modeled as complex Gaussian with zero mean and variance σ^2. Since $\mathbf{x}$ is obtained based only on η_{gsm} information bits, $\mathbf{x}$ and $\mathbf{H}$ are independent. Since $\mathbf{x} = \mathbf{A}\mathbf{s}$, $\mathbf{A}\mathbf{s}$ and $\mathbf{H}$ are independent. Also, $\mathbf{A}$ and $\mathbf{s}$ are independent. So, $\mathbf{A}$ and $\mathbf{H}$ are independent. So, the mutual information between $\mathbf{x}$ and $\mathbf{y}$ in GSM is given by $I(\mathbf{x}; \mathbf{y}|\mathbf{H}) = I(\mathbf{A}, \mathbf{s}; \mathbf{y}|\mathbf{H})$.

III. GSM-MIMO Capacity

The capacity of a spatially multiplexed $M \times N$ MIMO channel with channel state information at the receiver is

$$C = \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \left[\det \left(\mathbf{I}_M + \frac{1}{\sigma^2} \mathbf{H} \mathbf{\Phi} \mathbf{H}^H \right) \right] \right\}, \tag{6}$$

where $\mathbf{\Phi}$ is the covariance matrix of the transmit signal vector with elements from Gaussian codebook. For i.i.d. modulation symbols and total power σ_x^2, the capacity expression becomes

$$C = \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \left[\det \left(\mathbf{I}_M + \frac{\sigma_x^2}{N\sigma^2} \mathbf{H} \mathbf{H}^H \right) \right] \right\}. \tag{7}$$

Here, we are interested in the capacity of GSM-MIMO, where the symbols sent on the active antennas are from Gaussian codebook. The capacity of GSM-MIMO can be written as

$$\begin{aligned} C_{GSM} &= \mathbb{E}_{\mathbf{H}}(I(\mathbf{x}; \mathbf{y})) \\ &= \mathbb{E}_{\mathbf{H}}(h(\mathbf{y}) - h(\mathbf{y}|\mathbf{x})) = \mathbb{E}_{\mathbf{H}}(h(\mathbf{y}) - h(\mathbf{w})) \\ &= \mathbb{E}_{\mathbf{H}}(h(\mathbf{y}) - \log_2[\det(\pi e \sigma^2 \mathbf{I}_M)]), \end{aligned} \tag{8}$$

where $h(.)$ denotes the differential entropy. To compute C_{GSM}, we need to evaluate $h(\mathbf{y})$, which requires the knowledge of the distribution of $\mathbf{y}$. From (5), we can see that the distribution of $\mathbf{y}$ is a Gaussian mixture given by

$$p(\mathbf{y}) = \sum_{i=1}^{L} p(\mathbf{y}, \mathbf{A}_i) = \sum_{i=1}^{L} p(\mathbf{y}|\mathbf{A}_i) p(\mathbf{A}_i) = \sum_{i=1}^{L} \mathcal{N}(\boldsymbol{\mu}_i, \mathbf{\Phi}_i) p_i, \tag{9}$$

$$p_i = p(\mathbf{A}_i), \qquad \mathbf{A}_i \in \mathcal{A}, \qquad i = 1, 2, \ldots, L, \tag{10}$$

$$\boldsymbol{\mu}_i = \mathbb{E}(\mathbf{y}|\mathbf{A}_i) = \mathbb{E}(\mathbf{H}\mathbf{A}_i\mathbf{s} + \mathbf{w}) = \mathbf{0}, \tag{11}$$

$$\mathbf{\Phi}_i = \mathbb{E}(\mathbf{y}\mathbf{y}^H|\mathbf{A}_i) = \mathbf{H}\mathbf{A}_i \mathbb{E}_{\mathbf{s}}(\mathbf{s}\mathbf{s}^H) \mathbf{A}_i^H \mathbf{H}^H + \sigma^2 \mathbf{I}_M. \tag{12}$$

If all the possible antenna activation patterns are equally likely, then $p_i = \frac{1}{L}$. When the transmitted modulation symbols are independent of each other, $\mathbb{E}(\mathbf{s}\mathbf{s}^H) = \frac{\sigma_x^2}{R} \mathbf{I}_R$. Now, (9) becomes

$$\mathbf{y} \sim \frac{1}{L} \sum_{i=1}^{L} \mathcal{N} \left(\mathbf{0}, \frac{\sigma_x^2}{R} \mathbf{H}\mathbf{A}_i \mathbf{A}_i^H \mathbf{H}^H + \sigma^2 \mathbf{I}_M \right). \tag{13}$$

The differential entropy of $\mathbf{y}$ is given by

$$h(\mathbf{y}) = -\frac{1}{L} \sum_{i=1}^{L} \int_{\mathbf{y}} \mathcal{N}(\mathbf{0}, \mathbf{\Phi}_i) \log_2 \left(\frac{1}{L} \sum_{i=1}^{L} \mathcal{N}(\mathbf{0}, \mathbf{\Phi}_i) \right) d\mathbf{y}. \tag{14}$$

It is difficult to get a closed-form solution to (14). Hence, we bound it above and below by the following techniques.

Lower bound 1, L_1: Since $-\log(.)$ is a convex function, by Jensen's inequality, $\mathbb{E}[-\log p(y)] \geq -\log \mathbb{E}[p(y)]$. Hence, the differential entropy $h(\mathbf{y})$ can be lower bounded as

$$h(\mathbf{y}) \geq -\log_2 \int p^2(\mathbf{y}) d\mathbf{y} = -\log_2 \int \left(\frac{1}{L} \sum_{i=1}^{L} \mathcal{N}(\mathbf{0}, \mathbf{\Phi}_i) \right)^2 d\mathbf{y},$$

which can be simplified as

$$h(\mathbf{y}) \geq -\log_2 \left\{ \frac{1}{L^2 \pi^M} \sum_{i=1}^{L} \sum_{j=1}^{L} \frac{1}{\det(\mathbf{\Phi}_i + \mathbf{\Phi}_j)} \right\} \triangleq l_1. \tag{15}$$

It can be noted that $\det(\mathbf{\Phi}_i) > 0, \forall i$. From (8) and (15), a lower bound on C_{GSM} can be obtained as

$$C_{GSM} \geq L_1 \triangleq \mathbb{E}_{\mathbf{H}}(l_1 - \log_2 \det(\pi e \sigma^2 \mathbf{I}_M)). \tag{16}$$

Lower bound 2, L_2: Since differential entropy is a concave function, we can write

$$\begin{aligned} h(\mathbf{y}) &= h\left(\frac{1}{L} \sum_{i=1}^{L} \mathcal{N}(\mathbf{0}, \mathbf{\Phi}_i) \right) \\ &\geq \frac{1}{L} \sum_{i=1}^{L} h\left(\mathcal{N}(\mathbf{0}, \mathbf{\Phi}_i) \right) = \frac{1}{L} \sum_{i=1}^{L} \log_2 \det(\pi e \mathbf{\Phi}_i) \triangleq l_2. \end{aligned}$$

From the above equation, C_{GSM} can be lower bounded as

$$C_{GSM} \geq L_2 \triangleq \mathbb{E}_{\mathbf{H}}(l_2 - \log_2 \det(\pi e \sigma^2 \mathbf{I}_M)). \tag{17}$$

Based on the two lower bounds L_1 and L_2, a refined lower bound on GSM-MIMO capacity is given by

$$C_{GSM} \geq L \triangleq \max(L_1, L_2). \tag{18}$$

Upper bound 1, U_1: By the property of entropy,

$$h(\mathbf{y}) = h(\mathbf{y}, \mathbf{A}) - h(\mathbf{A}|\mathbf{y}) \leq h(\mathbf{y}, \mathbf{A}).$$

Using this property, an upper bound on $h(\mathbf{y})$ can be

$$\begin{aligned} h(\mathbf{y}) &\leq h(\mathbf{y}, \mathbf{A}) = h(\mathbf{y}|\mathbf{A}) + h(\mathbf{A}) \\ &= \int \sum_{\forall \mathbf{A}} p(\mathbf{y}, \mathbf{A}) \log_2 \frac{p(\mathbf{A})}{p(\mathbf{y}, \mathbf{A})} d\mathbf{y} + \sum_{i=1}^{L} -p_i \log_2 p_i \\ &= \frac{1}{L} \sum_{i=1}^{L} \log_2 \det(\pi e \mathbf{\Phi}_i) + \log_2 L \triangleq u_1. \end{aligned}$$

Now, an upper bound on C_{GSM} can be written as

$$C_{GSM} \leq U_1 \triangleq \mathbb{E}_{\mathbf{H}}(u_1 - \log_2 \det(\pi e \sigma^2 \mathbf{I}_M)). \tag{19}$$

Upper bound 2, U_2: Here, we approximate the probability distribution of $\mathbf{y}$ to a Gaussian distribution. This leads to an

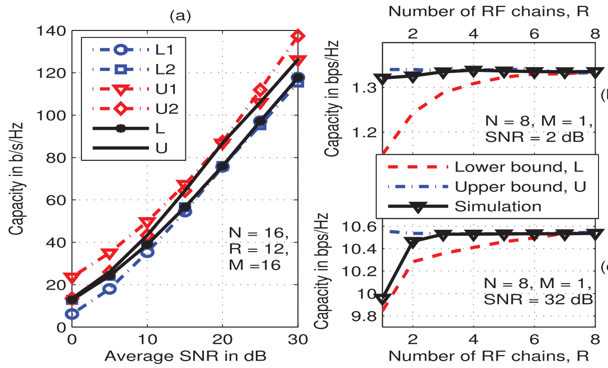

Fig. 1. Capacity bounds for different GSM-MIMO configurations.

upper bound because the entropy of any random variable is bounded above by the entropy of a Gaussian random variable with the same mean and variance. The mean of $\mathbf{y}$ is $\mathbf{0}$ and the covariance of $\mathbf{y}$ is given by

$$\begin{aligned}\mathbb{E}(\mathbf{y}\mathbf{y}^H) &= \mathbf{H}\mathbb{E}_{\mathbb{A}}[\mathbf{A}_i\mathbb{E}_{\mathbf{s}}(\mathbf{s}\mathbf{s}^H)\mathbf{A}_i^H]\mathbf{H}^H + \sigma^2\mathbf{I}_M \\ &= \mathbf{H}\left(\frac{1}{L}\sum_{i=1}^{L}\mathbf{A}_i\left(\frac{\sigma_x^2}{R}\mathbf{I}_R\right)\mathbf{A}_i^H\right)\mathbf{H}^H + \sigma^2\mathbf{I}_M \\ &= \frac{\sigma_x^2}{RL}\mathbf{H}\left(\sum_{i=1}^{L}\mathbf{D}_i\right)\mathbf{H}^H + \sigma^2\mathbf{I}_M, \end{aligned} \quad (20)$$

where $\mathbf{D}_i \triangleq \mathbf{A}_i\mathbf{A}_i^H$. Let $\{I_1^i, I_2^i, \ldots, I_r^i, \ldots, I_R^i\}$ be the set of active antenna indices that corresponds to the antenna activation pattern matrix $\mathbf{A}_i$. It can then be seen that $\mathbf{D}_i$ is a diagonal matrix, such that

$$(D_i)_{j,k} = \begin{cases} 1 & \text{if} j = k = I \in \{I_1^i, I_2^i, \ldots, I_R^i\} \\ 0 & \text{otherwise,} \end{cases}$$

where $(D_i)_{j,k}$ is the element in the jth row and kth column of $\mathbf{D}_i$. Assuming that all $\binom{N}{R}$ activation patterns are allowed, i.e., $|\mathcal{A}| = L = \binom{N}{R}$, the number of times any particular antenna will be active among the $\binom{N}{R}$ activation patterns is $\binom{N-1}{R-1}$. Therefore, $\sum_{i=1}^{L}\mathbf{D}_i = \binom{N-1}{R-1}\mathbf{I}_N = \frac{RL}{N}\mathbf{I}_N$, and (20) becomes

$$\mathbb{E}(\mathbf{y}\mathbf{y}^H) = \frac{\sigma_x^2}{N}\mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_M \triangleq \Phi'.$$

Now, an upper bound on GSM-MIMO capacity is given by

$$\begin{aligned} C_{GSM} &\le U_2 \triangleq \mathbb{E}_{\mathbf{H}}(\log_2\det(\pi e\Phi') - \log_2\det(\pi e\sigma^2\mathbf{I}_M)) \\ &= \mathbb{E}_{\mathbf{H}}\left\{\log_2\left[\det\left(\mathbf{I}_M + \frac{\sigma_x^2}{N\sigma^2}\mathbf{H}\mathbf{H}^H\right)\right]\right\}, \end{aligned} \quad (21)$$

which is the same as the capacity of a $M \times N$ spatially multiplexed MIMO system. Based on the two upper bounds U_1 and U_2, a refined upper bound on GSM capacity is

$$C_{GSM} \le U \triangleq \min(U_1, U_2). \quad (22)$$

Numerical results: We evaluated the lower and upper bounds on the GSM-MIMO capacity for different system configurations. Figure 1(a) shows the lower and upper bounds for GSM-MIMO systems with $N = 16$, $R = 12$, $M = 16$. We see that the lower bound L_2 and upper bound U_2 are tighter at low SNRs. Whereas, the lower bound L_1 and upper bound U_1 are tighter at high SNRs. It can be noted from the figure that the lower and upper bounds are very close at low SNRs; so, in this regime, the GSM-MIMO capacity is almost same as that of a spatially multiplexed MIMO system with the same N and M. In Figs. 1(b) and 1(c), we compare the bounds on GSM-MIMO capacity with the true GSM-MIMO capacity obtained through simulation. We consider GSM-MIMO with $N = 8$, $M = 1$, SNR = 2,32 dB, and varying R. In Fig. 1(b), we observe that, at SNR = 2 dB (top figure), the gap between the upper and lower bounds is $U - L = 0.188$ bps/Hz for $R = 1$ and $U - L < 10^{-2}$ bps/Hz for $R > 5$. Also, in Fig. 1(c), at SNR = 32 dB (bottom figure), the gap is $U - L = 0.782$ bps/Hz for $R = 1$ and $U - L < 10^{-2}$ bps/Hz for $R > 6$.

IV. Low-Complexity Encoding and Detection

A. GSM Encoding Using Combinadics

In GSM-MIMO, $\lfloor\log_2\binom{N}{R}\rfloor$ bits are used to choose an activation pattern matrix $\mathbf{A}$ from $\mathcal{A}$, and $R\lfloor\log_2\mathbb{A}\rfloor$ bits are used to generate $\mathbf{s}$ from $\mathbb{A}^R$. While the mapping of $R\lfloor\log_2\mathbb{A}\rfloor$ bits to modulation symbols in $\mathbf{s}$ is straight-forward, the mapping of $\lfloor\log_2\binom{N}{R}\rfloor$ bits to a choice of activation pattern is not. A table or a map of bit sequence to activation patterns has to be maintained both at the transmitter and receiver. For large values of N, M, R, the size of this map can become prohibitively large. For example, if $N = 64$, $R = 32$, then $|\mathcal{A}| = \binom{64}{32} \approx 1.83 \times 10^{18} \approx 2^{60}$. Implementation of an encoding map of this size is impractical. To address this issue, we use *combinadic* representations in combinatorial number system.

Definition: The combinadic of a number $n \in [0, \binom{N}{R} - 1]$ is the R-tuple $(N_1, N_2, \ldots, N_R)$ such that $n = \sum_{i=1}^{R}\binom{N_i}{i}$ and $N_1 < N_2 < \cdots < N_R < N$. The values of N_i for a given n can be obtained as [5]

$$N_i = \text{Largest non-negative integer s.t. } n - \sum_{j=i}^{R}\binom{N_j}{j} \ge 0.$$

The following encoding procedure maps the bits to antenna activation pattern. Let $\eta_a \triangleq \lfloor\log_2\binom{N}{R}\rfloor$.

1) Accumulate η_a bits to form the bit sequence $\mathbf{b} = [b_{\eta_a-1}, \ldots, b_1, b_0]$. Obtain $g(\mathbf{b}) = \sum_{i=0}^{\eta_a-1}2^i b_i$.
2) Find the combinadic of $g(\mathbf{b})$.
3) Construct $\mathbf{A}$ matrix such that the indices of the R non-zero rows of $\mathbf{A}$ are given by the combinadic of $g(\mathbf{b})$.

For example, for $N = 10$, $R = 4$, the combinadic of $n = 19$ can be computed as $(N_1, N_2, N_3, N_4) = (0, 1, 4, 6)$. The computational complexity to find the combinadic of a number is just $O(R)$, which enables GSM encoding for large values of N, R. A reverse procedure can perform the combinadic to information bits demapping at the receiver.

B. GSM Detection: Layered Message Passing Algorithm

The maximum a posteriori probability detection rule is

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{G}}{\operatorname{argmax}}\; p(\mathbf{x}|\mathbf{y}). \quad (23)$$

Note that $|\mathbb{G}| = 2^{\eta_{gsm}}$. So, exact computation of (23) requires exponential complexity in N, R. To address this problem, here we propose a low complexity layered message passing (LaMP) algorithm which gives an approximate solution to (23).

Definition: A variable a_i is called the antenna activity indicator if $a_i = 1$ if the ith antenna is active, else $a_i = 0$. Therefore, $x_i = a_i s$, $s \in \mathbb{A}$. Note that $\sum_{i=1}^{N} a_i = R$, which we call as the GSM system constraint G. Now, $p(\mathbf{x}|\mathbf{y})$ in (23) can be written as

$$\begin{aligned} p(\mathbf{x}, \mathbf{a}|\mathbf{y}) &\propto p(\mathbf{y}|\mathbf{x}, \mathbf{a})p(\mathbf{x}, \mathbf{a}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x}|\mathbf{a})p(\mathbf{a}) \\ &= \left\{\prod_{j=1}^{M}p(y_i|\mathbf{x})\prod_{i=1}^{N}p(x_i|a_i)\right\}p(\mathbf{a}). \end{aligned} \quad (24)$$

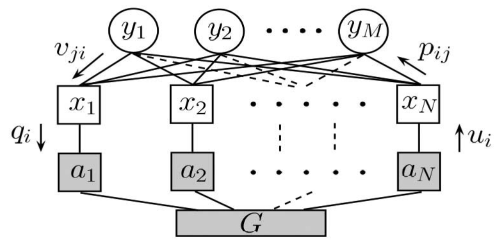

Fig. 2. Graphical model and messages passed in the proposed LaMP detector.

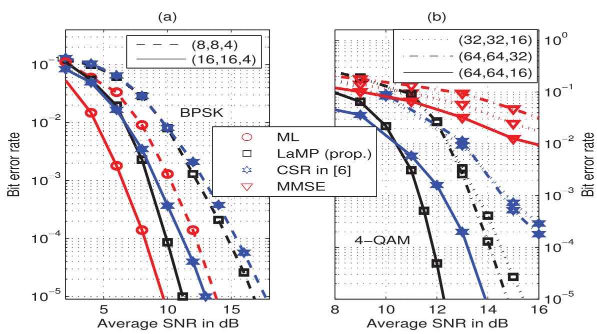

Fig. 3. BER performance comparison between proposed LaMP detection, CSR detection in [6], MMSE detection, and ML detection.

Thus, by defining a new layer of variables corresponding to antenna activity, we have effectively decoupled the dependences present among the elements of the transmit vector $\mathbf{x}$. Based on (24), we model the GSM-MIMO system as a graph with four types of nodes, namely, (i) M observation nodes corresponding to $\mathbf{y}$, (ii) N variable nodes corresponding to $\mathbf{x}$, (iii) N antenna activity nodes corresponding to $\mathbf{a}$, and (iv) a constraint node G. This is illustrated in Fig. 2. On this graph, we iteratively pass messages between nodes and obtain the marginal probabilities of the transmitted symbols. The different messages passed in this graph are (1) v_{ji}: from observation node y_j to variable node x_i, (2) p_{ij}: from variable node x_i to observation node y_j, (3) q_i: from variable node x_i to antenna activity node a_i, and (4) u_i: from antenna activity node a_i to variable node x_i. The messages are exchanged between two layers, namely, (i) Layer 1: observation nodes and variable nodes (denoted by unshaded nodes in Fig 2); these layers generate an approximate a posteriori probabilities of the individual elements of $\mathbf{x}$, and (ii) Layer 2: antenna activity nodes and GSM constraint node (denoted by shaded nodes in Fig 2); these layers generate an approximate a posteriori probabilities of the individual elements of $\mathbf{a}$. In constructing the messages p_{ij} at the variable nodes, we employ a Gaussian approximation of the interference as described below. This significantly reduces the detection complexity.

From (5), we can write

$$y_j = H_{ji}x_i + g_{ji}, \quad g_{ji} \triangleq \sum_{l=1, l\neq i}^{N} H_{jl}x_l + w_j. \tag{25}$$

Approximate g_{ji} to be Gaussian. Then,

$$\mu_{ji} \triangleq \mathbb{E}(g_{ji}) = \sum_{l\neq i} H_{jl}\mathbb{E}(x_l) = \sum_{l\neq i} H_{jl} \sum_{x\in\mathbb{A}\cup 0} x p_{ij}(x), \tag{26}$$

$$\sigma_{ji}^2 \triangleq \mathrm{Var}(g_{ji}) = \sigma^2 + \sum_{l\neq i} H_{jl}^2 \mathrm{Var}(x_l). \tag{27}$$

Using the above approximation, the messages are given by

$$v_{ji}(x) \triangleq p(x_i = x|y_j) \approx \frac{1}{\sigma_{ji}^2\sqrt{2\pi}} \exp\left(\frac{-(y_j - \mu_{ji} - H_{ji}x)^2}{2\sigma_{ji}^2}\right), \tag{28}$$

$$p_{ij}(x) \triangleq p(x_i = x|\mathbf{y}_{\setminus j})$$

$$\approx \prod_{k=1,k\neq j}^{M} p(x_i = x|y_k) \propto u_i(x^{\odot}) \prod_{k\neq j} v_{ki}(x),$$

$$q_i(b) \triangleq p(a_i = b|\mathbf{x}), \qquad u_i(b) \triangleq p(a_i = b|\mathbf{x}_{\setminus i}),$$

$$q_i(b) \approx \begin{cases} \sum_{x\in\mathbb{A}} \prod_{k=1}^{M} p(x_i = x|y_k) \propto \sum_{x\in\mathbb{A}} \prod_{k=1}^{M} v_{ki}(x) & \text{if } b = 1 \\ \prod_{k=1}^{M} p(x_i = 0|y_k) \propto \prod_{k=1}^{M} v_{ki}(0) & \text{if } b = 0, \end{cases}$$

$$u_i(b) \propto \begin{cases} p\left(\sum_{l\neq i} a_l = R - 1|\mathbf{a}_{\setminus i}\right) \approx \phi_i(R-1) & \text{if } b = 1 \\ p\left(\sum_{l\neq i} a_l = R|\mathbf{a}_{\setminus i}\right) \approx \phi_i(R) & \text{if } b = 0, \end{cases}$$

here $x^{\odot} = 0$ if $x = 0$ and $x^{\odot} = 1$ if $x \neq 0$. $\mathbf{x}_{\setminus i}$ denotes the set of all elements of $\mathbf{x}$ except x_i, and $\phi_i = \bigotimes_{l=1,l\neq i}^{N} q_l$, where $\bigotimes$ is the convolution operation.

Simulation results: Figure 3(a) presents a performance comparison between the proposed LaMP detection, the maximum likelihood (ML) detection, and the convex superset relaxation (CSR) based detection in [6] for different (N, M, R)- GSM MIMO systems with BPSK. It can be seen that the LaMP performance is away from ML performance by about 3 dB and 1.6 dB for (8, 8, 4)- and (16, 16, 4)-GSM MIMO systems, respectively, at 10^{-5} BER. Also, LaMP detection performs better than the CSR detection in [6] – e.g., by about 1.8 dB at 10^{-5} BER in (16, 16, 4)-GSM MIMO system. Figure 3(b) presents a performance comparison between the LaMP detection, MMSE detection (performed as $[\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}]^{-1}\mathbf{H}^H\mathbf{y}$), CSR detection in [6] for the following large-scale GSM-MIMO system configurations: (i) (32, 32, 16)-GSM, 4-QAM, $|\mathcal{A}| = 2^{29}$, 61 bpcu, (ii) (64, 64, 16)-GSM, 4-QAM, $|\mathcal{A}| = 2^{48}$, 80 bpcu, and (iii) (64, 64, 32)-GSM, 4-QAM, $|\mathcal{A}| = 2^{60}$, 124 bpcu. It can be seen that the LaMP algorithm performs better and its performance improves as the dimensionality of the GSM signal increases.

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