Communication and Radar Sensing using Pulsones

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Joint work with
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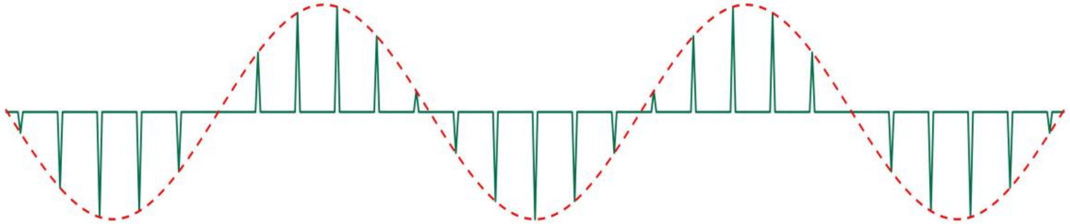

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Outline

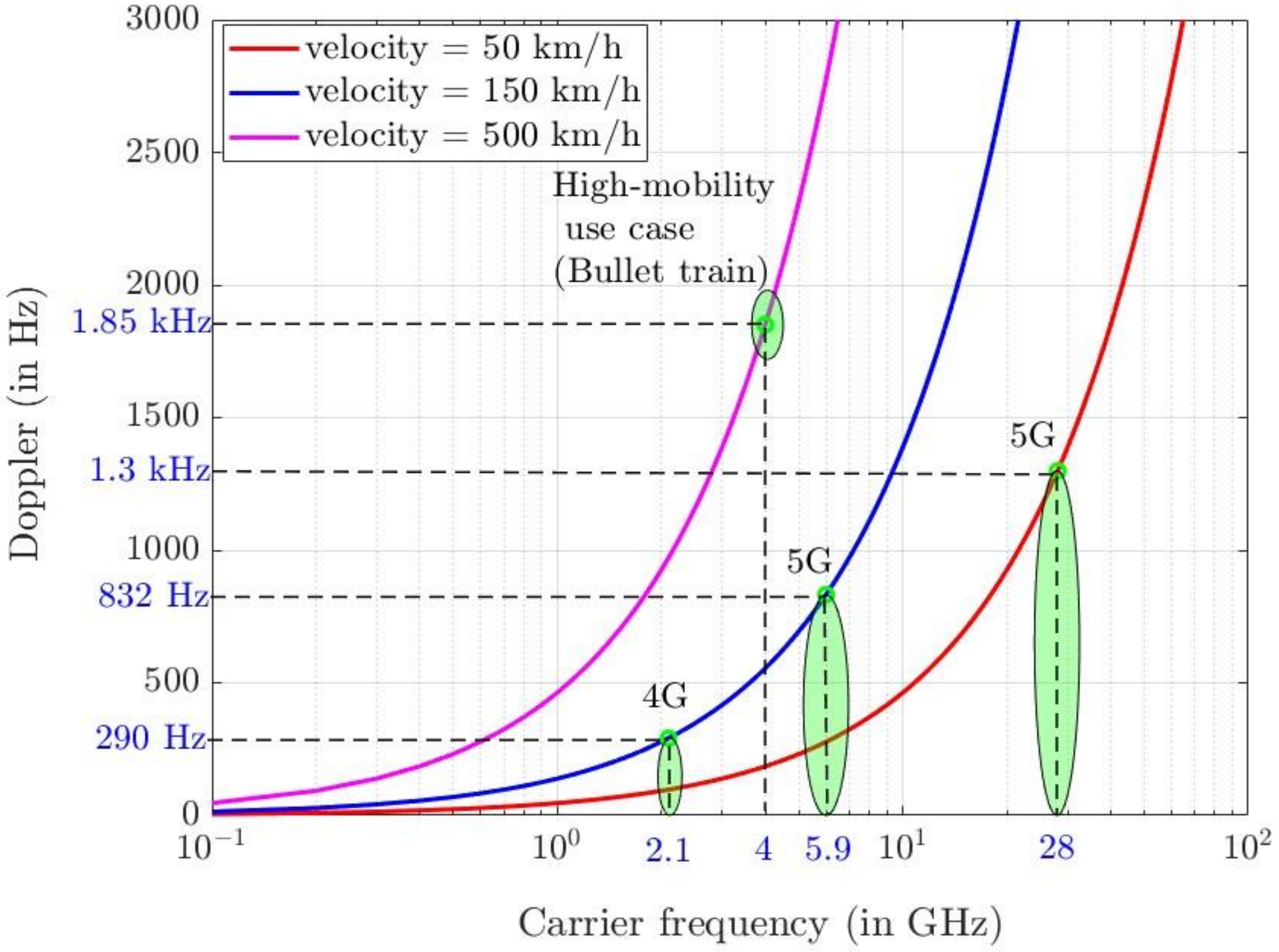
- 6G and beyond
- Pulsones – what and why
- Communication using pulsones
- Radar sensing using pulsones
- Concluding remarks

6G and beyond

- Operational
  - 3D communication
  - confluence of terrestrial, UAV/drones/aeroplanes, LEO satellites
  - high relative velocities
- Spectrum
  - 28 GHz limited success so far
  - sub-6 GHz will continue to be important
  - mmWave frequencies (30 to 300 GHz) yet to make a mark
- Application
  - popular use of AR/VR/XR
  - holographic communication
    - widely anticipated XR use case
  - Cloudification of the network

- Physical layer
  - Waveforms
    - robust to arbitrarily complex channels
      - high-mobility/high-Doppler
    - for integrated communication and radar sensing
      - Intelligent transportation (V2X)
      - high communication throughput
      - sense the environment in high resolution
      - optimality with respect to both goals
  - Reconfigurable intelligent surfaces

Doppler shifts on the rise

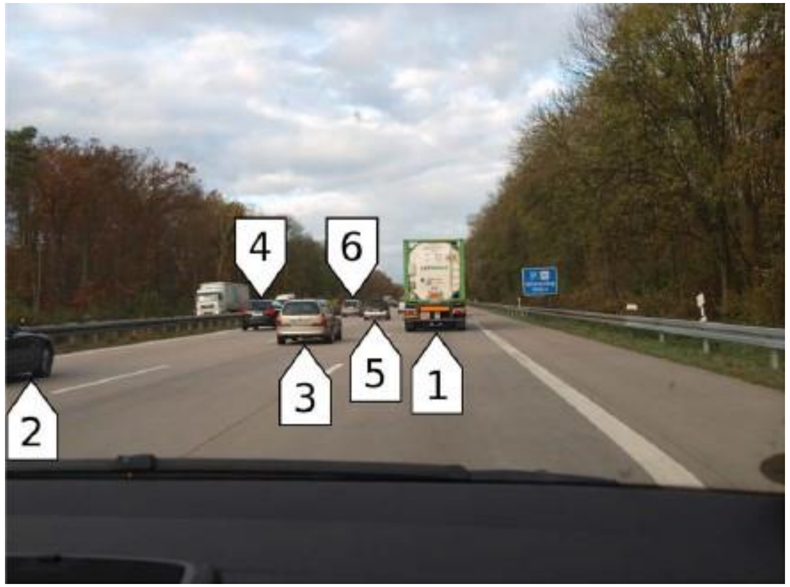


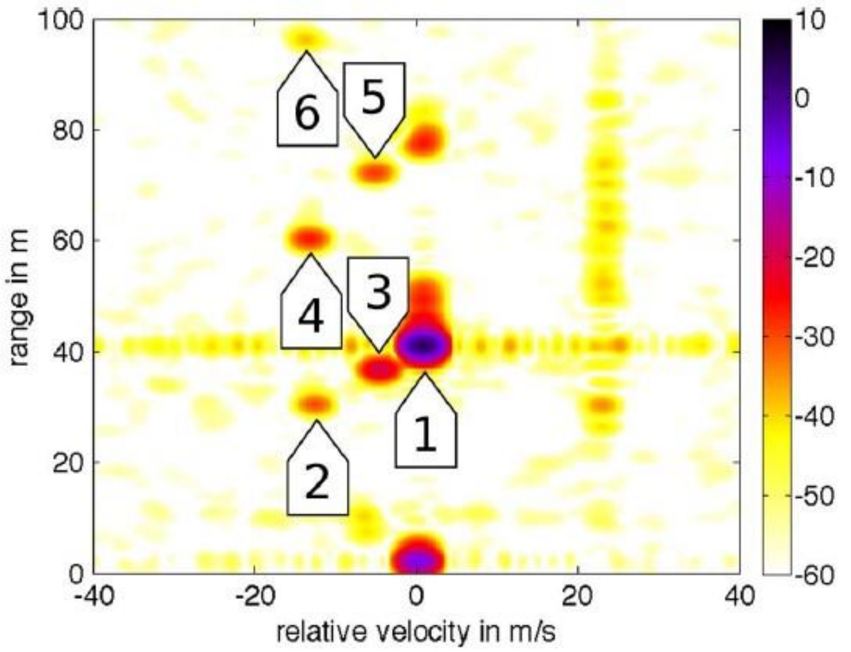
- Doppler shift = $\frac{f_c v}{c}$ Hz
  - f_c: Carrier frequency
  - v: Velocity
  - c: Speed of light

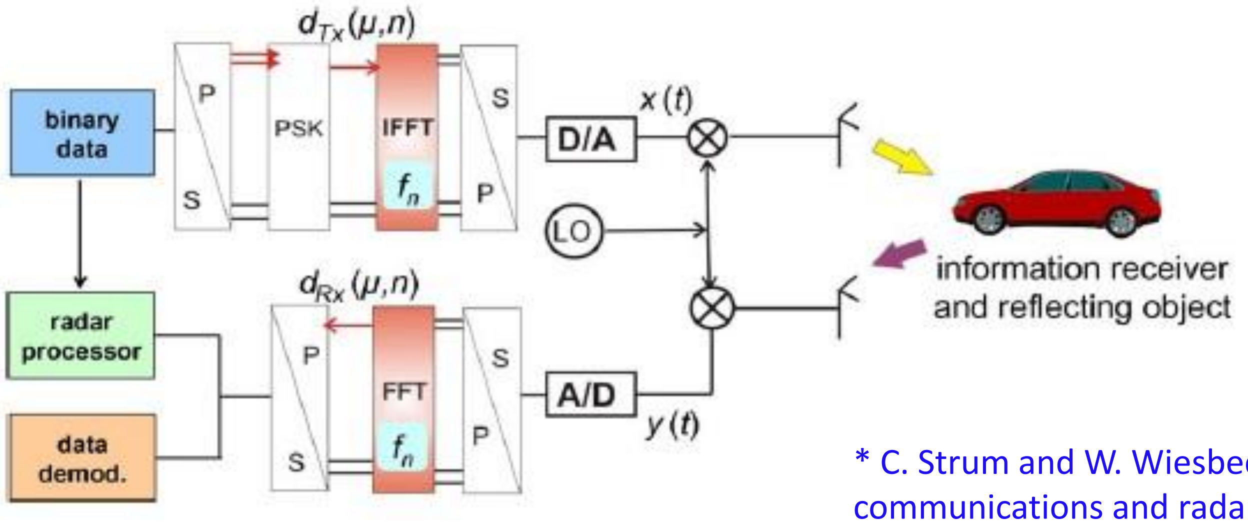
- Up to 4G/5G
  - Dopplers: tens to few hundred Hz range

- 6G and beyond
  - Dopplers: kHz range

Sensing the environment

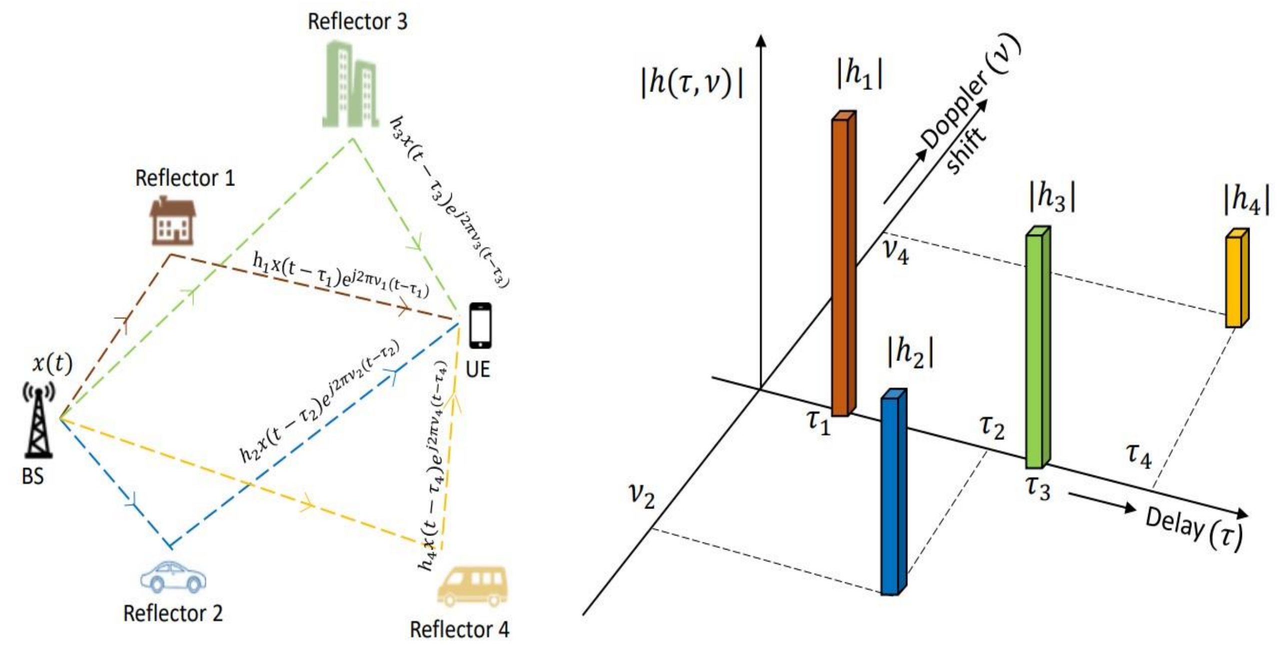




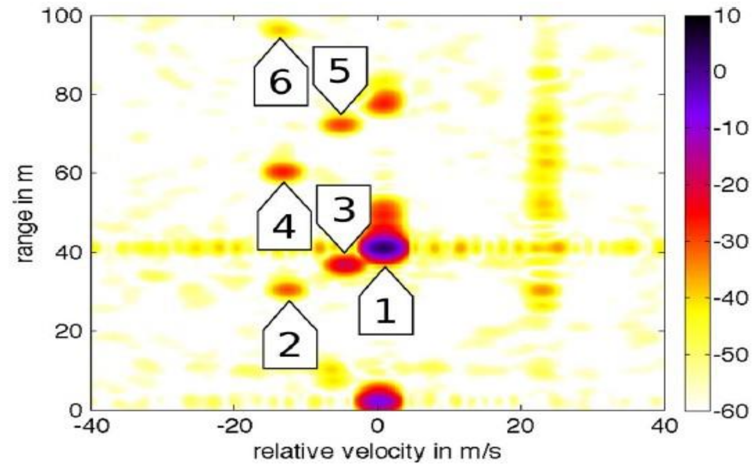


* C. Strum and W. Wiesbeck, "Waveform design and signal processing aspects of wireless communications and radar sensing," *Proceedings of the IEEE*, vol. 99, no. 7, pp. 1236-1259, Jul. 2011.

Delay-Doppler channel representation



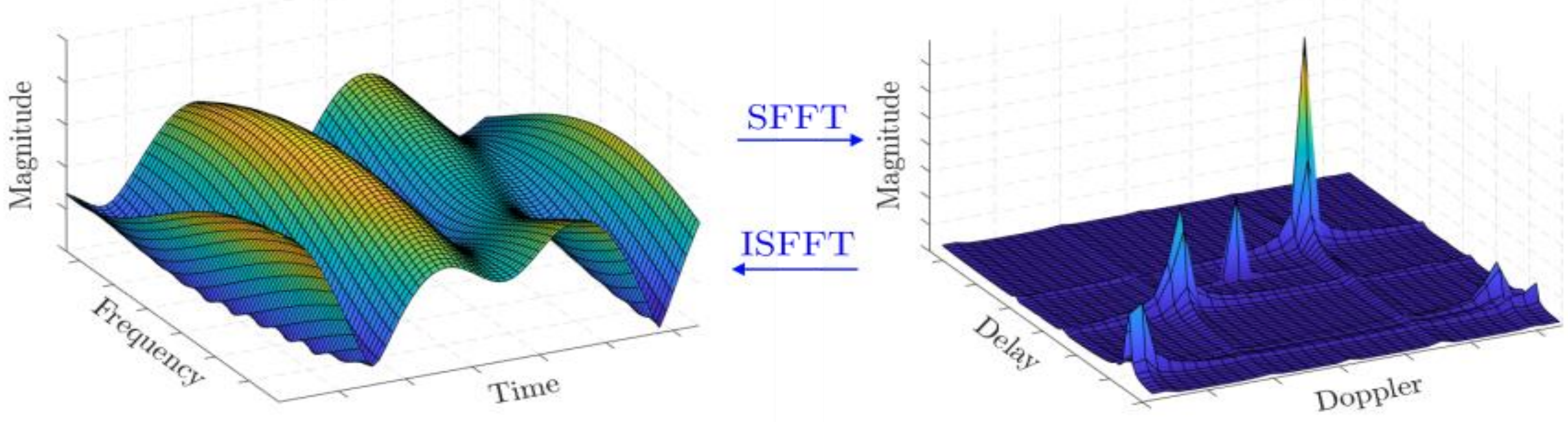
- Doubly-selective wireless channel characterization
  - P dominant reflectors in the environment
  - ith reflector delay (τ_i)
  - ith reflector Doppler (ϑ_i)
  - ith reflector gain (h_i)
- $(\tau_i, \vartheta_i, h_i)$ – tuple, $i = 1,2,\ldots,P$
- *$\{\tau_i, \vartheta_i, h_i\}$ change slowly in time*



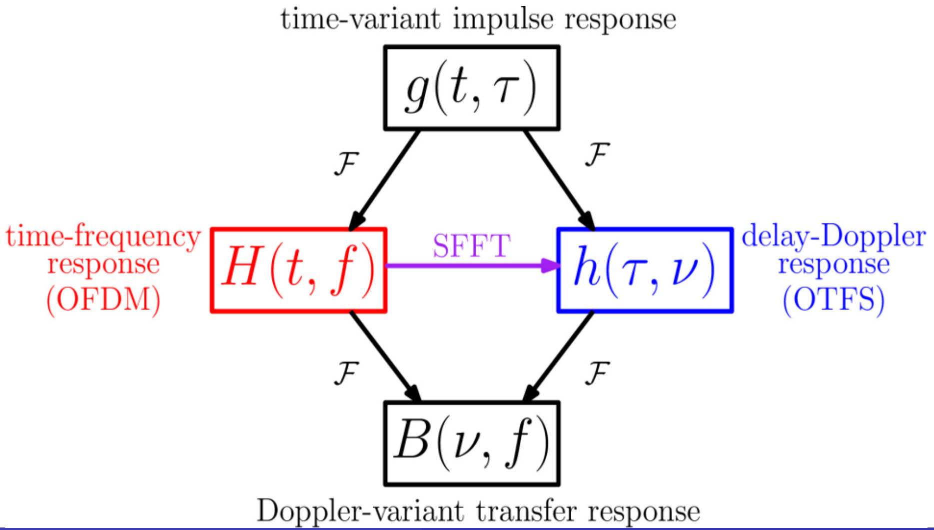
- Delay-Doppler channel response: $h(\tau,\nu) = \sum_{i=1}^{4} h_i \delta(\tau - \tau_i)\delta(\nu - \nu_i)$
- Signal received along ith path: $h_i \underbrace{x(t-\tau_i)}_{\text{delay: } \tau_i} \underbrace{e^{j2\pi\nu_i(t-\tau_i)}}_{\text{Doppler shift: } \nu_i}$
- Received signal: $y(t) = \iint h(\tau,\nu)x(t-\tau)e^{j2\pi\nu(t-\tau)}\, d\tau d\nu = \sum_{i=1}^{4} h_i x(t-\tau_i)\, e^{j2\pi\nu_i(t-\tau_i)}$

Doubly-selective channel

- Viewed in TF domain
- Viewed in DD domain

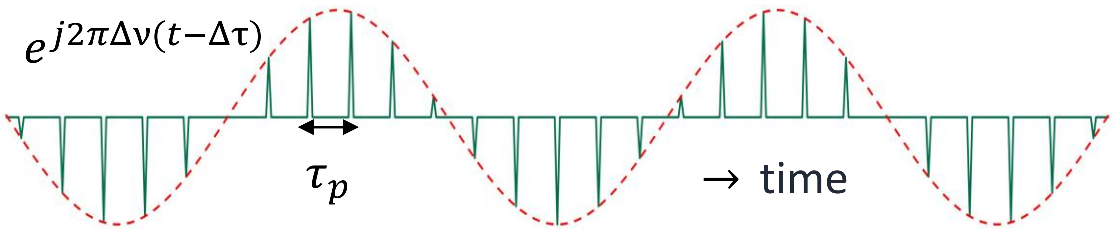


- Different representations to model LTV channels

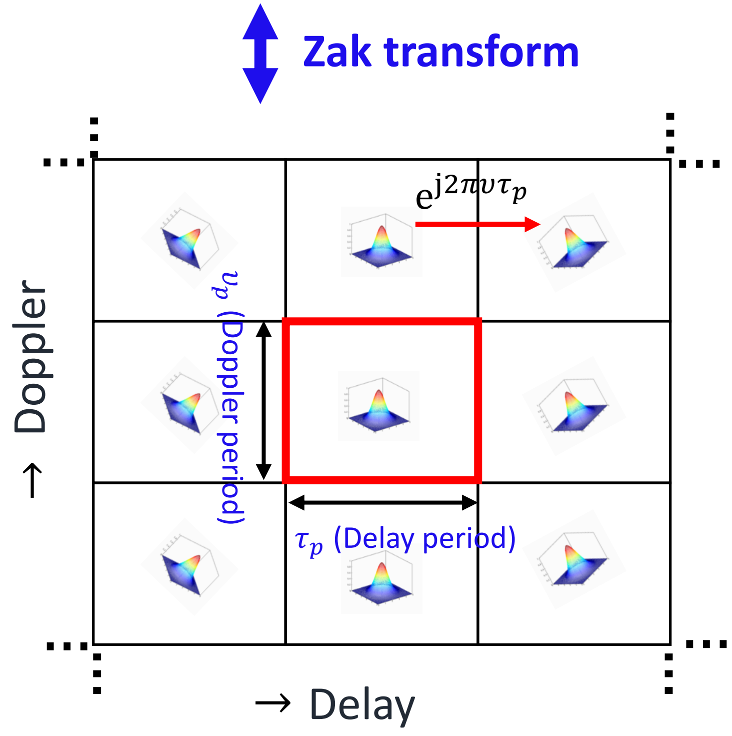


Pulsone

- Pulse train modulated by a frequency tone



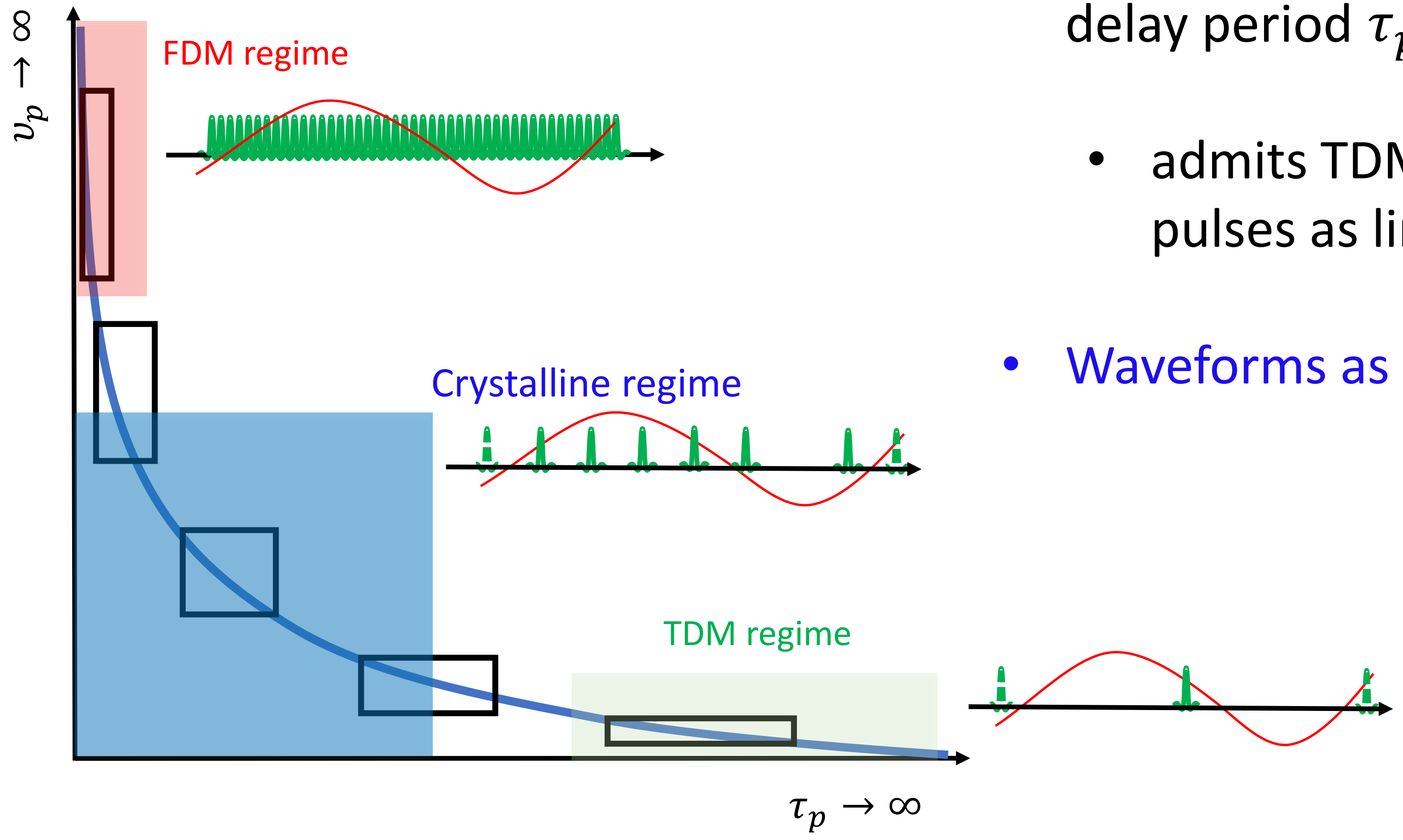
- Time domain realization of a quasi-periodic pulse in the DD domain
  - Delay period (τ_p)
  - Doppler period (υ_p) $\tau_p \upsilon_p = 1$



- Quasi-periodicity condition: For all $m, n \in \mathbb{Z}$

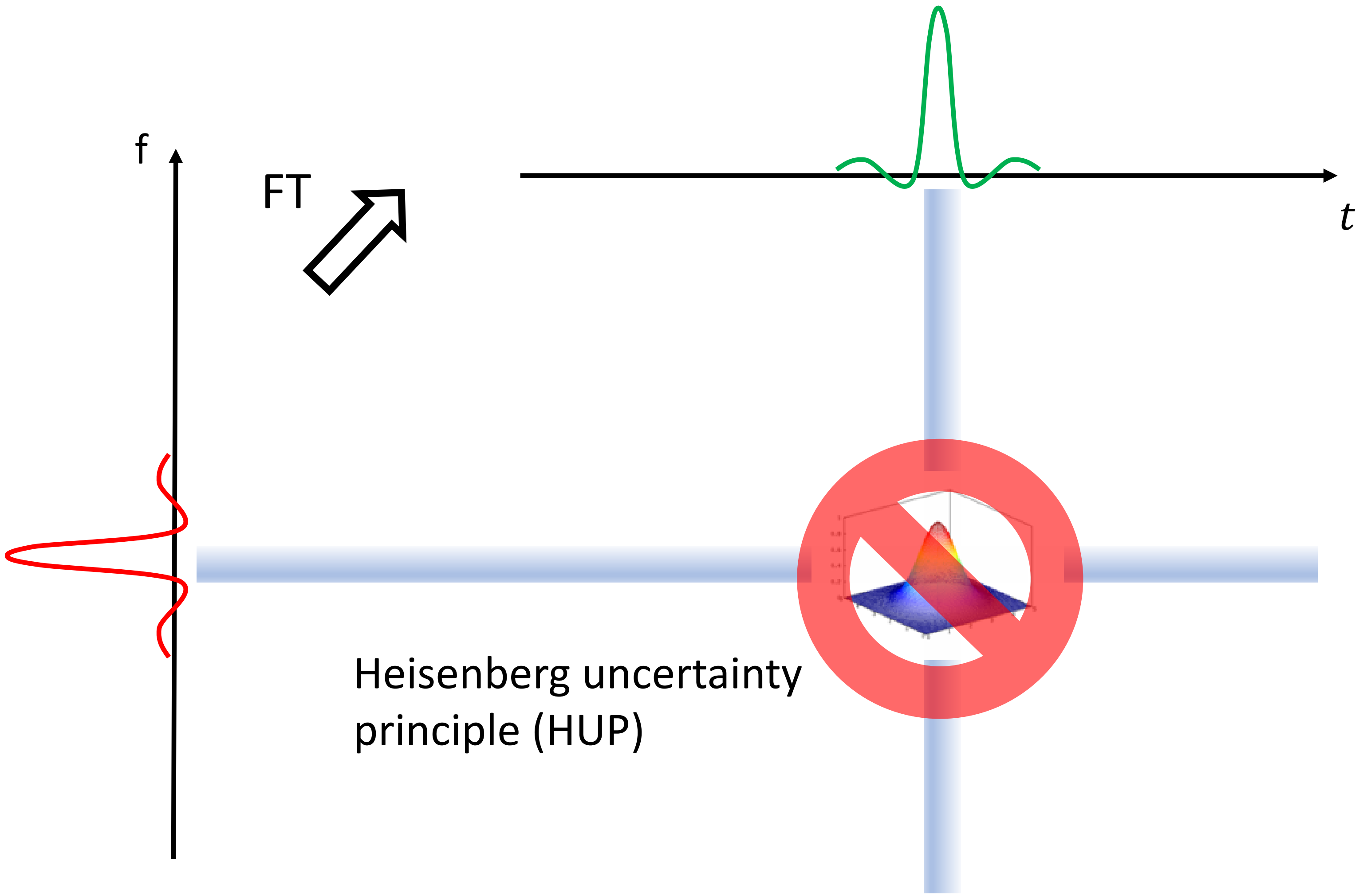
$$x_{dd}(\tau + n\tau_p, \nu + m\nu_p) = e^{j2\pi n\nu\tau_p} x_{dd}(\tau, \nu)$$

Family of waveforms on the period curve



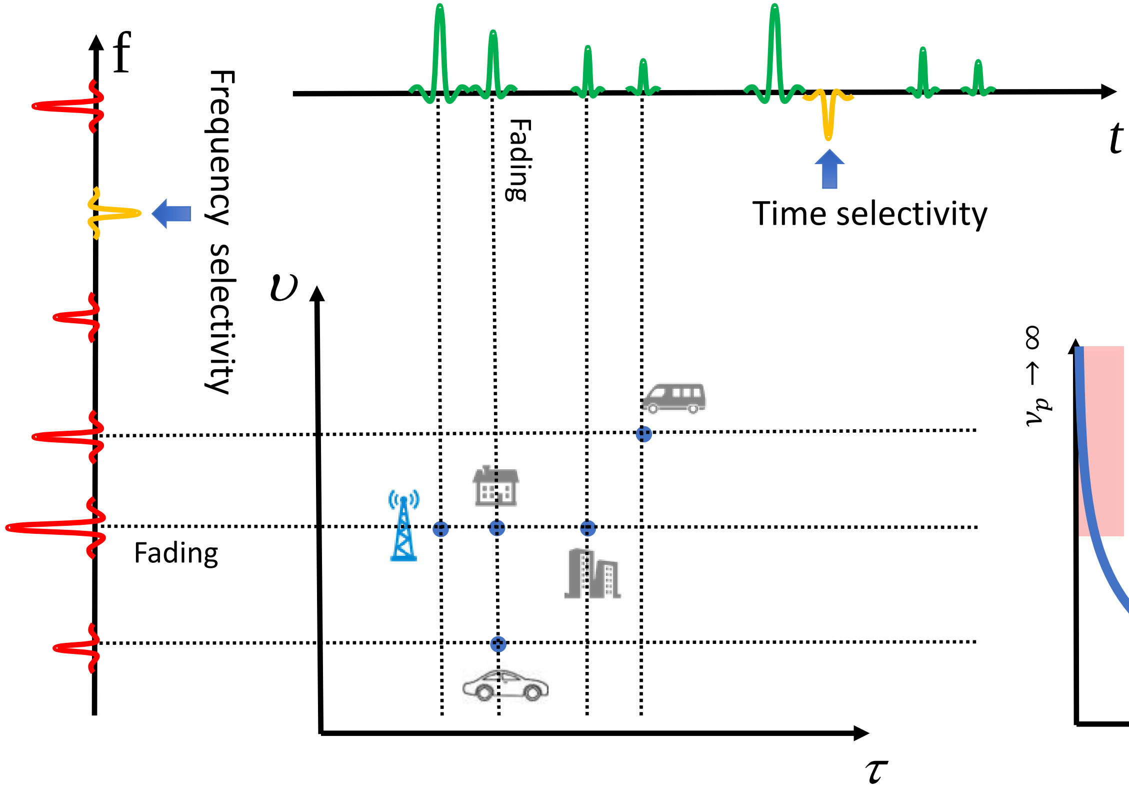
- Family of waveforms parameterized by the delay period τ_p s.t . $\tau_p . \upsilon_p = 1$
  - admits TDM ($\tau_p \to \infty$) and FDM ($\upsilon_p \to \infty$) pulses as limits
- Waveforms as information carriers?

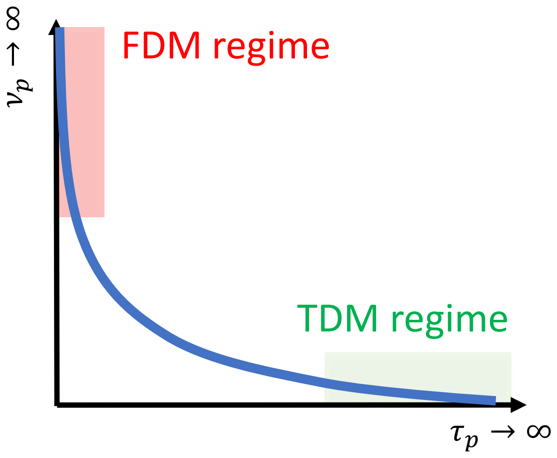
Information carriers in TDM, FDM



- The coupling of doubly-selective channel and TDM/FDM waveform is selective
  → fading and unpredictable

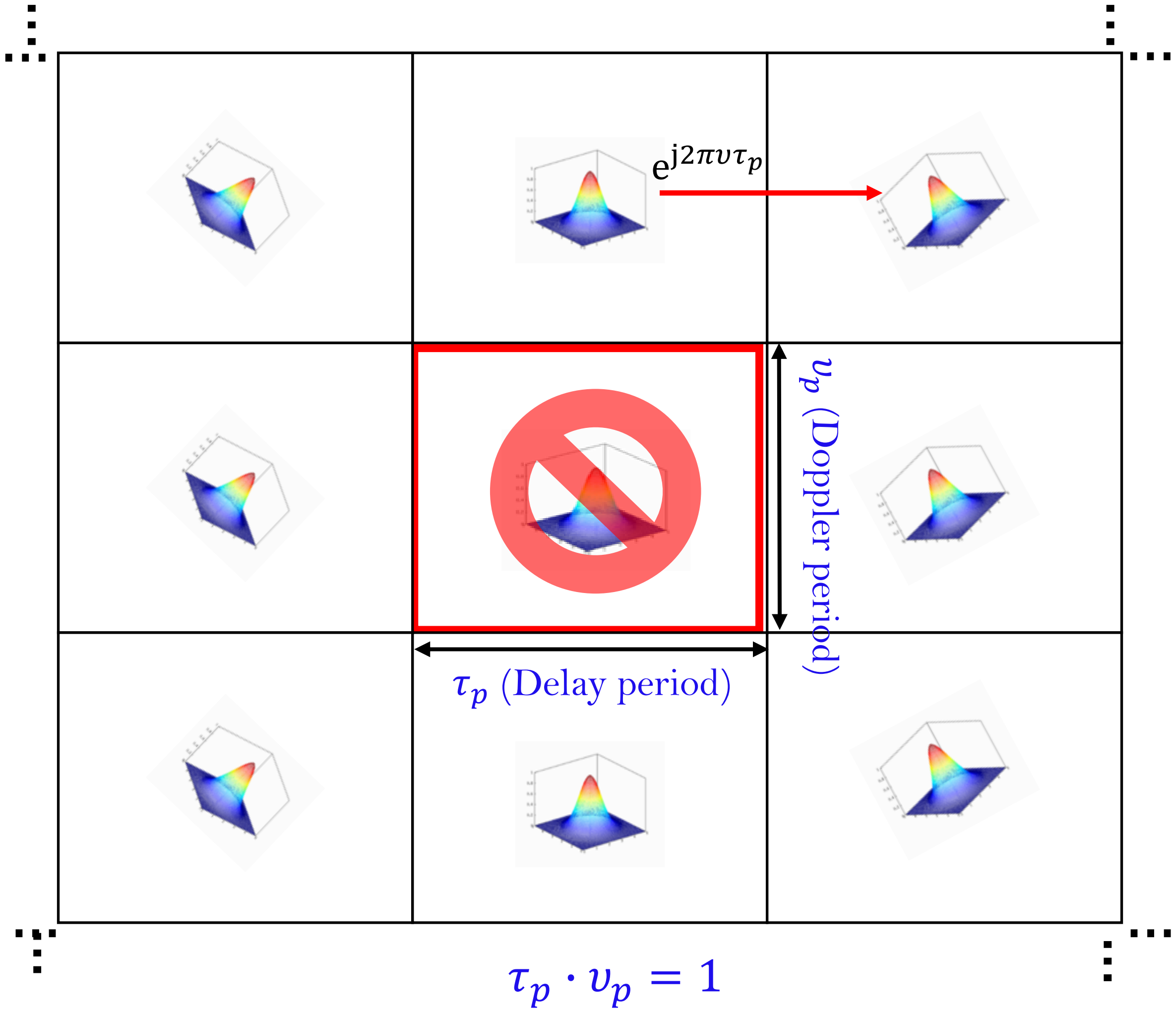
Channel coupling in the TDM/FDM regime





- The coupling of doubly-selective channel and TDM/FDM waveform is selective
  → fading and unpredictable

Work around HUP – Quasi-periodic DD pulses



- Quasi-periodicity condition: For all $m, n \in \mathbb{Z}$

$$x_{dd}(\tau + n\tau_p, \nu + m\nu_p) = e^{j2\pi n\nu\tau_p} x_{dd}(\tau, \nu)$$

- Delay period: τ_p Doppler period: $\nu_p = \frac{1}{\tau_p}$
- Pulse *effectively* localized in the fundamental DD period

$$\mathcal{D}_0 \triangleq \left\{(\tau, \nu) \,\middle|\, 0 \leq \tau < \tau_p,\, 0 \leq \nu < \nu_p\right\}$$

- Inverse time-Zak transform (DD → TD)
  - exists only for quasi-periodic DD signals

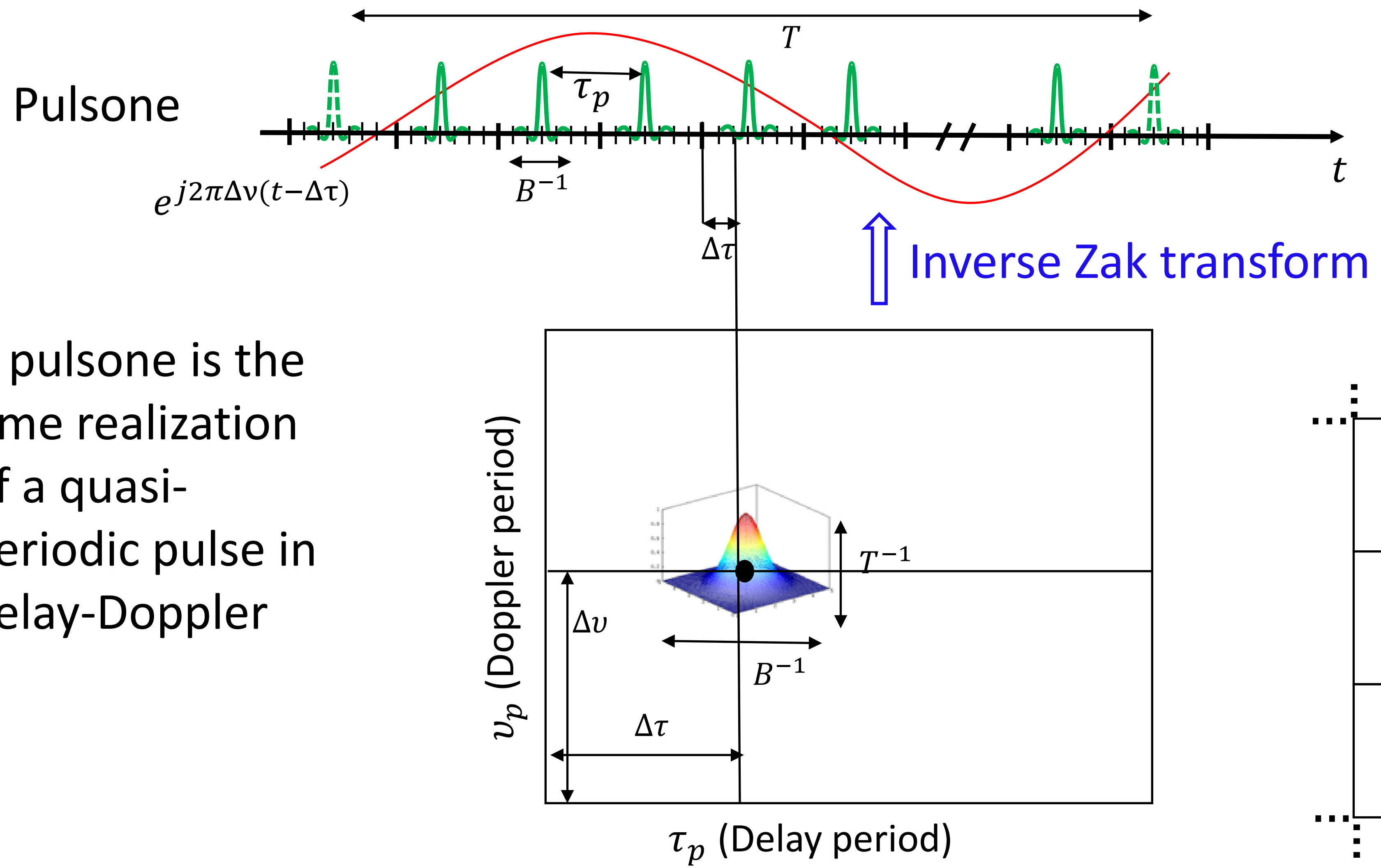
$$x_{dd}(\tau, \upsilon) \rightarrow x(t) = \sqrt{\tau_p} \int_0^{\upsilon_p} x_{dd}(t, \upsilon)\, d\upsilon$$

- Zak transform (TD → DD)
  - results in quasi-periodic DD signal

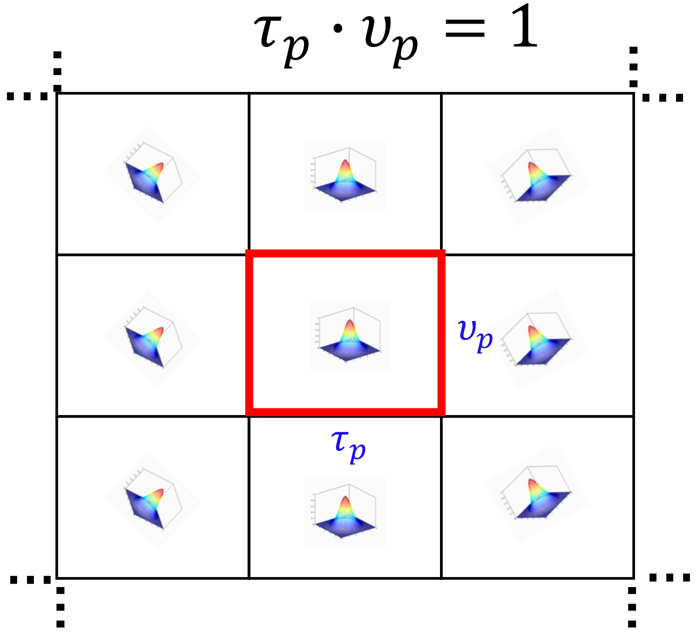
$$x(t) \rightarrow x_{dd}(\tau, \upsilon)$$

$$= \sqrt{\tau_p} \sum_{k \in \mathbb{Z}} x(\tau + k\tau_p) e^{-j2\pi k\upsilon\tau_p}$$

Pulsone as information carrier

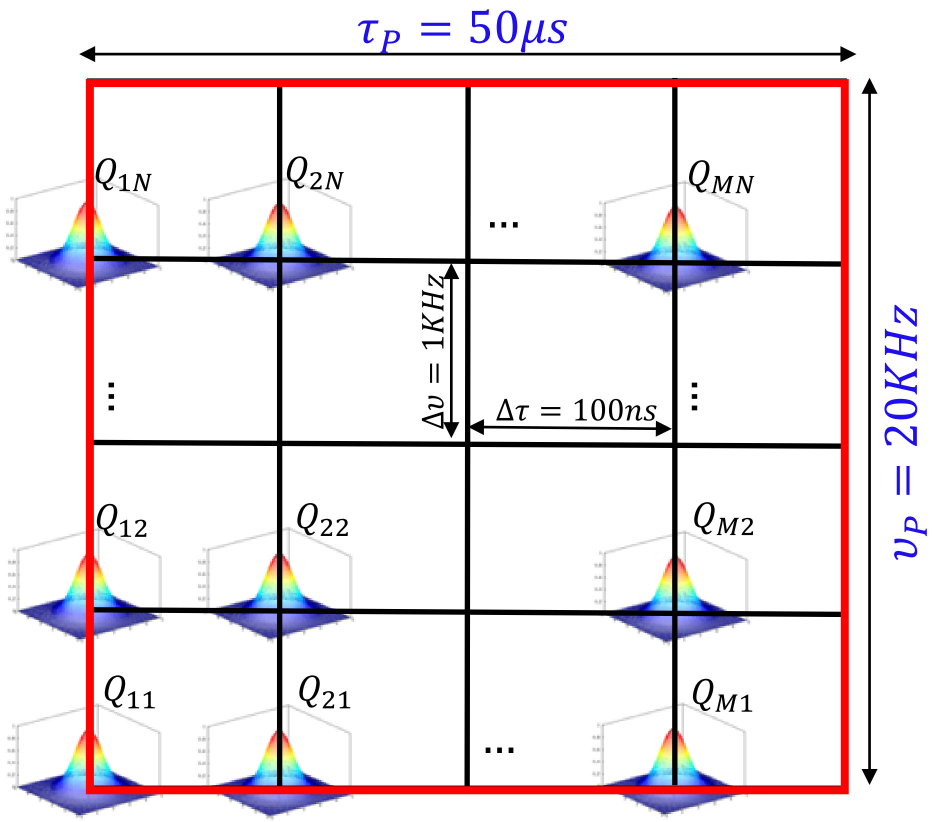


- A pulsone is the time realization of a quasi-periodic pulse in delay-Doppler



Communication using pulsones

- Information grid (information lattice)



$$B = \frac{1}{100\ ns} = 10\ MHz$$

$$T = \frac{1}{1\ KHz} = 1\ ms$$

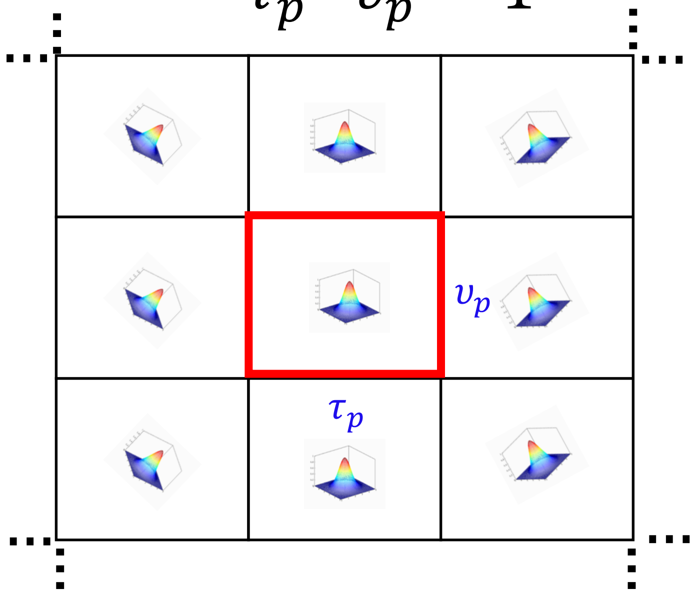
$$M = \frac{50\ \mu s}{100\ ns} = 500$$

$$N = \frac{20KHz}{1KHz} = 20$$

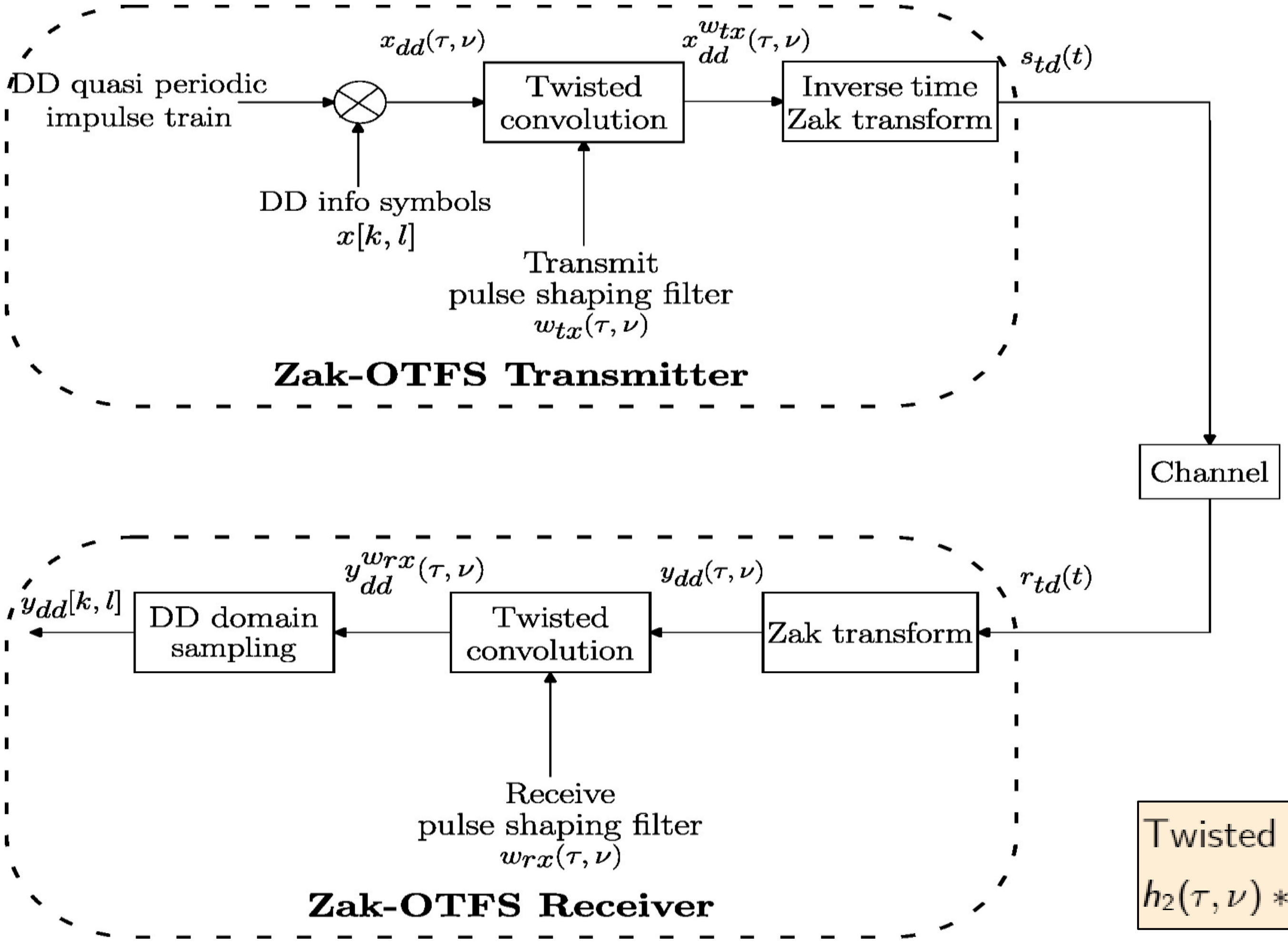
- *B:* Bandwidth
- *T:* Time duration
- *M*: number of delay bins
- *N*: number of Doppler bins
- *MN* information symbols

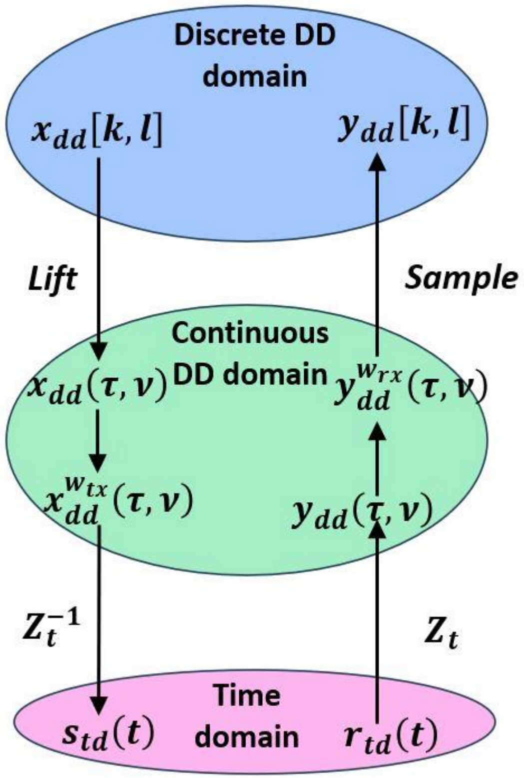
- Period grid (period lattice)

$$\tau_p \cdot \nu_p = 1$$



Communication using pulsones: Zak-OTFS modulation





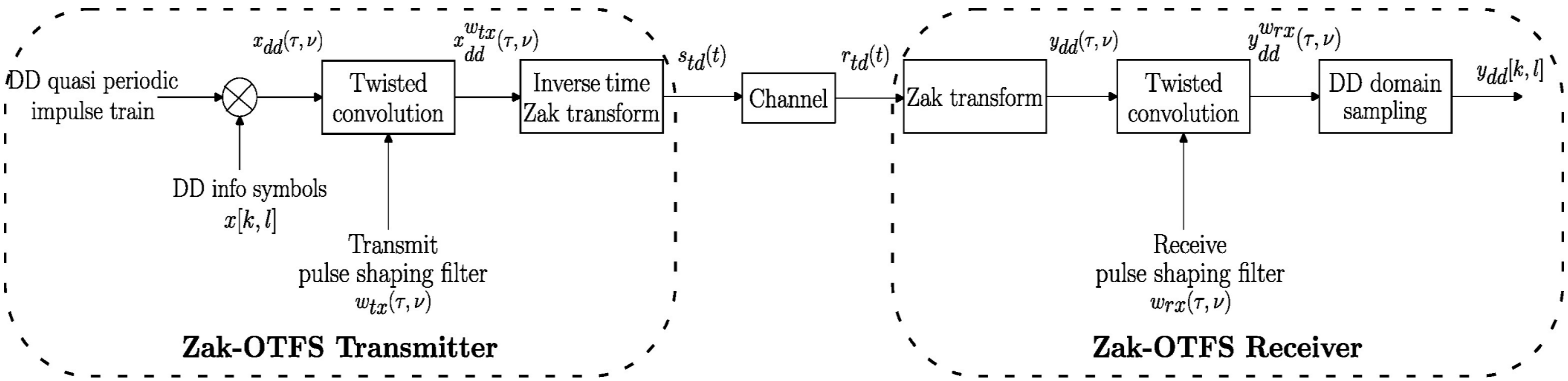
Twisted Convolution ($*_\sigma$):

$$h_2(\tau,\nu) *_\sigma h_1(\tau,\nu) = \iint h_2(\tau',\nu')\, h_1(\tau-\tau',\nu-\nu')\, e^{j2\pi\nu'(\tau-\tau')}\, d\tau'\, d\nu'$$

* S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, ``OTFS - A mathematical foundation for communication and radar sensing in the delay-Doppler domain,'' *IEEE Bits the Information Theory Magazine,* vol. 2, no. 2, pp. 36-55, Nov. 2022.

* S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, ``OTFS - Predictability in the delay-Doppler domain and its value to communication and radar sensing, *IEEE Bits the Information Theory Magazine,* doi: 10.1109/MBITS.2023.3319595.

Zak-OTFS transceiver signal processing & I/O relation



- I/O relation $y_{dd}^{w_{rx}}(\tau,\nu) = \underbrace{\left(w_{rx}(\tau,\nu) *_\sigma h(\tau,\nu) *_\sigma w_{tx}(\tau,\nu)\right)}_{h_{dd}(\tau,\nu)} *_\sigma x_{dd}(\tau,\nu)$
- Output $y_{dd}[k,l]$ is given by discrete twisted convolution of the input $x_{dd}[k,l]$ with the effective DD channel filter $h_{dd}[k,l]$

$$\begin{aligned} y_{dd}[k,l] &= \sum_{k',l' \in \mathbb{Z}} h_{dd}[k',l'] \, x_{dd}[k-k', l-l'] \, e^{j2\pi \frac{(k-k')}{M}\frac{l'}{N}} \\ &= h_{dd}[k,l] *_\sigma x_{dd}[k,l]. \end{aligned}$$

where $h_{dd}[k,l] \triangleq h_{dd}(\tau,\nu)\Big|_{\left(\tau=\frac{k\tau_p}{M},\, \nu=\frac{l\nu_p}{N}\right)}$

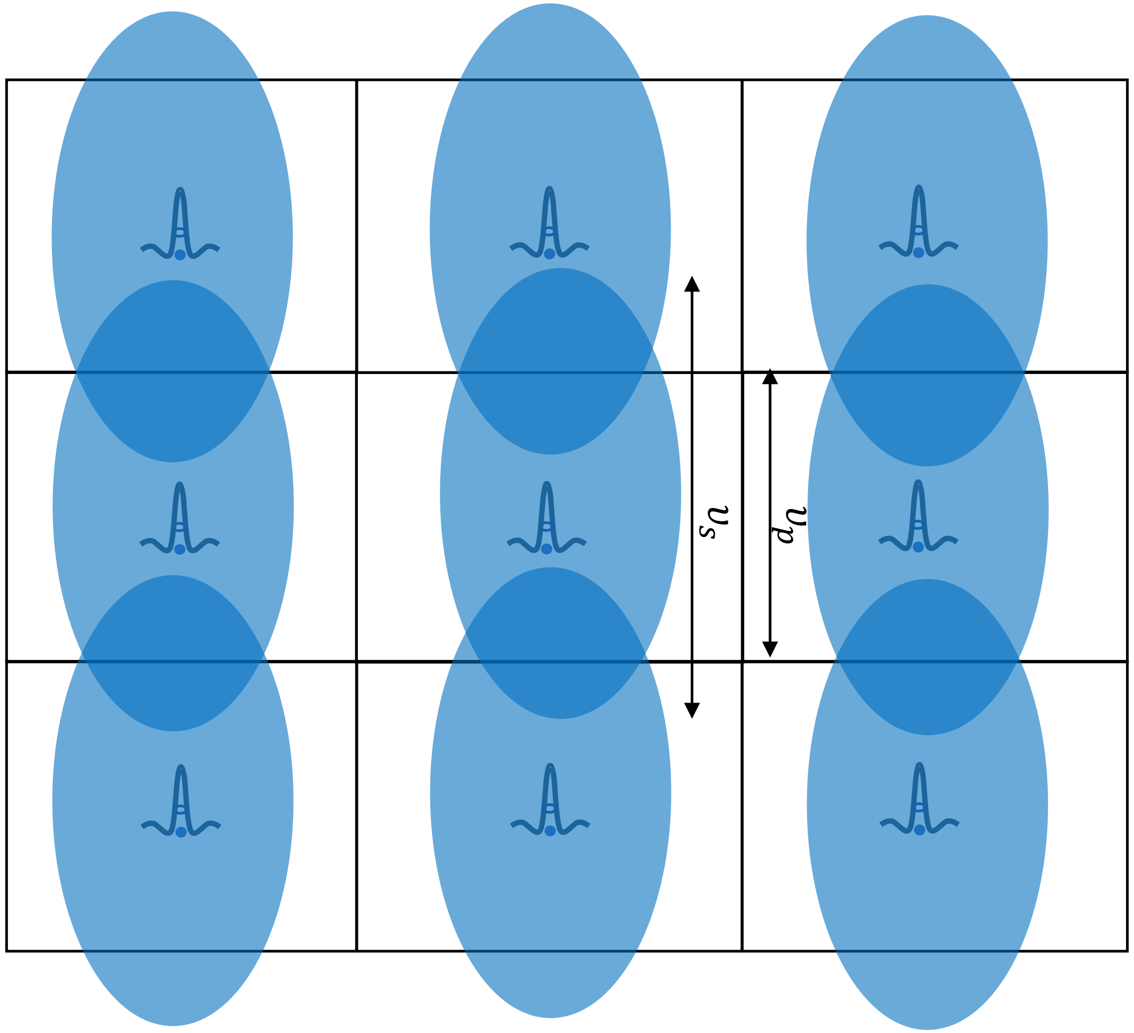
- Cascade of twisted convolution
  - Analogous to cascade of linear convolution in LTI systems

- In the vectorized form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

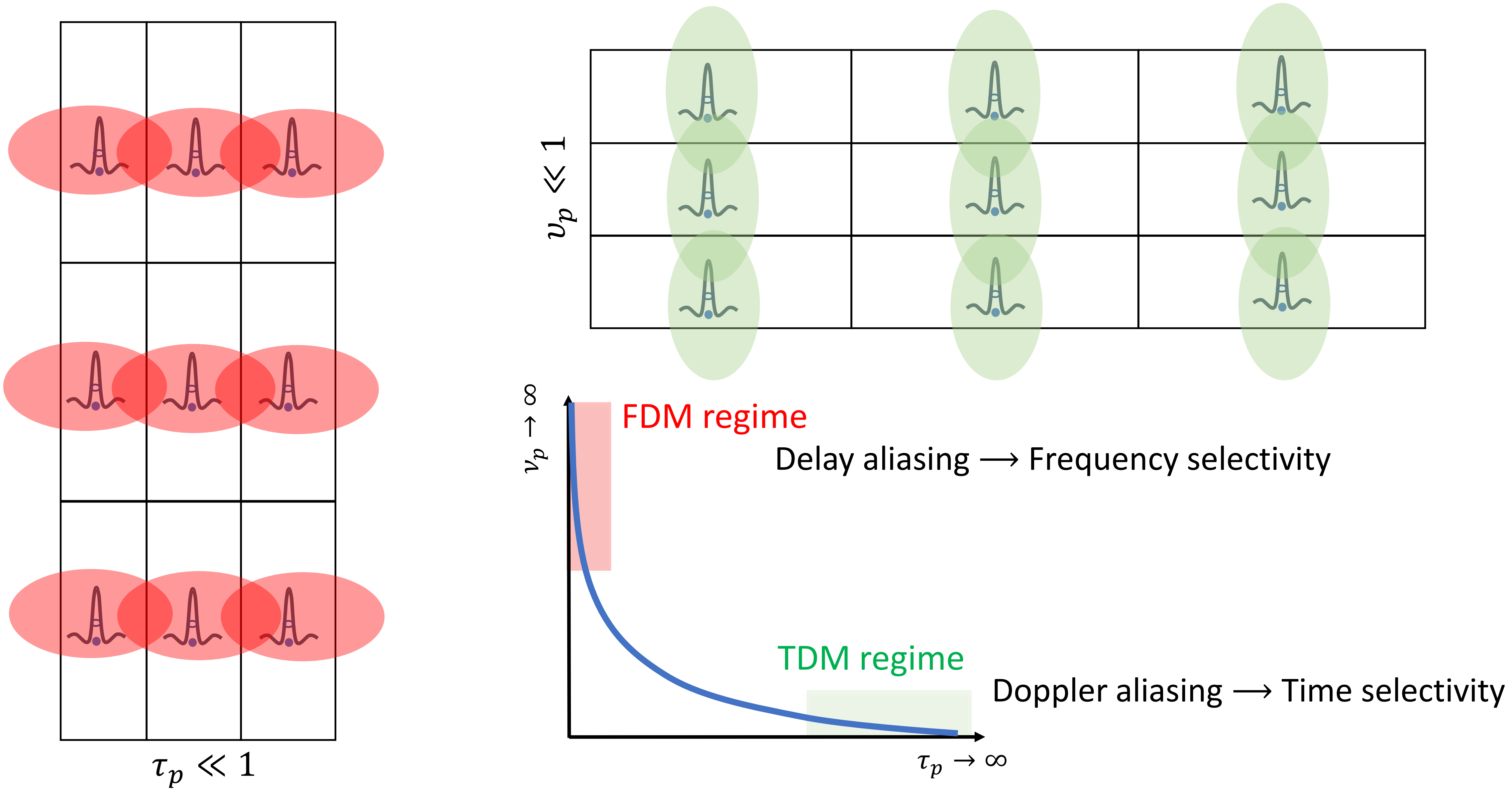
$\mathbf{x}, \mathbf{y}, \mathbf{n} \in \mathbb{C}^{MN\times 1}$ such that $x_{kN+l+1} = x_{dd}[k,l]$, $y_{kN+l+1} = y_{dd}[k,l]$, $\mathbf{H} \in \mathbb{C}^{MN\times MN}$ and $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$

Aliasing is the root cause of selectivity

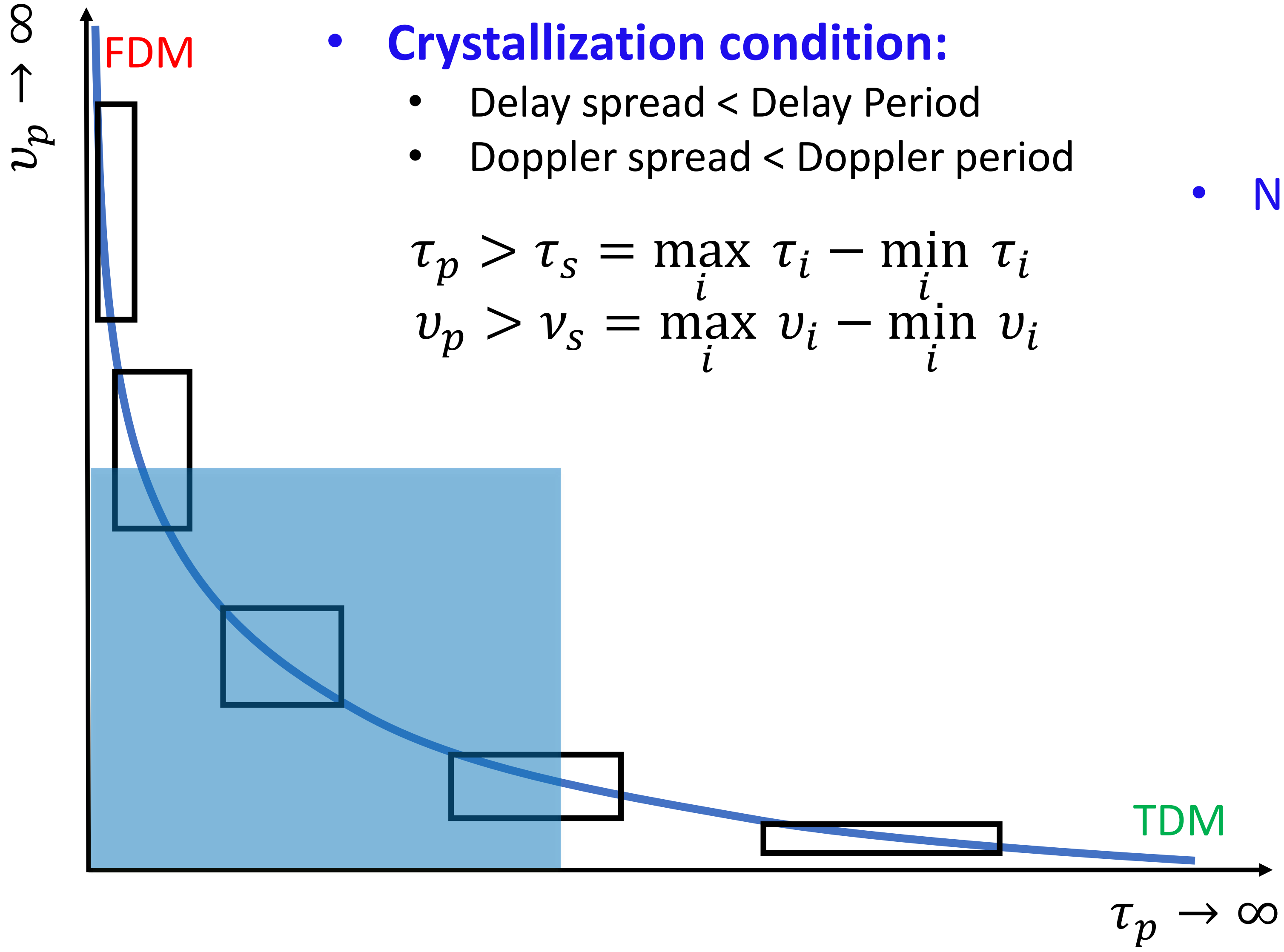


- Doppler spread > Doppler period
  (of the channel) (of the waveform)
- Fading and unpredictability occurs in the regions of self-interaction

Aliasing causes time/frequency selectivity



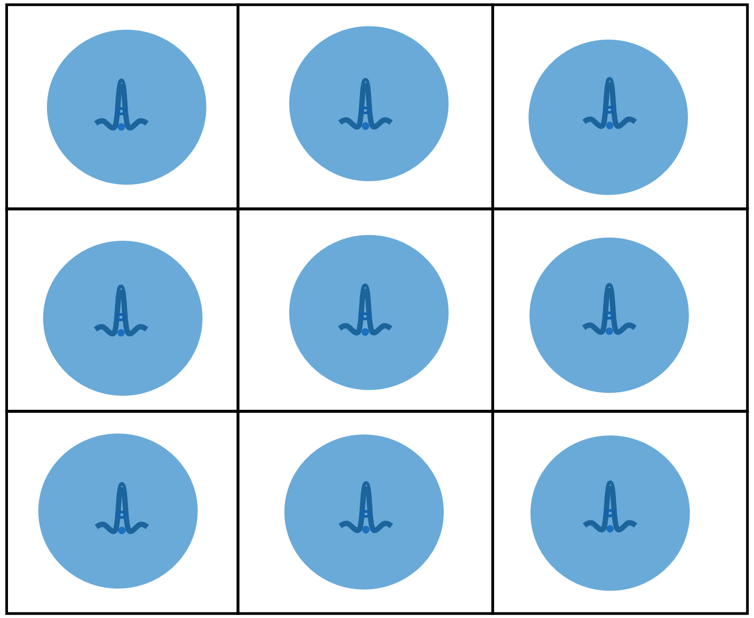
Crystallization condition



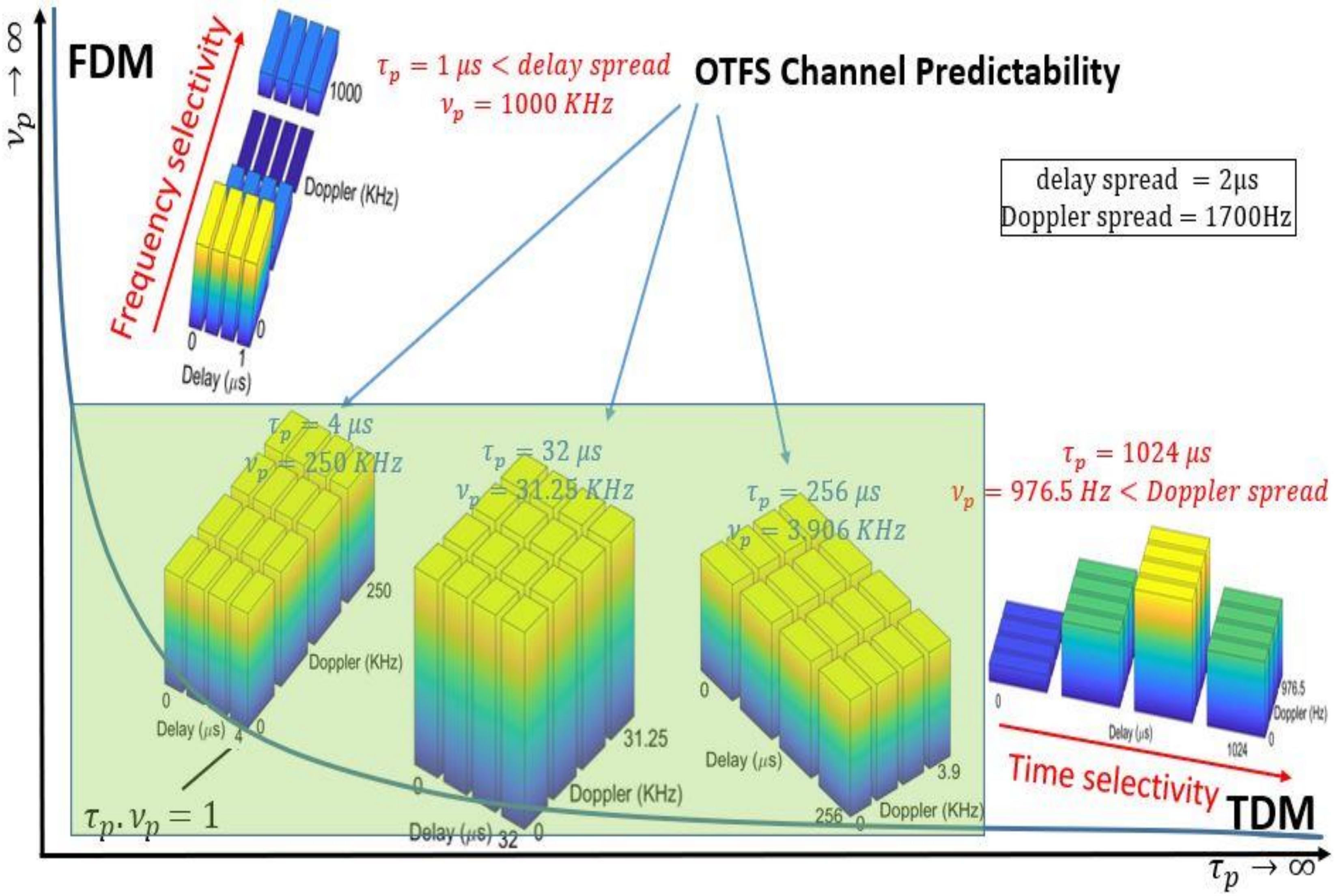
- **Crystallization condition:**
  - Delay spread < Delay Period
  - Doppler spread < Doppler period

$$\tau_p > \tau_s = \max_i \tau_i - \min_i \tau_i$$
$$\nu_p > \nu_s = \max_i \nu_i - \min_i \nu_i$$

- No aliasing in the crystalline regime

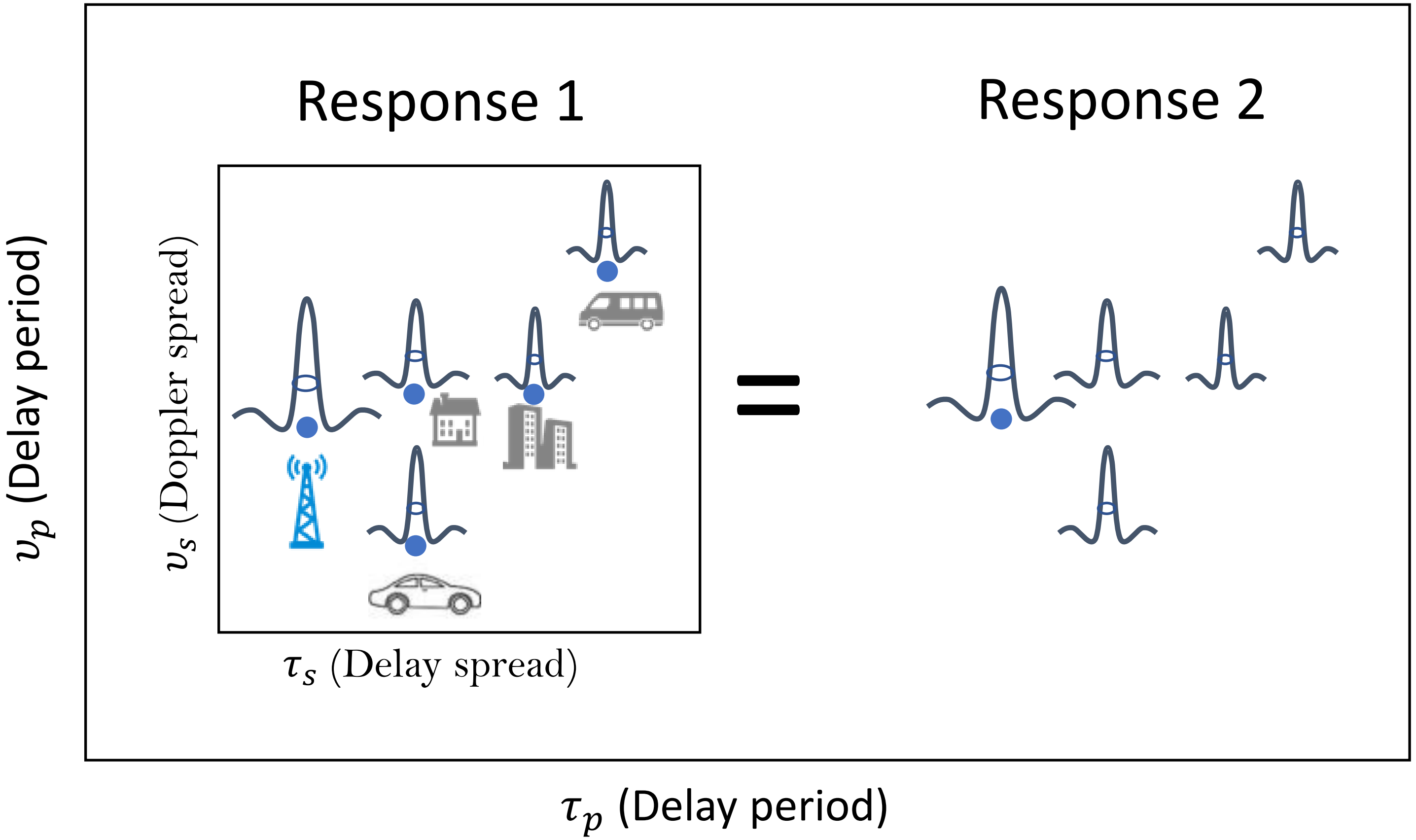


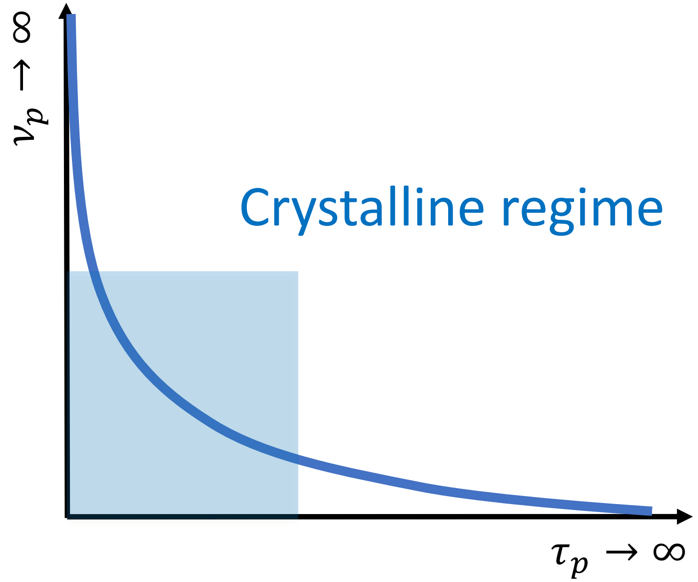
Non-fading in the crystalline regime



- In the crystalline regime,
  - No DD domain aliasing
  - Average received power profile becomes flat
  - Non-fading

Predictability in the crystalline regime



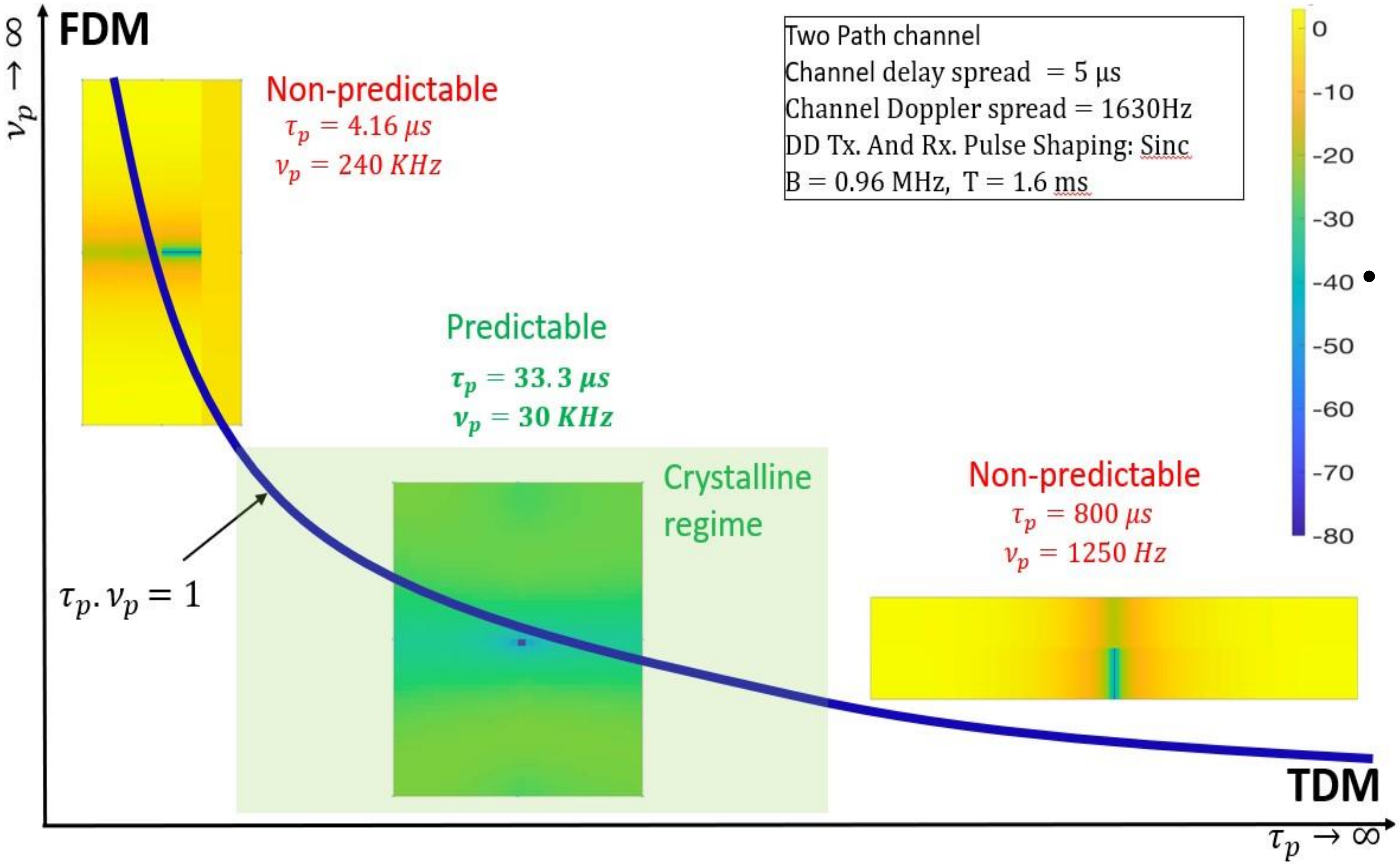


Crystallization condition:
Delay spread < Delay Period
Doppler spread < Doppler period

$$\nu_s = 500Hz < \nu_p = 20KHz$$
$$\tau_s = 5\mu s < \tau_p = 50\mu s$$

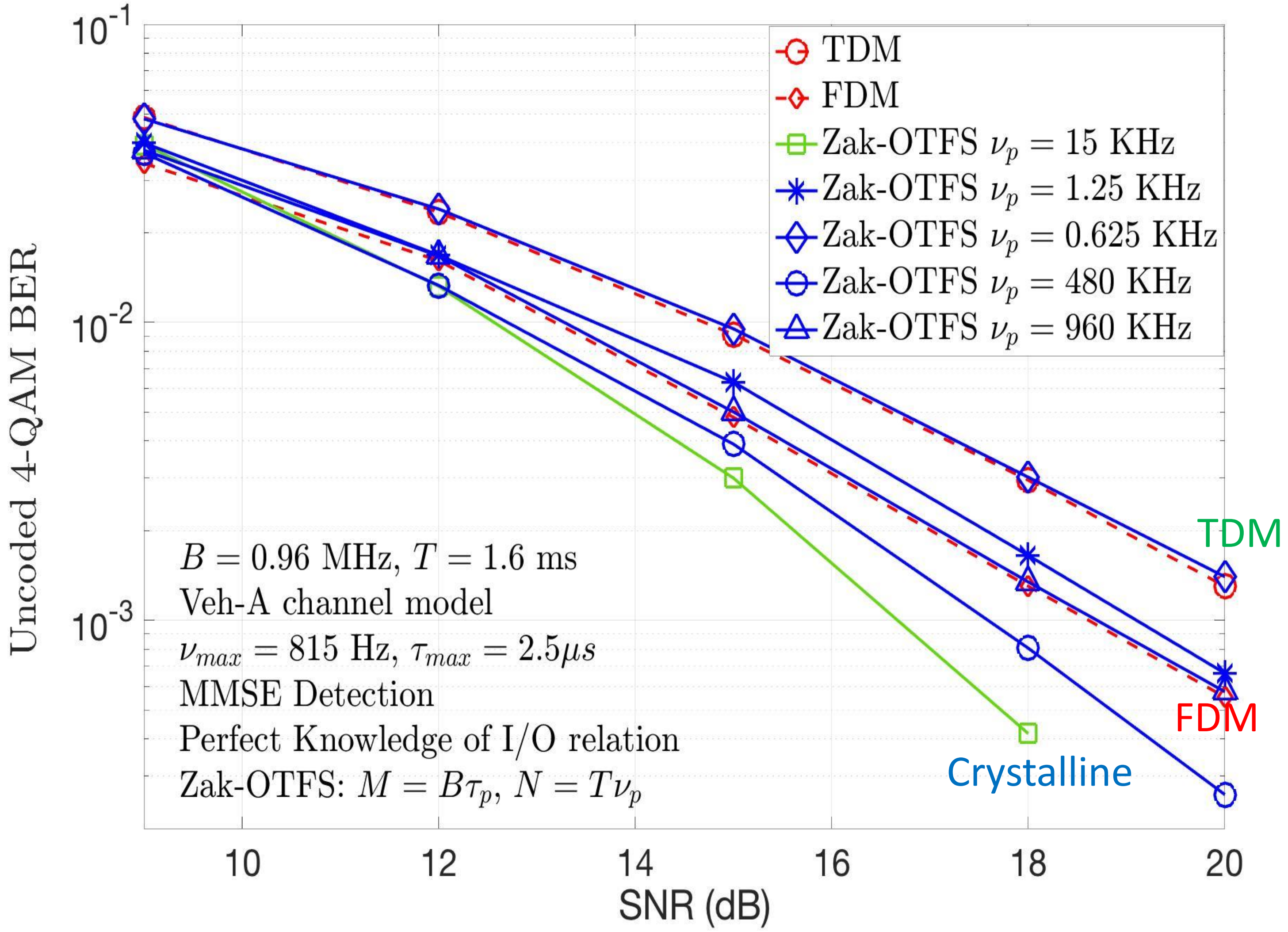
- In the crystalline regime, the interaction of the doubly spread channel with the OTFS waveform crystallizes - predictable and non-fading

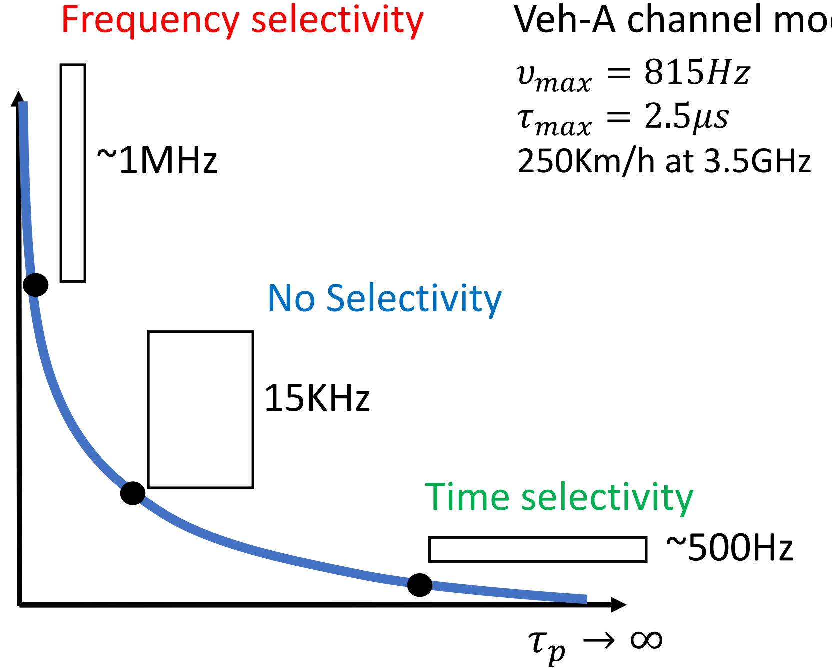
Prediction error



- In the crystalline regime,
  - Prediction error is small
  - Predictable I/O relation
  - Model-free operation

Performance advantage under perfect CSI





Veh-A channel model

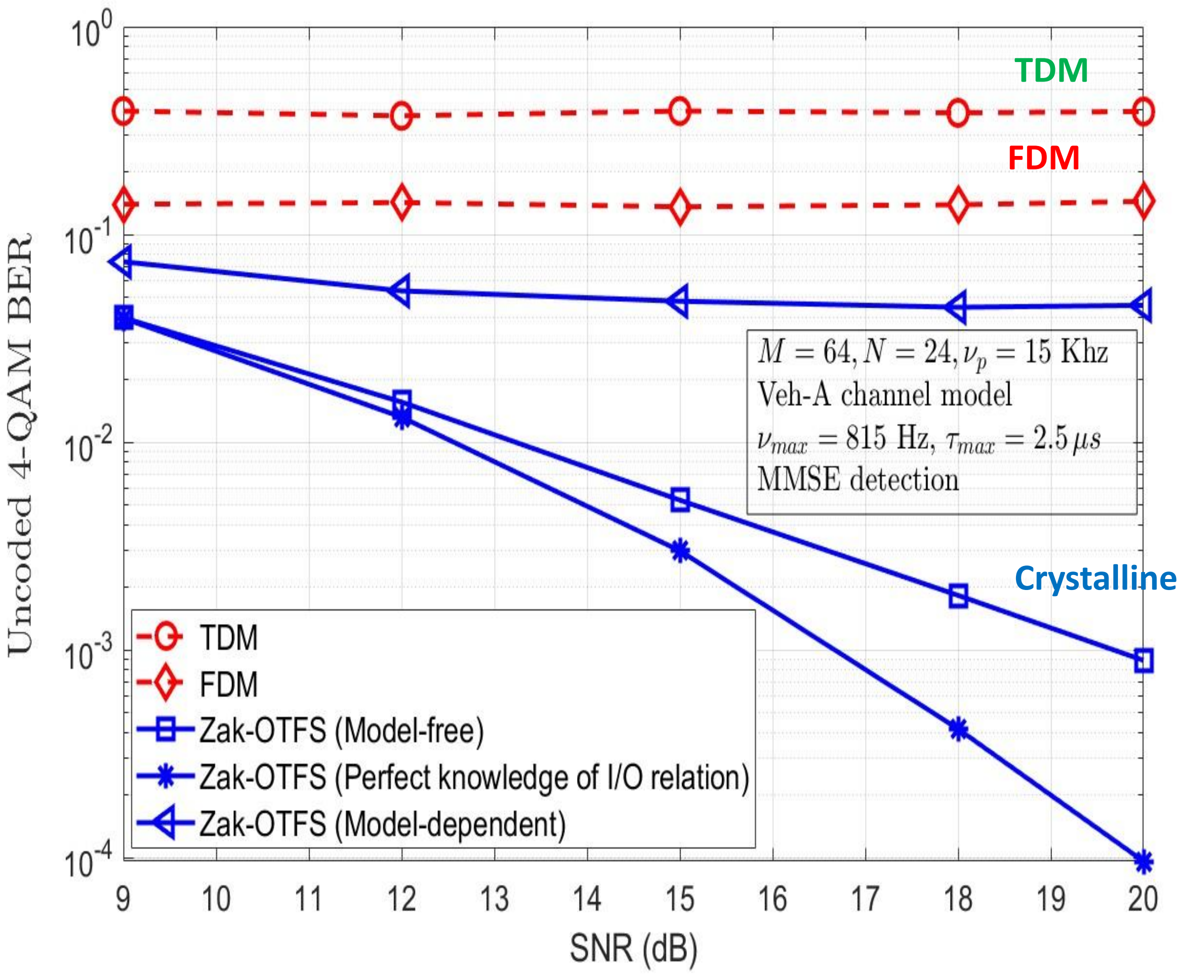
$\upsilon_{max} = 815Hz$

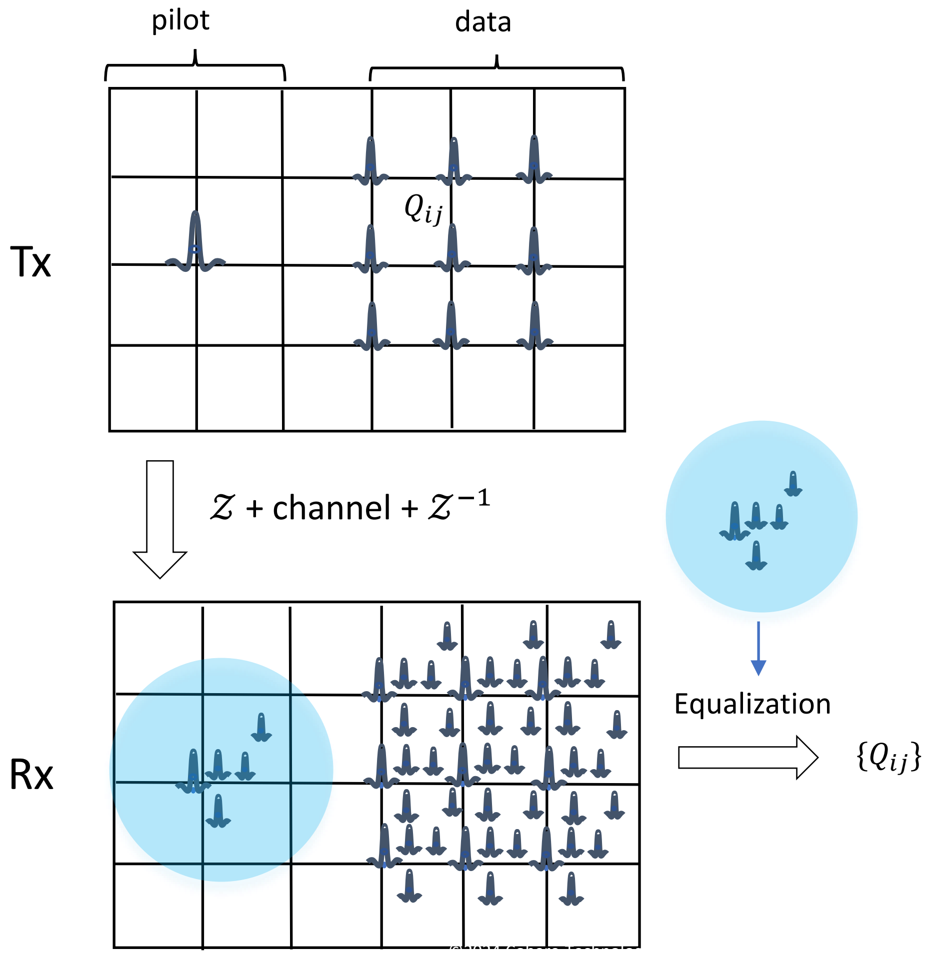
$\tau_{max} = 2.5\mu s$

250Km/h at 3.5GHz

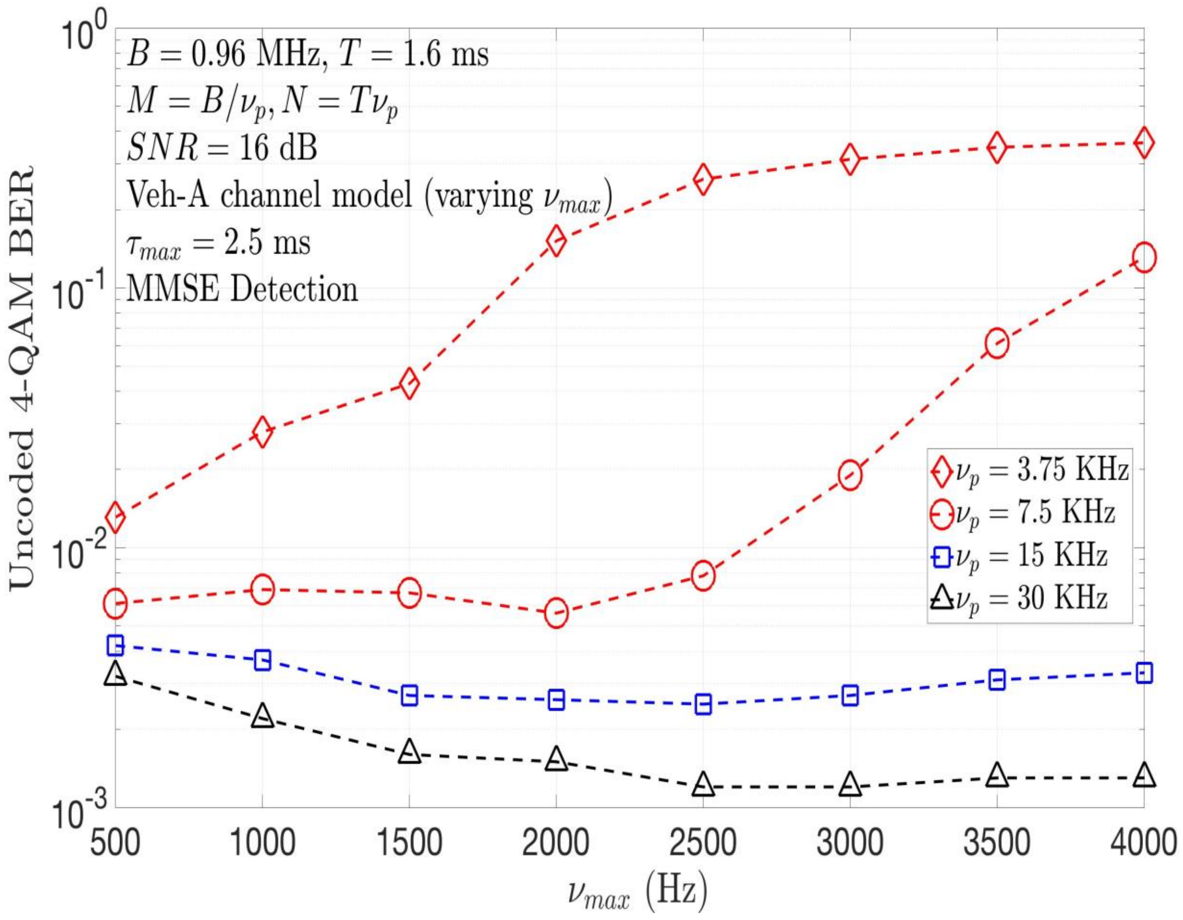
- Operating in the crystalline regime yields superior uncoded BER performance

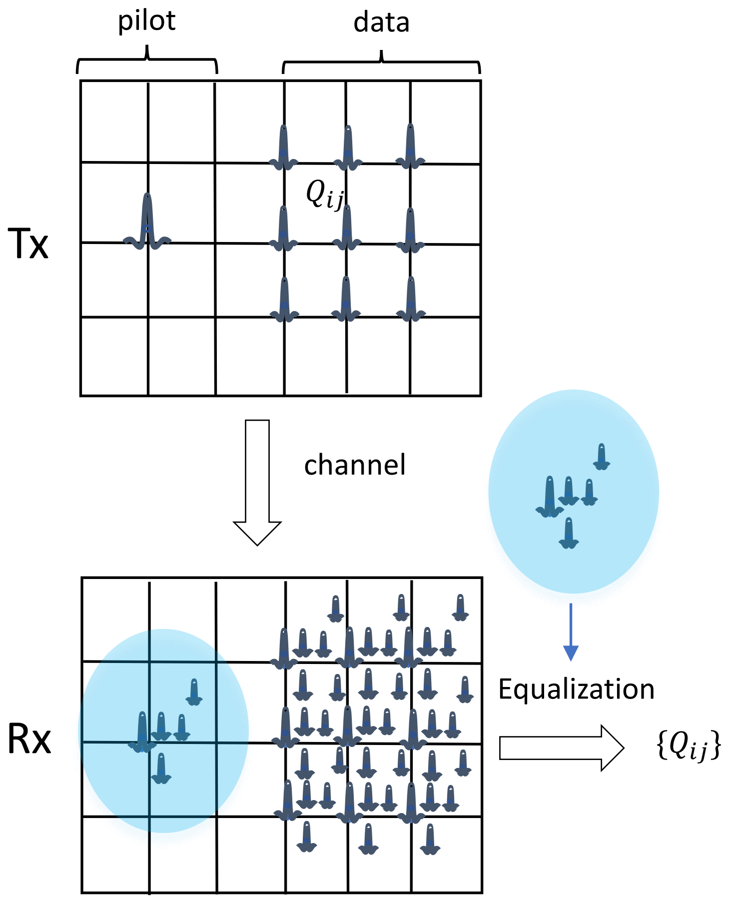
Performance advantage under non-perfect CSI



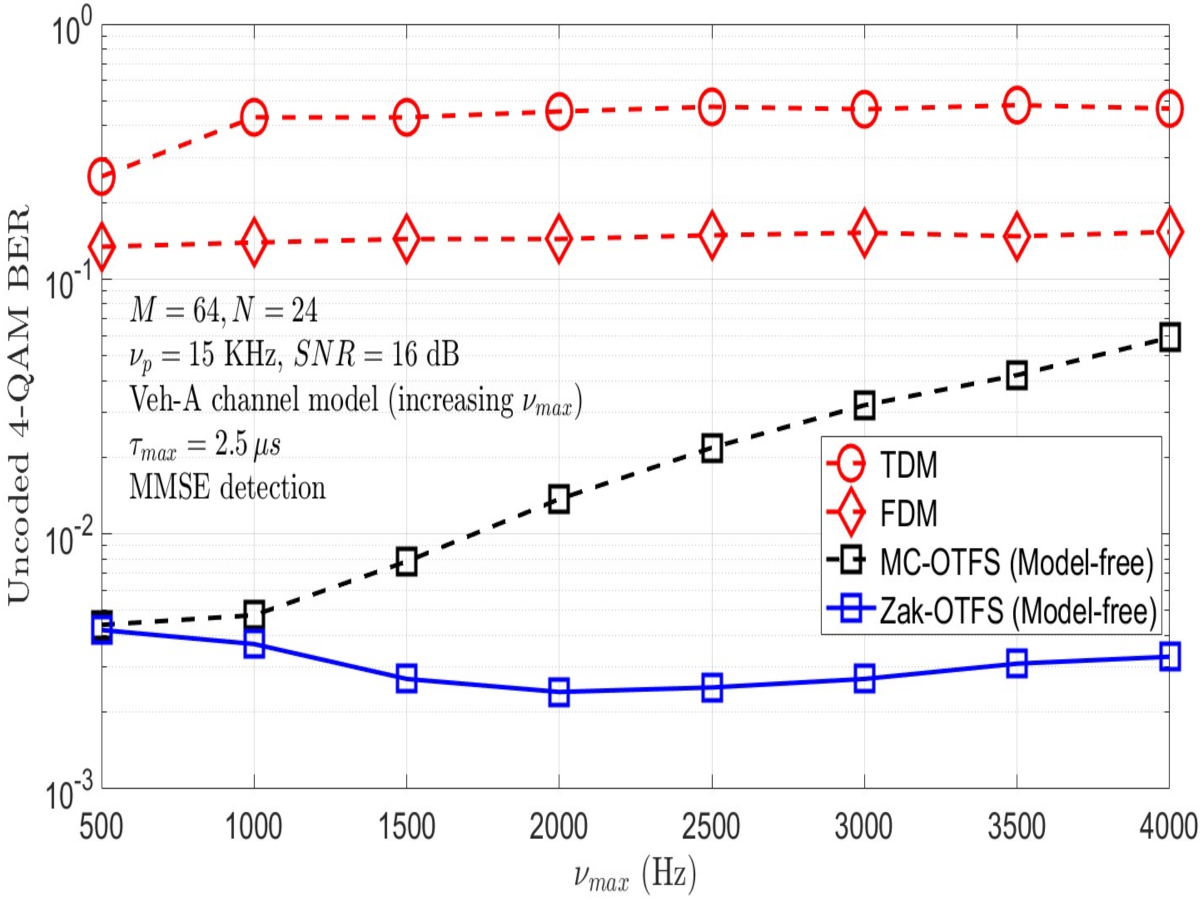


Performance advantage under non-perfect CSI



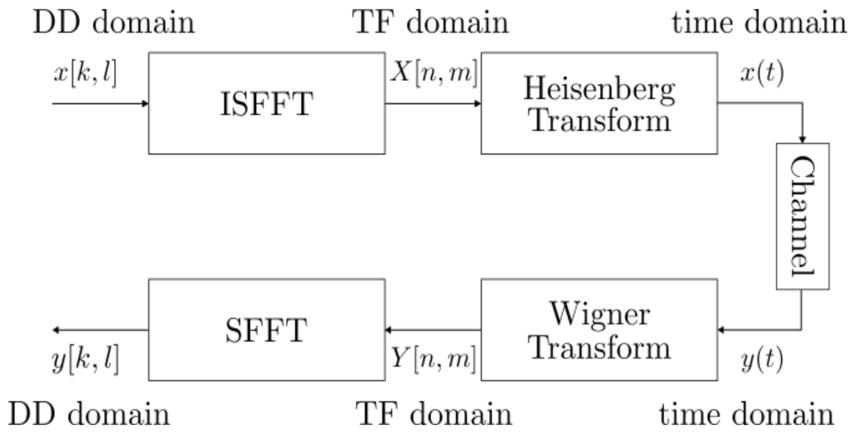


Zak-OTFS (OTFS 2.0) vs MC-OTFS (OTFS 1.0)

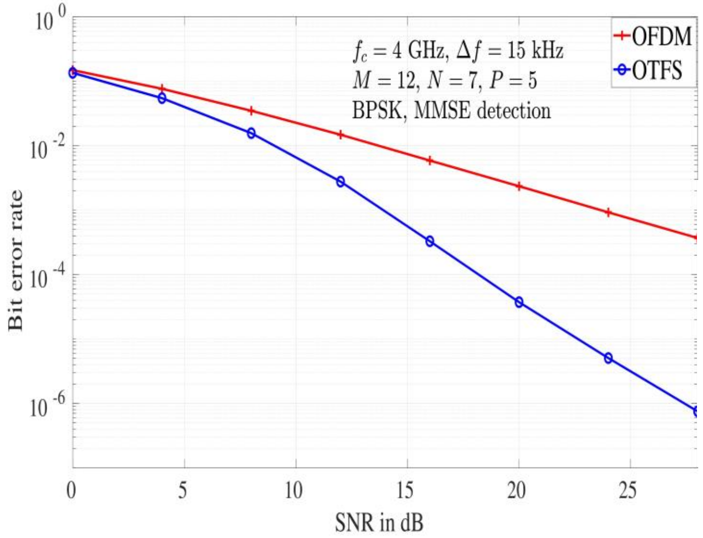


- Zak-OTFS (OTFS 2.0) more robust large channel spreads compared to MC-OTFS (OTFS 1.0)

- OTFS 1.0 (MC-OTFS)



* R. Hadani, S. Rakib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch, R. Calderbank, "Orthogonal time frequency space modulation," *IEEE WCNC'2017*, Mar. 2017.



Parameter	Value
Carrier frequency (GHz)	4
Subcarrier spacing (kHz)	15
Frame size (M, N)	(12, 7)
Number of paths (P)	5
Delay profile	Exponential
Maximum speed (km/h)	500
Maximum Doppler (Hz)	1875
Modulation scheme	BPSK

Near-optimal detection of Zak-OTFS

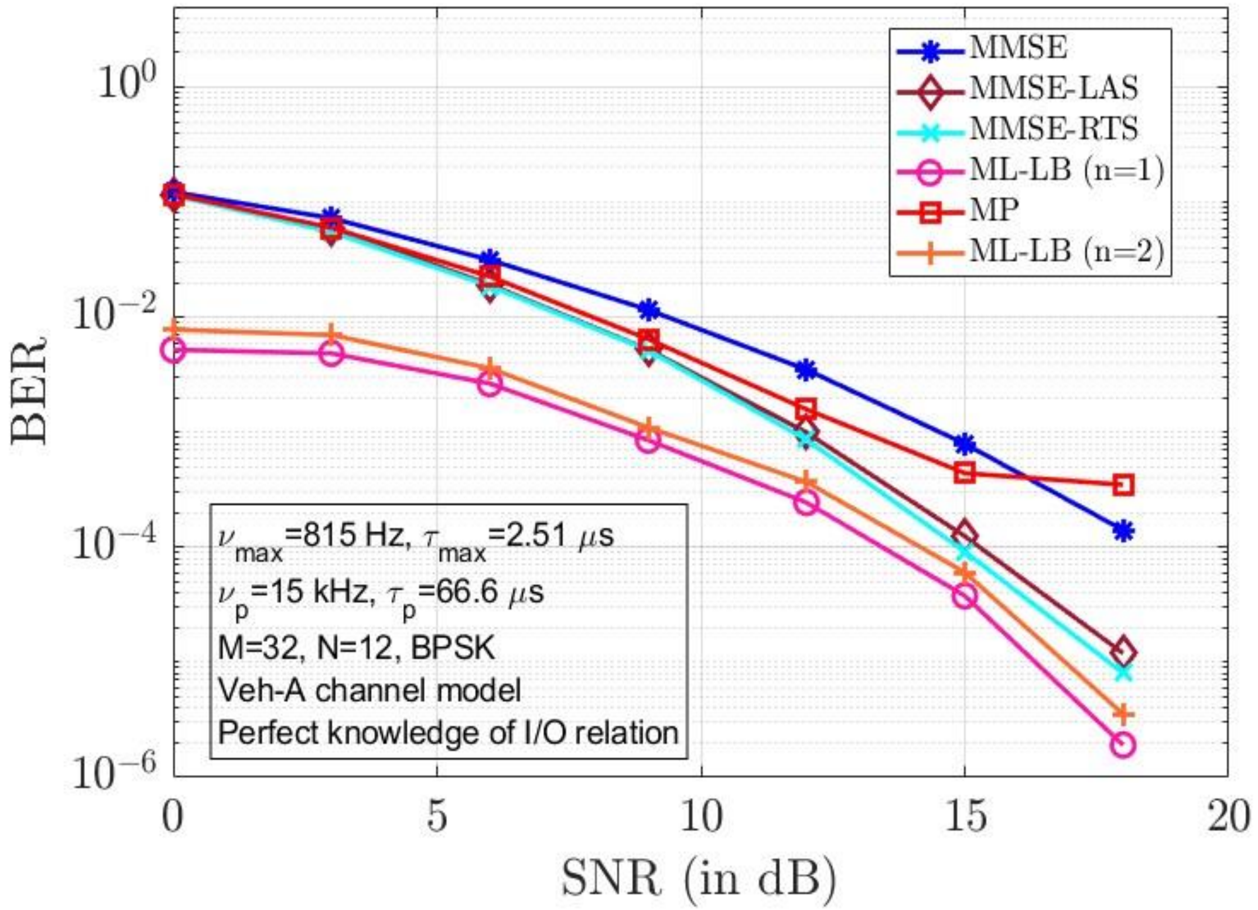
- MMSE detection
- Message passing (MP) detection
- Far from optimum ML performance

- Local search-based detection
  - Likelihood ascent search (LAS)
  - Reactive tabu search (RTS)

- Lower bound on ML performance using RTS simulations

- LAS, RTS perform better than MMSE and MP (and close to optimum)

- Efficient detection and channel estimation for Zak-OTFS – open for research

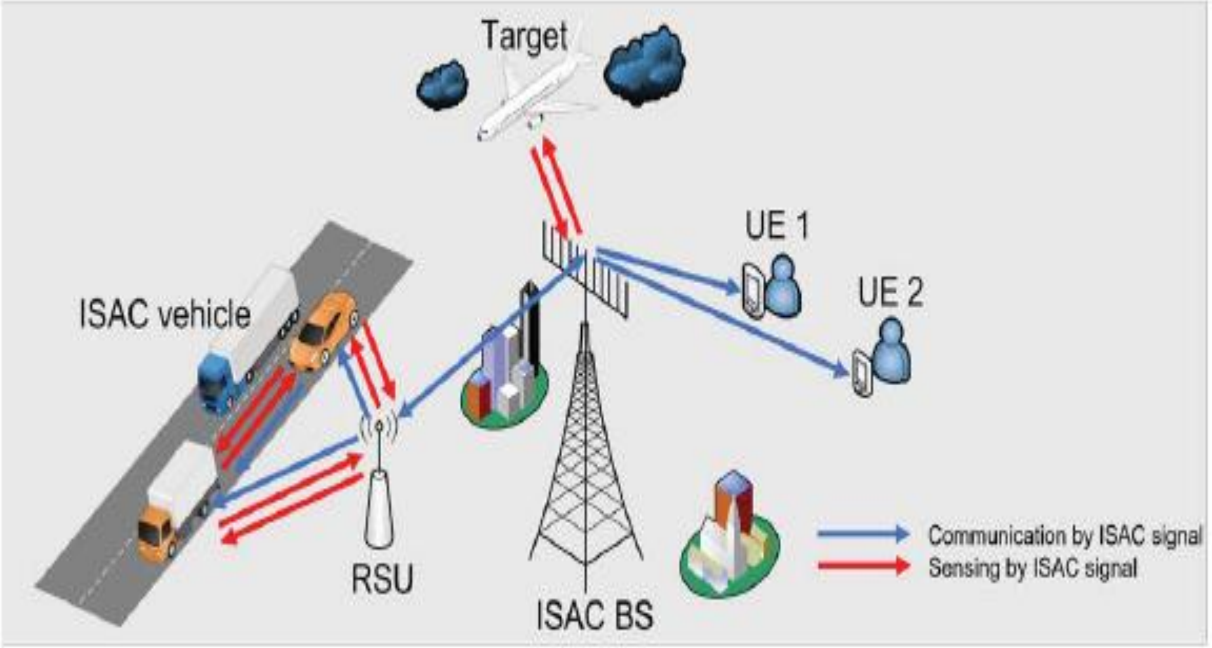


* F. Jesbin and A. Chockalingam, ``Near-optimal detection of Zak-OTFS signals,'' *IEEE ICC'2024*, Jun. 2024.

Radar sensing

- Transmit radar sensing signal
- Use the received echo to estimate delay and Doppler of multiple targets present in the radar scene
- Radar scene is completely described by the DD spreading function of the echo channel between the radar Tx and Rx
- DD spreading function has peaks at the target locations in the DD domain
- Good radar waveforms must be localized in the DD domain

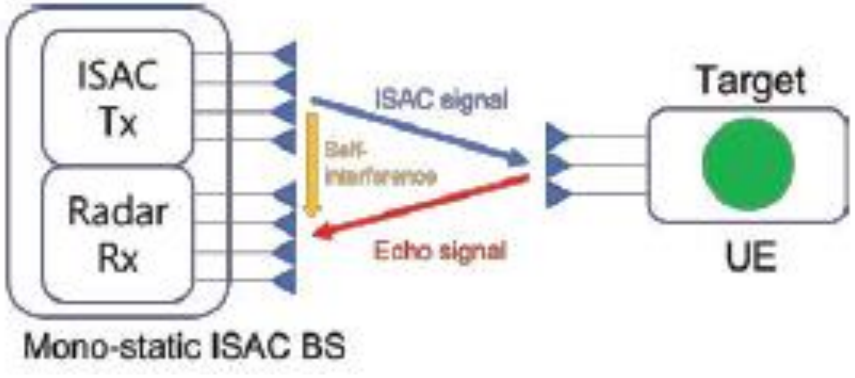
- Integrated communication and sensing (ISAC): an example scenario



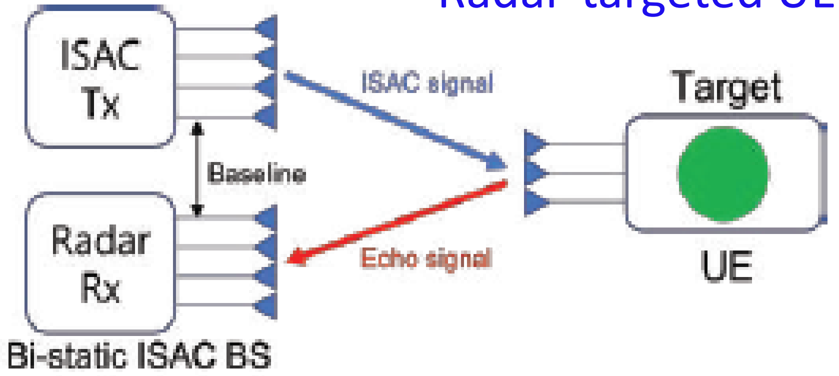
* K. Kim, J. Kim, and J. Joung, ``A survey on system configurations of integrated sensing and communication (ISAC) Systems,'' ICICTC, 2022.

ISAC system configurations

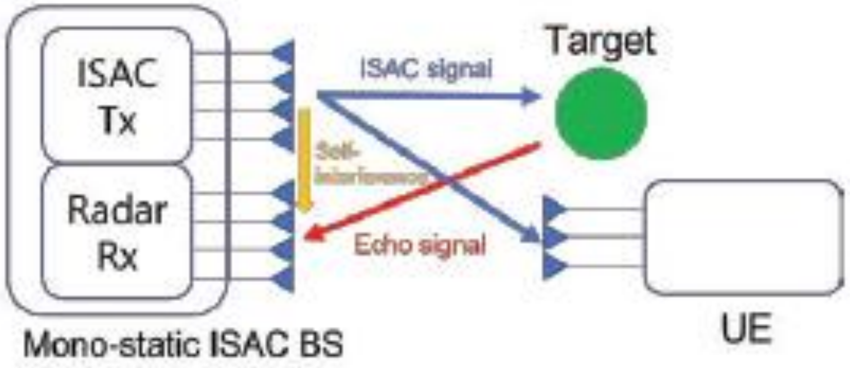
- Mono-static radar BS
- Radar-targeted UE



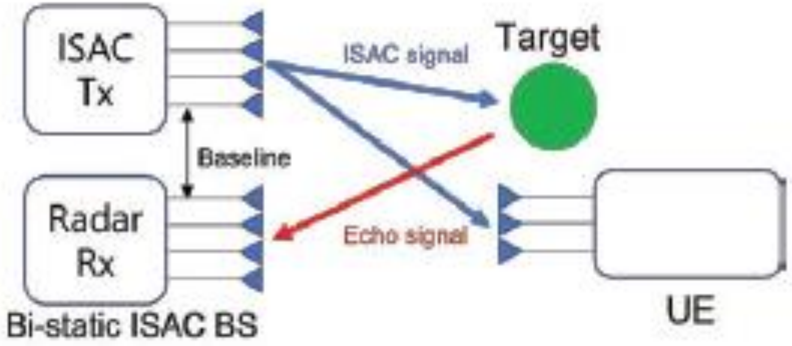
- Bi-static radar BS
- Radar-targeted UE



- Mono-static radar BS
- Non-targeted UE



- Bi-static radar BS
- Non-targeted UE



Radar ambiguity and resolution

- Cross-ambiguity function

- Radar scene with single target, no reflector
- Tx. radar waveform: $s_{\mathrm{td}}(t)$
- Received echo:

$$r_{\mathrm{td}}(t) = h\, s_{\mathrm{td}}(t-\tau)\, e^{j2\pi\nu(t-\tau)} + n_{\mathrm{td}}(t)$$

- ML estimate of delay and Doppler

$$(\widehat{\tau}, \widehat{\nu}) = \arg\max_{\tau,\nu} |A_{r,s}(\tau,\nu)|$$

$$A_{r,s}(\tau,\nu) \triangleq \int r_{\mathrm{td}}(t)\, s^*_{\mathrm{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt \qquad \text{(Cross-ambiguity)}$$

- Detection of multiple targets and reflector: Peaks of cross-ambiguity

- Cross-ambiguity for general radar scene:

$$A_{r,s}(\tau,\nu) = h(\tau,\nu) *_\sigma A_{s,s}(\tau,\nu) + \int n_{\mathrm{td}}(t)\, s^*_{\mathrm{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt$$

- Ambiguity function of $s_{\mathrm{td}}(t)$:

$$A_{s,s}(\tau,\nu) \triangleq \int s_{\mathrm{td}}(t)\, s^*_{\mathrm{td}}(t-\tau)\, e^{-j2\pi\nu(t-\tau)} dt$$

- Moyal's identity:

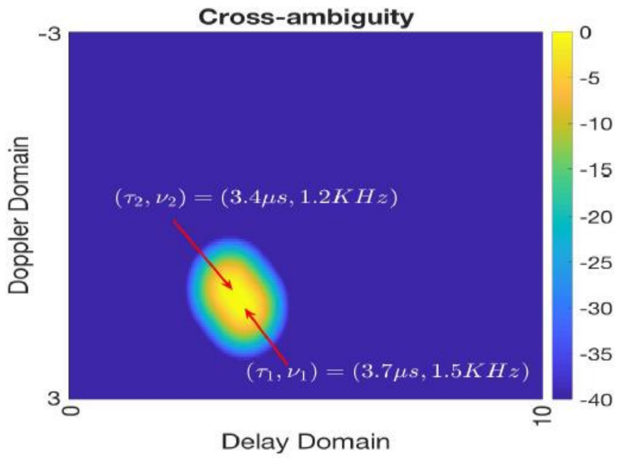
$$\iint |A_{s,s}(\tau,\nu)|^2\, d\tau d\nu = \left(\int |s_{\mathrm{td}}(t)|^2\, dt\right)^2$$

Radar ambiguity and resolution

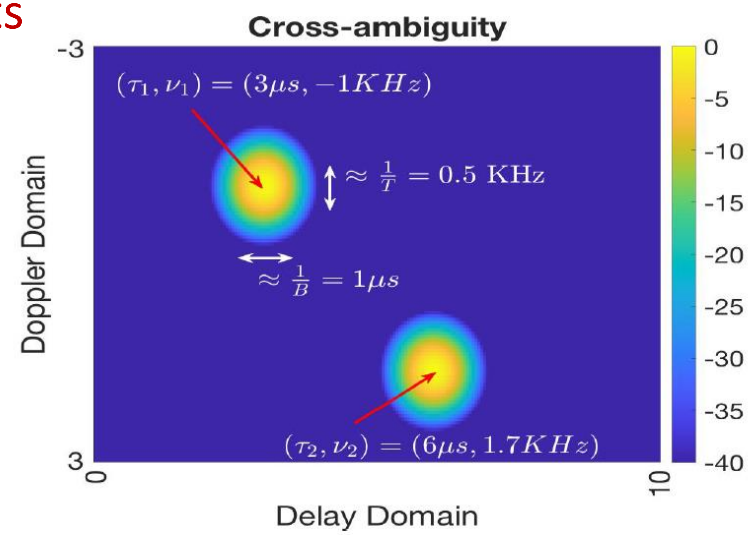
- **Ambiguity: an example**
  - One target at (τ_1, ν_1), $h(\tau,\nu) = h_1\delta(\tau - \tau_1)\delta(\nu - \nu_1)$
  - Cross-ambiguity function (AWGN free)

    $$A_{r,s}(\tau,\nu) = h_1\, A_{s,s}(\tau - \tau_1, \nu - \nu_1)\, e^{j2\pi\nu_1(\tau - \tau_1)}$$
    $$|A_{r,s}(\tau,\nu)|^2 = |h_1|^2\; |A_{s,s}(\tau - \tau_1, \nu - \nu_1)|^2$$
  - Ideal Ambiguity function $A_{s,s}(\tau,\nu)$: Dirac-delta impulse at $(0,0)$
  - Let $|A_{s,s}(\tau,\nu)|$ have more than one peak, e.g. at $(0,0)$ and (τ_0, ν_0)
  - Then, $|A_{r,s}(\tau,\nu)|^2$ has two peaks at (τ_1,ν_1) and $(\tau_1 + \tau_0, \nu_1 + \nu_0)$
  - Ambiguity in detection

- **Non-resolvable targets**



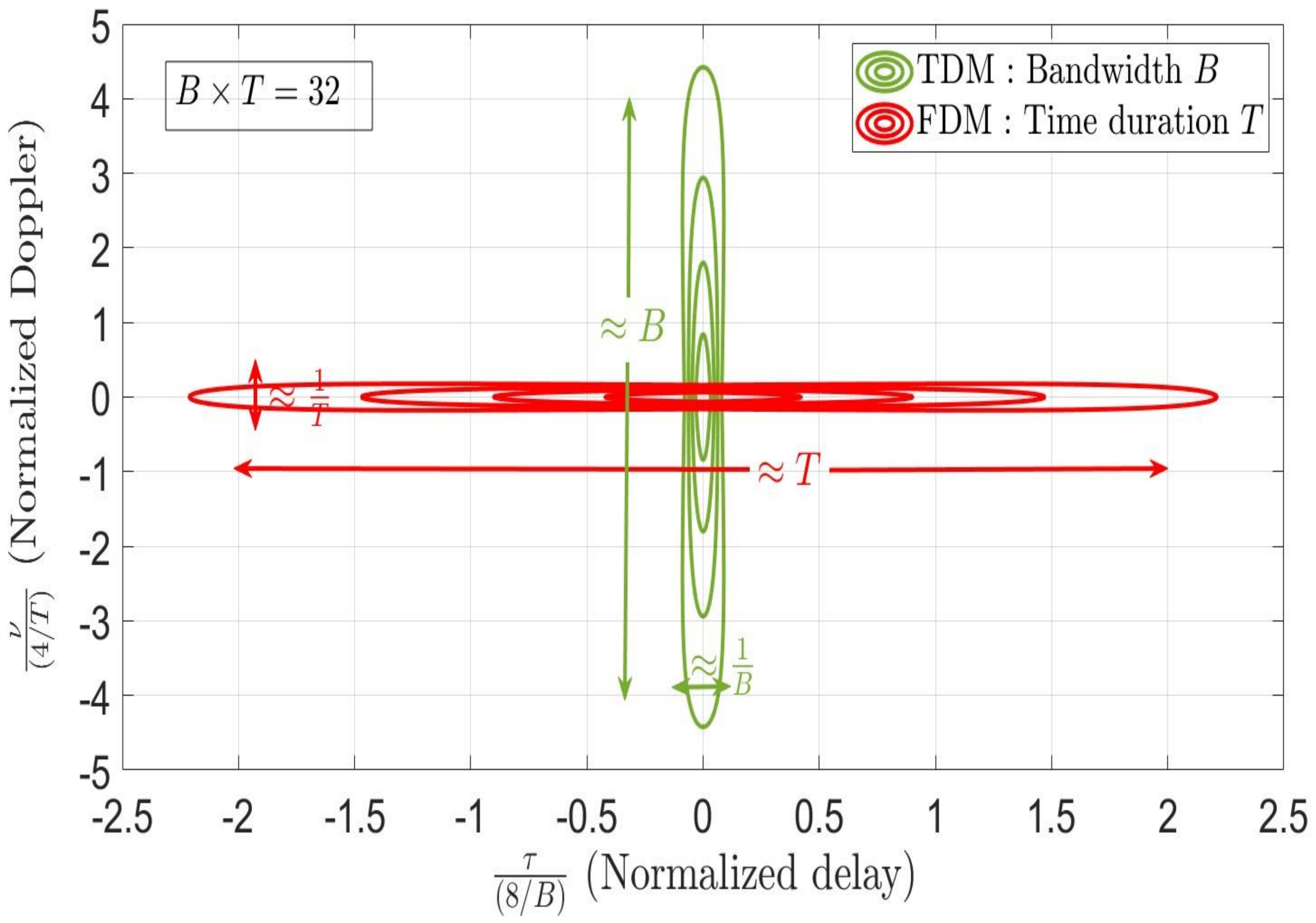
- **Resolvable targets**



- **Resolution: an example**
  - Two targets: $(\tau_i, \nu_i), i = 1, 2,\ h(\tau,\nu) = \sum_{i=1}^{2} h_i\delta(\tau - \tau_i)\delta(\nu - \nu_i)$
  - Noise free cross-ambiguity

    $$A_{r,s}(\tau,\nu) = h_1 A_{s,s}(\tau - \tau_1, \nu - \nu_1)e^{j2\pi\nu_1(\tau-\tau_1)} + h_2 A_{s,s}(\tau - \tau_2, \nu - \nu_2)e^{j2\pi\nu_2(\tau-\tau_2)}$$
  - Both targets can be detected simultaneously only if $|A_{s,s}(\tau - \tau_1, \nu - \nu_1)|$ and $|A_{s,s}(\tau - \tau_2, \nu - \nu_2)|$ have minimal overlap
  - In other words, delay domain and Doppler domain spread of $A_{s,s}(\tau,\nu)$ must be less than $|\tau_1 - \tau_2|$ and $|\nu_1 - \nu_2|$ respectively

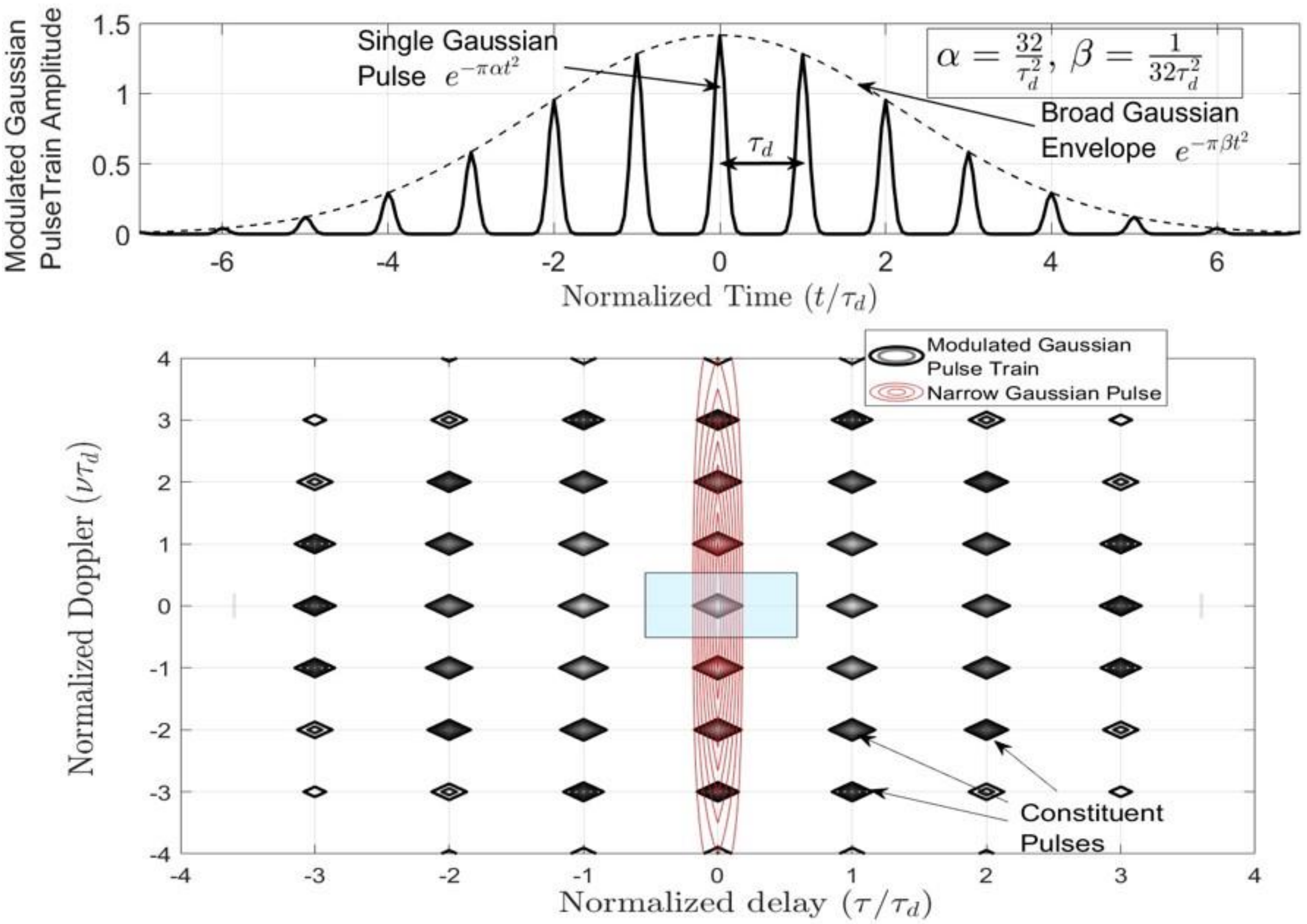
Ambiguity of TD and FD pulses



- TD/FD pulses can not resolve targets simultaneously along delay and Doppler

- A good radar waveform distributes "ambiguity" such that simultaneous delay-Doppler resolvability is achieved

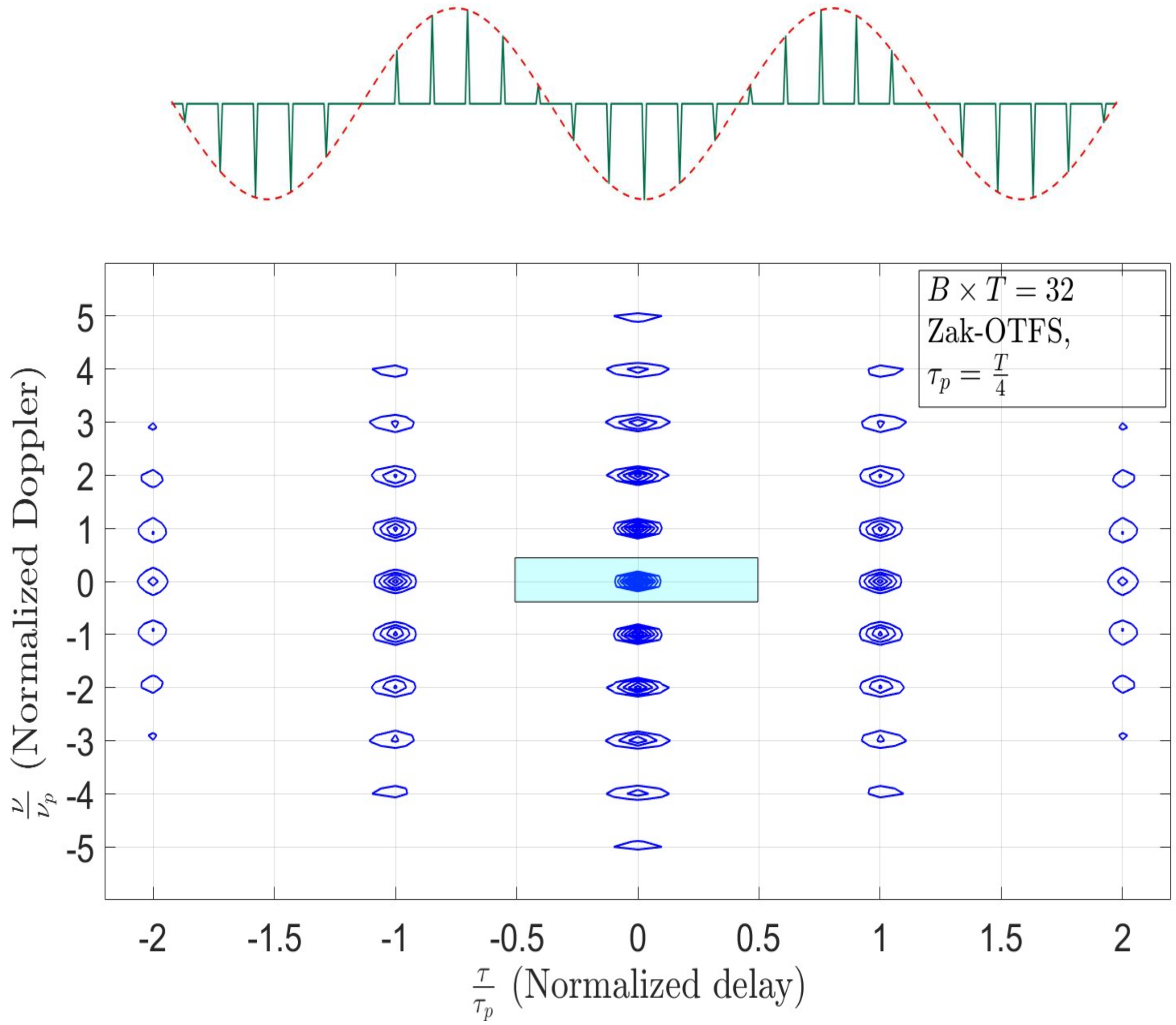
A good radar waveform (P. M. Woodward in 1953)



- Modulate a train of narrow TD Gaussian pulses with a broad Gaussian envelope
- Re-distribution of ambiguity
- Woodward's waveform is strikingly similar to the Zak-OTFS pulsone

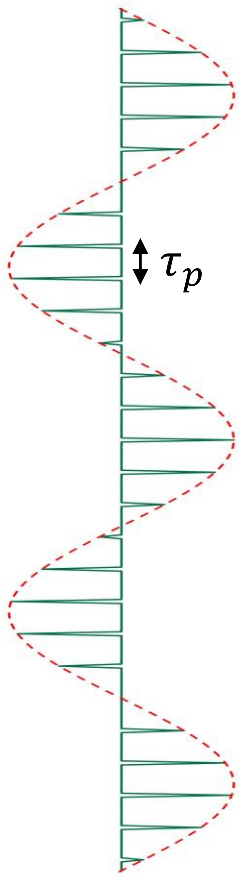
* P. M. Woodward, Probability and Information Theory with Applications to Radar, Pergamon Press, 1953.

Ambiguity function of Zak-OTFS pulsone



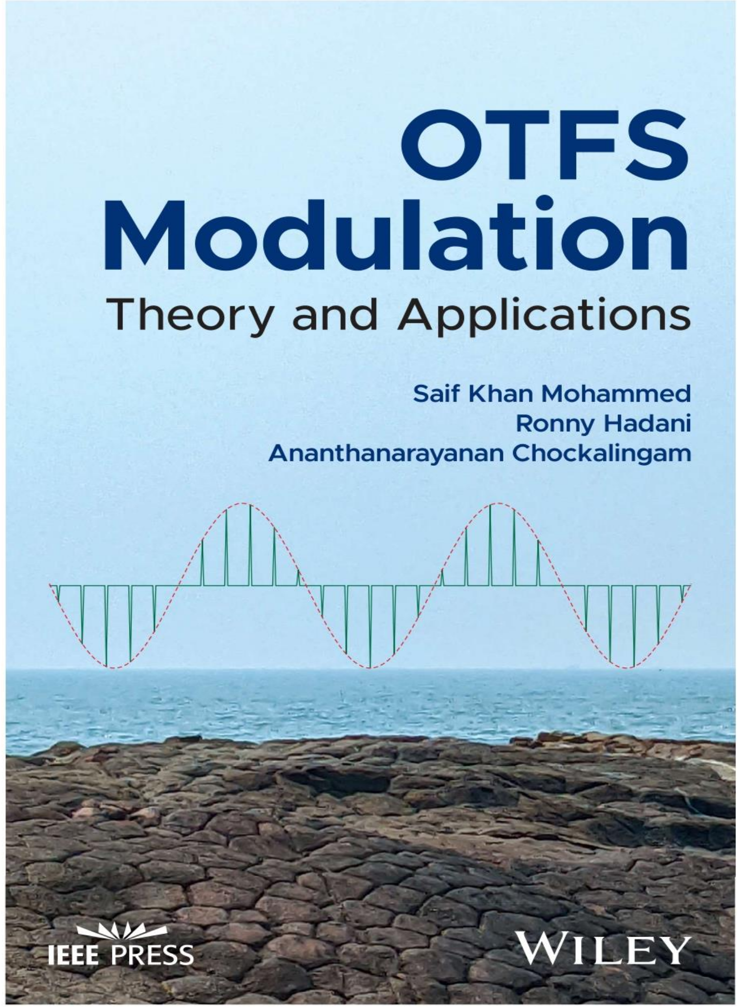
- No ambiguity when crystallization condition is satisfied
- Delay and Doppler domain resolutions are ∝ 1/B and 1/T, respectively
- Ambiguity function can be expressed analytically in terms of the tx. pulse wtx (τ, ν)
- Design of good radar waveforms therefore reduces to pulse design in the DD domain
- Zak theory provides a mathematical framework for design of good radar waveforms

Concluding remarks



- Pulsones
  - Universal family of time domain waveforms
  - Parameterized by τ_p s. t. $\tau_p \nu_p = 1$ (admits TDM and FDM as limits)
  - Quasi-periodic pulse in the DD domain
  - Zak transform connects TD ⇔ DD domain
    - Zak theory is to LTI systems as Fourier theory is to LTI systems
  - Information carrier for Zak-OTFS modulation
    - Information multiplexing and signal processing in DD domain
    - Channel interaction through twisted convolution
    - Robust communication in doubly-selective channels
  - Good localization properties as a radar sensing waveform
    - Natural waveform for integrated sensing and communication
  - Optimal operating regime
    - crystalline regime → no aliasing (non-fading and predictable)

Upcoming book on OTFS



Thank you