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Layered Tabu Search Algorithm for Large-MIMO Detection and a Lower Bound on ML Performance

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***Abstract*— In this paper, we are concerned with low-complexity detection in large multiple-input multiple-output (MIMO) systems with tens of transmit/receive antennas. Our new contributions in this paper are two-fold. First, we propose a low-complexity algorithm for large-MIMO detection based on a *layered low-complexity local neighborhood search*. Second, we obtain a lower bound on the maximum-likelihood (ML) bit error performance using the local neighborhood search. The advantages of the proposed ML lower bound are i) it is easily obtained for MIMO systems with large number of antennas because of the inherent low complexity of the search algorithm, ii) it is tight at moderate-to-high SNRs, and iii) it can be tightened at low SNRs by increasing the number of symbols in the neighborhood definition. Interestingly, the proposed detection algorithm based on the layered local search achieves bit error performances which are quite close to this lower bound *for large number of antennas and higher-order QAM*. For e.g., in a 32×32 V-BLAST MIMO system, the proposed detection algorithm performs close to within 1.7 dB of the proposed ML lower bound at 10^{-3} BER for 16-QAM (128 bps/Hz), and close to within 4.5 dB of the bound for 64-QAM (192 bps/Hz).**

Keywords – *Large-MIMO detection, local neighborhood search, QR decomposition, ML lower bound, higher-order QAM, high spectral efficiency.*

I. INTRODUCTION

Large-MIMO systems with tens of transmit and receive antennas are of interest because of the high capacities theoretically predicted in them [1],[2]. Research in low-complexity receive processing (e.g., MIMO detection) techniques that can lead to practical realization of large-MIMO systems is both nascent as well as promising. For e.g., a 12×12 V-BLAST system at 5 Gbps data rate and 50 bps/Hz spectral efficiency in 5 GHz band at a mobile speed of 10 Km/hr has been reported [3],[4]. Evolution of WiFi standards from IEEE 802.11n to IEEE 802.11ac to achieve multi-gigabit rate transmissions in 5 GHz band now considers 16×16 MIMO operation; see 16×16 MIMO indoor channel sounding measurements at 5 GHz reported in [5] for consideration in WiFi standards. Also, 64×64 MIMO channel sounding measurements at 5 GHz in indoor environments have been reported in [6]. With RF/antenna technologies/measurements for large-MIMO systems getting matured, there is an increasing need to focus on low-complexity receiver algorithms for large-MIMO systems to reap high spectral efficiency benefits.

Recently, certain algorithms from machine learning/artificial intelligence have been shown to achieve near-optimal performance in large-MIMO systems with tens of antennas at low complexities [7]-[15][1]. In [14], near-optimal detection in a 50×50 MIMO system with BPSK was reported using a Gibbs sampling based detection algorithm. In [15], near-optimal detection performance in a 64×64 MIMO system, again with BPSK modulation, was reported using a factor graph based belief propagation (BP) algorithm that employed a Gaussian approximation of the interference. In [12],[13], *tabu search algorithm*, which is a local neighborhood search algorithm, was shown to achieve near-optimal performance in large-MIMO systems for 4-QAM modulation, but its performance was far from optimum for higher-order QAM. Our first new contribution in this paper is that we adopt a *layering approach* in conjunction with the low-complexity tabu search which significantly improves higher-order QAM performance (e.g., 16-QAM, 64-QAM) in large-MIMO systems, bringing it much closer to the maximum-likelihood (ML) performance compared to the basic tabu search without layering.

In order to assess how well the proposed layered search algorithm performs w.r.t. to the true ML performance in large-MIMO systems (i.e., for large n_t, where n_t denotes the number of transmit antennas), we resort to obtaining bounds on the ML performance which are computable at low complexities. This is because predicting the ML performance either through a brute-force search or by using sphere decoding (SD) is prohibitively complex for large-MIMO systems (like a 32×32 V-BLAST system). Upper bounds on the ML performance based on union bounding are known [16]. But the tightness of these upper bounds for large n_t for a given SNR/BER is difficult to predict because of the lack of knowledge of true ML performance for those n_t and SNR/BER values. This then brings the need for good lower bounds on the ML performance for large n_t, so that the true ML performance can be predicted to be within the upper and lower bounds.

Consequently, our second new contribution in this paper is that we obtain a lower bound on the ML bit error performance using the neighborhood search in tabu search algorithm. The advantages of the proposed bound are i) it is easily obtained for MIMO systems with large n_t because of the inherent low complexity of the search algorithm, ii) it is tight at moderate-to-high SNRs, and iii) it can be tightened at low SNRs by increasing the number of symbols in the neighborhood definition. Interestingly, the proposed layered search algorithm for detection, termed as *layered tabu search (LTS)* algorithm, achieves bit error performances which are quite close to this lower bound for large n_t and higher-order QAM. For e.g., in a 32×32 V-BLAST MIMO system, the proposed LTS algorithm performs close to within 1.7 dB of the proposed ML lower bound at 10^{-3} BER for 16-QAM (128 bps/Hz), and within 4.5 dB of the bound for 64-QAM (192 bps/Hz).

The system model is presented in Sec. II. The proposed LTS algorithm is presented in Sec. III. The proposed ML lower bound is presented in Sec. IV. BER performance results are presented in Sec. V. Conclusions are presented in Sec. VI.

[1] Similar algorithms have been reported earlier in the context of multiuser detection in large CDMA systems.

II. System Model

Consider a V-BLAST MIMO system with n_t transmit and n_r receive antennas. The transmitted symbols take values from a modulation alphabet $\mathbb{A}$. Let $\mathbf{x} \in \mathbb{A}^{n_t}$ denote the transmitted vector. Let $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix, whose entries are assumed to be i.i.d. Gaussian with zero mean and unit variance. The received vector $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the noise vector whose entries are are modeled as i.i.d. $\mathbb{C}\mathcal{N}(0,\sigma^2)$. The ML detection rule is given by

$$\begin{aligned} \widehat{\mathbf{x}}_{ML} &= \arg\min_{\mathbf{x}\in\mathbb{A}^{n_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \\ &= \arg\min_{\mathbf{x}\in\mathbb{A}^{n_t}} \phi(\mathbf{x}), \end{aligned} \tag{2}$$

where

$$\phi(\mathbf{x}) \stackrel{\triangle}{=} \mathbf{x}^H\mathbf{H}^H\mathbf{H}\mathbf{x} - 2\Re\left(\mathbf{y}^H\mathbf{H}\mathbf{x}\right) \tag{3}$$

is the ML cost. The computational complexity in (2) is exponential in n_t, which is prohibitive for large n_t. Our interest is to achieve near-ML performance for large n_t at low complexities for modulation alphabets including higher-order QAM. Our proposed layered tabu search (LTS) algorithm, which is presented in the next section, essentially addresses these two performance and complexity objectives for large n_t and higher-order QAM.

III. Proposed Layered Tabu Search Algorithm

In this section, we present the proposed layered tabu search (LTS) algorithm for large-MIMO detection. The proposed algorithm involves a strategy of detecting symbols in a layered manner, where in each layer a low-complexity local neighborhood search detects a sub-vector of the transmitted symbol vector. The sub-vector size is increased from one layer to the next layer. In addition, the detected sub-vector in a given layer is used to form the initializing solution for the search in the next layer.

A. Proposed LTS Algorithm

Let $\mathbf{U}$ denote the upper triangular matrix obtained from the QR decomposition of the channel matrix $\mathbf{H}$. Then, the objective equivalent to (2) will be to find the transmitted vector $\mathbf{x}$ which minimizes $\|\mathbf{U}(\mathbf{x}-\bar{\mathbf{x}})\|^2$, where

$$\bar{\mathbf{x}} = \mathbf{H}^\dagger \mathbf{y}, \tag{4}$$

and $\mathbf{H}^\dagger$ is the Moore-Penrose pseudo inverse of $\mathbf{H}$. Let u_{ij} denote the element in the ith row and jth column of the $\mathbf{U}$ matrix, and x_i denote the ith element of the vector $\mathbf{x}$.

The algorithm processes one layer at a time. It starts with the n_tth layer first. In the kth layer, $k = n_t, (n_t-1), (n_t-2), \cdots, 1$, the algorithm detects the (n_t-k+1)-sized sub-vector $[x_k, x_{k+1}, \cdots, x_{n_t}]$. We detect the symbols of this sub-vector jointly because they interfere with each other due to the structure of the $\mathbf{U}$ matrix. For e.g., since $\mathbf{U}$ is upper triangular, there will be no interference to the symbol x_{n_t} in the n_tth layer. In the (n_t-1)th layer, there will be one interferer x_{n_t}. In the (n_t-2)th layer there will be two interferers x_{n_t-1} and x_{n_t}, and so on in the subsequent layers. The joint detection method employed in each layer is a low-complexity search described in Sec. III-C. We propose to reduce the complexity further by skipping the joint detection search in a layer if a simple cancellation of interference due to the already detected symbols in the previous layer results in a good quality output. The resulting algorithm is stated below.

Let $\check{\mathbf{x}}$ be the quantized version of $\bar{\mathbf{x}}$, i.e., each element in $\bar{\mathbf{x}}$ is rounded-off to its nearest symbol in the alphabet to get $\check{\mathbf{x}}$, so that $\check{\mathbf{x}} \in \mathbb{A}^{n_t}$. Let d_{min} be the minimum Euclidean distance between any two symbols in $\mathbb{A}$. The steps performed in the kth layer, $k = n_t, (n_t-1), \cdots, 1$, are as follows:

Step 1): Calculate

$$r_k = \bar{x}_k - \sum_{l=k+1}^{n_t} \frac{u_{kl}}{u_{kk}}(x_l - \bar{x}_l), \tag{5}$$

which is a cancellation operation that removes the interference due to the symbols detected in the previous layer.

Step 2): Find the symbol in the alphabet $\mathbb{A}$ which is closest to r_k in Euclidean distance. Let this symbol be a_q.

i) If $|r_k - a_q| < \frac{d_{min}}{4}$, then $\hat{x}_k = a_q$. Make $k = k-1$ and return to Step 1)[2].

ii) If $|r_k - a_q| \geq \frac{d_{min}}{4}$, then set $\hat{x}_k = \check{x}_k$. Run the local search algorithm described in Sec. III-C. The search algorithm needs the following matrix and vectors as inputs: $\tilde{\mathbf{H}}$, $\tilde{\mathbf{y}}$, and $\tilde{\mathbf{x}}^{(0)}$. These inputs for the kth layer are obtained as:

$$\tilde{\mathbf{x}}^{(0)} = [\hat{x}_k, \hat{x}_{k+1}, \cdots, \hat{x}_{n_t}], \tag{6}$$

$$\tilde{\mathbf{H}} = \begin{bmatrix} u_{kk} & u_{k(k+1)} & \cdots & u_{kn_t} \\ 0 & u_{(k-1)(k-1)} & \cdots & u_{(k-1)n_t} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{n_tn_t} \end{bmatrix}, \tag{7}$$

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\,[\bar{x}_k \ \ \bar{x}_{k+1} \ \cdots \ \bar{x}_{n_t}]^T. \tag{8}$$

The output of the search algorithm is made as the updated $[\hat{x}_k, \hat{x}_{k+1}, \cdots, \hat{x}_{n_t}]$ sub-vector. Make $k = k-1$ and return to Step 1).

B. Detection with Ordering

A way to improve performance is to follow an optimum order while detecting the symbols. We need to find an optimum order $(p_1, p_2, \cdots, p_{n_t})$ which is a permutation of $(1, 2, \cdots, n_t)$. We obtain the optimum ordering based on the post-detection SNR of the symbols as follows. Perform the following steps for $i = n_t, \cdots, 1$ with $\mathbf{H}_{n_t} = \mathbf{H}$: *i)* Find $\mathbf{H}_i^\dagger$, the Moore-Penrose pseudo-inverse of $\mathbf{H}_i$, where $\mathbf{H}_i$ is obtained by zeroing $(p_{i+1}, p_{i+2}, \cdots, p_{n_t})$ columns of $\mathbf{H}$; *ii)* Find p_i, the row index of $\mathbf{H}_i^\dagger$, that results in the minimum norm among all rows of $\mathbf{H}_i^\dagger$. Detection is then carried out in the following order: $p_{n_t}, p_{n_t-1}, p_{n_t-2}$, and so on.

[2]Execution of this part of the step essentially skips the joint detection using local search. Nearness of r_k to an element in $\mathbb{A}$ to within $\frac{d_{min}}{4}$ is used as the criterion to decide to carry out or skip the local search in layer k. We found this simple criterion to work well in our simulations.

C. Search Algorithm for Joint Detection in Layer k

Consider the linear vector channel model wherein a $d_r \times 1$ received vector $\tilde{\mathbf{y}}$ is given by

$$\tilde{\mathbf{y}} \quad = \quad \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}, \tag{9}$$

where $\tilde{\mathbf{x}}$ is a $d_t \times 1$ transmitted vector, $\tilde{\mathbf{H}}$ is a $d_r \times d_t$ channel matrix, and $\tilde{\mathbf{n}}$ is a $d_r \times 1$ zero mean Gaussian noise vector. The ML estimate of $\tilde{\mathbf{x}}$ is given by

$$\widehat{\tilde{\mathbf{x}}}_{ML} \quad = \quad \arg\min_{\tilde{\mathbf{x}} \in \mathbb{A}^{d_t}} \quad \phi(\tilde{\mathbf{x}}), \tag{10}$$

where $\phi(\tilde{\mathbf{x}}) \stackrel{\triangle}{=} \tilde{\mathbf{x}}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \tilde{\mathbf{x}} - 2\Re\big(\tilde{\mathbf{y}}^H \tilde{\mathbf{H}} \tilde{\mathbf{x}}\big)$. In this subsection, we briefly describe a low-complexity local neighborhood search algorithm to obtain an estimate of $\tilde{\mathbf{x}}$ based on tabu search [10]-[13]. In each layer of the proposed algorithm in Sec. III-A, the matrix and vectors in (7), (6), (8) are passed as inputs to this algorithm to jointly detect the symbol subvector in that layer. A detailed description of the tabu search algorithm for MIMO detection was presented in [12],[13]. A brief high level summary of this algorithm in given in the following paragraph.

The tabu search algorithm starts with an initial solution vector, defines a neighborhood around it (i.e., defines a set of neighboring vectors based on a neighborhood criteria), and moves to the best vector among the neighboring vectors (even if the best neighboring vector is worse, in terms of ML cost, than the current solution vector; this allows the algorithm to escape from local minima). This process is continued for a certain number of iterations, after which the algorithm is terminated and the best among the solution vectors in all the iterations is declared as the final solution vector. In defining the neighborhood of the solution vector in a given iteration, the algorithm attempts to avoid cycling by making the moves to solution vectors of the past few iterations as 'tabu' (i.e., prohibits these moves), which ensures efficient search of the solution space. The number of these past iterations is parametrized as the 'tabu period,' which is dynamically changed depending on the number of repetitions of the solution vectors observed in the search path. The per-symbol complexity of the tabu search algorithm is $O(d_t d_r)$, which is quite attractive for use in detection of large-MIMO signals.

The stopping criterion used in this paper has been simplified from that given in [12],[13]; the number of parameters used has been reduced to simplify the stopping criterion. The stopping criterion used is as follows.

Stopping criterion: The search algorithm is stopped if maximum number of iterations max_iter is reached. Also, if the current solution is a local minima ($lflag = 1$) and the total number of repetitions of solutions is greater than max_rep, the algorithm is stopped. The solution would then be the vector with the least ML cost which has been found before the algorithm was stopped. The solution vector is fed back to the main algorithm as the solution sub-vector for that layer.

IV. A Lower Bound on ML Performance

In this section, we obtain a lower bound on the ML bit error performance using the neighborhood search in the tabu search algorithm in Sec. III-C. To find the lower bound, we will use the actually transmitted vector $\mathbf{x}$ as the initial vector in the tabu search algorithm. Two vectors are said to be n-symbol neighbors of each other if they differ in exactly n coordinates. Define n-symbol neighborhood of a certain vector to be the set of all vectors which differ from that vector in i coordinates, $i \leq n$. With the transmitted vector $\mathbf{x}$ as the initial vector, tabu search algorithm is run and the output solution vector is obtained. Let $\mathbf{x}_{TS}$ denote the output solution vector obtained from the tabu search, and $\mathbf{x}_{ML}$ denote the true ML solution vector. Let e_{TS} denote the number of symbol errors in $\mathbf{x}_{TS}$, and e_{ML} denote the number of symbol errors in $\mathbf{x}_{ML}$. Now, one of the three cases below will be true.

1) $e_{TS} = 0$, i.e., $\mathbf{x}_{TS} = \mathbf{x}$, which may or may not be equal to $\mathbf{x}_{ML}$. So, $e_{ML} \geq 0$.
2) $e_{TS} = \kappa$, $\kappa \leq n$. In this case, $\mathbf{x}$ is not a local minima. Also, the global minima $\mathbf{x}_{ML}$ can not have less than κ symbol errors. This is because for $\mathbf{x}_{ML}$ to have less than κ symbol errors, $\mathbf{x}_{ML}$ has to be a q-symbol neighbor of $\mathbf{x}$ where $q < \kappa$, which is not possible because $\mathbf{x}_{TS}$ obviously has the best ML cost among all n-symbol neighbors of $\mathbf{x}$, and $\mathbf{x}_{TS}$ can not have better ML cost than $\mathbf{x}_{ML}$. Therefore, $e_{ML} \geq \kappa$.
3) $e_{TS} \geq n+1$. In this case, the global minima $\mathbf{x}_{ML}$ does not lie in the n-symbol neighborhood of $\mathbf{x}$. So, $e_{ML} \geq n+1$.

Based on the the above three cases, the tabu search algorithm simulation can take e_{ML} as 0 in case 1), as κ in case 2), and as $n+1$ in case 3), which gives a lower bound on the ML symbol error performance. Since the number of symbol errors is a lower bound on the number of bit errors, the above bound is a bit error bound as well.

A. Results and Discussions on the Lower Bound

We simulated the tabu search algorithm for a 16×16 V-BLAST MIMO system and obtained the proposed lower bounds for 4-, 16-, and 64-QAM. In Fig. 1, we plot these lower bounds for $n = 1, 2, 3, 4$ and compare them with the actual ML performance obtained by sphere decoding. We note that sphere decoding simulations for the considered 16×16 system took long simulation run time. In the interest of making a comparison with the proposed bound at such large n_t, we carried out these sphere decoder simulations and the results are shown in Fig. 1. From Fig. 1, it can be observed that the proposed bound is quite tight (within just 0.5 dB) for BERs less than 10^{-2}, and gets increasingly tighter for lesser BERs. Even in the lower SNR region, the bound gets increasingly tighter for increasing n.

An Approximate Prediction of ML Performance: The improved tightness of the bound for increasing n is observed to be quite significant at low SNRs in Fig. 1. However, a large n means increased complexity. As a low-complexity alternative, we approximate the true ML error performance to be the error performance of the tabu search solution when the transmitted vector $\mathbf{x}$ is used as the initial vector, i.e., we assume $e_{ML} = e_{TS}$. From the previous discussion on the lower bound, we note that e_{TS} indeed corresponds to an upper bound to the proposed ML lower bound. But this upper

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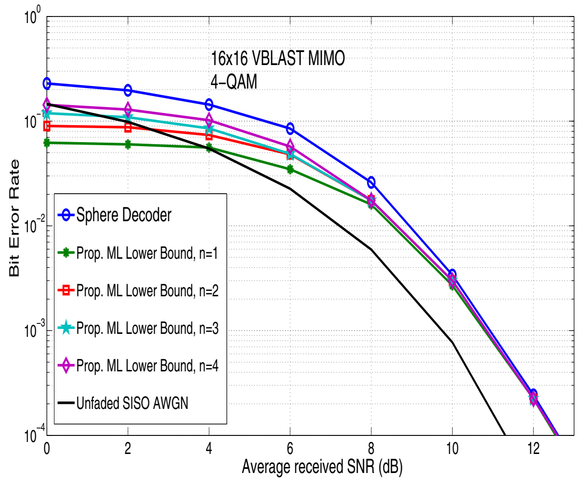

Fig. 1. Comparison of the proposed lower bound on ML performance for $n = 1, 2, 3, 4$ with the ML performance predicted by sphere decoder for 16×16 V-BLAST MIMO with 4-QAM.

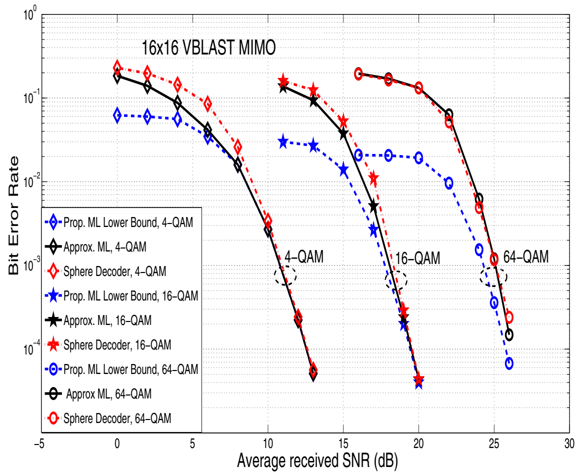

Fig. 2. Comparison of the proposed lower bound on ML performance for $n = 1$ and the 'approximate ML' performance with the ML performance predicted by sphere decoder for 16×16 V-BLAST MIMO with 4-QAM, 16-QAM, and 64-QAM.

bound need not be a lower or upper bound to true ML performance. So we refer to the performance obtained by equating e_{TS} to e_{ML} as an 'approximate ML performance.' It can be noted that, complexity-wise, like the proposed lower bound, the approximate ML performance is also easily obtained for large n_t. In Fig. 2, we compare the lower bound, approximate ML, and the sphere decoder performances for 16×16 V-BLAST with 4-, 16-, and 64-QAM. It is seen that the proposed approximate ML performance is quite close to the actual ML performance even at low SNRs.

V. BER Performance of the Proposed LTS Algorithm in Large-MIMO

In this section, we present the simulated BER performance of the proposed LTS algorithm for large-MIMO detection and compare with that of the tabu search (TS) without layering and with those of the proposed ML lower bound and the approximate ML. The following parameters are used in the tabu search algorithm: $max\_rep = 10$, $max\_iter = 20$, $\beta = 10$ for 4-QAM, $max\_rep = 10$, $max\_iter = 100$, $\beta = 100$ for 16-QAM and $max\_rep = 20$, $max\_iter = 200$, $\beta = 200$ for 64-QAM, and $P_0 = 1$.

Large-System Behavior of LTS: Figure 3 shows the performance of the LTS algorithm with ordering in $n_t \times n_r$ V-BLAST MIMO systems with $n_t = n_r = 4, 8, 32$ and 16-QAM. The LTS algorithm is found to exhibit large-system behavior, where the BER improves with increasing $n_t = n_r$.

BER/Complexity Comparison with TS with No Layering: Figure 4 shows a comparison between the BER performances of the proposed LTS algorithm without and with ordering, and the TS algorithm without layering in a 32×32 V-BLAST MIMO system with 16-QAM and 64-QAM. It can be seen that compared to TS without layering, the proposed layered TS approach significantly improves the BER performance. For e.g., TS without layering needs 24 dB SNR to achieve 10^{-3} BER for 16-QAM, whereas the proposed LTS algorithm with ordering achieves the same BER at 19 dB, which amounts to an SNR gain of 5 dB. For 64-QAM, this SNR gain is even higher. The layered approach achieves this better performance in about the same complexity as that of the TS without layering. This can be seen from Fig. 5, where we have plotted the average simulation run time as a function $n_t = n_r$ for 16-QAM. Though the order of complexity for TS without layering is less, the constant is high and at $n_t = 16$ and 32 the proposed LTS has comparable complexity. So the proposed LTS significantly outperforms TS without layering for $n_t = n_r = 32$ without any major increase in complexity. We do not give the performance of sphere decoder or its low-complexity variants for 32×32 system because of their prohibitive complexity to simulate them in such large dimensions (64 real dimensions in case of 32×32 MIMO with QAM modulation).

Nearness to the ML Lower Bound and Approximate ML: Finally, Fig. 5 shows the BER of the LTS algorithm in comparison with those of the ML lower bound and the approximate ML presented in Sec. IV in a 32×32 V-BLAST system with 4-, 16-, and 64-QAM. The LTS algorithm is found to perform quite close to the bound and the approximate ML performance. For 16-QAM the nearness at 10^{-3} BER is within just 1.7 dB, and for 64-QAM it is within 4.5 dB. These are very good performances considering the large number of antennas, high orders of modulation, high spectral efficiencies, and low complexities involved.

VI. Conclusions

We have made two new contributions in this paper. First, we presented a *layered detection approach in conjunction with a low-complexity local neighborhood tabu search*, and showed that it indeed works very well in terms of both performance as well as complexity in MIMO systems with large number of antennas. Performance-wise, we showed that it achieves close to ML performance, and complexity-wise it scales well for large number of antennas. Such good performance and complexity features of the proposed algorithm are quite attractive for large-MIMO system implementations. Second, we proposed a lower bound on ML bit error performance based on the neighborhood search of the tabu search algorithm, which is a novel and effective approach. The proposed

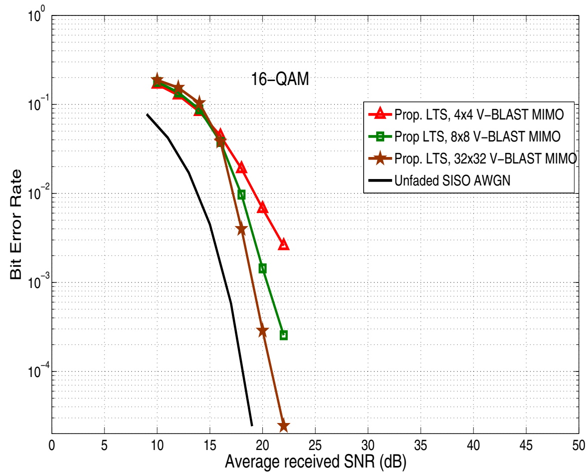

Fig. 3. BER performance of the proposed LTS algorithm with ordering in V-BLAST MIMO for $n_t = n_r = 4, 8, 32$ and 16-QAM.

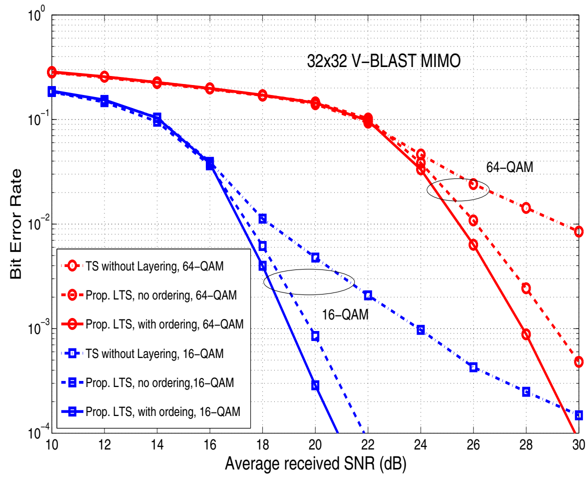

Fig. 4. BER comparison between the proposed LTS algorithm without and with ordering, and TS without layering for 32×32 V-BLAST MIMO with 16-QAM and 64-QAM.

bound is easy to obtain for large-MIMO systems, and it can serve as a good benchmark for evaluating the nearness to ML performance achieved by different large-MIMO detection algorithms.

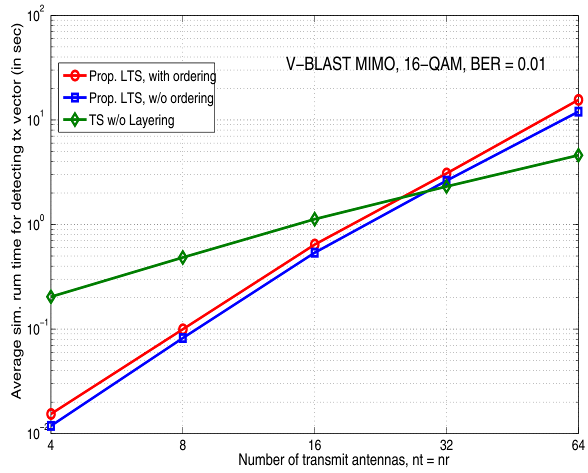

Fig. 5. Complexity comparison between LTS with ordering, LTS without ordering, and TS without layering in terms of average simulation run time for different $n_t = n_r$, 16-QAM.

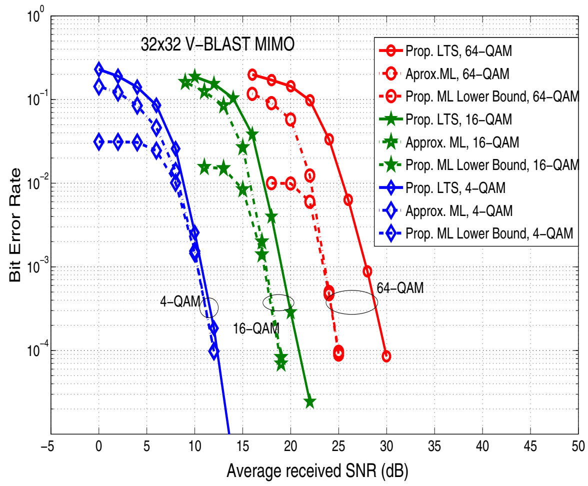

Fig. 6. Performance nearness of LTS with ordering to the proposed lower bound of ML and the approximate ML for 32×32 V-BLAST MIMO with 4-, 16-, and 64-QAM. $n = 1$ for the ML lower bound.

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