Generalized Space-Frequency Index Modulation: Low-Complexity Encoding and Detection

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***Abstract*—This paper is concerned with an attractive multi-antenna multi-carrier modulation scheme, termed as *generalized space-frequency index modulation (GSFIM)*, which is a promising modulation scheme for next generation wireless systems. GSFIM uses both spatial domain and frequency domain to encode bits through *indexing*. In GSFIM, information bits are mapped through antenna indexing in the spatial domain, subcarrier indexing in the frequency domain, and M-ary modulation. GSFIM can offer higher rates using fewer transmit radio frequency (RF) chains and better performance compared to conventional MIMO-OFDM. In this paper, we address the problem of low-complexity encoding and detection of large-dimensional GSFIM signals. The proposed encoding procedure exploits the low complexity computation of *combinadics* in combinatorial number system. This allows 'on-the-fly' computation of GSFIM encoding maps. For detecting GSFIM signals, we propose a low complexity detection algorithm based on a *multi-stage message passing* approach. The proposed low complexity encoding/detection algorithms allow practical implementation of large-dimension GSFIM systems.**

Keywords – *Space-frequency index modulation, multi-antenna systems, multi-carrier systems, low-complexity encoding, detection.*

I. Introduction

Large-scale MIMO technology is widely being viewed as a key component in the physical layer evolution towards 5G. Spatial modulation (SM) scheme which uses multiple antenna elements but only one radio frequency (RF) chain at the transmitter is attractive to achieve reduced RF hardware complexity in large-scale MIMO systems [1]. Also, SM in large-scale multiuser MIMO on the uplink has been found to outperform massive MIMO with conventional modulation [2]. This is because, due to additional bits conveyed through the active antenna index in SM, a smaller-sized modulation alphabet can be used in SM compared to that in conventional modulation to achieve the same spectral efficiency. Generalized spatial modulation (GSM) is a generalization of SM [3],[4]. The GSM transmitter can have more than one RF chain. This allows simultaneous transmission of multiple modulation symbols in addition to conveying bits through active antenna indices. In large-scale multiuser MIMO systems, GSM has been found to perform better than SM and massive MIMO with conventional modulation [5].

SM and GSM can be viewed as instances of the general idea of *index modulation*, where additional information bits are conveyed through indices of certain transmit entities that get involved in the transmission. Transmit antennas in multi-antenna systems and subcarriers in multi-carrier systems are examples of such transmit entities that can be used to convey index bits. SM and GSM can be essentially viewed as antenna index modulation schemes using single-carrier approach. In [6],[7], subcarrier index modulation in multi-carrier single-antenna systems has been proposed. It has been found in [6],[7] that subcarrier index modulation can offer better performance and rate compared to OFDM without subcarrier indexing.

Introduced in [8], *generalized space-frequency index modulation (GSFIM)* is a generalization of both GSM and subcarrier index modulation, in which information bits are conveyed as antenna index bits, subcarrier index bits, and M-ary modulation bits. Rate results in [8] have shown that GSFIM can achieve higher rates compared to conventional MIMO-OFDM. Also, bit error performance results using maximum-likelihood (ML) detection have shown the potential for GSFIM performing better than MIMO-OFDM. In [8], it has been pointed out that an issue that needs further investigation is low complexity encoding and detection methods for GSFIM. This is because the dimension of the GSFIM signal vector can become large when the number of antennas and subcarriers are large, and therefore the complexities of encoding antenna/frequency index bits and ML detection become prohibitive.

In this paper, we propose low complexity schemes for encoding and detection of large-dimension GSFIM signals. The encoding procedure exploits the low complexity computation of *combinadics* in combinatorial number system. This allows 'on-the-fly' computation of GSFIM encoding maps. For detecting GSFIM signals, we propose a detection algorithm based on a *multi-stage message passing* approach. Good bit error performance of the proposed detection algorithm is achieved by detecting the antenna indexing bits using the subcarrier activity in the frequency domain. Low complexity is achieved by detecting the modulation bits and the subcarrier indexing bits in two different layers. The proposed detection algorithm is shown to significantly outperform MMSE detection.

II. GSFIM System Model

A GSFIM system has n_t transmit antennas, n_{rf} transmit RF chains, $1 \leq n_{rf} \leq n_t$, N subcarriers, and n_r receive antennas. The channel between each pair of transmit and receive antennas is assumed to be frequency-selective fading with L paths. The block diagrams of the GSFIM transmitter and receiver are shown in Fig. 1. In a given channel use, only n_{rf} antennas out of n_t transmit antennas will be activated and the remaining $n_t - n_{rf}$ antennas remain silent. Information bits are conveyed as antenna index bits, frequency index bits, and M-ary modulation bits. The GSFIM encoder takes $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$ bits and maps to n_{rf} active antennas (antenna index bits). It also takes additional bits to index subcarriers (frequency index bits) and bits for M-ary modulation symbols on active subcarriers. The frequency and antenna indexing mechanisms are explained below.

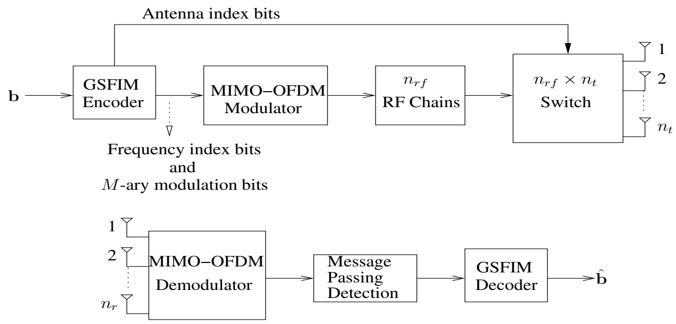

Fig. 1. GSFIM transmitter and receiver.

1) Frequency indexing: Consider a matrix $\mathbf{B}$ of size $n_{rf} \times N$ whose entries belong to $\mathbb{A}_0$, where $\mathbb{A}_0 = \mathbb{A} \cup 0$ with $\mathbb{A}$ denoting an M-ary modulation alphabet. The frequency index bits and M-ary modulation bits are embedded in $\mathbf{B}$ as follows. The matrix $\mathbf{B}$ is divided into n_b submatrices $\mathbf{B}_1, \mathbf{B}_2, \cdots \mathbf{B}_{n_b}$, each of size $n_{rf} \times n_f$, where $n_f = \frac{N}{n_b}$ is the number subcarriers per submatrix (see Fig. 2). Let k, $1 \leq k \leq n_{rf}n_f$ denote the number of non-zero elements in each submatrix, where each of the non-zero elements belong to $\mathbb{A}$. This k is a design parameter. Then, for each submatrix, there are $l_f = \binom{n_{rf}n_f}{k}$ possible 'frequency activation patterns'. A frequency activation pattern for a given submatrix refers to a possible combination of zero and non-zero entries in that submatrix. Note that not all l_f activation patterns are needed for frequency indexing. Any 2^{k_f} patterns out of them, where $k_f = \left\lfloor \log_2 \binom{n_{rf}n_f}{k} \right\rfloor$, are adequate. Take any 2^{k_f} patterns out of l_f patterns and form a set called the 'frequency activation pattern set', denoted by $\mathbb{S}_f$. The frequency activation pattern for a given submatrix is then formed by choosing one among the patterns in the set $\mathbb{S}_f$ using k_f bits. These k_f bits are the frequency index bits for that submatrix. So, there are a total of n_bk_f frequency index bits in the entire matrix $\mathbf{B}$. In addition to these frequency index bits, $kn_b \log_2 M$ bits are carried as M-ary modulation bits in the non-zero entries of $\mathbf{B}$.

Example: Let $n_{rf} = 2$, $N = 16$, $n_b = 4$, and $k = 7$. Then, $n_f = \frac{16}{4} = 4$, $l_f = \binom{8}{7} = 8$, $k_f = \lfloor \log_2 8 \rfloor = 3$, and $2^{k_f} = 8$. In this example, $l_f = 2^{k_f} = 8$, i.e., all the 8 possible patterns are in the frequency activation pattern set, given by

$$\begin{aligned}\mathbb{S}_f = \Bigg\{ & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \\ & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \\ & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \Bigg\}.\end{aligned}$$

Suppose $\mathbb{A}$ is 4-QAM. Let [00101001111000110] denote the information bit sequence for submatrix $\mathbf{B}_1$. The GSFIM encoder translates these bits to the submatrix $\mathbf{B}_1$ as follows: the first 3 bits are used to choose the frequency activity pattern (i.e., 001 chooses the activation pattern $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ in the set $\mathbb{S}_f$ above), and the next 14 bits are mapped to seven 4-QAM symbols so that one 4-QAM symbol gets mapped to one active subcarrier. The submatrix $\mathbf{B}_1$ then becomes

$$\mathbf{B}_1 = \begin{bmatrix} -1-\mathbf{j} & 0 & -1+\mathbf{j} & 1-\mathbf{j} \\ 1-\mathbf{j} & -1+\mathbf{j} & -1-\mathbf{j} & 1+\mathbf{j} \end{bmatrix},$$

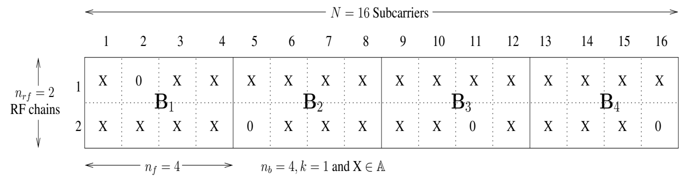

Fig. 2. Frequency indexing in GSFIM.

where $\mathbf{j} = \sqrt{-1}$. Likewise, the submatrices $\mathbf{B}_i$, $i = 2, 3, 4$ are formed. The full matrix $\mathbf{B}$ of size $n_{rf} \times N$ is then formed as $\mathbf{B} = [\mathbf{B}_1\ \mathbf{B}_2\ \mathbf{B}_3\ \mathbf{B}_4]$. Each row of the matrix $\mathbf{B}$ is of dimension $1 \times N$. There are n_{rf} rows. Each N-length row vector in $\mathbf{B}$ is fed to the IFFT block in the OFDM modulator to generate an N-length OFDM symbol. A total of n_{rf} such OFDM symbols, one for each row in $\mathbf{B}$, are generated. These n_{rf} OFDM symbols are then transmitted through n_{rf} active transmit antennas in parallel. The choice of these n_{rf} active transmit antennas among the n_t available antennas is made through antenna indexing as described below.

2) Antenna indexing: The selection of n_{rf} out of n_t antennas for transmission is made based on antenna index bits. The antenna index bits choose an 'antenna activation pattern', which tells which n_{rf} antennas out of n_t antennas are used for transmission. There are $l_a = \binom{n_t}{n_{rf}}$ antenna activation patterns possible, and $k_a = \left\lfloor \log_2 \binom{n_t}{n_{rf}} \right\rfloor$ bits are used to choose one among them. These k_a bits are the antenna index bits. Note that not all l_a activation patterns are needed, and any 2^{k_a} patterns out of them are adequate. Take any 2^{k_a} patterns out of l_a patterns and form a set called the 'antenna activation pattern set', denoted by $\mathbb{S}_a$.

Example: Let us illustrate this using the following example. Let $n_t = 3$, $n_{rf} = 2$. Then, $l_a = \binom{3}{2} = 3$, $k_a = \left\lfloor \log_2 \binom{3}{2} \right\rfloor = \lfloor \log_2 3 \rfloor = 1$, and $2^{k_a} = 2$. The possible antenna activation patterns are given by $\left\{[1,1,0]^T, [1,0,1]^T, [0,1,1]^T\right\}$. The set $\mathbb{S}_a$ is formed by selecting any two patterns out of the above three patterns. For example, $\mathbb{S}_a$ can be $\mathbb{S}_a = \left\{[1,1,0]^T, [1,0,1]^T\right\}$. An $n_{rf} \times n_t$ switch connects the RF chains to the transmit antennas. The chosen n_{rf} out of n_t antennas transmit the MIMO-OFDM symbol constructed using the frequency index bits and M-ary modulation bits. The active transmit antennas can change from one MIMO-OFDM symbol to the other.

Achievable rate in GSFIM: The achievable rate in GSFIM with n_t transmit antennas, n_{rf} transmit RF chains, N subcarriers, n_b submatrices, and M-ary modulation is given by

$$\begin{aligned} R_{\text{gsfim}} = \frac{1}{N-L+1} \Bigg(& \underbrace{\left\lfloor \log_2 \binom{n_t}{n_{rf}} \right\rfloor}_{\text{ant. index bits}} + \underbrace{\left\lfloor \log_2 \binom{n_{rf}n_f}{k} \right\rfloor n_b}_{\text{freq. index bits}} \\ & + \underbrace{kn_b \log_2 M}_{M\text{-ary symb. bits}} \Bigg) \ \text{bpcu.} \end{aligned} \tag{1}$$

III. Low complexity encoding of GSFIM signals

In this section, we first motivate the need for low complexity encoding methods for large-dimensional GSFIM. We then propose a encoding method that uses combinatorial number system and has very low complexity.

In GSFIM, $k_a = \lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$ bits are used for antenna indexing, $k_f = \lfloor \log_2 \binom{n_{rf} n_f}{k} \rfloor$ bits are used for subcarrier pattern selection per submatrix, and $k n_b \log_2 M$ bits are used for modulation symbols selection on active subcarriers. The mapping of $k n_b \log_2 M$ bits to modulation symbols is straightforward, but the mapping of k_a bits to antenna pattern and k_f bits to frequency pattern is not. For encoding and decoding purposes, a table/map of information bits to corresponding frequency/antenna activation pattern has to be maintained both at the transmitter and receiver. For small values of n_t, n_{rf}, n_f, and k, this table/map is practical. However, for large values of n_t, n_{rf}, n_f, and k, the size of the table/map can become prohibitively large. We illustrate this point using the following example. Consider $n_t = 8$, $n_{rf} = 4$, $N = 16$, $n_b = 1$, and $k = 56$, then $|\mathbb{S}_a| = 2^6$, $|\mathbb{S}_f| = 2^{32}$. An implementation of the encoding map of this size is impractical. This motivates the need for low complexity encoding methods. To solve this problem, we resort to *combinadic* representation of numbers in combinatorial number system [9] as follows.

Definition 1: The combinadic of a number $n \in [0, \binom{K}{R} - 1]$ is the R-tuple $(K_1, K_2, \cdots, K_R)$ such that $n = \sum_{i=1}^{R} \binom{K_i}{i}$ and $K_1 < K_2 < \cdots < K_R < K$. The value of K_i for a given n can be obtained as [9]

$$K_i = \text{Largest non-negative integer s.t. } n - \sum_{j=i}^{R} \binom{K_j}{j} \geq 0.$$

Using this definition, any number n in the range $[0, \binom{K}{R} - 1]$ can be uniquely mapped to a particular combination in $\binom{K}{R}$ and vice versa. The combinadic of the number n gives the R elements that are active/chosen in that particular combination. Given a particular combination, the active elements in that particular combination give an R-tuple combinadic, which, in-turn, can be used to obtain the number n uniquely. The computational complexity to find the combinadic of a number is just $O(R)$, which we exploit for large-dimensional GSFIM encoding.

The combinadic representation can be used for GSFIM encoding by taking $K = n_t$ and $R = n_{rf}$ for antenna selection, and $K = n_f n_{rf}$ and $R = k$ for subcarrier pattern selection for a submatrix. The following procedure maps k_a bits to antenna activation pattern.

- Accumulate k_a antenna indexing bits and form the bit sequence $\mathbf{b}_a = [b_{k_a-1}, \cdots, b_1, b_0]$.
- Obtain decimal equivalent of $\mathbf{b}_a$ as $d(\mathbf{b}_a) \triangleq \sum_{i=0}^{k_a-1} 2^i b_i$.
- Find the combinadic of $d(\mathbf{b}_a)$.
- Obtain the active antennas using the combinadic R-tuple.

Using a similar procedure, the mapping of k_f bits to subcarrier activity pattern per submatrix can be obtained.

Examples: Examples of encoding maps of antenna indexing and frequency indexing for $n_t = 4$, $n_{rf} = 2$, $N = 4$, $n_b = 1$, and $k = 7$ based on above procedure are illustrated in Tables I and II, respectively. Consider the case of $\mathbf{b}_a = [01]$ and $\mathbf{b}_f = [110]$. Observe that $d(\mathbf{b}_a) = 1$ and the combinadic of 1 is $(K_1, K_2) = (0, 2)$, and therefore the activated antennas are 1 and 3. Likewise, $d(\mathbf{b}_f) = 6$ and the combinadic of 6 is $(K_1, K_2, K_3, K_4, K_5, K_6, K_7) = (0, 1, 2, 3, 5, 6, 7)$, and the active subcarrier positions in the submatrix is (1,2,3,4,6,7,8). Encoding maps for large dimensions can be computed 'on-the-fly' based on the low complexity procedure given above. A reverse procedure can perform the combinadic to information bits demapping at the receiver. The above low complexity mapping/demapping procedure allows implementation of GSFIM encoding in large dimensions.

$\mathbf{b}_a$	$d(\mathbf{b}_a)$	Combinadic of $d(\mathbf{b}_a)$: (K_1, K_2)	Indices of active antennas
00	0	(0,1)	(1,2)
01	1	(0,2)	(1,3)
10	2	(1,2)	(2,3)
11	3	(0,3)	(1,4)

TABLE I
ANTENNA ACTIVITY PATTERN MAP. $n_t = 4$, $n_{rf} = 2$.

$\mathbf{b}_f$	$d(\mathbf{b}_f)$	Combinadic of $d(\mathbf{b}_f)$: $(K_1, K_2, \cdots, K_7)$	Indices of active subcarriers
000	0	(0,1,2,3,4,5,6)	(1,2,3,4,5,6,7)
001	1	(0,1,2,3,4,5,7)	(1,2,3,4,5,6,8)
010	2	(0,1,2,3,4,6,7)	(1,2,3,4,5,7,8)
011	3	(0,1,2,3,5,6,7)	(1,2,3,4,6,7,8)
100	4	(0,1,2,4,5,6,7)	(1,2,3,5,6,7,8)
101	5	(0,1,3,4,5,6,7)	(1,2,4,5,6,7,8)
110	6	(0,2,3,4,5,6,7)	(1,3,4,5,6,7,8)
111	7	(1,2,3,4,5,6,7)	(2,3,4,5,6,7,8)

TABLE II
SUBCARRIER ACTIVITY PATTERN MAP. $N = 4$, $n_b = 1$, $k = 7$, $n_{rf} = 2$.

IV. LOW COMPLEXITY DETECTION OF GSFIM SIGNALS

In this section, we propose a low-complexity algorithm based on multi-stage message passing for GSFIM signal detection.

A. Received signal

Let $\mathbf{H}_n$ denote the overall $n_r \times n_t$ channel matrix on subcarrier n. Let $\mathbf{H}_n^{\mathbf{a}}$ denote the $n_r \times n_{rf}$ channel matrix on subcarrier n corresponding to the chosen n_{rf} antennas. The superscript $\mathbf{a}$ in $\mathbf{H}_n^{\mathbf{a}}$ refers to the antenna activation pattern that tells which n_{rf} antennas are chosen. The received signal vector $\mathbf{y}_n$ of size $n_r \times 1$ at the GSFIM receiver is given by

$$\mathbf{y}_n = \mathbf{H}_n^{\mathbf{a}} \mathbf{z}_n + \mathbf{w}_n, \quad n = 1, 2, \cdots, N, \tag{2}$$

where $\mathbf{z}_n$ is the vector of size $n_{rf} \times 1$ transmitted by the n_{rf} active antennas on subcarrier n, and $\mathbf{w}_n$ is the noise vector of size $n_r \times 1$ on subcarrier n, $\mathbf{w}_n \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{n_r})$. Considering that zeros are transmitted from the inactive antennas, the vector $\mathbf{x}_n$ of size $n_t \times 1$ transmitted by all the n_t antennas on subcarrier n is obtained by adding zeros to the vector $\mathbf{z}_n$ at the positions of the inactive antennas. The received signal vector is

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n, \quad n = 1, 2, \cdots, N. \tag{3}$$

Equation (3) can be written as

$$\mathbf{y}^i = \mathbf{G}_i \mathbf{x}^i + \mathbf{w}^i, \quad i = 1, 2, \cdots, n_b, \tag{4}$$

$$\mathbf{y}^i = \begin{bmatrix} \mathbf{y}_{i_1} \\ \mathbf{y}_{i_2} \\ \vdots \\ \mathbf{y}_{i_{n_f}} \end{bmatrix}, \mathbf{x}^i = \begin{bmatrix} \mathbf{x}_{i_1} \\ \mathbf{x}_{i_2} \\ \vdots \\ \mathbf{x}_{i_{n_f}} \end{bmatrix}, \mathbf{G}_i = \begin{bmatrix} \mathbf{H}_{i_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{i_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_{i_{n_f}} \end{bmatrix}, \mathbf{w}^i = \begin{bmatrix} \mathbf{w}_{i_1} \\ \mathbf{w}_{i_2} \\ \vdots \\ \mathbf{w}_{i_{n_f}} \end{bmatrix}.$$

in which $i_j = (i-1)n_f + j$. Equation (4) is a system model for each submatrix. For the case of $n_b = 1$, (4) becomes

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{w}, \tag{5}$$

$$\mathbf{y}=\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\\ \vdots\\ \mathbf{y}_N\end{bmatrix},\mathbf{x}=\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \vdots\\ \mathbf{x}_N\end{bmatrix},\mathbf{G}=\begin{bmatrix}\mathbf{H}_1 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{H}_2 & \dots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}_N\end{bmatrix},\mathbf{w}=\begin{bmatrix}\mathbf{w}_1\\ \mathbf{w}_2\\ \vdots\\ \mathbf{w}_N\end{bmatrix}.$$

We assume that the receiver has the knowledge of the channel matrix $\mathbf{H}_n$ on every subcarrier and noise variance σ^2. The detection algorithm proposed in the following subsection works on a per-submatrix basis. We define the following variables.

Definition 2: Antenna activity indicator a_i is defined such that $a_i = 1$ whenever ith antenna is active, else $a_i = 0$. Note that $\sum_{i=1}^{n_t} a_i = n_{rf}$, which we will call as the GSFIM system antenna constraint $\mathbf{A}$. Define $\mathbf{a} \triangleq [a_1, a_2, \cdots, a_{n_t}]$.

Definition 3: Subcarrier activity indicator $f_{(i-1)N+j}$ is defined such that $f_{(i-1)N+j} = 1$ whenever jth subcarrier of ith antenna is active, else $f_{(i-1)N+j} = 0$. Therefore, $x_i = f_i\alpha$, $\alpha \in \mathbb{A}$. Note that $\sum_{i=1}^{n_tN} f_i = k$, which we will call as the GSFIM system frequency constraint $\mathbf{F}$. Define $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_{n_tN}]$.

B. Proposed detection algorithm

We call the proposed detection algorithm as 'multi-stage message passing (MSMP)' algorithm. In the proposed algorithm, we carry out the first stage message passing on the system model in (5) and obtain the a posteriori probabilities (APPs) of the elements of $\mathbf{x}$ and $\mathbf{f}$, using which $\hat{\mathbf{a}}$ (i.e., an estimate of $\mathbf{a}$) is obtained. The antenna index bits are recovered by demapping $\hat{\mathbf{a}}$. The $\hat{\mathbf{a}}$ is also fed as input to the second stage message passing on the system model in (2). The modulation and frequency index bits are recovered using the APPs of the modulation symbol variables and frequency constraint variables at the end of the second stage message passing.

1) First stage message passing:

In this stage, the APPs of the elements of $\mathbf{x}$ and $\mathbf{f}$, and an estimate of $\mathbf{a}$ are obtained using message passing on the system model in (5). For this, we define four sets of nodes and exchange messages between them in layers. There are two layers of message exchanges corresponding to modulation symbols and subcarrier activity pattern. Now, $\Pr(\mathbf{x}|\mathbf{y})$ is

$$\begin{aligned}\Pr(\mathbf{x}|\mathbf{y}) = \Pr(\mathbf{x},\mathbf{f}|\mathbf{y}) \;\propto\; \Pr(\mathbf{y}|\mathbf{x},\mathbf{f})\Pr(\mathbf{x},\mathbf{f}) = \Pr(\mathbf{y}|\mathbf{x})\Pr(\mathbf{x}|\mathbf{f})\Pr(\mathbf{f})\\ = \Big\{\prod_{j=1}^{n_rN}\Pr(y_j|\mathbf{x})\prod_{i=1}^{n_tN}\Pr(x_i|f_i)\Big\}\Pr(\mathbf{f}). \qquad (6)\end{aligned}$$

Thus, by defining a new layer of variables f_is corresponding to subcarrier activity, we have effectively decoupled the dependencies present among the elements of the transmit vector $\mathbf{x}$. Based on (6), we model the system as a graph with four types of nodes, namely,

- n_rN *observation nodes* corresponding to elements of $\mathbf{y}$,
- n_tN *variable nodes* corresponding to elements of $\mathbf{x}$,
- n_tN *subcarrier activity nodes* corresponding to elements of $\mathbf{f}$,
- a *constraint node* corresponding to frequency constraint $\mathbf{F}$.

Figure 3 shows the graphical model with the nodes defined above. On this graph, we iteratively pass messages between the nodes and obtain the APPs of the elements of $\mathbf{x}$ and $\mathbf{f}$. The different messages passed in the graph are:

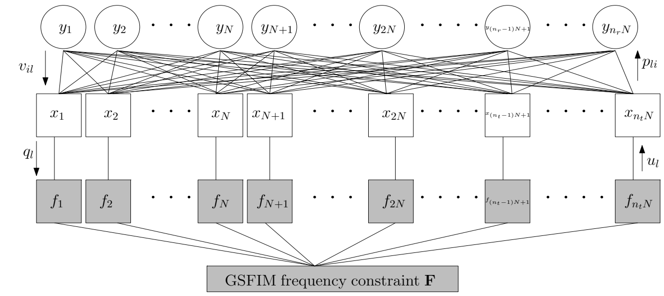

Fig. 3. Graphical model and messages in the proposed detector.

- v_{il}: message from observation node y_i to variable node x_l,
- p_{li}: message from variable node x_l to observation node y_i,
- q_l: message from variable node x_l to subcarrier activity node f_l,
- u_l: message from subcarrier activity node f_l to variable node x_l.

The messages are exchanged between two layers, namely,

- *Layer 1:* observation nodes and variable nodes (denoted by unshaded nodes in Fig. 3); this layer generates approximate APPs of the individual elements of $\mathbf{x}$, and
- *Layer 2:* subcarrier activity nodes and GSFIM frequency constraint node (denoted by shaded nodes in Fig. 3); this layer generates approximate APPs of the elements of $\mathbf{f}$.

In constructing the messages at the variable nodes, we employ a Gaussian approximation of interference as described below. This approximation significantly reduces the detection complexity. From (5), we can write

$$y_i \;=\; G(i,l)x_l + \underbrace{\sum_{j=1,j\neq l}^{n_tN} G(i,j)x_j + w_i}_{\triangleq g_{i,l}}, \qquad (7)$$

where $i = 1, 2, \cdots, n_rN$ and $l = 1, 2, \cdots, n_tN$. We approximate $g_{i,l}$ to be Gaussian with mean $\mu_{i,l}$ and variance $\sigma^2_{i,l}$, which can be calculated as

$$\mu_{i,l} \;=\; \mathbb{E}\Bigg[\sum_{\substack{j=1,\\ j\neq l}}^{n_tN} G(i,j)x_j + w_i\Bigg] \;=\; \sum_{\substack{j=1,\\ j\neq l}}^{n_tN}\sum_{x\in\mathbb{A}_0} p_{j,i}(x)G(i,j)x, \qquad (8)$$

$$\begin{aligned}\sigma^2_{i,l} = \mathrm{Var}\Bigg(\sum_{\substack{j=1,\\ j\neq l}}^{n_tN} G(i,j)x_j + w_i\Bigg) = \sum_{\substack{j=1,\\ j\neq l}}^{n_tN}\Bigg(\sum_{x\in\mathbb{A}_0} p_{j,i}(x)G(i,j)xx^H G(i,j)^H\\ - \Big|\sum_{x\in\mathbb{A}_0} p_{j,i}(x)G(i,j)x\Big|^2\Bigg) + \sigma^2. \qquad (9)\end{aligned}$$

Using this approximation, we derive the messages as follows.

Layer 1: The message v_{il} is obtained as

$$\begin{aligned}v_{il}(x) &\triangleq \Pr(x_l = x|y_i)\\ &\approx \frac{1}{\sigma_{i,l}\sqrt{2\pi}}\exp\Bigg(\frac{-(y_i-\mu_{i,l}-G(i,l)x)^2}{2\sigma^2_{i,l}}\Bigg). \qquad (10)\end{aligned}$$

The APP of the individual elements of $\mathbf{x}$ is given by

$$\begin{aligned}p_{li}(x) \triangleq \Pr(x_l = x|\mathbf{y}_{/i}) &\approx \prod_{j=1,j\neq i}^{n_rN}\Pr(x_l = x|y_j)\\ &\propto u_l(x^{\odot})\prod_{j=1,j\neq i}^{n_rN} v_{jl}(x), \qquad (11)\end{aligned}$$

where $\mathbf{y}_{/i}$ denotes the set of all elements of $\mathbf{y}$ except y_i, and $x^{\odot} = 0$, if $x = 0$ and $x^{\odot} = 1$, if $x \neq 0$. The inclusion of $u_i(.)$

in the computation of this APP helps us to relieve the elements of $\mathbf{x}$ from the dependencies on the subcarrier activity pattern.

Layer 2: The APP estimate of f_l from the variable nodes is

$$
\begin{aligned}
q_l(b) &\triangleq \Pr(f_l = b|x) \\
&\approx \begin{cases} \sum_{x\in\mathbb{A}} \prod_{j=1}^{n_rN} \Pr(x_l = x|y_j), & \text{if } b=1 \\ \prod_{j=1}^{n_rN} \Pr(x_l = 0|y_j), & \text{if } b=0 \end{cases} \\
&\propto \begin{cases} \sum_{x\in\mathbb{A}} \prod_{j=1}^{n_rN} v_{jl}(x), & \text{if } b=1 \\ \prod_{j=1}^{n_rN} v_{jl}(0) & \text{if } b=0. \end{cases}
\end{aligned} \tag{12}
$$

The APP estimate of f_l after processing GSFIM frequency constraint $\mathbf{F}$ is

$$
\begin{aligned}
u_l(b) &= \Pr(f_l = b|\mathbf{x}_{/l}) \\
&\propto \begin{cases} \Pr(\sum_{j=1,j\neq l}^{n_tN} f_j = k-1|\mathbf{f}_{/l}), & \text{if } b=1 \\ \Pr(\sum_{j=1,j\neq k}^{n_tN} f_j = k|\mathbf{f}_{/l}), & \text{if } b=0 \end{cases} \\
&\approx \begin{cases} \phi_{1_l}(k-1) & \text{if } b=1 \\ \phi_{1_l}(k) & \text{if } b=0, \end{cases}
\end{aligned} \tag{13}
$$

where $\Pr(\sum_{j=1,j\neq(i-1)N+l}^{n_tN} f_j = k-1|\mathbf{f}_{/(i-1)N+l})$ denotes the probability that the subcarrier activity pattern satisfies the GSFIM frequency constraint $\mathbf{F}$ $\big($i.e., $\sum_{i=1}^{n_tN} f_i = k\big)$ given that lth subcarrier of ith antenna is active, and $\Pr(\sum_{j=1,j\neq(i-1)N+l}^{n_tN} f_j = k|\mathbf{f}_{/(i-1)N+l})$ denotes the probability that the subcarrier activity pattern satisfies $\mathbf{F}$ given that the lth subcarrier of the ith antenna is not active. Since this probability involves the summation of n_tN-1 variables, it is evaluated as $\phi_{1_l} = \otimes_{j=1,j\neq l}^{n_tN} q_j$, where $\otimes$ is the convolution operator, and $\phi_1(.)$ is a probability mass function with probability masses at n_tN points $(0,1,\cdots,n_tN-1)$.

Message passing: The message passing schedule is as follows.

1. Initialize $p_{li}(x) = \frac{1}{|\mathbb{A}\cup 0|}$ and $q_i(b) = \frac{k}{n_tN}$, $\forall x,b,i,l$.
2. Compute $v_{il}(x)$, $\forall x,l,i$.
3. Compute $u_l(b)$, $\forall l,b$.
4. Compute p_{li}, $\forall l,i,b$.
5. Compute $q_l(b)$, $\forall l,b$.

The above steps are repeated for a fixed number of iterations.

After the fixed number of iterations, the APP estimates of the antenna activity indicators a_ls are computed from the APPs of f_ls as follows. If an antenna is not active, then the subcarriers are not active on that antenna, and if the antenna is active, then at least one subcarrier on that antenna will be active. The APP estimate of a_l from the subcarrier activity nodes is

$$
\begin{aligned}
c_l(b) &\propto \begin{cases} \Pr(\sum_{j=(l-1)N+1}^{lN} f_j \neq 0), & \text{if } b=1 \\ \Pr(\sum_{j=(l-1)N+1}^{lN} f_j = 0), & \text{if } b=0 \end{cases} \\
&\approx \begin{cases} 1-\phi_{2_l}(0), & \text{if } b=1 \\ \phi_{2_l}(0), & \text{if } b=0, \end{cases}
\end{aligned} \tag{14}
$$

where $\Pr(\sum_{j=(l-1)N+1}^{lN} f_j \neq 0)$ denotes the probability that at least one subcarrier is active on the lth antenna, and $\Pr(\sum_{j=(l-1)N+1}^{lN} f_j = 0)$ denotes the probability that all subcarriers are not active on the lth antenna. $\phi_{2_l} = \otimes_{j=(l-1)N+1}^{lN} u_j$, where ϕ_2 is the probability mass function with probability masses at $N+1$ points $(0,1,\cdots,N)$.

The APP estimate of a_l after processing GSFIM antenna constraint $\mathbf{A}$ is

$$
\begin{aligned}
d_l(b) &\propto \begin{cases} \Pr(\sum_{j=1,j\neq l}^{n_t} a_j = n_{rf}-1|\mathbf{a}_{/l}), & \text{if } b=1 \\ \Pr(\sum_{j=1,j\neq k}^{n_t} a_j = n_{rf}|\mathbf{a}_{/l}), & \text{if } b=0 \end{cases} \\
&\approx \begin{cases} \phi_{3_l}(n_{rf}-1) & \text{if } b=1 \\ \phi_{3_l}(n_{rf}) & \text{if } b=0, \end{cases}
\end{aligned} \tag{15}
$$

where $\Pr(\sum_{j=1,j\neq l}^{n_t} a_j = n_{rf}-1|\mathbf{a}_{/l})$ denotes the probability that the antenna pattern satisfies the GSFIM antenna constraint $\mathbf{A}$ $\big($i.e., $\sum_{i=1}^{n_t} a_i = n_{rf}\big)$ given that the lth antenna is active, and $\Pr(\sum_{j=1,j\neq k}^{n_t} a_j = n_{rf}|\mathbf{a}_{/l})$ denotes the probability that the antenna activity pattern satisfies $\mathbf{A}$ given that the lth antenna is not active. $\phi_{3_l} = \otimes_{j=1,j\neq l}^{n_t} c_j$, where ϕ_3 is the probability mass function with probability masses at n_t points $(0,1,\cdots,n_t-1)$.

Finally, an estimate of the antenna activity pattern $\hat{\mathbf{a}}$ is obtained by declaring those n_{rf} antennas having the largest APPs as the active antennas. $\hat{\mathbf{a}}$ is demapped to recover the antenna index bits. $\hat{\mathbf{a}}$ is also fed as an input to the second stage message passing that recovers the modulation and frequency index bits as follows.

2) Second stage message passing:

To recover the modulation and frequency index bits, we consider the system model in (2), i.e.,

$$\mathbf{y}_n = \mathbf{H}_n^{\mathbf{a}}\mathbf{z}_n + \mathbf{w}_n, \quad n = 1,2,\cdots,N.$$

The above equations for all n can be combined into one as

$$\mathbf{y} = \mathbf{G}^{\mathbf{a}}\mathbf{z} + \mathbf{w}, \tag{16}$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_N \end{bmatrix}, \quad \mathbf{G}^{\mathbf{a}} = \begin{bmatrix} \mathbf{H}_1^{\mathbf{a}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2^{\mathbf{a}} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}_N^{\mathbf{a}} \end{bmatrix},$$

and $\mathbf{y}$ and $\mathbf{w}$ are as defined in (5). Note that (16) is similar to (5) with smaller dimensions; while the sizes of $\mathbf{G}$ and $\mathbf{x}$ in (5) are $n_rN\times n_tN$ and $n_tN\times 1$, the sizes of $\mathbf{G}^{\mathbf{a}}$ and $\mathbf{z}$ in (16) are $n_rN\times n_{rf}N$ and $n_{rf}N\times 1$. Therefore, for a given $\mathbf{a}$, the same first stage message passing presented above can be applied to the system model in (16) to get the APPs of z_ls and f_l's, where f_l's are the subcarrier activity indicators for the system model in (16). That is, $z_l = f_l's, s\in\mathbb{A}$, and the corresponding frequency constraint $\mathbf{F}'$ is given by $\sum_{l=1}^{n_{rf}N} f_l' = k$, and the subcarrier activity indicator vector is $\mathbf{f}' \triangleq [f_1', f_2', \cdots, f_{n_{rf}N}']$. We use $\hat{\mathbf{a}}$ as the antenna activity indicator vector in (16) and perform the first stage message passing. At the end, $\hat{\mathbf{f}}'$ is obtained by declaring those k subcarriers having the largest APPs as the active subcarriers. Finally, z_ls and $\hat{\mathbf{f}}'$ are demapped to get the modulation and frequency index bits.

V. Results and discussions

In this section, we present the BER performance of the proposed MSMP algorithm for GSFIM signal detection obtained through simulations. For this, we have taken the channel to be Rayleigh fading frequency-selective channel with $L=4$ paths.

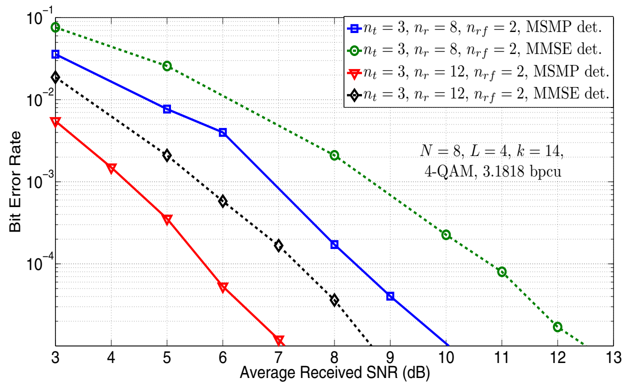

Fig. 4. BER performance of GSFIM system with $n_t = 3$, $n_{rf} = 2$, $n_r = 8, 12$, $n_f = 8$, $N = 8$, $L = 4$, $k = 14$, 4-QAM, 3.1818 bpcu, using proposed MSMP detection and MMSE detection.

The power delay profile of the channel is assumed to follow an exponential decaying model, i.e., $\mathbb{E}[|h^{(j,i)}(l)|^2] = \exp(-l)$, $l = 0, 1, \cdots, L-1$.

In Fig. 4, we present the BER performance of GSFIM with $n_t = 3$, $n_{rf} = 2$, $n_r = 8, 12$, $n_b = 1$, $N = 8$, $k = 14$, 4-QAM, using the proposed MSMP detection algorithm. Minimum mean square error (MMSE) detection performance is also shown for comparison. The rate achieved by the above system is 3.1818 bpcu. Note that ML detection is prohibitively complex for this GSFIM system. It is observed that the proposed MSMP algorithm performs better compared to MMSE detection. For example, at 10^{-4} BER, the proposed detection algorithm outperforms MMSE detection by about 2 dB. The better performance of the proposed algorithm compared to MMSE detection is preserved when we move to higher-order modulation alphabets, i.e., 4-QAM to 8-QAM, keeping the other system parameters same. This is illustrated in Fig. 5, where we show the BER performance of GSFIM with $n_t = 3$, $n_{rf} = 2$, $n_r = 8, 12$, $n_b = 1$, $N = 8$, $k = 14$, and 8-QAM. The rate achieved by this system is 4.45 bpcu. In this case also, at 10^{-4} BER, the proposed MSMP algorithm performs better compared to the MMSE detection by about 2 dB.

In Fig. 6, we show the BER performance of a larger GSFIM system with $n_t = 8$, $n_{rf} = 4$, $n_r = 12, 16$, $n_b = 2$, $N = 32$, $k = 56$, and 4-QAM. The rate achieved by the above system is 8.4 bpcu. In this system, the proposed MSMP algorithm outperforms MMSE detection by about 5 dB at 10^{-4} BER, illustrating the performance advantage achieved by the proposed low complexity detection approach.

VI. Conclusions

Previous studies had shown that GSFIM can offer higher rates and better performance compared to conventional MIMO-OFDM. However, buffer requirements for storing the encoding map and computational complexity involved in ML detection of large-dimension GSFIM signals were prohibitive for practical use. Our contribution in this paper is an early attempt to remove these two implementation bottlenecks. Specifically, we proposed low complexity encoding and detection methods that scale well for large dimensions in GSFIM. The encoding method exploited the low complexity of computing combinadics in combinatorial number systems. This approach enables 'on-the-fly' computation of GSFIM encoding maps. The proposed detection algorithm achieved good performance by using a multi-layer message passing approach. Investigation of GSFIM in multiuser MIMO can be taken up for further study.

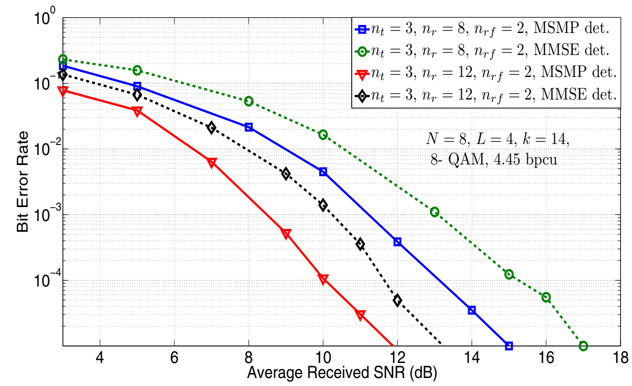

Fig. 5. BER performance of GSFIM system with $n_t = 3$, $n_{rf} = 2$, $n_r = 8, 12$, $n_f = 8$, $N = 8$, $L = 4$, $k = 14$, 8-QAM, 4.45 bpcu, using proposed MSMP detection and MMSE detection.

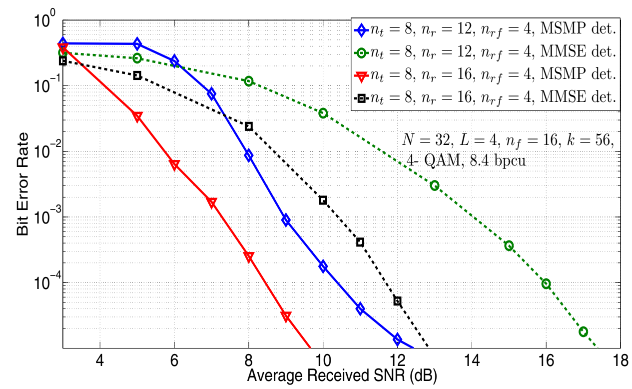

Fig. 6. BER performance of GSFIM system with $n_t = 8$, $n_{rf} = 4$, $n_r = 12, 16$, $n_f = 16$, $N = 32$, $L = 4$, $k = 56$, 4-QAM, 8.4 bpcu, using proposed MSMP detection and MMSE detection.

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