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Large MIMO Systems: A Low-Complexity Detector at High Spectral Efficiencies

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***Abstract*— We consider large MIMO systems, where by '*large*' we mean number of transmit and receive antennas of the order of tens to hundreds. Such large MIMO systems will be of immense interest because of the very high spectral efficiencies possible in such systems. We present a low-complexity detector which achieves uncoded near-exponential diversity performance for hundreds of antennas in V-BLAST (i.e., achieves near SISO AWGN performance in a large MIMO fading environment) with an average per-bit complexity of just $O(N_tN_r)$, where N_t and N_r denote the number of transmit and receive antennas, respectively. With an outer turbo code, the proposed detector achieves good coded bit error performance as well. For example, in a 600 transmit and 600 receive antennas V-BLAST system with a high spectral efficiency of 200 bps/Hz (using BPSK and rate-1/3 turbo code), our simulation results show that the proposed detector performs close to within about 4.6 dB of the theoretical capacity. We also adopt the proposed detector for the low-complexity decoding of high-rate non-orthogonal space-time block codes (STBC) from division algebras (DA). We have decoded the 16×16 full-rate STBC from DA using the proposed detector and show that it performs close to within about 5.5 dB of the capacity using 4-QAM and rate-3/4 turbo code at a spectral efficiency of 24 bps/Hz. The practical feasibility of the proposed high-performance low-complexity detector could trigger wide interest in the implementation of large MIMO systems.**

Keywords *– Large MIMO systems, V-BLAST, non-orthogonal space-time block codes, low-complexity detection, high spectral efficiency.*

I. INTRODUCTION

MIMO techniques offer transmit diversity and high data rates through the use of multiple transmit antennas [1]-[5]. A key component of a MIMO system is the MIMO detector at the receiver, which, in practice, is often the bottleneck for the overall performance and complexity. MIMO detectors including sphere decoder and several of its variants [6]-[10] achieve near-maximum likelihood (ML) performance at the cost of high complexity. Other well known detectors including ZF (zero forcing), MMSE (minimum mean square error), and ZF-SIC (ZF with successive interference cancellation) detectors [3] are attractive from a complexity view point, but achieve relatively poor performance. For example, the ZF-SIC detector (i.e., the well known V-BLAST detector with ordering [11]) does not achieve the full diversity in the system. The MMSE-SIC detector has been shown to achieve optimal performance [3]. However, the order of per-bit complexity involved in MMSE-SIC and ZF-SIC detectors is cubic in number of antennas. Even reduced complexity detectors (e.g., [12]) are prohibitively complex for large number of antennas of the order of hundreds. With small number of antennas, the high capacity potential of MIMO is not fully exploited. A key issue with using large number of antennas, however, is the high detection complexities involved.

Our focus in this paper is on large MIMO systems, where by '*large*' we mean number of transmit and receive antennas of the order of tens to hundreds. Such large MIMO systems will be of immense interest because of the very high spectral efficiencies possible in such systems. For example, in a V-BLAST system, increased number of transmit antennas means increased data rate without bandwidth increase. However, major bottlenecks in realizing such large MIMO systems include $i)$ physical placement of such a large number of antennas in communication terminals[1], $ii)$ lack of practical low-complexity detectors for such large systems, and $iii)$ channel estimation issues. In this paper, we address the second problem in the above (i.e., low-complexity large MIMO detection). Specifically, we present a low-complexity detector/decoder for large MIMO systems, including V-BLAST and high-rate non-orthogonal STBCs [13].

The proposed detector has its roots in past work on Hopfield neural network (HNN) based algorithms for image restoration [14],[15], which are meant to handle large digital images. HNN based image restoration algorithms in [15] are applied to multiuser detection (MUD) in CDMA systems on AWGN channels in [15]. This detector, referred to as the likelihood ascent search (LAS) detector, essentially searches out a sequence of bit vectors with monotonic likelihood ascent and converges to a fixed point in finite number of steps [16]. The power of the LAS detector for CDMA lies in $i)$ its linear average per-bit complexity in number of users, and $ii)$ its ability to perform very close to ML detector for large number of users. Taking the cue from LAS detector's complexity and performance superiority in *large systems*, we, in this paper, successfully adopt the LAS detector for large MIMO systems and report interesting results.

We refer to the proposed detector as MF/ZF/MMSE-LAS[2] detector depending on the initial vector used in the algorithm; MF-LAS uses the matched filter output as the initial vector, and ZF-LAS and MMSE-LAS employ ZF and MMSE outputs, respectively, as the initial vector. Our major findings in this paper are summarized as follows:

Detection in Large V-BLAST Systems:

- In an uncoded V-BLAST system with BPSK, the proposed detector achieves *near-exponential diversity* with hundreds of antennas (i.e., achieves near SISO AWGN performance). For e.g., the detector nearly renders a 200×200 MIMO fading channel into 200 parallel, non-interfering SISO AWGN channels. The detector achieves

[1] We, however, point out that there can be several large MIMO applications where antenna placement need not be a major issue. An example of such an scenario is to provide high-speed backbone connectivity between base stations using large MIMO links, where large number of antennas can be placed at the backbone base stations. Also, tens of antennas can be placed in moderately sized terminals (e.g., laptops, set top boxes) that can enable interesting spectrally efficient, high data rate applications like wireless IPTV.

[2] Throughout the paper, whenever we write MF/ZF/MMSE-LAS, we mean MF-LAS, ZF-LAS, and MMSE-LAS.

this excellent performance with an average per-bit complexity of just $O(N_tN_r)$, where N_t and N_r denote the number of transmit and receive antennas, respectively.
- With an outer turbo code, the proposed detector achieves good coded bit error performance as well. For e.g., in a 600 transmit and 600 receive antennas V-BLAST system with a high spectral efficiency of 200 bps/Hz (using BPSK and rate-1/3 turbo code), our simulation results show that the proposed detector performs close to within about 4.6 dB of the theoretical capacity. We note that performance with such closeness to capacity has not been reported in the literature so far for such large number of antennas using a practical complexity detector.

Decoding of Large High-Rate Non-Orthogonal STBCs:

- We have adopted the proposed detector for the low-complexity decoding of large high-rate, non-orthogonal space-time block codes (STBC) from division algebras (DA) in [13]. We decode the 16×16 full-rate STBC from DA using the proposed detector and show that it performs close to within about 5.5 dB of the capacity using 4-QAM and rate-3/4 turbo code at a spectral efficiency of 24 bps/Hz.
- We point out that because of the high complexities involved in the decoding of large non-orthogonal STBCs using other known detectors (e.g., sphere decoder and its variants), the BERs of such high-rate large STBCs have not been reported in the literature so far. The very fact that we could show the simulated BER plots (both uncoded as well as turbo coded) for a 16×16 full-rate non-orthogonal STBC with 256 complex symbols in one code matrix in itself is a clear indication of the superior low-complexity attribute of the proposed detector. To our knowledge, this is the first time that simulated BER plots for a full-rate 16×16 STBC from DA are reported in the literature; this became feasible due to the low-complexity of the proposed detector.

II. Proposed LAS Detector for Large MIMO

Consider a V-BLAST system with N_t transmit antennas and N_r receive antennas, $N_t \leq N_r$, where N_t symbols are transmitted from N_t transmit antennas simultaneously. Let $b_j \in \{+1,-1\}$ be the symbol[3] transmitted by the jth transmit antenna. Each transmitted symbol goes through the wireless channel to arrive at each of N_r receive antennas. Denote the path gain from transmit antenna j to receive antenna k by h_{kj}. Considering a flat-fading MIMO channel model, the signal received at antenna k, denoted by y_k, is given by

$$y_k \;=\; \sum_{j=1}^{N_t} h_{kj} b_j + n_k. \qquad (1)$$

The $\{h_{kj}\}$, $\forall k \in \{1,2,\cdots,N_r\}$, $\forall j \in \{1,2,\cdots,N_t\}$, are assumed to be i.i.d. complex Gaussian r.v's with zero mean and $E\Big[\big(h^I_{kj}\big)^2\Big] = E\Big[\big(h^Q_{kj}\big)^2\Big] = 0.5$, where h^I_{kj} and h^Q_{kj} are the real and imaginary parts of h_{kj}. The noise sample at the kth receive antenna, n_k, is assumed to be complex Gaussian with zero mean, and $\{n_k\}$, $k = 1,2,\cdots,N_r$, are assumed to be independent with $E[n_k^2] = N_0 = \frac{N_tE_s}{\gamma}$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna [2].

Collecting the received signals from all receive antennas, we write[4]

$$\mathbf{y} \;=\; \mathbf{H}\mathbf{b} + \mathbf{n}, \qquad (2)$$

where $\mathbf{y} = [\,y_1\ y_2\ \cdots\ y_{N_r}\,]^T$ is the N_r-length received signal vector, $\mathbf{b} = [\,b_1\ b_2\ \cdots\ b_{N_t}\,]^T$ is the N_t-length transmitted bit vector, $\mathbf{H}$ denotes the $N_r \times N_t$ channel matrix with channel coefficients $\{h_{kj}\}$, and $\mathbf{n} = [\,n_1\ n_2\ \cdots\ n_{N_r}\,]^T$ is the N_r-length noise vector. $\mathbf{H}$ is assumed to be known at the receiver, but not at the transmitter.

A. Proposed LAS Algorithm

The proposed algorithm essentially searches out a sequence of bit vectors until a fixed point is reached; this sequence is decided based on an update rule. In the V-BLAST system considered, for ML detection [3], the most likely $\mathbf{b}$ is taken as that $\mathbf{b}$ which maximizes

$$\Lambda(\mathbf{b}) \;=\; \mathbf{b}^T\mathbf{H}^H\mathbf{y} + \mathbf{b}^T\Big(\mathbf{H}^H\mathbf{y}\Big)^* - \mathbf{b}^T\mathbf{H}^H\mathbf{H}\mathbf{b}. \quad (3)$$

The likelihood function in (3) can be written as

$$\Lambda(\mathbf{b}) \;=\; \mathbf{b}^T\mathbf{y}_{eff} - \mathbf{b}^T\mathbf{H}_{eff}\mathbf{b}, \qquad (4)$$

where $\mathbf{y}_{eff} \;=\; \mathbf{H}^H\mathbf{y} + \Big(\mathbf{H}^H\mathbf{y}\Big)^*, \quad \mathbf{H}_{eff} \;=\; \mathbf{H}^H\mathbf{H}.$ (5)

Update Criterion in the Search Procedure: Let $\mathbf{b}(n)$ denote the bit vector tested by the LAS algorithm in the search step n. The starting vector $\mathbf{b}(0)$ can be the output vector from any known detector. When the output vector of the MF detector is taken as the $\mathbf{b}(0)$, we call the resulting LAS detector as the MF-LAS detector. We define ZF-LAS and MMSE-LAS detectors likewise. Given $\mathbf{b}(n)$, the algorithm obtains $\mathbf{b}(n+1)$ through an update rule until a fixed point is reached. The update is made in such a way that the change in likelihood from step n to $n+1$, denoted by $\Delta\Lambda\left(\mathbf{b}(n)\right)$, is positive, i.e.,

$$\Delta\Lambda\left(\mathbf{b}(n)\right) \;\stackrel{\triangle}{=}\; \Lambda\left(\mathbf{b}(n+1)\right) - \Lambda\left(\mathbf{b}(n)\right) \;\geq\; 0. \quad (6)$$

An expression for the above change in likelihood can be obtained in terms of the gradient of the likelihood function as follows. Let $\mathbf{g}(n)$ denote the gradient of the likelihood function evaluated at $\mathbf{b}(n)$, i.e.,

$$\mathbf{g}(n) \;\stackrel{\triangle}{=}\; \frac{\partial\left(\Lambda(\mathbf{b}(n))\right)}{\partial\left(\mathbf{b}(n)\right)} \;=\; \mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n), \quad (7)$$

where $\mathbf{H}_{real} \;=\; \mathbf{H}_{eff} + \left(\mathbf{H}_{eff}\right)^* \;=\; 2\,\Re\left(\mathbf{H}_{eff}\right).$ (8)

Using (4) in (6), we can write

$$\begin{aligned}
\Delta\Lambda\left(\mathbf{b}(n)\right) \;&=\; \mathbf{b}^T(n+1)\mathbf{y}_{eff} - \mathbf{b}^T(n+1)\mathbf{H}_{eff}\mathbf{b}(n+1) \\
&\quad -\Big(\mathbf{b}^T(n)\mathbf{y}_{eff} - \mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n)\Big) \\
&=\; \Big(\mathbf{b}^T(n+1) - \mathbf{b}^T(n)\Big)\Big(\mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n)\Big) \\
&\quad -\Big(\mathbf{b}^T(n+1) - \mathbf{b}^T(n)\Big)\Big(-\mathbf{H}_{real}\mathbf{b}(n)\Big) \\
&\quad -\ \mathbf{b}^T(n+1)\mathbf{H}_{eff}\mathbf{b}(n+1) + \mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n). \qquad (9)
\end{aligned}$$

[3] Although we present the detector for BPSK here, we have adopted it for M-QAM/M-PAM as well.

[4] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $(.)^*$, and $[.]^H$ denote transpose, conjugate, and conjugate transpose operations, respectively, and $\Re(.)$ denotes the real part of the complex argument.

Now, defining

$$\Delta \mathbf{b}(n) \stackrel{\triangle}{=} \mathbf{b}(n+1) - \mathbf{b}(n), \tag{10}$$

and $i)$ observing that $\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n) = 2\mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n)$, $ii)$ adding & subtracting the term $\frac{1}{2}\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n+1)$ to the RHS of (9), and $iii)$ further observing that $\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n+1) = \mathbf{b}^T(n+1)\mathbf{H}_{real}\mathbf{b}(n)$, we can simplify (9) as

$$\begin{aligned} \Delta\Lambda\left(\mathbf{b}(n)\right) &= \Delta\mathbf{b}^T(n)\Big(\mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n)\Big) \\ &\quad -\frac{1}{2}\Delta\mathbf{b}^T(n)\mathbf{H}_{real}\Delta\mathbf{b}(n) \\ &= \Delta\mathbf{b}^T(n)\Big(\mathbf{g}(n) + \frac{1}{2}\mathbf{z}(n)\Big), \end{aligned} \tag{11}$$

where

$$\mathbf{z}(n) = -\mathbf{H}_{real}\Delta\mathbf{b}(n). \tag{12}$$

Now, given $\mathbf{y}_{eff}$, $\mathbf{H}_{eff}$, and $\mathbf{b}(n)$, the objective is to obtain $\mathbf{b}(n+1)$ from $\mathbf{b}(n)$ such that $\Delta\Lambda(\mathbf{b}(n))$ in (11) is positive. Potentially any one or several bits in $\mathbf{b}(n)$ can be flipped (i.e., changed from +1 to -1 or vice versa) to get $\mathbf{b}(n+1)$. We refer to the set of bits to be checked for possible flip in a step as a *check candidate set*. Let $L(n) \subseteq \{1, 2, \cdots, N_t\}$ denote the check candidate set at step n. With the above definitions, it can be seen that the likelihood change at step n, given by (11), can be written as

$$\Delta\Lambda(\mathbf{b}(n)) = \sum_{j\in L(n)} \Big(b_j(n+1) - b_j(n)\Big)\Big[g_j(n) + \frac{1}{2}z_j(n)\Big], \tag{13}$$

where $b_j(n)$, $g_j(n)$, and $z_j(n)$ are the jth elements of the vectors $\mathbf{b}(n)$, $\mathbf{g}(n)$, and $\mathbf{z}(n)$, respectively. As shown in [16] for synchronous CDMA on AWGN, the following update rule can be shown to achieve monotonic likelihood ascent (i.e., $\Delta\Lambda(\mathbf{b}(n)) > 0$ if there is at least one bit flip) in the V-BLAST system as well.

LAS Update Algorithm: Given $L(n) \subseteq \{1, 2, \cdots, N_t\}, \forall n \geq 0$ and an initial bit vector $\mathbf{b}(0) \in \{-1, +1\}^{N_t}$, bits in $\mathbf{b}(n)$ are updated as per the following update rule:

$$b_j(n+1) = \begin{cases} +1, & \text{if } j \in L(n),\ b_j(n) = -1 \\ & \text{and } g_j(n) > t_j(n), \\ -1, & \text{if } j \in L(n),\ b_j(n) = +1 \\ & \text{and } g_j(n) < -t_j(n), \\ b_j(n), & \text{otherwise}, \end{cases} \tag{14}$$

where $t_j(n)$ is a threshold for the jth bit in the nth step, which, similar to the threshold in [16], is taken to be

$$t_j(n) = \sum_{i\in L(n)} \Big|(\mathbf{H}_{real})_{j,i}\Big|, \quad \forall j \in L(n), \tag{15}$$

where $(\mathbf{H}_{real})_{j,i}$ is the element in the jth row and ith column of the matrix $\mathbf{H}_{real}$. It can be shown, as in [16], that $t_j(n)$ in (15) is the minimum threshold that ensures monotonic likelihood ascent.

It is noted that different choices can be made to specify the sequence of $L(n), \forall n \geq 0$. One of the simplest sequences correspond to checking one bit in each step for a possible flip, which is termed as a sequential LAS (SLAS) algorithm with constant threshold, $t_j = \Big|\Big(\mathbf{H}_{real}\Big)_{j,j}\Big|$. The sequence of $L(n)$ in SLAS can be such that the indices of bits checked in successive steps are chosen circularly or randomly. Checking of multiple bits for possible flip is also possible. Let $L_f(n) \subseteq L(n)$ denote the set of indices of the bits flipped according to the update rule in (14) at step n. Then the updated bit vector $\mathbf{b}(n+1)$ can be written as

$$\mathbf{b}(n+1) = \mathbf{b}(n) - 2\sum_{i\in L_f(n)} b_i(n)\mathbf{e}_i, \tag{16}$$

where $\mathbf{e}_i$ is the ith coordinate vector. Using (16) in (7), the gradient vector for the next step can be obtained as

$$\begin{aligned} \mathbf{g}(n+1) &= \mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n+1) \\ &= \mathbf{g}(n) + 2\sum_{i\in L_f(n)} b_i(n)\Big(\mathbf{H}_{real}\Big)_i, \end{aligned} \tag{17}$$

where $(\mathbf{H}_{real})_i$ denotes the ith column of the matrix $\mathbf{H}_{real}$. The LAS algorithm keeps updating the bits in each step based on the update rule given in (14) until $\mathbf{b}(n) = \mathbf{b}_{fp}, \forall n \geq n_{fp}$ for some $n_{fp} \geq 0$, in which case $\mathbf{b}_{fp}$ is a fixed point, and it is taken as the detected bit vector and the algorithm terminates.

B. *Complexity of the Proposed LAS Detector*

In terms of complexity, given an initial vector, the LAS operation part alone has an average per-bit complexity of $O(N_tN_r)$. This can be explained as follows. The complexity involved in the LAS operation is due to three components: $i)$ initial computation of $\mathbf{g}(0)$ in (7), $ii)$ update of $\mathbf{g}(n)$ in each step as per (17), and $iii)$ the average number of steps required to reach a fixed point. Computation of $\mathbf{g}(0)$ requires the computation of $\mathbf{H}^H\mathbf{H}$ for each MIMO fading channel realization (see Eqns. (7), (8), and (5)), which requires a per-bit complexity of order $O(N_tN_r)$. Update of $\mathbf{g}(n)$ in the nth step as per (17) using sequential LAS requires a complexity of $O(N_t)$, and hence a constant per-bit complexity. We obtained the average number of steps required to reach a fixed point for sequential LAS through simulations. We observed that the average number of steps required is linear in N_t, i.e., constant per-bit complexity where the constant c depends on SNR, N_t, N_r, and the initial vector [17]. Putting the complexities of $i)$, $ii)$, and $iii)$ in the above together, we see that the average per-bit complexity of LAS operation alone is $O(N_tN_r)$. In addition to the above, the initial vector generation also contributes to the overall complexity. The average per-bit complexity of generating initial vectors using MF, ZF, and MMSE are $O(N_r)$, $O(N_tN_r)$, and $O(N_tN_r)$, respectively. The higher complexity of ZF and MMSE compared to MF is because of the need to perform matrix inversion operation in ZF/MMSE. Again, putting the complexities of the LAS part and the initial vector generation part together, we see that the overall average per-bit complexity of the proposed MF/ZF/MMSE-LAS detector is $O(N_tN_r)$. Several known detectors including ZF-SIC, and detectors based on sphere decoding and its variants [7],[8]. Markov Chain Monte Carlo techniques [9], QR decomposition [10], have higher complexity than $O(N_tN_r)$, and hence are prohibitively complex for hundreds of antennas.

III. LAS Detector Performance in V-BLAST

In this section, we present the uncoded/coded BER performance of the proposed LAS detector in V-BLAST obtained through simulations, and compare with those of other known detectors. The LAS algorithm used is the sequential LAS with circular checking of bits starting from the first antenna bit. We also quantify how far is the proposed detector's turbo coded BER performance away from the theoretical capacity. The SNRs in all the BER performance figures are the average received SNR per received antenna, γ, defined in Sec. II [2].

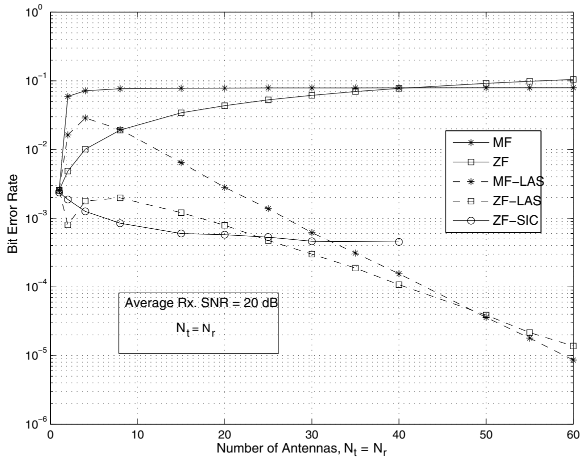

Fig. 1. Uncoded BER performance of MF/ZF-LAS detectors as a function of number of transmit/receive antennas ($N_t = N_r$) in V-BLAST at an average received SNR of 20 dB and BPSK. N_t bps/Hz spectral efficiency.

A. Uncoded BER Performance

MF/ZF-LAS performs increasingly better than ZF-SIC for increasing $N_t = N_r$: In Fig. 1, we plot the uncoded BER of the MF-LAS, ZF-LAS and ZF-SIC detectors in V-BLAST as a function of $N_t = N_r$ at an average received SNR of 20 dB and BPSK. The BER of MF and ZF detectors are also plotted for comparison. From Fig. 1, we observe the following:

- The BER at $N_t = N_r = 1$ is nothing but the SISO flat Rayleigh fading BER, given by $\frac{1}{2}\left[1-\sqrt{\frac{\gamma}{1+\gamma}}\right]$, which is equal to 2.5×10^{-3} for $\gamma = 20$ dB [20]. While the BER of MF and ZF detectors degrade as $N_t = N_r$ is increased, the performance of ZF-SIC improves for antennas up to $N_t = N_r = 15$, beyond which a flooring effect occurs. This improvement is likely due to the diversity in the ordering (selection) in ZF-SIC, whereas the flooring for $N_t > 15$ is likely due to interference being large beyond the cancellation ability of the ZF-SIC.
- The behavior of MF-LAS and ZF-LAS for increasing $N_t = N_r$ is interesting. Starting with the MF output as the initial vector, the MF-LAS always achieves better performance than MF. More interestingly, this improved performance of MF-LAS compared to that of MF increases remarkably as $N_t = N_r$ increases. For example, for $N_t = N_r = 15$, the improvement is an order of BER (i.e., 7.5×10^{-2} BER for MF versus 7×10^{-3} BER for MF-LAS), whereas for $N_t = N_r = 60$ the performance improvement is a remarkable 4 orders of BER (i.e., 8×10^{-2} BER for MF versus 9×10^{-6} BER for MF-LAS). This is due to the large system effect in the LAS algorithm which is able to successfully pick up much of the diversity possible in the system. This large system performance superiority of the LAS is in line with the observations/results reported in [16] for a large CDMA system $\big($large number of antennas in our case, whereas it was large number of users in [16]$\big)$.
- While the ZF-LAS performs slightly better than ZF-SIC for antennas less than 4, ZF-SIC performs better than ZF-LAS for antennas in the range 4 to 24. This is likely because, for antennas less than 4, the BER of ZF is small enough for the LAS to clean up the ZF initial vector better than the output of ZF-SIC. However, for antennas in the range of 4 to 24, the BER of ZF gets high to an extent that the ZF-LAS is less effective in cleaning the initial vector beyond the diversity performance achieved by the ZF-SIC. A more interesting observation, however, is that for antennas greater than 25, the large system effect of the ZF-LAS starts showing up. So, in the large system setting (e.g., antennas more than 25 in Fig. 2), the ZF-LAS performs increasingly better than ZF-SIC for increasing $N_t = N_r$. We found the number of antennas at which the cross-over between ZF-SIC and ZF-LAS occurs) to be different for different SNRs.
- Another observation in Fig. 1 is that for antennas greater than 50, MF-LAS performs better than ZF-LAS. This behavior can be explained by observing the performance comparison between MF and ZF detectors given in the same figure. For more than 50 antennas, MF performs slightly better than ZF. It is known that ZF detector can perform worse than MF detector under high noise/interference conditions [19] (here high interference due to large N_t). Hence, starting with a better initial vector, MF-LAS performs better than ZF-LAS.

ZF-LAS outperforms ZF-SIC in large V-BLAST systems both in complexity & diversity: In Fig. 2, we present an interesting comparison of the uncoded BER performance between ZF, ZF-LAS and ZF-SIC, as a function of average SNR for a 200×200 V-BLAST system. This system being a large system, the ZF-LAS has a huge complexity advantage over ZF-SIC as pointed out before in Sec. II-B. In fact, although we have taken the effort to show the performance of ZF-SIC at such a large number of antennas like 200, we had to obtain these simulation points for ZF-SIC over days of simulation time, whereas the same simulation points for ZF-LAS were obtained in just few hours. This is due to the $O(N_t^2 N_r)$ complexity of ZF-SIC versus $O(N_t N_r)$ complexity of ZF-LAS. More interestingly, in addition to this significant complexity advantage, ZF-LAS is able to achieve a much higher order of diversity (in fact, near-exponential diversity) in BER performance compared to ZF-SIC (which achieves only a little better than first order diversity). This is clearly evident from the slopes of the BER curves of ZF-LAS and ZF-SIC. *Note that the BER curve for ZF-LAS is almost same as the uncoded BER curve for a SISO AWGN channel, given by* $Q(\sqrt{\gamma})$ *[20]. This means that the proposed detector nearly renders a* 200×200 *MIMO fading channel into 200 parallel, non-interfering SISO AWGN channels.*

LAS Detector's performance with hundreds of antennas: As pointed out in the above, obtaining ZF-SIC results for more than even 50 antennas requires very long simulation run times, which is not the case with ZF-LAS. In fact, we could easily generate BER results for up to 400 antennas for ZF-LAS, which are plotted in Fig. 3. The key observations in Fig. 3 are that i) the average SNR required to achieve a certain BER performance keeps reducing for increasing number of antennas for ZF-LAS, and ii) increasing the number of antennas results in increased orders of diversity achieved (close to SISO AWGN performance for 200 and 400 antennas). We have also observed from our simulations that for large number of antennas, the LAS algorithm converges to almost the same near-ML performance regardless of the initial vector chosen.

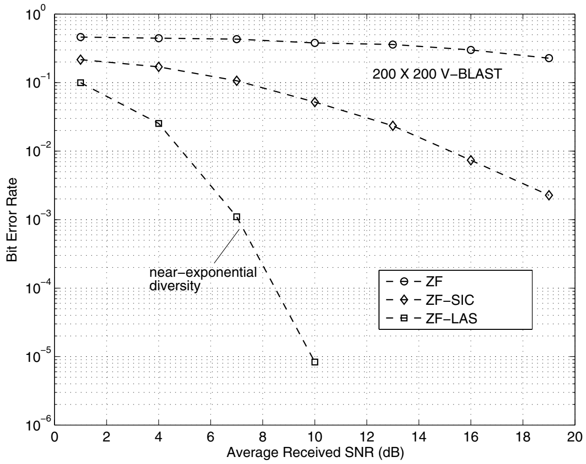

Fig. 2. Uncoded BER performance of ZF-LAS versus ZF-SIC as a function of average received SNR for a 200×200 V-BLAST system. BPSK, 200 bps/Hz spectral efficiency. ZF-LAS achieves higher order diversity (near-exponential diversity) than ZF-SIC at a much lesser complexity.

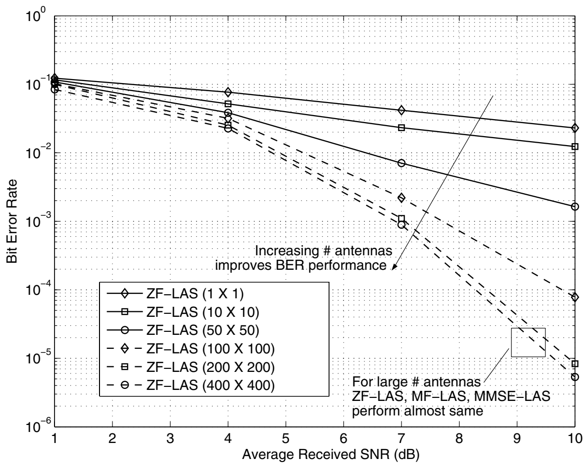

Fig. 3. Uncoded BER performance of ZF-LAS for V-BLAST as a function of average received SNR for increasing values of $N_t = N_r$. BPSK, N_t bps/Hz spectral efficiency. For large N_t, N_r, (e.g., $N_t = N_r = 200, 400$), the BER of ZF-LAS, MF-LAS, and MMSE-LAS are almost the same.

For example, for the case of 200 and 400 antennas in Fig. 3, the BER performance achieved by ZF-LAS, MF-LAS, and MMSE-LAS are almost the same (although we have not explicitly plotted the BER curves for MF-LAS and MMSE-LAS in Fig. 3). So, in such large MIMO system settings, MF-LAS may be preferred over ZF-LAS and MMSE-LAS since ZF-LAS and MMSE-LAS require matrix inverse operation whereas MF-LAS does not.

Observation i) in the above is explicitly brought out in Fig. 4, where we have plotted the average received SNR required to achieve a target uncoded BER of 10^{-3} as a function of $N_t = N_r$ for ZF-LAS and ZF-SIC. It can be seen that the SNR required to achieve 10^{-3} BER with ZF-LAS significantly reduces for increasingly large $N_t = N_r$. For example, the required SNR reduces from about 25 dB for a SISO system to about 7 dB for a 400×400 V-BLAST system using ZF-LAS; it is noted that the SNR required to achieve 10^{-3} BER in a SISO AWGN channel is also 7 dB [20], i.e., $20 \log \left(Q^{-1}(10^{-3})\right) \approx 7$ dB.

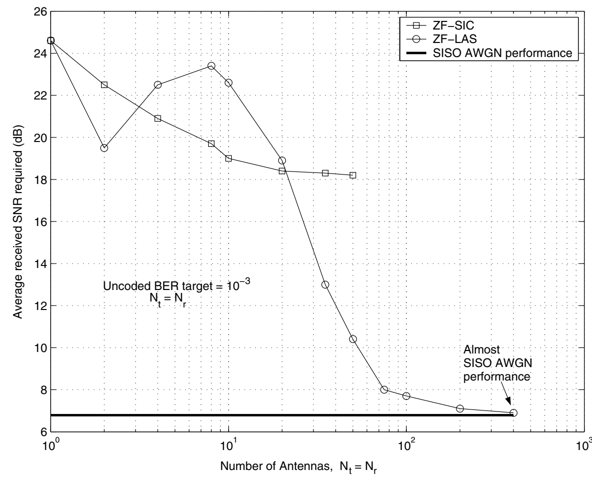

Fig. 4. Average received SNR required to achieve a target uncoded BER of 10^{-3} in V-BLAST for increasing values of $N_t = N_r$. BPSK. ZF-LAS vs ZF-SIC. ZF-LAS achieves near SISO AWGN performance for large N_t.

B. Turbo Coded BER Performance

In this subsection, we present the turbo coded BER performance of the proposed LAS detector. We also quantify how far is the proposed detector's performance away from the theoretical capacity. For a $N_t \times N_r$ MIMO system model in Sec. II with perfect channel state information (CSI) at the receiver, the ergodic capacity is given by [4]

$$C = E\left[\log \det \left(\mathbf{I}_{N_r} + (\gamma/N_t)\,\mathbf{H}\mathbf{H}^H\right)\right], \qquad (18)$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix and γ is the average SNR per receive antenna. We evaluated the capacity in (18) for a 600×600 MIMO system through Monte-Carlo simulations and plotted it as a function of average SNR in Fig. 5. Figure 6 shows the simulated BER of the proposed LAS detector for a 600×600 MIMO system with BPSK, rate-1/3 turbo code at a spectral efficiency of 200 bps/Hz. The following interesting observations can be made from Fig. 6:

- In terms of uncoded BER, the performance of MF, ZF, and MMSE are different, with ZF and MMSE performing the worst and best, respectively. But the performance of MF-LAS, ZF-LAS, and MMSE-LAS are almost the same (near-exponential diversity performance) with the number of antennas being large ($N_t = N_r = 600$).
- With a rate-1/3 turbo code, all the LAS detectors considered (i.e., MF-LAS, ZF-LAS, MMSE-LAS) achieve almost the same performance, which is about 4.6 dB away from capacity (i.e., near-vertical fall of coded BER occurs at about -0.8 dB). Turbo coded MF/MMSE without LAS also achieve good performance in this case (i.e., less than only 2 dB away from turbo coded MF/ZF/MMSE-LAS performance). This is because the uncoded BERs of MF and MMSE at around 0 to 2 dB SNR are small enough for the turbo code to be effective. However, this is not the case with turbo coded ZF without LAS. As can be seen, in the range of SNRs shown, the uncoded BER of ZF without LAS is so high (close to 0.5) that the vertical fall of coded BER can happen only at very high SNRs, because of which we have not shown the turbo coded ZF (without LAS) performance.

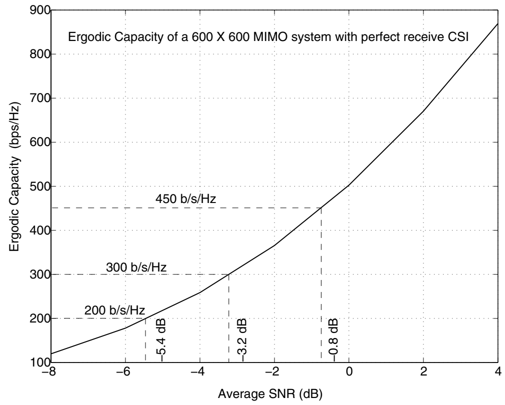

Fig. 5. Ergodic capacity for 600×600 MIMO system with receive CSI.

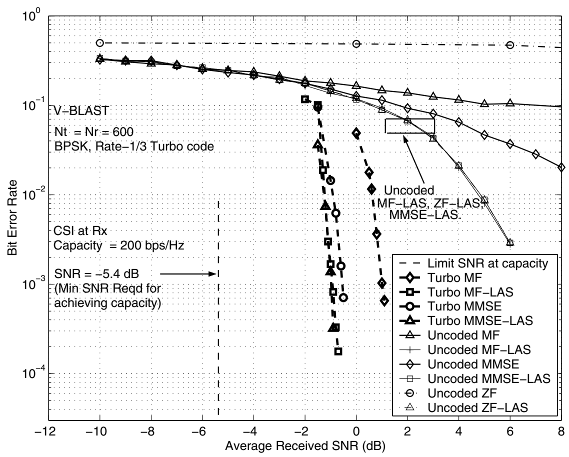

Fig. 6. Coded BER performance of various detectors for rate-1/3 turbo-encoded data using BPSK in a 600×600 V-BLAST system. 200 bps/Hz spectral efficiency. Proposed MF/ZF/MMSE-LAS detectors' performance is away from capacity by 4.6 dB.

In Table I, we summarize the performance of various detectors in terms of their nearness to capacity[5] in 600×600 V-BLAST using BPSK, for rate-1/3, 1/2, and 3/4 turbo codes. From Table-I, it can be seen that there is a clear superiority of the proposed MF/ZF/MMSE-LAS over MF/MMSE without LAS in terms of coded BER (nearness to capacity) when high-rate turbo codes are used. For example, when a rate-3/4 turbo code is used the MF/ZF/MMSE-LAS performs to within about 5.6 dB from capacity, whereas the performance of rate-3/4 turbo coded MF/MMSE without LAS are much farther away from capacity. With channel estimation errors, our simulation results show that in a 200×200 V-BLAST system with BPSK, rate-1/2 turbo code and LAS detection, the coded BER degradation compared to perfect channel estimation is only 0.2 dB and 0.6 dB for channel estimation error variances of 1% and 5%, respectively [17].

IV. LAS DECODING OF NON-ORTHOGONAL STBCS

High-rate non-orthogonal STBCs[6] from division algebras (DA) [13] are attractive for achieving high spectral efficiencies in addition to achieving full transmit diversity, using large number of transmit antennas. Well known orthogonal STBCs have

Code Rate, Spect. Eff.	Min. SNR at capacity	Vertical fall of coded BER occurs at			
		Proposed LAS	ZF	MF	MMSE
Rate-1/3, 200 bps/Hz	-5.4 dB	-0.8 dB	high	1.2 dB	-0.3 dB
Rate-1/2 300 bps/Hz	-3.2 dB	1.5 dB	high	high	3 dB
Rate-3/4 450 bps/Hz	-0.8 dB	4.75 dB	high	high	high

TABLE I: Nearness to capacity of various detectors for 600×600 V-BLAST with BPSK and various turbo code rates. Proposed LAS detector performs to within about 4.6 dB, 4.7 dB, 5.6 dB from capacity for 200, 300, and 450 bps/Hz spectral efficiencies, respectively.

the advantages of low decoding complexity and full transmit diversity, but suffer from rate loss for increasing number of transmit antennas [2]. Non-orthogonal STBCs which achieve full-rate[7] can be constructed from DA for arbitrary number of transmit antennas, n, [13]. High spectral efficiencies can be achieved using these STBCs from DA with large n. For example, with $n = 16$ transmit antennas, the 16×16 STBC from DA in [13] with 4-QAM and rate-3/4 turbo code achieves a high spectral efficiency of 24 bps/Hz. This high spectral efficiency is achieved along with the full-diversity of order nN_r. However, since the code is non-orthogonal, ML decoding gets increasingly impractical for large n (there are n^2 symbols in a code matrix). Consequently, a key challenge in realizing the benefits of these large STBC codes in practice is that of achieving near-ML performance for large n at low decoding complexities. In this context, a significant contribution in this paper is that we have successfully adopted the proposed LAS detector to decode large STBCs[8] from DA, and show that the it achieves near capacity performance with low decoding complexity for large n.

Uncoded/Coded BER Performance of Large STBCs from DA: In Fig. 7, we present the uncoded BER of the LAS detector in decoding $n \times n$ full-rate non-orthogonal STBCs from DA in [13] for $n = 4, 8, 16$ and 4-QAM. It can be observed that as the STBC code size n increases, the LAS performs increasingly better such that it achieves close to SISO AWGN performance (within 0.5 dB at 10^{-3} BER and less) with the 16×16 STBC. We point out that due to the high complexities involved in decoding large size STBCs using other known detectors, the BER performance of STBCs with large n has not been reported in the literature so far. The very fact that we could show the simulated BER plots (both uncoded as well as turbo coded) for a 16×16 STBC with 256 complex symbols in one code matrix in itself is a clear indication of the superior low-complexity attribute of the proposed LAS detector. To our knowledge, we are the first to report the simulated BER performance of a 16×16 STBC from DA; this became feasible because of the low-complexity feature of the proposed detector. In addition, the achievement of near SISO AWGN performance with 16×16 STBC is a significant result from an implementation view point as well, since 16 antennas can be easily placed in communication terminals of moderate size, which can make large MIMO systems practical.

Turbo Coded BER Performance: In Fig. 8, we show the coded BER performance of the 16×16 STBC using dif-

[5] We point out that the turbo coded BER curves shown in Figs. 7 to 11 in [17] have been plotted erroneously with an SNR shift of $-10 \log r$ dB, where r is the turbo code rate, which amounted to a pessimistic prediction of nearness to capacity. We have corrected this plotting error now in Fig. 6 and the nearness to capacity results in Table-I shown in this page.

[6] An STBC is represented by a $p \times n$ matrix with complex entries, where n and p denote the number of transmit antennas and time slots, respectively.

[7] An $n \times n$ STBC is said to be full-rate if the number of complex symbols transmitted per channel use is equal to $min(N_t, N_r)$.

[8] We write the STBC received signal model in an equivalent V-BLAST form, and apply the LAS algorithm on this equivalent signal model.

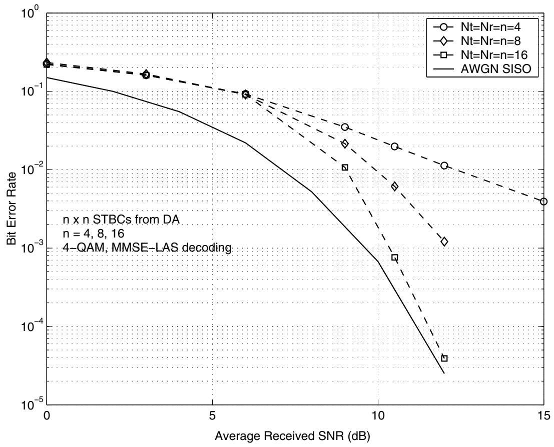

Fig. 7. Uncoded BER performance of the proposed LAS detector in decoding $n \times n$ full-rate non-orthogonal STBCs from DA for $n = 4, 8, 16$. MMSE initial vector, 4-QAM, $N_t = N_r = n$. 16×16 STBC with 256 complex symbols in each code matrix achieves close to SISO AWGN performance.

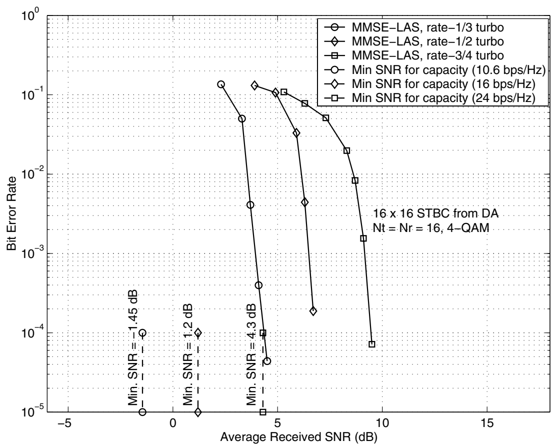

Fig. 8. Coded BER performance of the proposed LAS detector in decoding 16×16 full-rate non-orthogonal STBC from DA. $N_t = N_r = 16$. MMSE initial vector, 4-QAM. Rates of turbo codes: 1/3, 1/2, 3/4. Proposed LAS detector performs close to within about 5.5 dB of the theoretical capacity.

ferent turbo code rates of 1/3, 1/2, and 3/4. With 4-QAM, these turbo code rates along with the 16×16 STBC from DA correspond to spectral efficiencies of 10.6 bps/Hz, 16 bps/Hz and 24 bps/Hz, respectively. The minimum SNRs required to achieve these capacities are also shown in Fig. 8. It can be observed that the proposed detector performs to within about 5.5 dB of the capacity, which is an impressive result[9].

V. Conclusions

We presented a near-ML performance achieving, low-complexity detector for large MIMO systems having tens to hundreds of antennas, and showed its uncoded/coded BER performance in the detection of V-BLAST and in the decoding of full-rate, non-orthogonal STBCs. The proposed detector was shown to have excellent attributes in terms of both low complexity as well as nearness to theoretical capacity performance, achieving high spectral efficiencies of the order of tens to hundreds of bps/Hz. To our knowledge, our reporting of the decoding of a large full-rate STBC like 16×16 STBC from DA and its uncoded/coded BER performance in this paper is for the first time in the literature. We are investigating issues in channel estimation, pilot symbols allocation, and antenna/RF technologies in the large MIMO context. The low-complexity feature of the proposed detector can allow the inclusion of 4×4, 8×8, and 16×16 full-rate STBCs from DA into wireless standards like IEEE 802.11n and IEEE 802.16e, and achieve much higher spectral efficiencies than those possible in these standards.

We conclude by pointing to the following remark made by the author of [2] in its preface in 2005: *"It was just a few years ago, when I started working at AT&T Labs – Research, that many would ask 'who would use more than one antenna in a real system?' Today, such skepticism is gone."* Extending this sentiment, we believe large MIMO systems would be practical in the future, and the practical feasibility of low-complexity detectors like the one we presented in this paper could be a potential trigger to create wide interest in the implementation of large MIMO systems.

[9] In all the turbo coded BER plots in this paper, we have used hard decision outputs from the LAS algorithm. In [18], we have proposed a method to generate soft decision outputs from the LAS for the individual bits that form the QAM/PAM symbols. With the proposed soft decision LAS outputs in [18], the coded performance is found to move closer to capacity by an additional 1 to 1.5 dB compared to that achieved using hard decision LAS outputs.

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