Large-Scale Multiuser SM-MIMO Versus Massive MIMO

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Abstract-Spatial modulation (SM) is attractive for multiantenna wireless communications. SM uses multiple transmit antenna elements but only one transmit radio frequency (RF) chain. In SM, in addition to the information bits conveyed through conventional modulation symbols (e.g., QAM), the index of the active transmit antenna also conveys information bits. In this paper, we establish that SM has significant signal-to-noise (SNR) advantage over conventional modulation in large-scale multiuser (multiple-input multiple-output) MIMO systems. Our new contribution in this paper addresses the key issue of largedimension signal processing at the base station (BS) receiver (e.g., signal detection) in large-scale multiuser SM-MIMO systems, where each user is equipped with multiple transmit antennas (e.g., 2 or 4 antennas) but only one transmit RF chain, and the BS is equipped with tens to hundreds of (e.g., 128) receive antennas. Specifically, we propose two novel algorithms for detection of large-scale SM-MIMO signals at the BS; one is based on message passing and the other is based on local search. The proposed algorithms achieve very good performance and scale well. For the same spectral efficiency, multiuser SM-MIMO outperforms conventional multiuser MIMO (recently being referred to as massive MIMO) by several dBs. The SNR advantage of SM-MIMO over massive MIMO can be attributed to: (i) because of the spatial index bits, SM-MIMO can use a lower-order QAM alphabet compared to that in massive MIMO to achieve the same spectral efficiency, and (ii) for the same spectral efficiency and QAM size, massive MIMO will need more spatial streams per user which leads to increased spatial interference.

Keywords – Large-scale MIMO systems, spatial modulation, SM-MIMO, massive MIMO, message passing, local search.

I. INTRODUCTION

Large-scale MIMO systems with tens to hundreds of antennas are getting increased research attention [1]-[4]. Because of their advantages in terms of very high spectral efficiencies/sum rates, increased reliability, and power efficiency, large-scale MIMO technology is being considered as a potential technology for future fifth generation (5G) wireless systems. The following two characteristics are typical in conventional MIMO systems: (i) there will be one transmit RF chain for each transmit antenna (i.e., if n_t is the number of transmit antennas, the number of transmit RF chains will also be n_t), and (*ii*) information bits are carried only on the modulation symbols (e.g., QAM). Spatial modulation MIMO (SM-MIMO) systems [5] differ from conventional MIMO systems in the following two aspects: (i) in SM-MIMO there will be multiple transmit antennas but only one transmit RF chain, and (ii) the index of the active transmit antenna will also convey information bits in addition to information bits conveyed through modulation symbols like QAM symbols. SM-MIMO offers the advantages of reduced RF hardware complexity, size, and cost.

Conventional multiuser MIMO systems with a large number (tens to hundreds) of antennas at the base station (BS) are referred to as 'massive MIMO' systems in the recent literature [4]. The users in a massive MIMO system can have one or more transmit antennas with equal number of transmit RF chains. In large-scale multiuser SM-MIMO systems also, the number of BS antennas will be large. The users in SM-MIMO will have multiple transmit antennas but only one transmit RF chain. Figures 1(a) and 1(b) illustrate the large-scale multiuser SM-MIMO system (with K users, N BS antennas, n_t transmit antennas per user, and $n_{rf} = 1$ transmit RF chain per user) and massive MIMO system (with K users, N BS antennas, 1 transmit antenna per user, and 1 transmit RF chain per user), respectively.

Several works have focused on single user point-to-point SM-MIMO systems – e.g., [6] and the references therein. Some works on multiuser SM-MIMO have also been reported [7]-[9]. An interesting result reported in [7] is that multiuser SM-MIMO outperforms conventional multiuser MIMO by several dBs for the same spectral efficiency. This work is limited to 3 users (with 4 antennas each) and 4 antennas at BS receiver. Also, only maximum likelihood (ML) detection is considered, which is prohibitive (because of its exponential complexity) for large-scale systems. This superiority of SM-MIMO over conventional MIMO calls for further investigations on multiuser SM-MIMO. In particular, investigations in the following two directions are of interest: (i) large-scale SM-MIMO (with large number of users and BS antennas), and (*ii*) detection algorithms that can scale and perform well in such large-scale SM-MIMO systems. In this paper, we make new contributions in these two directions.

We investigate multiuser SM-MIMO with similar number of users and BS antennas envisaged in massive MIMO, e.g., tens of users and hundreds of BS antennas. Our contributions in this paper can be summarized as follows.

• Proposal of two novel algorithms for detection of largescale SM-MIMO signals at the BS. One algorithm is based on message passing referred to as MPD-SM (*message passing detection for spatial modulation*) algorithm, and the other is based on local search referred to as LSD-SM (*local search detection for spatial modulation*)



Fig. 1. Large-scale multiuser SM-MIMO and massive MIMO system architectures.

algorithm. Simulation results show that these proposed algorithms achieve very good performance and scale well.

- Performance comparison between SM-MIMO and massive MIMO on the uplink for the same spectral efficiency. Simulation results show that SM-MIMO outperforms massive MIMO by several dBs; e.g., SM-MIMO has a 4 to 5 dB SNR advantage over massive MIMO at 10^{-3} BER for 16 users, 128 BS antennas, and 4 bpcu per user. The SNR advantage of SM-MIMO over massive MIMO is attributed to the following reasons: (*i*) because of the spatial index bits, SM-MIMO can use a lower-order QAM alphabet compared to that in massive MIMO to achieve the same spectral efficiency, and (*ii*) for the same spectral efficiency and QAM size, massive MIMO will need more spatial streams per user which leads to increased spatial interference.
- Signal detection and performance of large-scale multiuser SM-MIMO on frequency-selective channels.

The rest of the paper is organized as follows. The system model for multiuser SM-MIMO is presented in Section II. The proposed MPD-SM algorithm for detection of SM-MIMO signals and its performance in frequency-flat fading are presented in Section III. In Section IV, the proposed LSD-SM algorithm and its performance in frequency-flat fading are presented. Sections III and IV also present performance comparisons between SM-MIMO and massive MIMO. In Section V, the SM-MIMO system model and performance in frequencyselective fading are presented. Conclusions are presented in Section VI.

II. MULTIUSER SM-MIMO SYSTEM MODEL

Consider a multiuser system with K uplink users communicating with a BS having N receive antennas, where N is in the

order of tens to hundreds. The ratio $\alpha = K/N$ is the system loading factor. Each user employs spatial modulation (SM) for transmission, where each user has n_t transmit antennas but only one transmit RF chain (see Fig. 1(a)). In a given channel use, each user selects any one of its n_t transmit antennas, and transmits a symbol from a modulation alphabet \mathbb{A} on the selected antenna. The number of bits conveyed per channel use per user through the modulation symbols is $\lfloor \log_2 |\mathbb{A}| \rfloor$. In addition, $\lfloor \log_2 n_t \rfloor$ bits per channel use (bpcu) per user is conveyed through the index of the chosen transmit antenna (see Fig. 2). Therefore, the overall system throughput is

$$K(\lfloor \log_2 |\mathbb{A}| \rfloor + \lfloor \log_2 n_t \rfloor)$$
 bpcu.

For e.g., in a system with K = 3, $n_t = 4$, 4-QAM, the system throughput is 12 bpcu.



Fig. 2. SM-MIMO transmitter at the user terminal with n_t transmit antennas and one transmit RF chain.

The SM signal set $\mathbb{S}_{n_t,\mathbb{A}}$ for each user is given by

$$S_{n_t,\mathbb{A}} = \{ \mathbf{s}_{j,l} : j = 1, \cdots, n_t, \quad l = 1, \cdots, |\mathbb{A}| \},$$

s.t. $\mathbf{s}_{j,l} = [0, \cdots, 0, \underbrace{s_l}_{j\text{th coordinate}}, 0, \cdots, 0]^T, \quad s_l \in \mathbb{A}.$ (1)

For e.g., for $n_t = 2$ and 4-QAM, $\mathbb{S}_{n_t,\mathbb{A}}$ is given by

$$\mathbb{S}_{2,+\text{QAM}} = \left\{ \begin{bmatrix} +1+j\\0 \end{bmatrix}, \begin{bmatrix} +1-j\\0 \end{bmatrix}, \begin{bmatrix} -1+j\\0 \end{bmatrix}, \begin{bmatrix} -1-j\\0 \end{bmatrix}, \begin{bmatrix} 0\\+1+j \end{bmatrix}, \begin{bmatrix} 0\\+1-j \end{bmatrix}, \begin{bmatrix} 0\\-1+j \end{bmatrix}, \begin{bmatrix} 0\\-1-j \end{bmatrix} \right\}.$$
(2)

Let $\mathbf{x}_k \in \mathbb{S}_{n_t,\mathbb{A}}$ denote the transmit vector from user k. Let $\mathbf{x} \triangleq [\mathbf{x}_1^T \ \mathbf{x}_2^T \cdots \mathbf{x}_k^T \cdots \mathbf{x}_K^T]^T$ denote the vector comprising of transmit vectors from all the users. Note that $\mathbf{x} \in \mathbb{S}_{n_t,\mathbb{A}}^K$.

Let $\mathbf{H} \in \mathbb{C}^{N \times Kn_t}$ denote the channel gain matrix, where $H_{i,(k-1)n_t+j}$ denotes the complex channel gain from the *j*th transmit antenna of the *k*th user to the *i*th BS receive antenna. The channel gains are assumed to be independent Gaussian with zero mean and variance σ_k^2 , such that $\sum_k \sigma_k^2 = K$. The σ_k^2 models the imbalance in the received power from user *k* due to path loss etc., and $\sigma_k^2 = 1$ corresponds to the case of perfect power control. Assuming perfect synchronization, the received signal at the *i*th BS antenna is given by

$$y_i = \sum_{k=1}^{K} x_{l_k} H_{i,(k-1)n_t + j_k} + n_i,$$
(3)

where x_{l_k} is the l_k th symbol in \mathbb{A} , transmitted by the j_k th antenna of the kth user, and n_i is the noise modeled as a complex Gaussian random variable with zero mean and variance σ^2 . The received signal at the BS antennas can be written in vector form as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{4}$$

where $\mathbf{y} = [y_1, y_2, \cdots, y_N]^T$ and $\mathbf{n} = [n_1, n_2, \cdots, n_N]^T$. We assume perfect channel knowledge at the BS receiver.

For this system model, the maximum-likelihood (ML) detection rule is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}_{n_t, \mathbb{A}}^K}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2,$$
(5)

where $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$ is the ML cost. The maximum a posteriori probability (MAP) decision rule, is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}_{n_t,\mathbb{A}}^K}{\operatorname{argmax}} \operatorname{Pr}(\mathbf{x} \mid \mathbf{y}, \mathbf{H}).$$
(6)

Since $|\mathbb{S}_{n_t,\mathbb{A}}^K| = (|\mathbb{A}|n_t)^K$, the exact computation of (5) and (6) requires exponential complexity in *K*. We propose two low complexity detection algorithms for multiuser SM-MIMO; one based on message passing (in Section III) which gives an approximate solution to (6), and another based on local search (in Section IV) which gives an approximate solution to (5). To our knowledge, such low complexity algorithms for large-scale multiuser SM-MIMO signal detection have not been reported.

We note that the condition for the spectral efficiencies of SM-MIMO (with n_t transmit antennas, 1 transmit RF chain, and modulation alphabet \mathbb{A}) and massive MIMO to be the same is given by

$$|\mathbb{A}|n_t = |\mathbb{B}|^{m_t},$$

where \mathbb{B} is the modulation alphabet used in massive MIMO, and m_t is the number of transmit antennas (also the number of transmit RF chains) at each user in massive MIMO. For e.g., SM-MIMO with 4-QAM, $n_t = 4$ and 1 transmit RF chain, and massive MIMO with 4-QAM, $m_t = 2$ and 2 transmit RF chains have the same spectral efficiency of 4 bpcu per user. Similarly, massive MIMO systems with (*i*) BPSK, $m_t = 4$ and 4 transmit RF chains, and (*ii*) 16-QAM, $m_t = 1$ and 1 transmit RF chain also have the same 4 bpcu per user spectral efficiency. In massive MIMO, the vector $\mathbf{x} \in \mathbb{B}^{Km_t}$ and the channel matrix $\mathbf{H} \in \mathbb{C}^{N \times Km_t}$.

III. MESSAGE PASSING DETECTION FOR SM-MIMO

In this section, we propose a message passing based algorithm for detection in SM-MIMO systems. We refer to the proposed algorithm as the MPD-SM (message passing detection for spatial modulation) algorithm. We model the system as a fully connected factor graph with K variable nodes (or factor nodes) corresponding to \mathbf{x}_k 's and N observation nodes corresponding to y_i 's, as shown in Fig. 3(a).

Messages: We derive the messages passed in the factor graph as follows. Equation (4) can be written as

$$y_{i} = \mathbf{h}_{i,[k]} \mathbf{x}_{k} + \underbrace{\sum_{j=1, j \neq k}^{K} \mathbf{h}_{i,[j]} \mathbf{x}_{j} + n_{i}}_{\triangleq g_{ik}}, \tag{7}$$

where $\mathbf{h}_{i,[j]}$ is a row vector of length n_t , given by $[H_{i,(j-1)n_t+1} \quad H_{i,(j-1)n_t+2} \cdots H_{i,jn_t}]$, and $\mathbf{x}_j \in \mathbb{S}_{n_t,\mathbb{A}}$.

We approximate the term g_{ik} to have a Gaussian distribution¹ with mean μ_{ik} and variance σ_{ik}^2 as follows:

$$\mu_{ik} = \mathbb{E}\left[\sum_{j=1, j \neq k}^{K} \mathbf{h}_{i,[j]} \mathbf{x}_{j} + n_{i}\right] = \sum_{j=1, j \neq k}^{K} \sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) \mathbf{h}_{i,[j]} \mathbf{s}$$
$$= \sum_{j=1, j \neq k}^{K} \sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) s_{l_{s}} H_{i,(j-1)n_{t}+l_{s}}, \tag{8}$$

where s_{l_s} is the only non-zero entry in s and l_s is its index, and $p_{ki}(s)$ is the message from kth variable node to the *i*th observation node. The variance is given by

$$\sigma_{ik}^{2} = \operatorname{Var}\left(\sum_{j=1, j \neq k}^{K} \mathbf{h}_{i,[j]} \mathbf{x}_{j} + n_{i}\right)$$

$$= \sum_{j=1, j \neq k}^{K} \sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) \mathbf{h}_{i,[j]} \mathbf{s} \mathbf{s}^{H} \mathbf{h}_{i,[j]}^{H}$$

$$- \left|\sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) \mathbf{h}_{i,[j]} \mathbf{s}\right|^{2} + \sigma^{2}$$

$$= \sum_{j=1, j \neq k}^{K} \sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) \left|s_{l_{s}} H_{i,(j-1)n_{t}+l_{s}}\right|^{2}$$

$$- \left|\sum_{\mathbf{s} \in \mathbb{S}_{n_{t},\mathbb{A}}} p_{ji}(\mathbf{s}) s_{l_{s}} H_{i,(j-1)n_{t}+l_{s}}\right|^{2} + \sigma^{2}.$$
(9)

¹This Gaussian approximation will be accurate for large K; e.g., in systems with tens of users.



Fig. 3. The factor graph and messages passed in MPD-SM algorithm.

The message $p_{ki}(s)$ is given by

$$p_{ki}(\mathbf{s}) \propto \prod_{m=1, m \neq i}^{N} \exp\left(-\frac{\left|y_m - \mu_{mk} - \mathbf{h}_{m,[k]}\mathbf{s}\right|^2}{2\sigma_{mk}^2}\right).$$
(10)

Message passing: The message passing is done as follows. Step 1: Initialize $p_{ki}(\mathbf{s})$ to $1/|\mathbb{S}_{n_t,\mathbb{A}}|$ for all i, k and \mathbf{s} . Step 2: Compute μ_{ik} and σ_{ik}^2 from (8) and (9), respectively.

Step 3: Compute p_{ki} from (10). To improve the convergence rate, damping [10] of the messages in (10) is done with a damping factor $\delta \in (0, 1]$, as shown in the algorithm listing in Algorithm 1.

Repeat Steps 2 and 3 for a certain number of iterations. Figures 3(b) and 3(c) illustrate the exchange of messages between observation and variable nodes, where the vector message $\mathbf{p}_{ki} = [p_{ki}(\mathbf{s}_1), p_{ki}(\mathbf{s}_2), \cdots, p_{ki}(\mathbf{s}_{|\mathbb{S}_{n_i,\mathbb{A}}|})]$. The final symbol probabilities at the end are given by

$$p_k(\mathbf{s}) \propto \prod_{m=1}^N \exp\left(-\frac{\left|y_m - \mu_{mk} - \mathbf{h}_{m,[k]}\mathbf{s}\right|^2}{2\sigma_{mk}^2}\right).$$
(11)

The detected vector of the kth user at the BS is obtained as

$$\hat{\mathbf{x}}_k = \underset{\mathbf{s} \in \mathbb{S}_{n_t,\mathbb{A}}}{\operatorname{argmax}} p_k(\mathbf{s}).$$
(12)

The non-zero entry in $\hat{\mathbf{x}}_k$ and its index are then demapped to obtain the information bits of the kth user. The algorithm listing is given in Algorithm 1.

Complexity: From (8), (9), and (10), we see that the total complexity of the MPD-SM algorithm is $O(NK|\mathbb{S}_{n_{t},\mathbb{A}}|)$. This complexity is less than the MMSE detection complexity of $O(N^2Kn_t)$. Also, the computation of double summation in (8) and (9) can further be simplified by using FFT, as the double summation can be viewed as a convolution operation.

Performance: We evaluated the performance of multiuser SM-MIMO using the proposed MPD-SM algorithm and compared it with that of massive MIMO with ML detection (using

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Output: $p_k(\mathbf{s})$ as per (11) and $\hat{\mathbf{x}}_k$ as per (12), $\forall k$

Algorithm 1: Listing of the proposed MPD-SM algorithm.

sphere decoder) for the same spectral efficiency with K = 16, $n_{rf} = 1$ transmit RF chain at each user, and N = 64, 128. For SM-MIMO, we consider the number of transmit antennas at each user to be $n_t = 2, 4$. Figure 4 shows the performance comparison between SM-MIMO with $(n_t = 2, n_{rf} = 1, 4$ -QAM) and massive MIMO² with $(m_t = 1, n_{rf} = 1, 8$ -QAM), both having 3 bpcu per user. From Fig. 4, we can see that SM-MIMO outperforms massive MIMO by several dBs. For example, at a BER of 10^{-3} , SM-MIMO has a 2.5 to 3.5 dB SNR advantage over massive MIMO. In Fig. 5, we observe a performance advantage of about 3 to 4 dB in favor of SM-MIMO with $(n_t = 4, n_{rf} = 1, 4$ -QAM) compared to massive MIMO with $(m_t = 1, n_{rf} = 1, 16$ -QAM), both at 4 bpcu per user. This SNR advantage in favor of SM-MIMO can be explained as follows. Since SM-MIMO conveys information bits through antenna indices in addition to carrying bits on QAM symbols, SM-MIMO can use a smaller-sized QAM compared to that used in massive MIMO to achieve the same spectral efficiency, and a small-sized QAM is more power efficient than a larger one.

²In all the figures, massive MIMO is abbreviated as M-MIMO.



Fig. 4. BER performance of multiuser SM-MIMO ($n_t = 2$, $n_{rf} = 1$, 4-QAM) using MPD-SM algorithm and massive MIMO ($m_t = 1$, $n_{rf} = 1$, 8-QAM) with sphere decoding, at 3 bpcu per user, K = 16, N = 64, 128.



Fig. 5. BER performance of multiuser SM-MIMO ($n_t = 4$, $n_{rf} = 1$, 4-QAM) using MPD-SM algorithm and massive MIMO ($m_t = 1$, $n_{rf} = 1$, 16-QAM) with sphere decoding, at 4 bpcu per user, K = 16, N = 64, 128.

IV. LOCAL SEARCH DETECTION FOR SM-MIMO

In this section, we propose another algorithm for SM-MIMO detection. The algorithm is based on local search. The algorithm finds a local optimum (in terms of ML cost) as the solution through a local neighborhood search. We refer to this algorithm as LSD-SM (local search detection for spatial modulation) algorithm. A key to the LSD-SM algorithm is the definition of a neighborhood suited for SM. This is important since SM carries information bits in the antenna indices also.

Neighborhood definition: For a given vector $\mathbf{x} \in \mathbb{S}_{n_t,\mathbb{A}}^K$, we define the neighborhood $\mathcal{N}(\mathbf{x})$ to be the set of all vectors in $\mathbb{S}_{n_t,\mathbb{A}}^K$ that differ from the vector \mathbf{x} in either one spatial index position or in one modulation symbol. That is, a vector \mathbf{w} is

said to be a neighbor of \mathbf{x} if and only if $\mathbf{w}_k \in \{\mathbb{S}_{n_t,\mathbb{A}} \setminus \mathbf{x}_k\}$ for exactly one k, and $\mathbf{w}_k = \mathbf{x}_k$ for all other k, i.e., the neighborhood $\mathcal{N}(\mathbf{x})$ is given by

$$\mathcal{N}(\mathbf{x}) \triangleq \big\{ \mathbf{w} | \mathbf{w} \in \mathbb{S}_{n_t,\mathbb{A}}^K, \mathbf{w}_k \neq \mathbf{x}_k \text{ for exactly one } k \big\}, \quad (13)$$

where $\mathbf{x}_k, \mathbf{w}_k \in \mathbb{S}_{n_t,\mathbb{A}}$ and $k \in 1, 2, \cdots, K$. Thus the size of this neighborhood is given by $|\mathcal{N}(\mathbf{x})| = (|\mathbb{S}_{n_t,\mathbb{A}}| - 1)K$.

For example, consider K = 2, $n_t = 2$, and BPSK (i.e., $\mathbb{A} = \{\pm 1\}$). We then have

$$\mathbb{S}_{2,\mathrm{BPSK}} = \left\{ \begin{bmatrix} +1\\ 0 \end{bmatrix}, \begin{bmatrix} -1\\ 0 \end{bmatrix}, \begin{bmatrix} 0\\ +1 \end{bmatrix}, \begin{bmatrix} 0\\ -1 \end{bmatrix} \right\},\$$

and

$$\mathcal{N}\left(\begin{bmatrix} +1\\ 0\\ 0\\ -1 \end{bmatrix} \right) = \left\{ \begin{bmatrix} -1\\ 0\\ 0\\ -1 \end{bmatrix}, \begin{bmatrix} 0\\ +1\\ 0\\ -1 \end{bmatrix}, \begin{bmatrix} 0\\ -1\\ 0\\ -1 \end{bmatrix}, \begin{bmatrix} +1\\ 0\\ 0\\ +1 \end{bmatrix}, \begin{bmatrix} +1\\ 0\\ 0\\ +1 \end{bmatrix}, \begin{bmatrix} +1\\ 0\\ -1\\ 0 \end{bmatrix}, \begin{bmatrix} +1\\ 0\\ +1\\ 0 \end{bmatrix} \right\}$$

LSD-SM algorithm: The LSD-SM algorithm for SM-MIMO detection starts with an initial solution vector $\hat{\mathbf{x}}^{(0)}$ as the current solution. For example, $\hat{\mathbf{x}}^{(0)}$ can be the MMSE solution vector $\hat{\mathbf{x}}_{\text{MMSE}}$. Using the neighborhood definition in (13), it considers all the neighbors of $\hat{\mathbf{x}}^{(0)}$ and searches for the best neighbor with least ML cost which also has a lesser ML cost than the current solution. If such a neighbor is found, then it declares this neighbor as the current solution. This completes one iteration of the algorithm. This process is repeated for multiple iterations till a local minimum is reached (i.e., no neighbor better than the current solution is found). The vector corresponding to the local minimum is declared as the final output vector $\hat{\mathbf{x}}$. The non-zero entry in the *k*th user's subvector in $\hat{\mathbf{x}}$ and its index are then demapped to obtain the information bits of the *k*th user.

Multiple restarts: The performance of the basic LSD-SM algorithm in the above can be further improved by using multiple restarts, where the LSD-SM algorithm is run several times, each time starting with a different initial solution and declaring the best solution among the multiple runs. The proposed LSD-SM algorithm with multiple restarts is listed in Algorithm 2.

Complexity: The LSD-SM algorithm complexity consists of two parts. The first part involves the computation of the initial solution. The complexity for computing the MMSE initial solution is $\mathcal{O}(Kn_tN^2)$. The second part involves the search complexity, where, in order to compute the ML cost, we require to compute (*i*) $\mathbf{H}^H \mathbf{H}$ which has $\mathcal{O}(K^2n_t^2N)$ complexity, and (*ii*) $\mathbf{H}^H \mathbf{y}$ which has $\mathcal{O}(Kn_tN)$ complexity. In addition, the complexity per iteration and the number of iterations to reach the local minima contribute to the search complexity, where the search complexity per iteration is $\mathcal{O}(K|\mathbb{S}_{n_t,\mathbb{A}}|)$.

Reducing the search complexity: From the above discussion on the complexity of the LSD-SM algorithm, we saw that the computation of the ML cost requires a complexity of order $O(K^2 n_t^2 N)$ which is greater than the MMSE complexity of $O(K n_t N^2)$ for systems with $K n_t > N$, i.e., with loading

1: Input : \mathbf{y} , \mathbf{H} , r: no. of restarts 2: for j = 1 to r do compute $\mathbf{c}^{(j)}$ (initial vector at *i*th restart) 3: find $\mathcal{N}(\mathbf{c}^{(j)})$ 4: $\mathbf{z}^{(j)} = \operatorname{argmin} \|\mathbf{y} - \mathbf{H}\mathbf{q}\|^2$ 5: $\mathbf{q} \in \mathcal{N}(\mathbf{c}^{(j)})$ if $\|\mathbf{y} - \mathbf{H}\mathbf{z}^{(j)}\|^2 < \|\mathbf{y} - \mathbf{H}\mathbf{c}^{(j)}\|^2$ then 6: $\mathbf{c}^{(j)} = \mathbf{z}^{(j)}$ 7: goto step 4 8: else 9: $\hat{\mathbf{x}}^{(j)} = \mathbf{c}^{(j)}$ 10: 11: end if 12: end for $i = \underset{1 \le j \le r}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}^{(j)}\|^2$ 13: 14: **Output** : $\hat{\mathbf{x}} = \hat{\mathbf{x}}^{(i)}$

Algorithm 2: Listing of the proposed LSD-SM algorithm with multiple restarts.

factor $\alpha > 1/n_t$. We propose to reduce the search complexity by the following method, which consists of the following three parts:

- 1) The channel gain matrix **H** can be written as $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_{Kn_t}]$, where \mathbf{h}_i is the *i*th column of **H**, which is a $N \times 1$ column vector. Before we start the search process in the LSD-SM algorithm, compute the set of vectors $\mathbb{J} \triangleq \{\mathbf{h}_i s\}_{\forall s \in \mathbb{A}, \forall i \in 1, 2, \cdots, Kn_t}$. The complexity of this computation is $O(|\mathbb{A}|Kn_tN)$.
- 2) Compute the vector $\mathbf{z}^{(0)}$, which is defined as

$$\mathbf{z}^{(0)} \triangleq \mathbf{y} - \mathbf{H}\hat{\mathbf{x}}^{(0)} = \mathbf{y} - \sum_{k=1}^{K} \hat{x}_{l_k}^{(0)} \mathbf{h}_{(k-1)n_t+j_k},$$
 (14)

where the terms $\hat{x}_{l_k}^{(0)} \mathbf{h}_{(k-1)n_t+j_k}$ belong to \mathbb{J} which is precomputed. The computation of $\mathbf{z}^{(0)}$ requires a complexity of O(KN).

Because of the way the neighborhood is defined, every neighbor of z⁽⁰⁾ can be computed from z⁽⁰⁾ by exactly adding a single vector from J and subtracting another vector from J. Thus the complexity of computing the ML cost of every neighbor is O(N).

In this method, the total number of operations performed for the search is $|\mathbb{A}|Kn_tN+KN+(2N-1)+K(|\mathbb{A}|n_t-1)(4N-1)T$, where T is the number of iterations performed to reach the local minima which depends on the transmit vector and the operating SNR (T is determined through simulations). Therefore, the total complexity of the algorithm in this method is given by $O(|\mathbb{A}|Kn_tNT)$, whereas, the total complexity without search complexity reduction is $O(K^2n_t^2N)$.

Performance: We evaluated the performance of multiuser SM-MIMO using the proposed LSD-SM algorithm and compared it with that of massive MIMO using ML detection for the same spectral efficiency. Figure 6 shows the performance comparison between SM-MIMO with $(n_t = 4, n_{rf} = 1, n_{rf})$



Fig. 6. BER performance of multiuser SM-MIMO ($n_t = 4$, $n_{rf} = 1$, 4-QAM) using LSD-SM and MPD-SM algorithms, and massive MIMO ($m_t = 1$, $n_{rf} = 1$, 16-QAM) using sphere decoding, at 4 bpcu per user, K = 16, N = 64, 128.

4-QAM) and massive MIMO with ($m_t = 1$, $n_{rf} = 1$, 16-QAM), both having 4 bpcu per user. For SM-MIMO, detection performance of both LSD-SM (presented in this section) and MPD-SM (presented in the previous section) are shown. In LSD-SM, the number of restarts used is r = 2. The initial vectors used in the first and second restarts are MMSE solution vector and random vector, respectively. For massive MIMO, ML detection performance using sphere decoder is plotted. It can be seen that SM-MIMO using LSD-SM and MPD-SM algorithms outperform massive MIMO using sphere decoding. Specifically, SM-MIMO using LSD-SM performs better than massive MIMO by about 5 dB at 10^{-3} BER. Also, comparing the performance of LSD-SM and MPD-SM algorithms in SM-MIMO, we see that LSD-SM performs better than MPD-SM by about 1 dB at 10^{-3} BER.

MPD initialized LSD-SM detection: The LSD-SM algorithm proposed in this section offers good performance but has higher complexity because of the need to compute the initial MMSE solution vector. The high complexity of MMSE is due to the matrix inversion. We can overcome this need for MMSE computation by using a hybrid detection scheme. In the hybrid detection scheme, we first run the MPD-SM algorithm (proposed in the previous section) and the output of the MPD-SM algorithm is fed as the initial solution vector to the LSD-SM algorithm (proposed in this section). We refer to this hybrid scheme as the 'MPD-LSD-SM' scheme. The MPD-LSD-SM scheme does not need the MMSE solution and hence avoids the associated matrix inversion.

Performance as a function of loading factor: In Fig. 7, we compare the performance of SM-MIMO (with $n_t = 4$, $n_{rf} = 1$, 4-QAM) and massive MIMO (with $m_t = 1$, $n_{rf} = 1$, 16-QAM), both at 4 bpcu per user, as a function of system loading factor α , at an average SNR of 9 dB. For SM-



Fig. 7. BER performance of SM-MIMO ($n_t = 4$, $n_{rf} = 1$, 4-QAM) and massive MIMO ($m_t = 1$, $n_{rf} = 1$, 16-QAM) as a function of system loading factor, α . N = 128, SNR = 9 dB, and 4 bpcu per user.

MIMO, the detectors considered are MMSE, MPD-SM, LSD-SM, and MPD-LSD-SM. The detectors considered for massive MIMO are MMSE detector and MMSE-LAS detector in [1], [2] with 2 restarts. From Fig. 7, we observe that SM-MIMO performs significantly better than massive MIMO at low-tomoderate loading factors. For the same SM-MIMO system settings, we show the complexity plots for various SM-MIMO detectors at different loading factors in Fig. 8. It can be seen that the proposed MPD-SM detector has less complexity than MMSE detector; yet, MPD-SM detector outperforms MMSE detector (as can be seen in Fig. 7). The proposed LSD-SM detector performs better than the MPD-SM detector with some additional computational complexity (as can be seen in Fig. 8). Among the considered detection schemes, the MPD-LSD-SM detection scheme gives the best performance with near-MMSE complexity.

Performance for same spectral efficiency and QAM size: We note that if both spectral efficiency and QAM size are to be kept same in SM-MIMO and massive MIMO, then the number of spatial streams per user in massive MIMO has to increase. For example, SM-MIMO can achieve 4 bpcu per user with 4-QAM using $n_t = 4$ and $n_{rf} = 1$. Massive MIMO can achieve the same spectral efficiency of 4 bpcu per user using one spatial stream (i.e., $m_t = 1$, $n_{rf} = 1$) with 16-QAM. But to achieve the same spectral efficiency using 4-QAM in massive MIMO, we have to use $m_t = 2$, $n_{rf} = 2$, i.e., two spatial streams per user with 4-QAM on each stream are needed. This increase in number of spatial streams per user increases the spatial interference.

The effect of increase in number of spatial streams per user in massive MIMO for the same spectral efficiency on the performance is illustrated in Fig. 9 for K = 16 and N = 128. In Fig. 9, we compare the performance of the following four systems with the same spectral efficiency of 4 bpcu per user:



Fig. 8. Complexity comparison between MMSE, MPD-SM, LSD-SM and MPD-LSD-SM detection algorithms in multiuser SM-MIMO as a function of system loading factor, α . N = 128, $n_t = 4$, $n_{rf} = 1$, 4-QAM, 4 bpcu per user and SNR = 9 dB.



Fig. 9. BER performance of SM-MIMO with $(n_t = 4, n_{rf} = 1, 4$ -QAM), massive MIMO with $(m_t = 1, n_{rf} = 1, 16$ -QAM), massive MIMO with $(m_t = 2, n_{rf} = 2, 4$ -QAM), and massive MIMO with $(m_t = 4, n_{rf} = 4, BPSK)$ for K = 16, N = 128, 4 bpcu per user.

1) SM-MIMO with $(n_t = 4, n_{rf} = 1, 4\text{-QAM})$, 2) massive MIMO with $(m_t = 1, n_{rf} = 1, 16\text{-QAM})$, 3) massive MIMO with $(m_t = 2, n_{rf} = 2, 4\text{-QAM})$, and 4) massive MIMO with $(m_t = 4, n_{rf} = 4, \text{BPSK})$. It can be seen that among the four systems considered in Fig. 9, SM-MIMO performs the best. This is because massive MIMO loses performance because of higher-order QAM or increased spatial interference from increased number of spatial streams per user.

V. SM-MIMO IN FREQUENCY SELECTIVE FADING

In this section, we assume the multiuser SM-MIMO system model described in Section II, except for the channel model which is now assumed to be frequency selective. Let L denote the number of multipath components between each pair of transmit antenna at the user and receive antenna at the BS. Let $H_{i,(k-1)n_t+j}^{(l)}$ represent the channel gain from the *j*th transmit antenna of the *k*th user to the *i*th BS receive antenna on the *l*th multipath component. The channel gains for the *l*th multipath component are modeled as Gaussian r.v. with zero mean and variance Ω_l . The power-delay profile of the channel is given by

$$\Omega_l = \mathbb{E}[|H_{i,(k-1)n_t+j}^{(l)}|^2] = \Omega_0 10^{-\beta l/10}, \quad l = 0, \cdots, L-1,$$
(15)

where β denotes the decay-rate of the average power in each of the multipath components in dB. The total power gain of the channel is assumed to be unity, i.e., $\sum_{l=0}^{L-1} \Omega_l = 1$. Further, we use cyclic prefixed single carrier (CPSC) transmission at each user terminal [11], [12],[13].

CPSC transmission: The CPSC transmission scheme has the advantage of low peak-to-average power ratio (PAPR) [11], [12]. Transmission is carried out in data blocks. Each data block consists of Q+L-1 channel uses, where Q information symbol vectors, each of length Kn_t , prefixed by (L-1)-length cyclic prefix from each user is transmitted.

Signal detection: Each data block (DB) consists of a CP followed by data symbols. Let $\mathbf{x}_k^{(t)} \in \mathbb{S}_{n_t,\mathbb{A}}$ denote the transmit vector from the *k*th user in the *t*th channel use in a DB. The *k*th user's transmit vector in a DB is of the form

$$[\underbrace{\mathbf{x}_{k}^{(Q-L)T} \mathbf{x}_{k}^{(Q-L+1)T} \cdots \mathbf{x}_{k}^{(Q-1)T}}_{CP} \underbrace{\mathbf{x}_{k}^{(0)T} \mathbf{x}_{k}^{(1)T} \cdots \mathbf{x}_{k}^{(Q-1)T}}_{Data}]^{T}$$

Assuming perfect synchronization and discarding the CP at the BS receiver, the received signal vector can be written as

$$\mathbf{y}' = \mathbf{H}'\mathbf{x}' + \mathbf{n}',\tag{16}$$

where \mathbf{y}' is $[\mathbf{y}^{(0)T} \mathbf{y}^{(1)T} \cdots \mathbf{y}^{(Q-1)T}]^T \in \mathbb{C}^{NQ \times 1}, \mathbf{y}^{(t)} \in \mathbb{C}^{N \times 1}$ denotes the received vector at the *t*th channel use in a DB, \mathbf{x}' is $[\mathbf{x}^{(0)T} \mathbf{x}^{(1)T}, \cdots \mathbf{x}^{(Q-1)T}]^T \in \mathbb{C}^{Kn_tQ \times 1}, \mathbf{x}^{(t)} \in \mathbb{C}^{Kn_t \times 1}$ is the vector comprising of transmit vectors of all the users in the *t*th channel use in a DB, \mathbf{H}' is the channel gain matrix of dimension $NQ \times Kn_tQ$, and \mathbf{n}' is the additive white Gaussian noise vector given by $[\mathbf{n}^{(0)T} \mathbf{n}^{(1)T} \cdots \mathbf{n}^{(Q-1)T}]^T \in \mathbb{C}^{NQ \times 1}$. The received signal vector in the *t*th channel use in a DB can be written as

$$\mathbf{y}^{(t)} = \sum_{l=0}^{L-1} \mathbf{H}^{(l)} \mathbf{x}^{(t-l)} + \mathbf{n}^{(t)}, \quad t = 0, 1, \cdots, Q-1, \quad (17)$$

where $\mathbf{H}^{(l)} \in \mathbb{C}^{N \times Kn_t}$ is the channel gain matrix corresponding to the *l*th multipath component such that $H_{i,(k-1)n_t+j}^{(l)}$ represents the channel gain from the *j*th transmit antenna of the *k*th user to the *i*th BS receive antenna in the *l*th multipath. For this system, the ML detection rule is given by

$$\hat{\mathbf{x}}' = \underset{\mathbf{x}' \in \mathbb{S}_{n_{\star},\mathbb{A}}^{KQ}}{\operatorname{argmin}} \|\mathbf{y}' - \mathbf{H}'\mathbf{x}'\|^2, \tag{18}$$

whose exact computation requires exponential complexity in KQ. We shall formulate the system model in (16) into an equivalent system model in the frequency domain, and employ

the MPD-SM and LSD-SM algorithms for signal detection in the resulting equivalent system model.

It is noted that because of the addition of CP, the matrix \mathbf{H}' is a block circulant matrix. Therefore, \mathbf{H}' can be transformed into a block diagonal matrix \mathbf{D} as

$$\mathbf{D} = (\mathbf{F} \otimes \mathbf{I}_N) \mathbf{H}' (\mathbf{F}^H \otimes \mathbf{I}_{Kn_t}), \tag{19}$$

where I_n denotes $n \times n$ identity matrix, and F is the $Q \times Q$ DFT matrix, given by

$$\mathbf{F} = \frac{1}{\sqrt{Q}} \begin{bmatrix} \rho_{0,0} & \rho_{0,1} & \cdots & \rho_{0,Q-1} \\ \rho_{1,0} & \rho_{1,1} & \cdots & \rho_{1,Q-1} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{Q-1,0} & \rho_{Q-1,1} & \cdots & \rho_{Q-1,Q-1} \end{bmatrix}$$

where $\rho_{u,v} = \exp(-j\frac{2\pi uv}{Q})$. **D** is a block diagonal matrix of the form

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{D}_{Q-1} \end{bmatrix},$$
(20)

where \mathbf{D}_t is of dimension $N \times Kn_t$. The $(i, (k-1)n_t + j)$ th element of \mathbf{D}_t is the *t*th element of the DFT of the vector $[H_{i,(k-1)n_t+j}^{(0)} H_{i,(k-1)n_t+j}^{(1)} \cdots H_{i,(k-1)n_t+j}^{(L-1)} 0 \cdots 0]^T$.

Performing DFT operation on the received vector \mathbf{y}' at the receiver, we get

$$\mathbf{z}' = (\mathbf{F} \otimes \mathbf{I}_N)\mathbf{y}' = (\mathbf{F} \otimes \mathbf{I}_N)\mathbf{H}'\mathbf{x}' + \mathbf{w}', \qquad (21)$$

where $\mathbf{w}' = (\mathbf{F} \otimes \mathbf{I}_N)\mathbf{n}'$. Further, \mathbf{z}' can be written as

$$\mathbf{z}' = \mathbf{D}(\mathbf{F} \otimes \mathbf{I}_{Kn_t})\mathbf{x}' + \mathbf{w}'$$
$$= \mathbf{H}\mathbf{x}' + \mathbf{w}', \qquad (22)$$

where $\mathbf{\tilde{H}} = \mathbf{D}(\mathbf{F} \otimes \mathbf{I}_{Kn_t})$ is the equivalent channel matrix. Now, detection can be performed on the system model in (22) using the MPD-SM and LSD-SM algorithms described in Sections III and IV.

Performance: We evaluated the performance of multiuser SM-MIMO CPSC system using the proposed detection algorithms in frequency selective channel with $L = 3, Q = 6, \beta = 3$ dB, and N = 128.

Figure 10 shows the performance comparison between SM-MIMO with ($n_t = 4$, $n_{rf} = 1$, 4-QAM) and massive MIMO with ($m_t = 1$, $n_{rf} = 1$, 16-QAM), both having 4 bpcu per user and K = 16. From Fig. 10, we can see that SM-MIMO (with LSD-SM and MPD-LSD-SM detector) outperforms massive MIMO (with MMSE-LAS detector) by about 5.5 dB at an uncoded BER of 10^{-4} . We have also plotted the SM-MIMO and massive MIMO system performance with MMSE detector for comparison. The LSD-SM and MPD-LSD-SM detectors outperform the MMSE detector by about 2.5 dB at an uncoded BER of 10^{-4} .

In Fig. 11, we show the performance of SM-MIMO CPSC with $(n_t = 4, n_{rf} = 1, 4$ -QAM) and massive MIMO CPSC with $(m_t = 1, n_{rf} = 1, 16$ -QAM), both at 4 bpcu per user, as a function of system loading factor α , at an average SNR of 9



Fig. 10. BER performance of SM-MIMO with $(n_t = 4, n_{rf} = 1, 4\text{-QAM})$ and massive MIMO with $(m_t = 1, n_{rf} = 1, 16\text{-QAM})$ for K = 16, N = 128, 4 bpcu per user in frequency selective fading using CPSC transmission for $(Q = 6, L = 3, \beta = 3 \text{ dB})$.



Fig. 11. BER performance of SM-MIMO with $(n_t = 4, n_{rf} = 1, 4\text{-QAM})$ and massive MIMO with $(m_t = 1, n_{rf} = 1, 16\text{-QAM})$ for N = 128, 4 bpcu per user in frequency selective fading using CPSC transmission for $(Q = 6, L = 3, \beta = 3 \text{ dB})$ as function of system loading factor, α .

dB, for the frequency selective channel. It can be observed that the SM-MIMO system outperforms massive MIMO at low-tomoderate loading factors. The proposed LSD-SM and MPD-LSD-SM detectors achieve better performance compared to the MMSE detector for all different loading factors.

VI. CONCLUSIONS

We proposed low complexity detection algorithms for largescale SM-MIMO systems. These algorithms, based on message passing and local search, scaled well in complexity and achieved very good performance. An interesting observation from the simulation results is that SM-MIMO outperforms massive MIMO by several dBs for the same spectral efficiency. Similar performance gains were shown in favor of SM-MIMO on frequency selective fading. The SNR advantage of SM-MIMO over massive MIMO is attributed to the following reasons: (*i*) because of the spatial index bits, SM-MIMO can use a lower-order QAM alphabet compared to that in massive MIMO to achieve the same spectral efficiency, and (*ii*) for the same spectral efficiency and QAM size, massive MIMO will need more spatial streams per user which leads to increased spatial interference. With such performance advantage at low RF hardware complexity, large-scale multiuser SM-MIMO is an attractive technology for future wireless systems.

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