Large-MIMO: A Technology Whose Time Has Come

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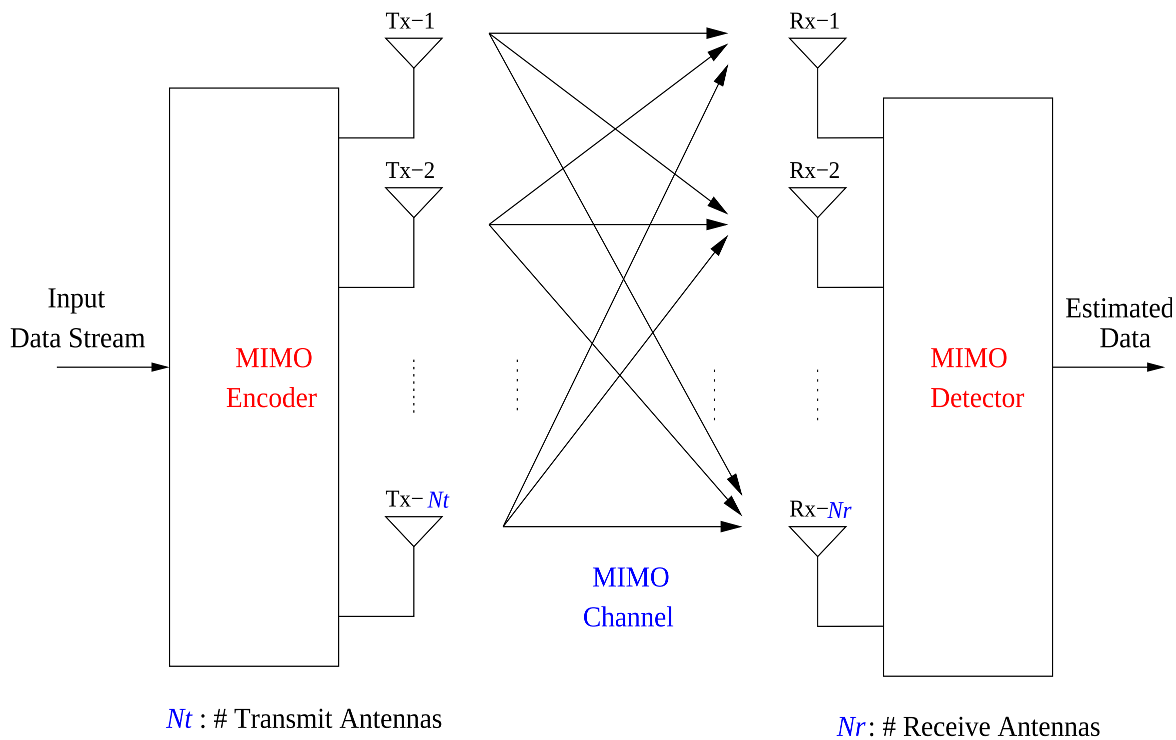
MIMO System



- Transmit Side Receive Side
- (e.g., Base Station, Access Point, Set top box) (e.g., Set top box, Laptop, HDTV)

Why Multiple Antennas?

N_t: No. of transmit antennas, N_r: No. of receive antennas

# Antennas	Error Probability (P_e)	Capacity (C), bps/Hz
$N_t = N_r = 1$ (SISO)	$P_e \propto SNR^{-1}$	$C = \log(SNR)$
$N_t = 1, N_r > 1$ (SIMO)	$P_e \propto SNR^{-N_r}$	$C = \log(SNR)$
$N_t > 1, N_r > 1$ (MIMO)	$P_e \propto SNR^{-N_t N_r}$	$C = \min(N_t, N_r) \log(SNR)$
	$N_t N_r$: Diversity Gain	$\min(N_t, N_r)$: Spatial Mux Gain

- Large N_t, N_r $\longrightarrow$ increased spectral efficiency

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[1] I. E. Telatar, Capacity of multi-antenna Gaussian channels, *European Trans. Telecommun.*, vol. 10, no. 6, pp. 585-595, November 1999.

[2] G. J. Foschini and M. J. Gans, On limits of wireless communications in a fading environment when using multiple antennas, *Wireless Pers. Commun.*, vol. 6, pp. 311-335, March 1998.

Large-MIMO Approach

- Employ large number (several tens) of antennas at the Tx and Rx
- Achieve high spectral efficiencies (tens to hundreds of bps/Hz)
  - Data rate (bps) = Spectral efficiency (bps/Hz) $\times$ Bandwidth (Hz)
  - e.g., 100 bps/Hz $\Longrightarrow$ 1 Gbps rate in just 10 MHz bandwidth
- Limitation in current MIMO standards
  - spectral efficiency: ~ 10 bps/Hz only
  - 2 to 4 transmit antennas
  - e.g., 750 Mbps in 80 MHz in 802.11n using 4 Tx antennas
  - do not exploit the potential of large spatial dimensions

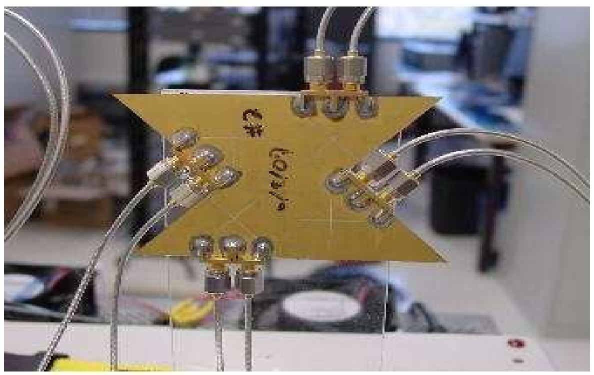
Technological Challenges in Realizing Large-MIMO

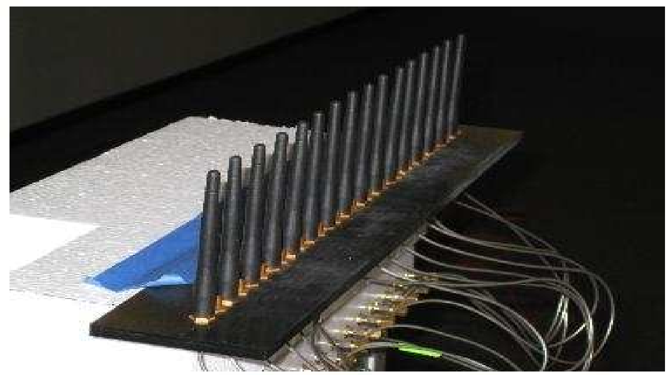
- Placement of large number of antennas in communication terminals
  - Feasible in moderately sized communication terminals (e.g., Set top boxes, Laptops, BS towers)
  - use high carrier frequencies for small carrier wavelengths (e.g., 5 GHz, 60 GHz)
- RF technologies
  - Multiple IF/RF transmit and receive chains
- Large-MIMO detection
  - Need low-complexity detectors
- Channel estimation
  - Estimation of large number of channel coefficients

16-Antenna Channel Sounding for IEEE 802.11ac (5 GHz)

Jan 19, 2009 doc.:IEEE 802.11-09/0088r0

Example Antenna Configurations Used





8 Slot Antenna Array

Two V-H and two ±45° pairs
λ/2 separation between slot pairs

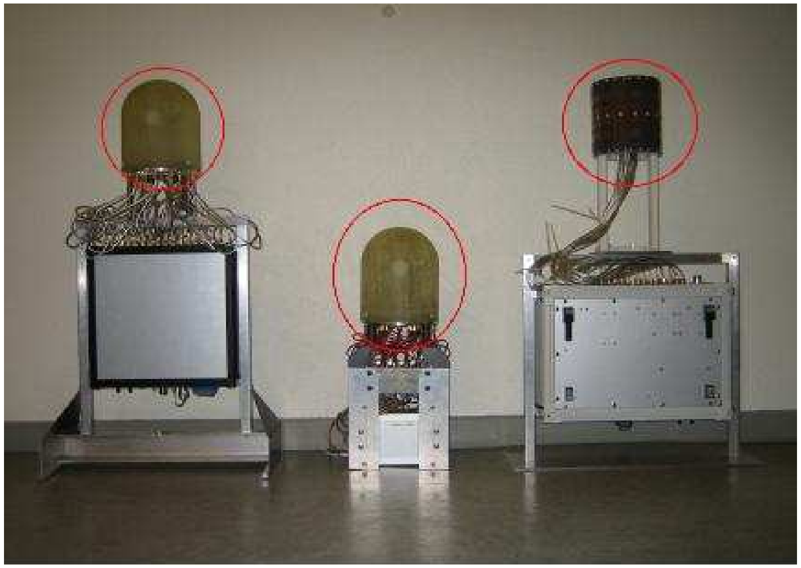
16 Linear Dipole Antenna Array

λ/2 separation between elements

Submission Slide 1 Names

[3] Gregory Breit *et al*, 802.11ac Channel Modeling, doc. IEEE 802.11-09/0088r0, submission to Task Group TGac, 19 January 2009.

64×64 MIMO Indoor Channel Sounding (5 GHz)



(a) 64-Antenna/RF hardware at 5 GHz

(b) LOS setup

[4] Jukka Koivunen, "Characterisation of MIMO propagation channel in multi-link scenarios," MS Thesis, Helsinki University of Technology, December 2007.

Some Recent Wireless Products

- Can see the trend in packaging increasing number of antennas/RF chains in wireless products

Source: Internet

Detection Complexity: A Key Challenge in Large-MIMO

- Optimal detection has exponential complexity in # transmit antennas
- Need low-complexity algorithms that are near-optimal
- A possible approach to low-complexity solutions
  - Seek algorithms from machine learning (ML)
  - Large-dimension problems are routinely addressed in other areas (e.g., computer vision, web search) using ML algorithms
  - Communications area too has benefited from ML algorithms
    - ∗ MUD in CDMA (large # users), Turbo/LDPC decoding (large frame sizes)

Linear Vector Channels

- Several communication systems can be characterized by the following linear vector channel model

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c$$

$\mathbf{x}_c \in \mathbb{C}^{d_t}$, $\mathbf{H}_c \in \mathbb{C}^{d_r \times d_t}$, $\mathbf{y}_c \in \mathbb{C}^{d_r}$, $\mathbf{n}_c \in \mathbb{C}^{d_r}$

- Examples
  - MIMO
    - $d_t = N_t$, # Tx antennas; $d_r = N_r$, # Rx antennas; $\mathbf{x}_c$: Tx symbol vector
    - $\mathbf{H}_c$: Channel gain matrix; $\mathbf{y}_c$: Rx signal vector; $\mathbf{n}_c$: Noise vector
  - Coding
    - $d_t = k$, # Information bits; $d_r = n$, # Coded bits; $\mathbf{x}_c$: Information bit vector
    - $\mathbf{H}_c$: Generator matrix; $\mathbf{y}_c$: Rx signal vector; $\mathbf{n}_c$: Noise vector
  - CDMA
    - $d_t = d_r = K$, # users; $\mathbf{x}_c$: Tx. bit vector; $\mathbf{H}_c$: Cross correlation matrix,
    - $\mathbf{y}_c$: Rx signal vector; $\mathbf{n}_c$: Noise vector

Optimum Detection

- Problem
  - Obtain an estimate of $\mathbf{x}_c$, given $\mathbf{y}_c$ and $\mathbf{H}_c$
- Maximum likelihood (ML) solution

$$\mathbf{x}_{ML} = \underset{\mathbf{x}_c \in \mathbb{A}^{d_t}}{\text{arg min}} \underbrace{\|\mathbf{y}_c - \mathbf{H}_c\mathbf{x}_c\|^2}_{\triangleq \phi(\mathbf{x}_c)} \quad (1)$$

$\mathbb{A}$: signaling alphabet; $\phi(\mathbf{x}_c)$: ML cost

- ML cost: $\phi(\mathbf{x}_c) = \mathbf{x}_c^H\mathbf{H}_c^H\mathbf{H}_c\mathbf{x}_c - 2\Re\left(\mathbf{y}_c^H\mathbf{H}_c\mathbf{x}_c\right)$

Optimum Detection

- Let $\mathbb{A}$ be M-PAM or M-QAM (Two PAMs in quadrature)
  - M-PAM symbols take values from $\{A_m, m = 1, \cdots, M\}$, $A_m = (2m - 1 - M)$
  - $\mathbf{y}_c = \mathbf{y}_I + j\mathbf{y}_Q$, $\mathbf{x}_c = \mathbf{x}_I + j\mathbf{x}_Q$, $\mathbf{n}_c = \mathbf{n}_I + j\mathbf{n}_Q$, $\mathbf{H}_c = \mathbf{H}_I + j\mathbf{H}_Q$
- Convert (1) into a real-valued system model

$$\mathbf{y} \;=\; \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

$\mathbf{H} \in \mathbb{R}^{2d_r \times 2d_t}$, $\mathbf{y} \in \mathbb{R}^{2d_r}$, $\mathbf{x} \in \mathbb{R}^{2d_t}$, $\mathbf{n} \in \mathbb{R}^{2d_r}$

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_I & -\mathbf{H}_Q \\ \mathbf{H}_Q & \mathbf{H}_I \end{pmatrix}, \quad \mathbf{y} = [\mathbf{y}_I^T \;\; \mathbf{y}_Q^T]^T, \quad \mathbf{x} = [\mathbf{x}_I^T \;\; \mathbf{x}_Q^T]^T, \quad \mathbf{n} = [\mathbf{n}_I^T \;\; \mathbf{n}_Q^T]^T.$$

- ML solution

$$\mathbf{x}_{ML} \;=\; \underset{\mathbf{x} \in \mathbb{S}}{\text{arg min}} \; \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \;\; \mathbf{x}^T\mathbf{H}^T\mathbf{H}\mathbf{x} - 2\mathbf{y}^T\mathbf{H}\mathbf{x}, \tag{3}$$

$\mathbb{S}$: $2d_t$-dimensional signal space (Cartesian product of $\mathbb{A}_1$ to $\mathbb{A}_{2d_t}$; $\mathbb{A}_i$: M-PAM signal set from which x_i takes values, $i = 1, \cdots, 2d_t$). ML Complexity:Exponential in d_t

Optimum Detection

- Maximum a posteriori (MAP) solution
  - Consider square M-QAM
  - Each entry of $\mathbf{x}$ belongs to a $\sqrt{M}$-PAM constellation
  - Let $b_i^{(0)}, b_i^{(1)}, \cdots, b_i^{(q-1)}$ denote the $q = \log_2(\sqrt{M})$ constituent bits of x_i
  - x_i can be written as

$$x_i = \sum_{j=0}^{q-1} 2^j \, b_i^{(j)}, \qquad i = 0, 1, \cdots, 2d_t - 1$$

  - Let the bit vector $\mathbf{b} \in \{\pm 1\}^{2qd_t}$ be written as

$$\mathbf{b} \triangleq \left[b_0^{(0)} \cdots b_0^{(q-1)} b_1^{(0)} \cdots b_1^{(q-1)} \cdots b_{2d_t-1}^{(0)} \cdots b_{2d_t-1}^{(q-1)} \right]^T$$

  - Defining $\mathbf{c} \triangleq [2^0 \, 2^1 \cdots 2^{q-1}]$, $\mathbf{x}$ can be written as

$$\mathbf{x} = (\mathbf{I}_{2d_t} \otimes \mathbf{c})\mathbf{b}$$

Optimum Detection

- Rx signal model can be written as

$$\mathbf{y} = \underbrace{\mathbf{H}(\mathbf{I}_{2d_t} \otimes \mathbf{c})}_{\triangleq \mathbf{H}' \in \mathbb{R}^{2d_r \times 2qd_t}}\mathbf{b} + \mathbf{n}$$

- MAP estimate of $b_i^{(j)}$, $\quad i = 0, \cdots, 2d_t - 1, \quad j = 0, \cdots, q-1$ is

$$\widehat{b}_i^{(j)} = \underset{a \in \{\pm 1\}}{\text{arg max}} \; p\big(b_i^{(j)} = a \,|\, \mathbf{y}, \mathbf{H}'\big)$$

- Complexity: Exponential in qd_t

Sub-optimum Solutions

- Matched filter (MF)

$$\mathbf{x}_{MF} = \mathbf{H}^T\mathbf{y}$$

- Zero-forcing (ZF) solution

$$\mathbf{x}_{ZF} = \mathbf{H}^{-1}\mathbf{y}$$

- Minimum mean square error (MMSE) solution

$$\mathbf{x}_{MMSE} = (\mathbf{H} + \sigma^2\mathbf{I})^{-1}\mathbf{y}$$

- These suboptimum solution vectors can be used as initial vectors in search algorithms to improve performance further

Near-Optimal Algorithms for Large d_t

- Near-ML algorithms
  - Local neighborhood search based
  - Likelihood ascent search (LAS)
  - Reactive tabu search (RTS)
- Near-MAP algorithms
  - Message passing based
  - Belief propagation (BP)
  - Probabilistic association (PDA)

LAS Algorithm

- Search for good solution vectors in the local neighborhood
- Neighborhood definition
  - Neighbors that differ in one coordinate
    - e.g., Consider $\mathbb{A} = \{\pm 1\}$; $\quad \mathbf{x} = [-1, 1, 1, -1]$
    - 1-bit away neighbors of $\mathbf{x}$:

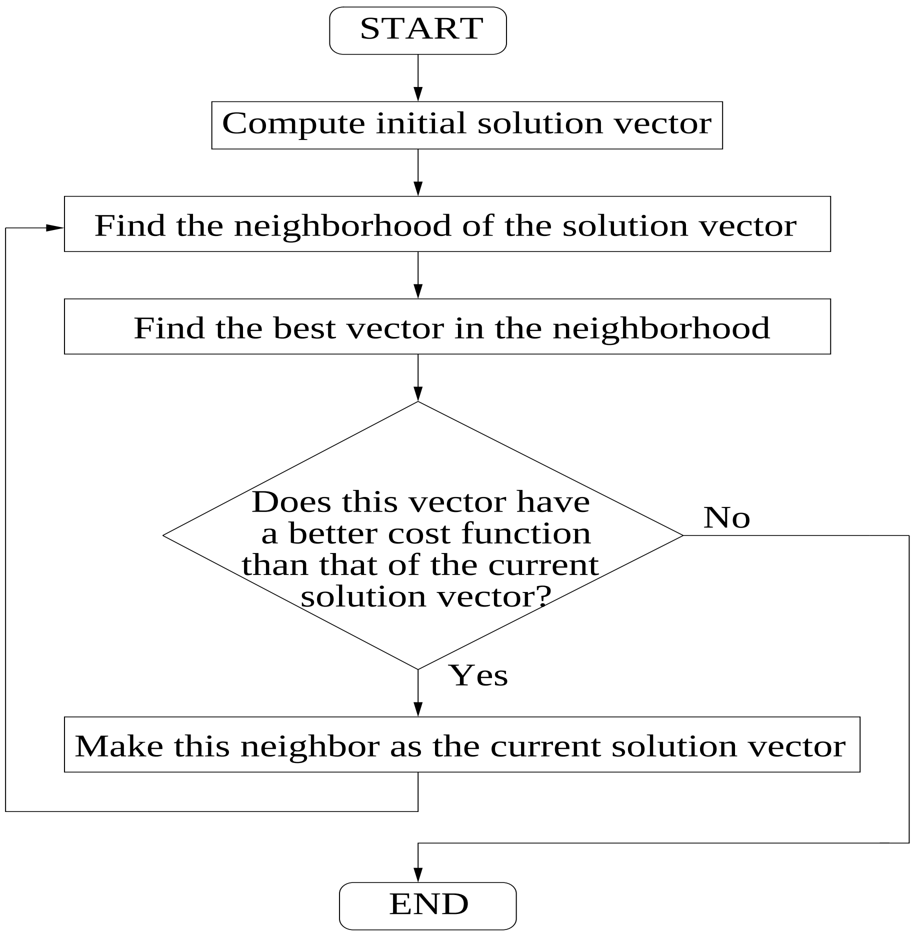
$$\mathcal{N}_1(\mathbf{x}) = \Big\{[-1, 1, 1, 1], [-1, 1, -1, -1], [-1, -1, 1, -1], [1, 1, 1, -1]\Big\}$$

  - Neighbors that differ in two coordinates
  - 2-bit away neighbors of $\mathbf{x}$:

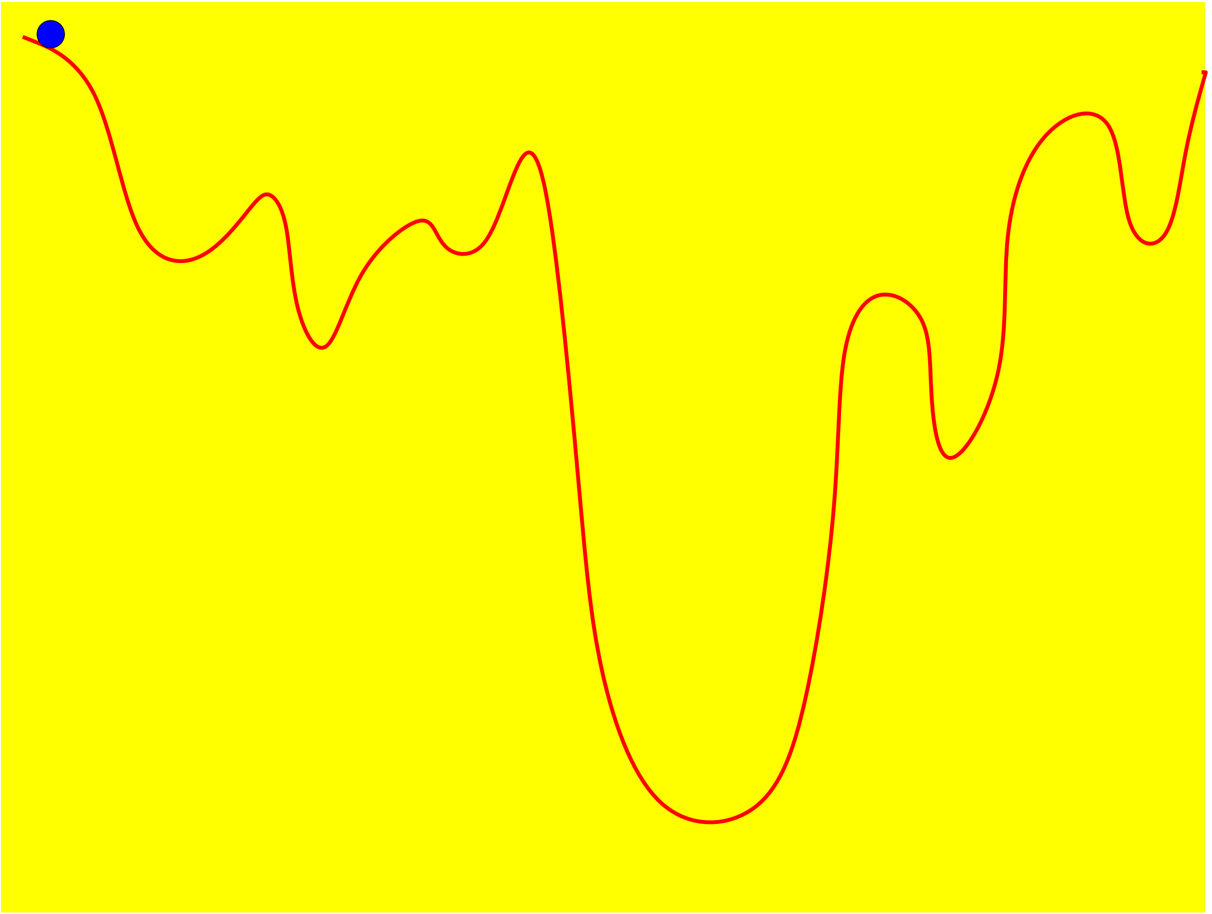
$$\mathcal{N}_2(\mathbf{x}) = \Big\{[-1, 1, -1, 1], [-1, -1, -1, -1], [1, -1, 1, -1], [1, 1, 1, 1], [1, 1, 1, -1], [1, -1, 1, 1]\Big\}$$

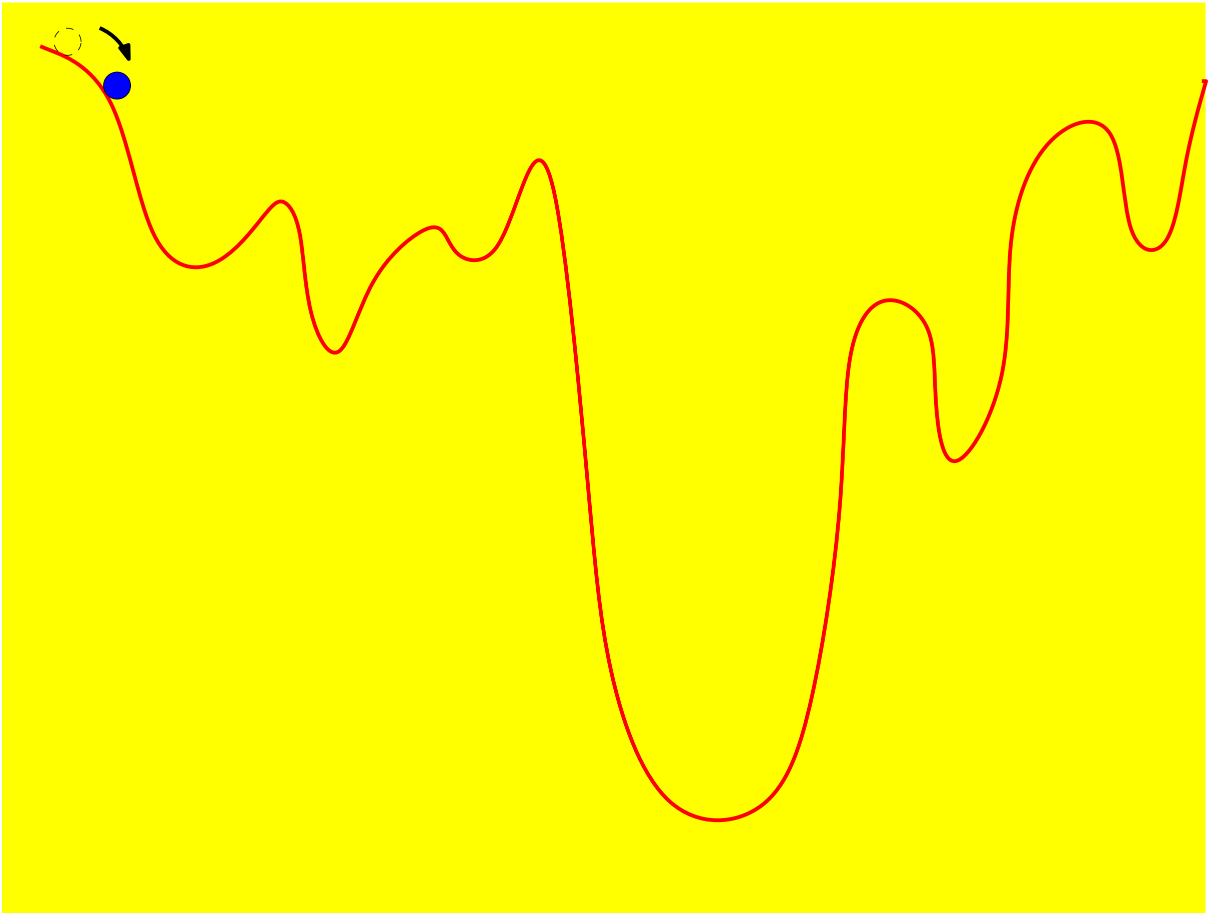
- Choose best neighbor based on ML cost: $\phi(\tilde{\mathbf{x}}) \; = \; \tilde{\mathbf{x}}^T\mathbf{H}^T\mathbf{H}\tilde{\mathbf{x}} - 2\mathbf{y}^T\mathbf{H}\tilde{\mathbf{x}}$

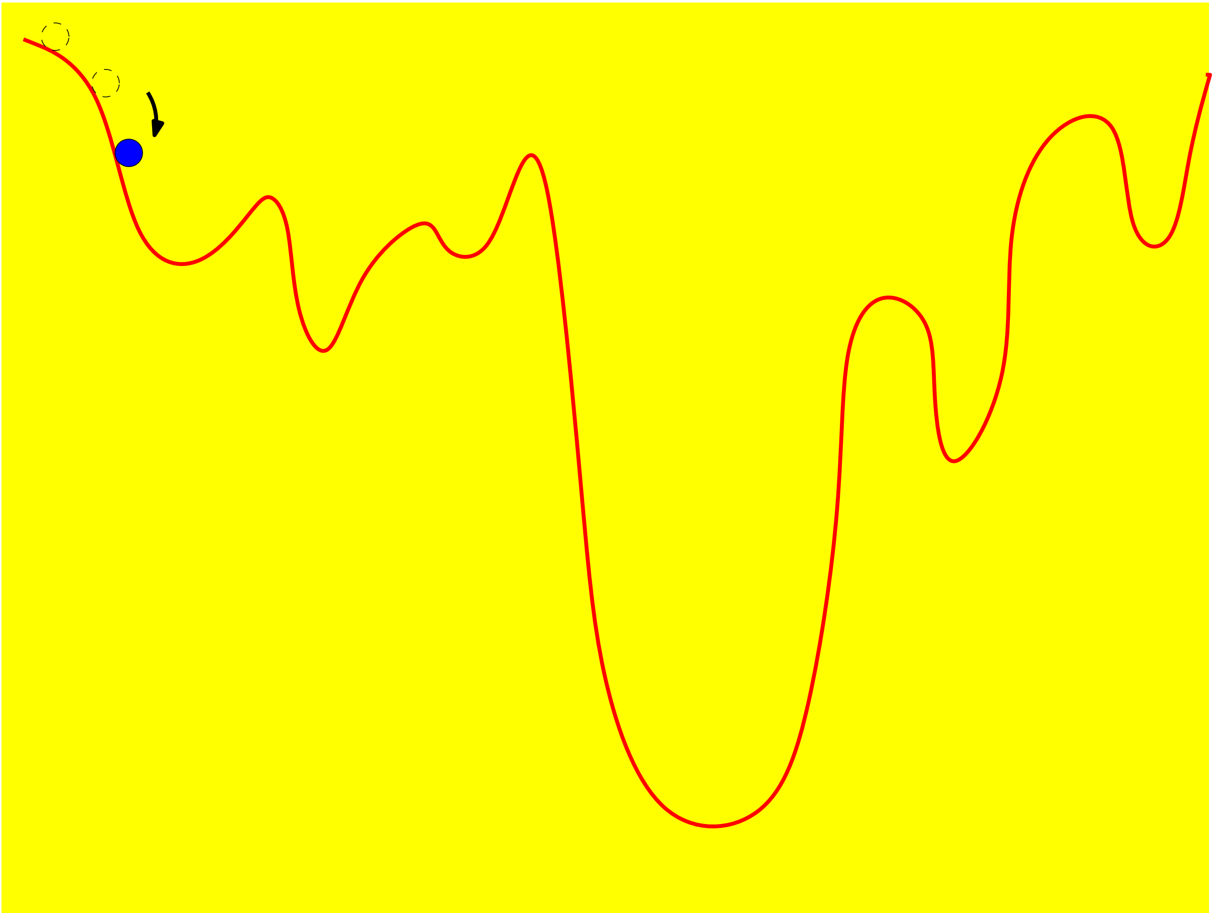
LAS Algorithm

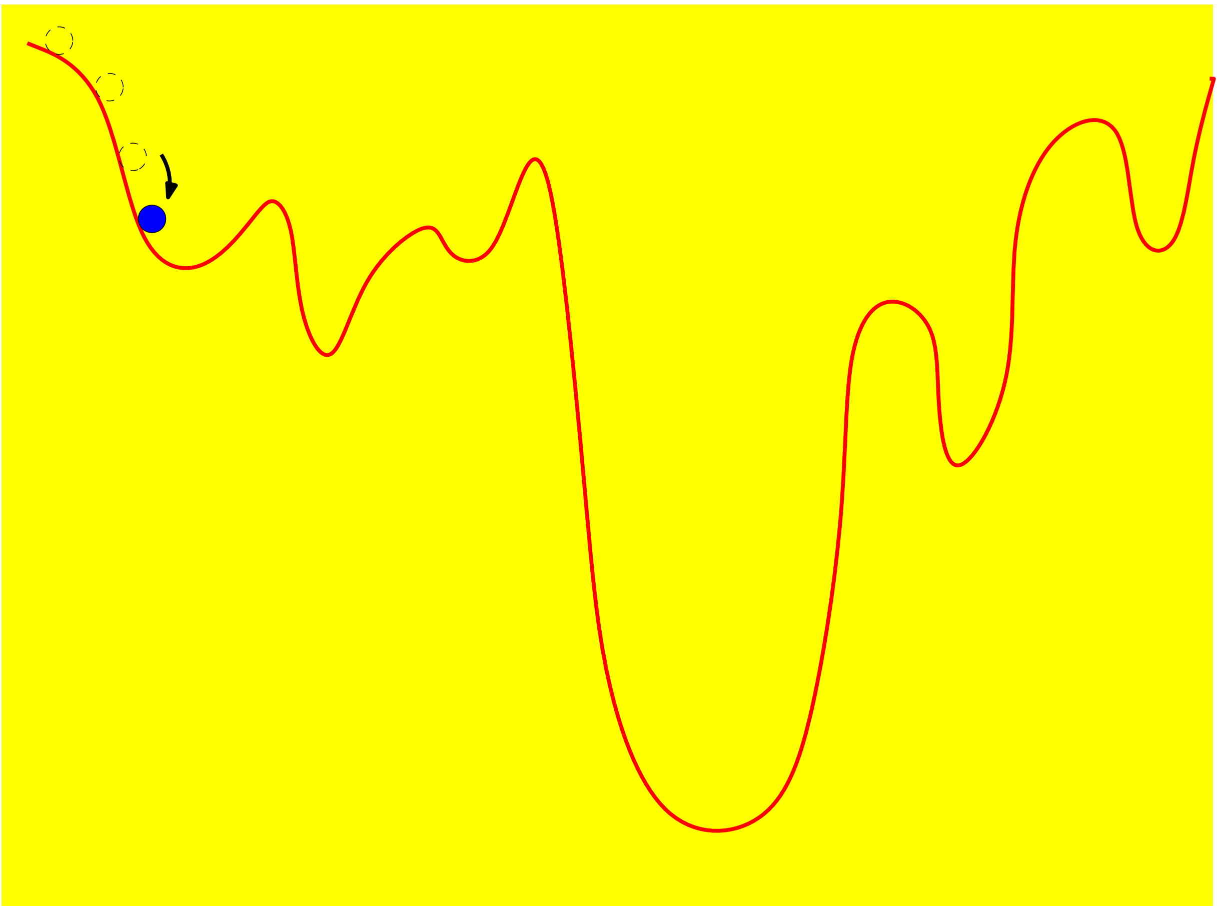


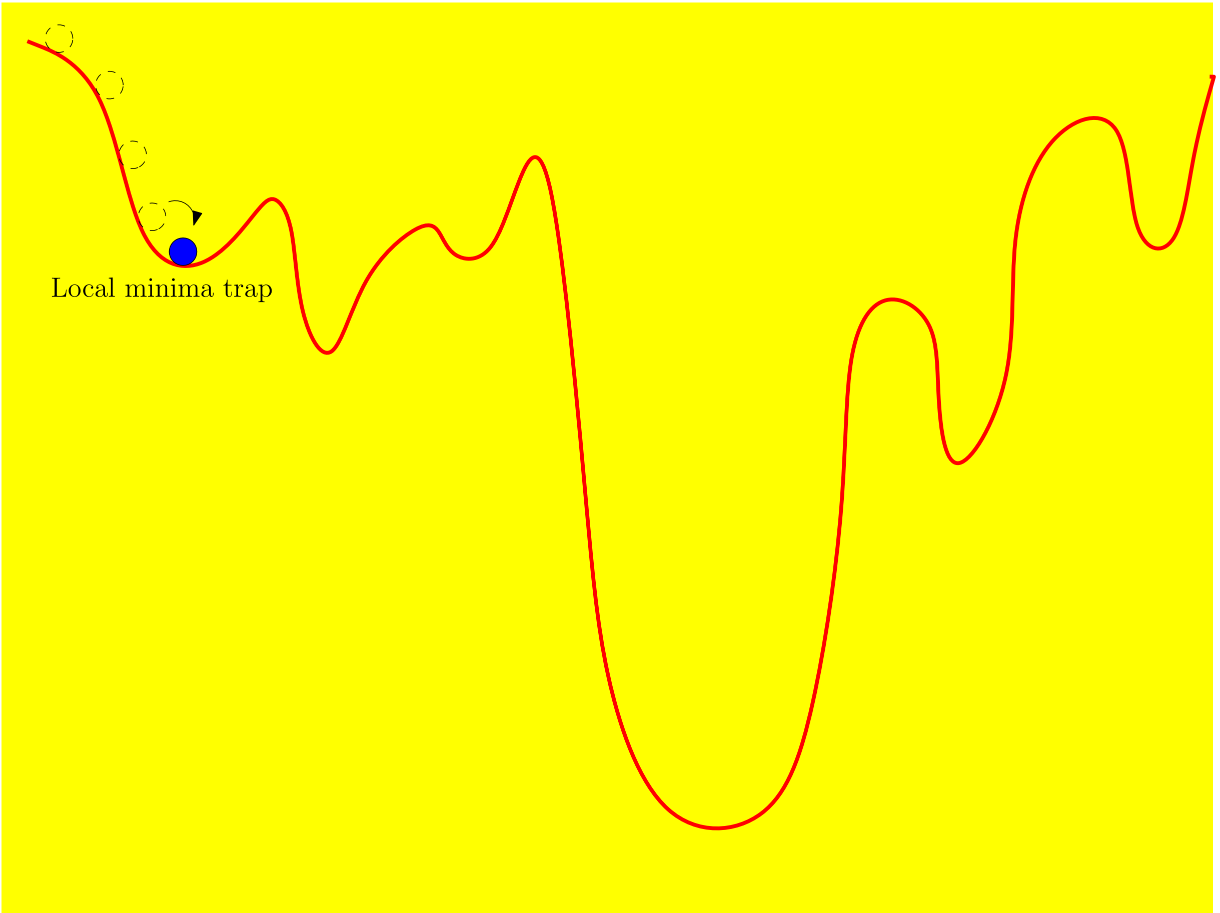
An Illustration of LAS Search Path







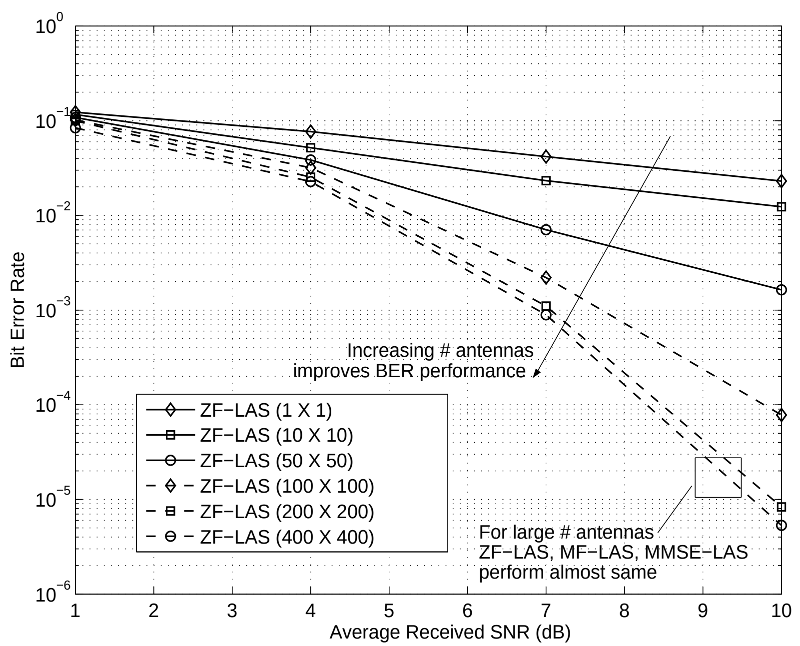


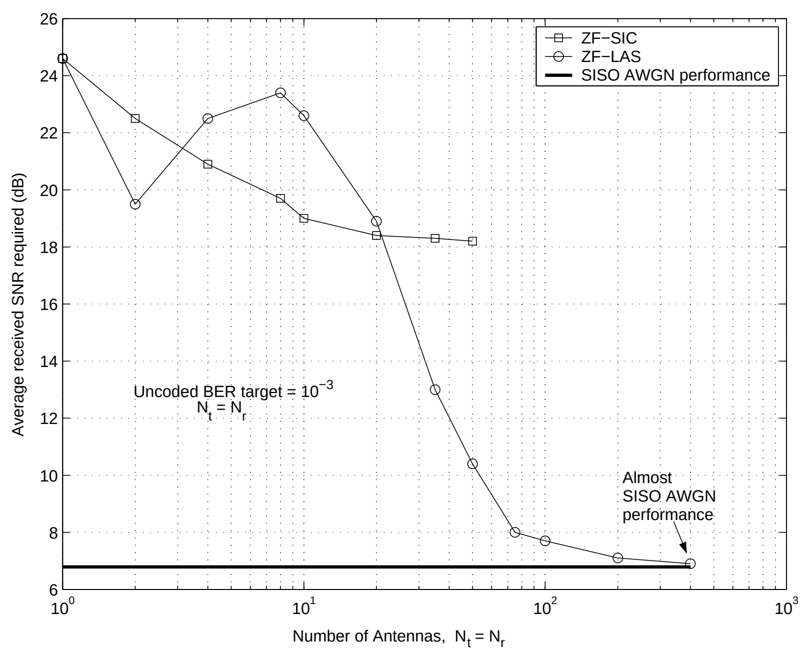

Local minima trap

Large-Dimension Behavior of LAS in V-BLAST [5],[6]

* 1-LAS: 1-symbol away neighborhood

* BER improves with increasing N_t (large-dimension effect)





[5] K. V. Vardhan, S. K. Mohammed, A. Chockalingam, B. S. Rajan, A low-complexity detector for large MIMO systems and multicarrier CDMA systems, *IEEE Jl. Sel. Areas in Commun. (JSAC)*, vol. 26, no.3, pp. 473-485, April 2008.

[6] S. K. Mohammed, K. V. Vardhan, A. Chockalingam, B. Sundar Rajan, Large MIMO systems: A low-complexity detector at high spectral efficiencies, *IEEE ICC'2008*, Beijing, May 2008.

Complexity of 1-LAS in V-BLAST

- Consider $N_t = N_r$
- Total complexity comprises of 3 main parts
  1. Computing initial vector (e.g., ZF, MMSE): $O(N_t^2)$ per symbol
  2. Computing $\mathbf{H}^T\mathbf{H}$: $O(N_t^2)$ per symbol
  3. Search operation: $O(N_t)$ per symbol (through simulations)
- So, overall average per-symbol complexity: $O(N_t^2)$
- This low-complexity allows detection of V-BLAST signals in hundreds of spatial dimensions

Large # Dimensions: The Key

- Observation
  - In V-BLAST, LAS algorithm achieves near-ML performance, but only when the # antennas is in hundreds
  - hundreds of antennas may not be practical
- Note
  - LAS requires large # dimensions to perform well
  - but, all dimensions need not be in space alone
- Q1: Can large # dimensions be created with less # Tx antennas?
- A1: Yes. Use time dimension as well. Approach: Non-orthogonal STBCs
- Q2: Can LAS modified to work well for smaller (tens) dimensions?
- A2: Yes. Approach: Escape strategies from local minima

An Escape Strategy from Local Minima [7]

- Multistage LAS (M-LAS)
  - Start the algorithm as 1-LAS
  - On reaching the local minima,
    - find 2-symbol away neighbors of the local minima
    - choose the best 2-symbol away neighbor if it has lesser cost than local minima
    - run 1-LAS from this best neighbor till a local minima is reached
  - Expect better performance. Complexity is increased a little, but not by an order
  - Escape strategy with 3-symbol away neighborhood on reaching local minima
- Another promising strategy is reactive tabu search

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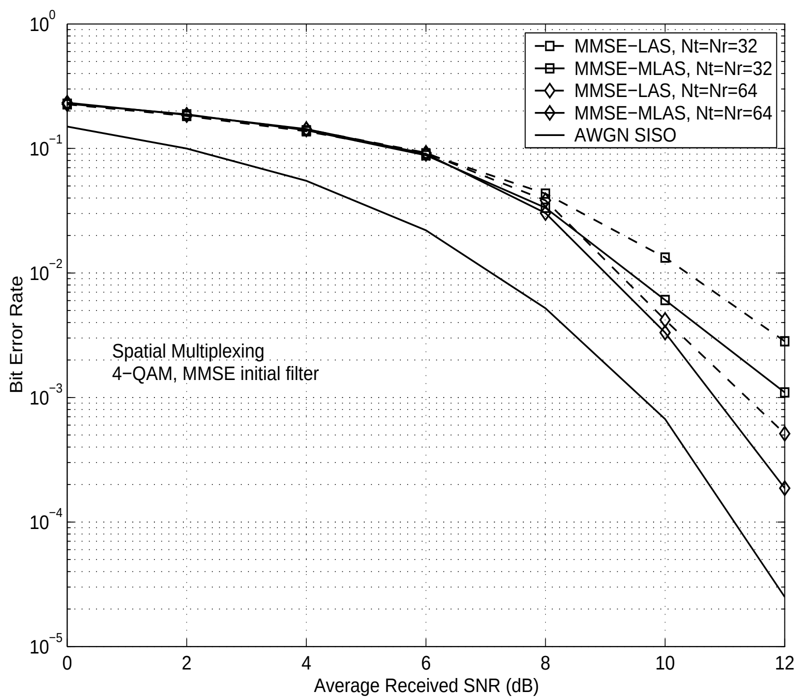
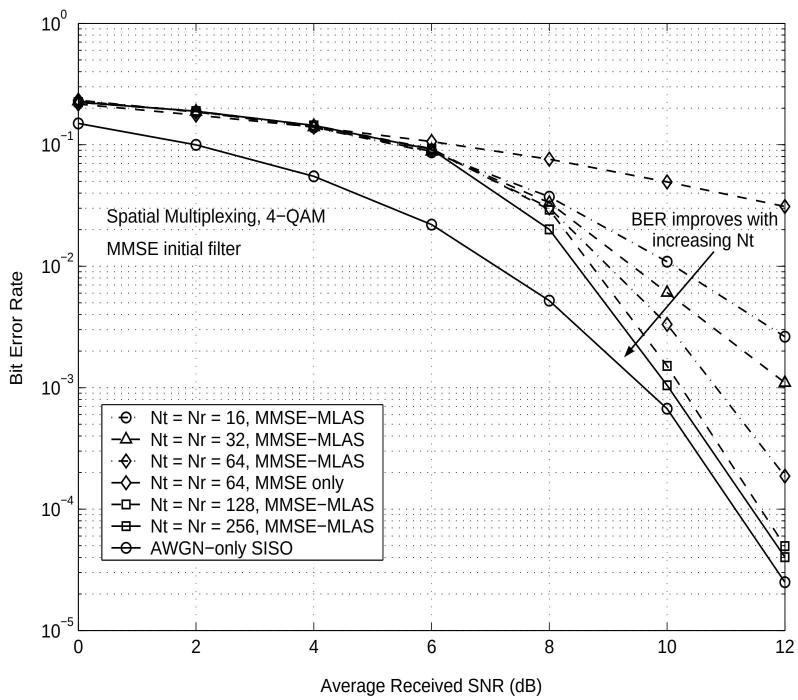
[7] S. K. Mohammed, A. Chockalingam, B. S. Rajan, A low-complexity near-ML performance achieving algorithm for large MIMO detection, *IEEE ISIT'2008*, Toronto, June 2008.

Performance of M-LAS

* 3-LAS performs better than 1-LAS



(e) 3-LAS versus 1-LAS



(f) 3-LAS

Space-Time Block Codes

- Provide redundancy across space and time
- Goal of space-time coding
  - Achieve the maximum Tx-diversity of N_t (i.e., full-diversity), high rate, decoding at low-complexity
- An STBC is usually represented by a $p \times n_t$ matrix
  - rows: time slots; p: # time slots
  - columns: Tx. antennas; n_t: # Tx. antennas

$$\mathbf{X} = \begin{bmatrix} s_{11} & s_{12} & \cdot & s_{1n_t} \\ s_{21} & s_{22} & \cdot & s_{2n_t} \\ \cdot & \cdot & \cdot & \cdot \\ s_{p1} & s_{42} & \cdot & s_{pn_t} \end{bmatrix}$$

- s_{ij} denotes the complex number transmitted in the ith time slot on the jth Tx antenna

Space-Time Block Codes

- Rate of an STBC, $r = \frac{k}{p}$
  - k: number of information symbols sent in one STBC
  - p: number of time slots in one STBC
    - Higher rate means more information carried by the code
- A matrix $\mathbf{X}$ is said to be a Orthogonal STBC if

$$\mathbf{X}^H \mathbf{X} = \left(|x_1|^2 + |x_2|^2 + \cdots + |x_k|^2\right) \mathbf{I}_{n_t}$$

  - Elements of $\mathbf{X}$ are linear combinations of $x_1, \cdots, x_k$ and their conjugates
  - $x_1, x_2, \cdots, x_k$ are information symbols
- 2-Tx Antennas Codes (2×2 Alamouti Code)

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \quad k = 2, p = 2, r = 1, \text{ orthogonal STBC}$$

Linear-Complexity Decoding of OSTBCs

- Consider Alamouti code with $n_t = 2$, $n_r = 1$
- Received signal in ith slot, y_i, $i = 1, 2$, is

$$\begin{aligned} y_1 &= h_1 x_1 + h_2 x_2 + n_1 \\ y_2 &= -h_1 x_2^* + h_2 x_1^* + n_2 \end{aligned}$$

- ML decoding amounts to
  - computing

$$\begin{aligned} \tilde{x}_1 &= y_1 h_1^* + y_2^* h_2 \\ \tilde{x}_2 &= y_1 h_2^* - y_2^* h_1 \end{aligned}$$

  - decoding x_1 by finding the symbol in the constellation that is closest to $\tilde{x}_1$
  - and decoding x_2 by finding the symbol that is closest to $\tilde{x}_2$
- This decoding feature is called Single-Symbol Decodability (SSD)

Orthogonal vs Non-Orthogonal STBCs

- Orthogonal STBCs are more widely known
  - e.g., 2×2 Alamouti code (Rate-1; 2 symbols in 2 channel uses)
  - advantages
    - ∗ linear complexity ML decoding, full transmit diversity
  - major drawback
    - ∗ rate falls linearly with increasing number of transmit antennas
- Non-orthogonal STBCs: less widely known
  - e.g., 2×2 Golden code (Rate-2; 4 symbols in 2 chl uses; same as V-BLAST)
    - ∗ advantages
      - · High-rate (same as V-BLAST, i.e., N_t symbols/channel use)
      - · Full Transmit diversity
      - · best of both worlds (in terms of data rate and transmit diversity)
    - ∗ What is the catch
      - · decoding complexity

Non-Orthogonal STBCs

- Golden code [8] (2×2 non-orthogonal STBC)

$$\mathbf{X} = \begin{bmatrix} x_1 + \tau x_2 & x_3 + \tau x_4 \\ i(x_3 + \mu x_4) & x_1 + \mu x_2 \end{bmatrix}, \quad k = 4, p = 2, r = 2$$

where $\tau = \frac{1+\sqrt{5}}{2}$ and $\mu = \frac{1-\sqrt{5}}{2}$

- Features
  - Information Losslessness (ILL)
  - Full Diversity (FD)
  - Coding Gain (CG)
- 'Perfect codes' [9] achieve all the above three features
  - Golden code is a perfect code

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[8] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The golden code: A 2×2 full-rate space-time code with non-vanishing determinants," *IEEE Trans. on Information Theory*, vol. 51, no. 4, pp. 1432-1436, April 2005.

[9] F. E. Oggier, G. Rekaya, J.-C. Belfiore, and E. Viterbo, "Perfect space-time block codes," *IEEE Trans. on Information Theory*, vol. 52, no. 9, pp. 3885-3902, September 2006.

High-Rate Non-Orthogonal STBCs from CDA for any N_t

- High-rate non-orthogonal STBCs from Cyclic Division Algebras (CDA) for arbitrary # transmit antennas, n, is given by the $n \times n$ matrix [10]

$$\mathbf{X} = \begin{bmatrix} \sum_{i=0}^{n-1} x_{0,i}\, t^i & \delta \sum_{i=0}^{n-1} x_{n-1,i}\, \omega_n^i\, t^i & \delta \sum_{i=0}^{n-1} x_{n-2,i}\, \omega_n^{2i}\, t^i & \cdots & \delta \sum_{i=0}^{n-1} x_{1,i}\, \omega_n^{(n-1)i}\, t^i \\ \sum_{i=0}^{n-1} x_{1,i}\, t^i & \sum_{i=0}^{n-1} x_{0,i}\, \omega_n^i\, t^i & \delta \sum_{i=0}^{n-1} x_{n-1,i}\, \omega_n^{2i}\, t^i & \cdots & \delta \sum_{i=0}^{n-1} x_{2,i}\, \omega_n^{(n-1)i}\, t^i \\ \sum_{i=0}^{n-1} x_{2,i}\, t^i & \sum_{i=0}^{n-1} x_{1,i}\, \omega_n^i\, t^i & \sum_{i=0}^{n-1} x_{0,i}\, \omega_n^{2i}\, t^i & \cdots & \delta \sum_{i=0}^{n-1} x_{3,i}\, \omega_n^{(n-1)i}\, t^i \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{n-1} x_{n-2,i}\, t^i & \sum_{i=0}^{n-1} x_{n-3,i}\, \omega_n^i\, t^i & \sum_{i=0}^{n-1} x_{n-4,i}\, \omega_n^{2i}\, t^i & \cdots & \delta \sum_{i=0}^{n-1} x_{n-1,i}\, \omega_n^{(n-1)i} t^i \\ \sum_{i=0}^{n-1} x_{n-1,i}\, t^i & \sum_{i=0}^{n-1} x_{n-2,i}\, \omega_n^i\, t^i & \sum_{i=0}^{n-1} x_{n-3,i}\, \omega_n^{2i}\, t^i & \cdots & \sum_{i=0}^{n-1} x_{0,i}\, \omega_n^{(n-1)i}\, t^i \end{bmatrix}$$

- $\omega_n = e^{\frac{\mathbf{j}2\pi}{n}}$, $\mathbf{j} = \sqrt{-1}$, and $x_{u,v}$, $0 \leq u, v \leq n-1$ are the data symbols from a QAM alphabet
- n^2 complex data symbols in one STBC matrix (i.e., n complex data symbols per channel use)
- $\delta = t = 1$: Information-lossless (ILL); $\delta = e^{\sqrt{5}\,\mathbf{j}}$ and $t = e^{\mathbf{j}}$: Full diversity and ILL
- Ques: Can large (e.g., 32×32) STBCs from CDA decoded?   Ans: LAS algorithm can.

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[10] B. A. Sethuraman, B. Sundar Rajan, V. Shashidhar, "Full-diversity high-rate space-time block codes from division algebras," *IEEE Trans. on Information Theory*, vol. 49, no. 10, pp. 2596-2616, October 2003.

Linear Vector Channel Model for NO-STBC

- (n, p, k) STBC is a matrix $\mathbf{X}_c \in \mathbb{C}^{n \times p}$, n: # time slots, p: # tx antennas, k: # data symbols in one STBC; $(n = p$ and $k = n^2$ for NO-STBC from CDA)
- Received space-time signal matrix

$$\mathbf{Y}_c = \mathbf{H}_c \mathbf{X}_c + \mathbf{N}_c,$$

- Consider linear dispersion STBCs where $\mathbf{X}_c$ can be written in the form

$$\mathbf{X}_c = \sum_{i=1}^{k} x_c^{(i)} \mathbf{A}_c^{(i)}$$

where $\mathbf{A}_c^{(i)} \in \mathbb{C}^{N_t \times p}$ is the weight matrix corresponding to data symbol $x_c^{(i)}$

- Applying $vec(.)$ operation

$$\begin{aligned} vec\left(\mathbf{Y}_c\right) &= \sum_{i=1}^{k} x_c^{(i)} vec\left(\mathbf{H}_c \mathbf{A}_c^{(i)}\right) + vec\left(\mathbf{N}_c\right) \\ &= \sum_{i=1}^{k} x_c^{(i)} \left(\mathbf{I}_{p \times p} \otimes \mathbf{H}_c\right) vec\left(\mathbf{A}_c^{(i)}\right) + vec\left(\mathbf{N}_c\right) \end{aligned}$$

Linear Vector Channel Model for NO-STBC

- Define $\mathbf{y}_c \stackrel{\triangle}{=} vec\,(\mathbf{Y}_c) \in \mathbb{C}^{N_r p}$, $\widehat{\mathbf{H}}_c \stackrel{\triangle}{=} (\mathbf{I} \otimes \mathbf{H}_c) \in \mathbb{C}^{N_r p \times N_t p}$, $\mathbf{a}_c^{(i)} \stackrel{\triangle}{=} vec\,(\mathbf{A}_c^{(i)}) \in \mathbb{C}^{N_t p}$, $\mathbf{n}_c \stackrel{\triangle}{=} vec\,(\mathbf{N}_c) \in \mathbb{C}^{N_r p}$
- System model can then be written in vector form as

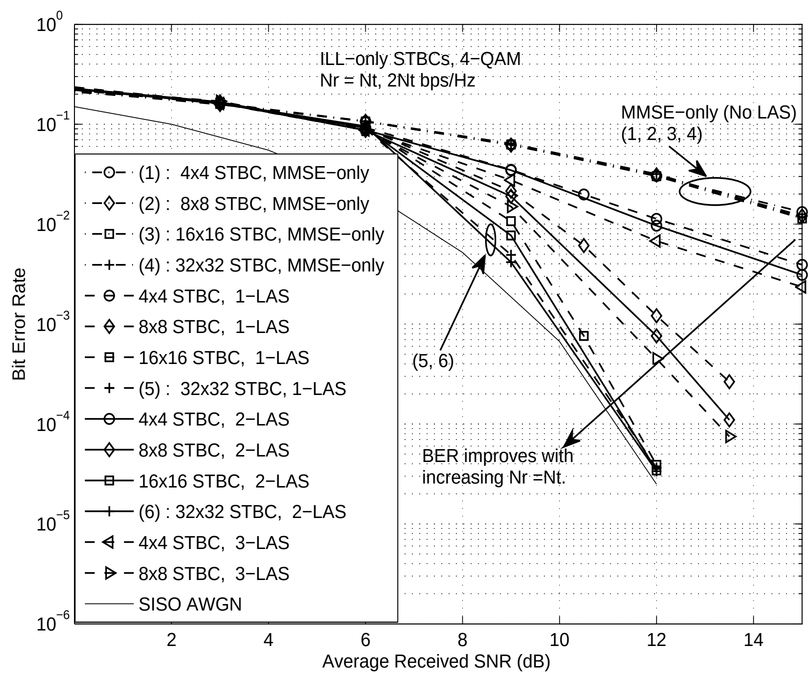
$$\begin{aligned} \mathbf{y}_c &= \sum_{i=1}^{k} x_c^{(i)} \, (\widehat{\mathbf{H}}_c \, \mathbf{a}_c^{(i)}) + \mathbf{n}_c \\ &= \widetilde{\mathbf{H}}_c \mathbf{x}_c + \mathbf{n}_c \end{aligned} \tag{4}$$

$\widetilde{\mathbf{H}}_c \in \mathbb{C}^{N_r p \times k}$, whose ith column is $\widehat{\mathbf{H}}_c \ \mathbf{a}_c^{(i)}$, $i = 1, \cdots, k$

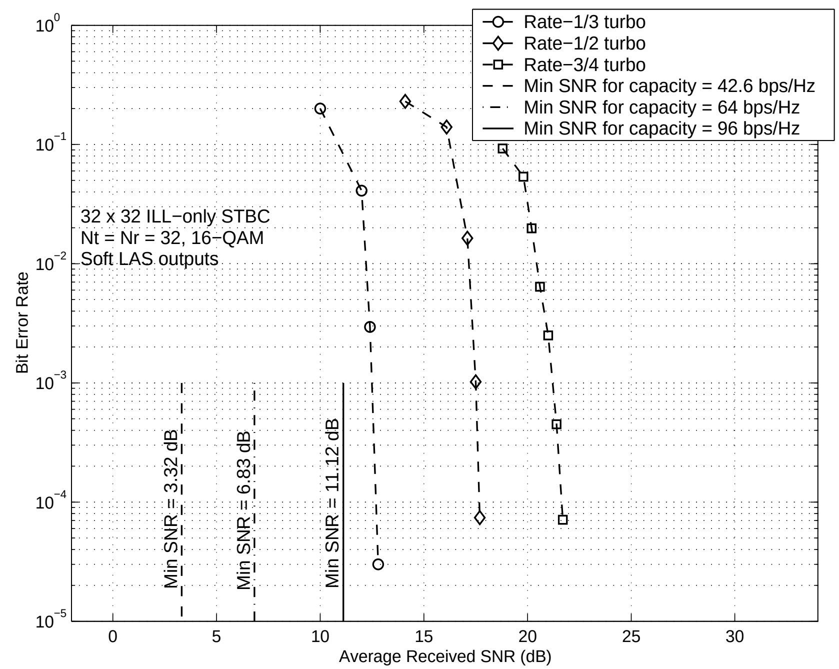
$\mathbf{x}_c \in \mathbb{C}^k$, whose ith entry is the data symbol $x_c^{(i)}$

- Convert the complex system model in (4) into real system model as before
- Apply LAS algorithm on the resulting real system model

LAS Performance in Decoding NO-STBCs [11]



(g) Uncoded 8×8, 16×16, 32×32 NO-STBC, 4-QAM



(h) Turbo Coded 32×32 NO-STBC, 16-QAM

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[11] S. K. Mohammed, A. Zaki, A. Chockalingam, B. Sundar Rajan, "High-Rate Space-Time Coded Large-MIMO Systems: Low-Complexity Detection and Channel Estimation," *IEEE Journal on Sel. Topics in Signal Processing (IEEE JSTSP): Special Issue on Managing Complexity in Multiuser MIMO Systems*, vol. 3, no. 6, pp. 958-974, December 2009.

Comparison with Other Architectures/Detectors [11]

No.	MIMO Architecture/Detector Combinations (fixed $N_t = N_r = 16$ and **32 bps/Hz** for all combinations)	Complexity (in # real operations per bit) at 5×10^{-2} uncoded BER	SNR required to achieve 5×10^{-2} uncoded BER
$i)$	**16×16 ILL-only CDA STBC (rate-16), 4-QAM and 1-LAS detection (Proposed scheme [11])**	$\mathbf{3.473 \times 10^3}$	**6.8 dB**
$ii)$	16×16 ILL-only CDA STBC (rate-16), 4-QAM and ISIC algorithm [Choi, Cioffi]	1.187×10^5	11.3 dB
$iii)$	Four 4×4 stacked rate-1 QOSTBCs, 256-QAM and IC algorithm [Jafarkhani]	5.54×10^6	24 dB
$iv)$	Eight 2×2 stacked rate-1 Alamouti codes, 16-QAM and IC algorithm [Jafarkhani]	8.719×10^3	17 dB
$v)$	16×16 V-BLAST (rate-16) scheme, 4-QAM and sphere decoding	4.66×10^4	7 dB
$vi)$	16×16 V-BLAST (rate-16) scheme, 4-QAM and V-BLAST detector (ZF-SIC)	1.75×10^4	13 dB

Effect of Spatial Correlation?

- Spatially correlated MIMO fading channel model by Gesbert et al [12]

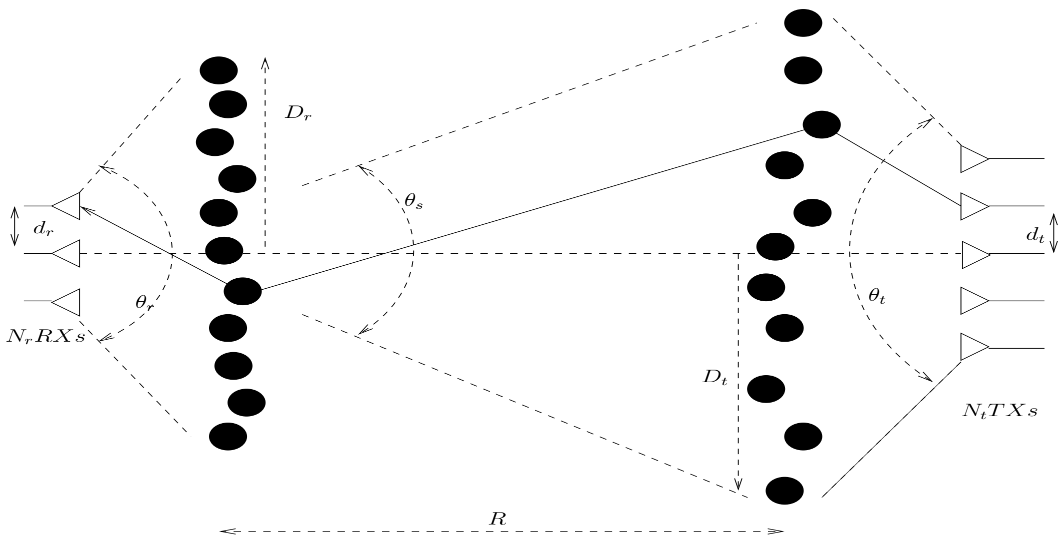


Figure 1: Propagation scenario for the MIMO fading channel model

correlated channel matrix: $$\mathbf{H} = \frac{1}{\sqrt{S}} \mathbf{R}^{1/2}_{\theta_r,d_r} \mathbf{G}_r \mathbf{R}^{1/2}_{\theta_S,2D_r/S} \mathbf{G}_t \mathbf{R}^{1/2}_{\theta_t,d_t}$$

[12] D. Gesbert, H. Bolcskei, D. A. Gore, A. J. Paulraj, "Outdoor MIMO wireless channels: Models and performance prediction," *IEEE Trans. on Commun.,* vol. 50, pp. 1926-1934, December 2002.

Spatial Correlation Degrades Performance [11]

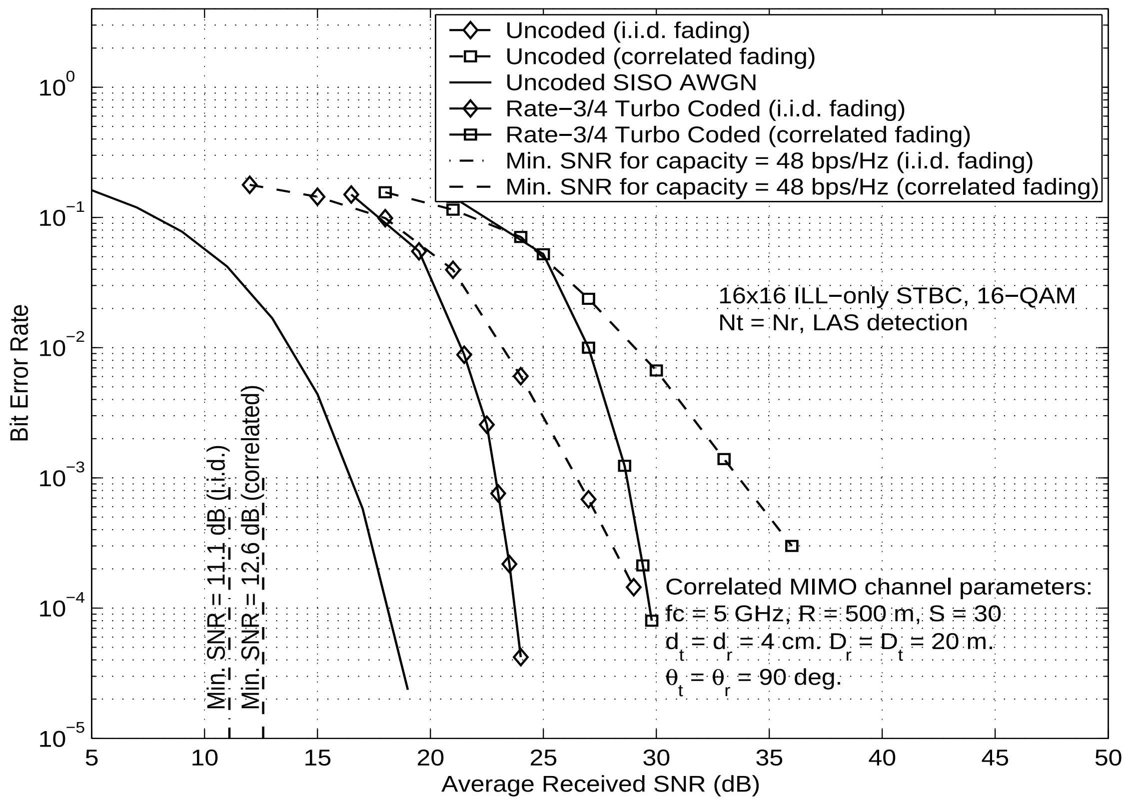


Figure 2: Uncoded/coded BER performance of 1-LAS detector $i)$ in i.i.d. fading, and $ii)$ in correlated MIMO fading in [3] with $f_c = 5$ GHz, $R = 500$ m, $S = 30$,$D_t = D_r = 20$ m, $\theta_t = \theta_r = 90^\circ$, and $d_t = d_r = 2\lambda/3 = 4$ cm. 16×16 STBC, $N_t = N_r = 16$, 16-QAM, rate-3/4 turbo code, 48 bps/Hz.

Spatial correlation degrades performance.

Increasing # Receive Dimensions Helps! [11]

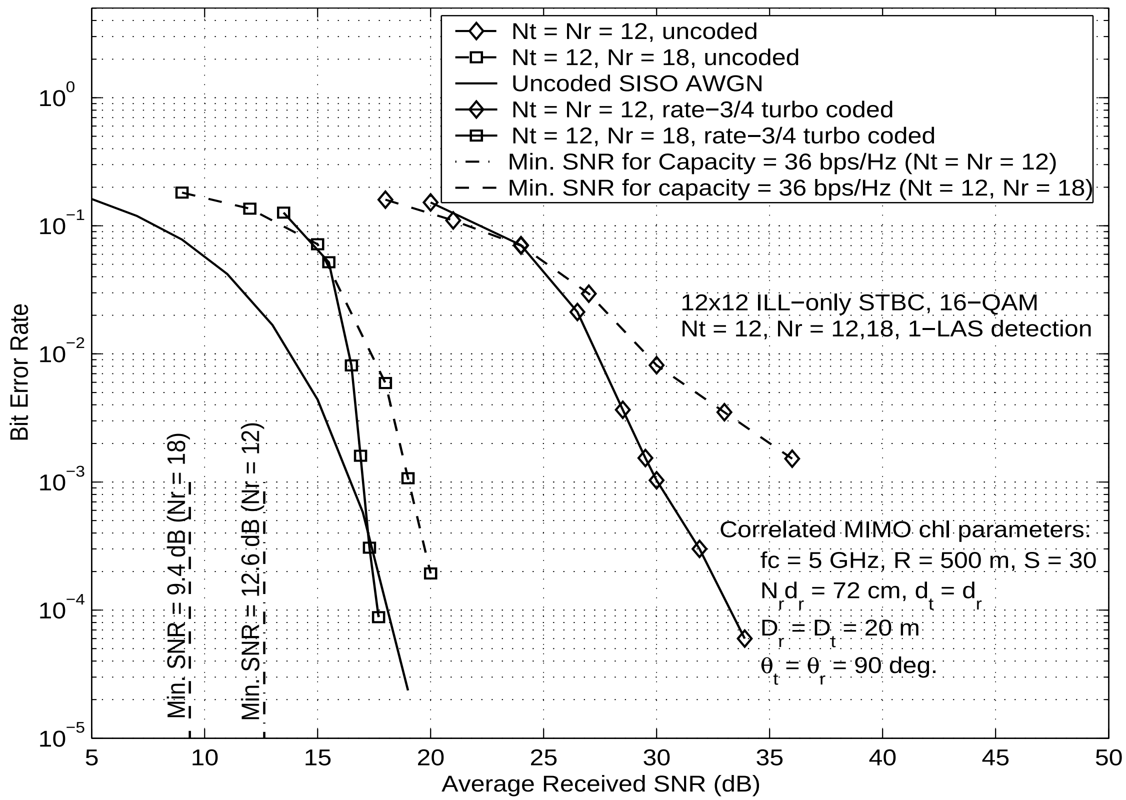
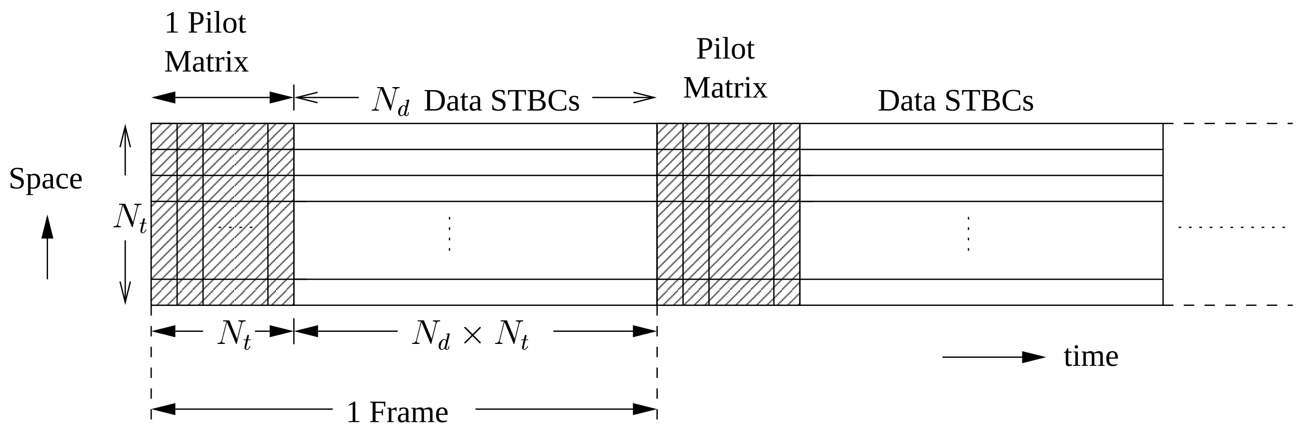


Figure 8: Effect of $N_r > N_t$ in correlated MIMO fading in [3] keeping $N_r d_r$ constant and $d_t = d_r$. $N_r d_r = 72$ cm, $f_c = 5$ GHz, $R = 500$ m, $S = 30$, $D_t = D_r = 20$ m, $\theta_t = \theta_r = 90°$, 12×12 ILL-only STBC, $N_t = 12$, $N_r = 12, 18$, 16-QAM, rate-3/4 turbo code, 36 bps/Hz. Increasing # receive dimensions alleviates the loss due to spatial correlation.

Channel Estimation in Large-MIMO?

- Training based channel estimation [11]
  - Send 1 Pilot matrix followed by N_d data STBC matrices



    - ∗ 1 frame length (in # of channel uses), $T = (N_d + 1)N_t$ [coherence time]
    - ∗ 1 pilot matrix length (in # of channel uses), $\tau = N_t$
  - Obtain an MMSE estimate of the channel matrix during pilot phase
  - Use estimated channel matrix to detect data matrices using LAS detection
  - Iterate between detection and channel estimation

MIMO Capacity with Estimated CSIR

Hassibi-Hochwald (H-H) bound [13] on capacity with estimated CSIR:

$$C \geq \frac{T-\tau}{T}\,\mathbb{E}\left[\text{logdet}\left(\mathbf{I}_{N_t} + \frac{\gamma^2\beta_d\beta_p\tau}{N_t(1+\gamma\beta_d)+\gamma\beta_p\tau}\,\frac{\hat{\mathbf{H}}_c\hat{\mathbf{H}}_c^H}{N_t\sigma^2_{\hat{\mathbf{H}}_c}}\right)\right]$$

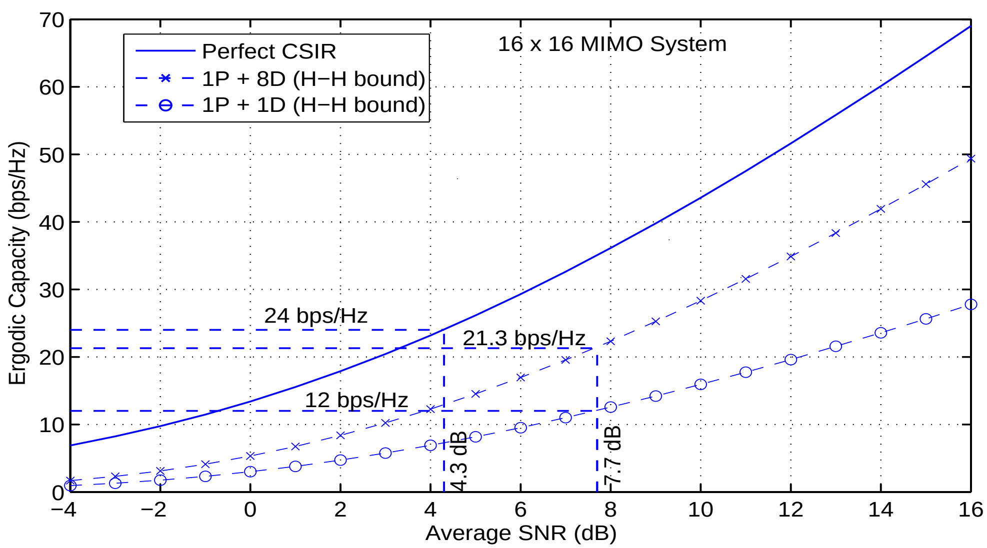


Figure 4: H-H capacity bound [13] for 1P+8D ($T = 144, \tau = 16, \beta_p = \beta_d = 1$) and 1P+1D ($T = 32, \tau = 16, \beta_p = \beta_d = 1$) training for a 16×16 MIMO channel.

[13] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?," *IEEE Trans. on Information Theory*, vol. 49, no. 4, pp. 951-963, April 2003.

How Much Training is Required?

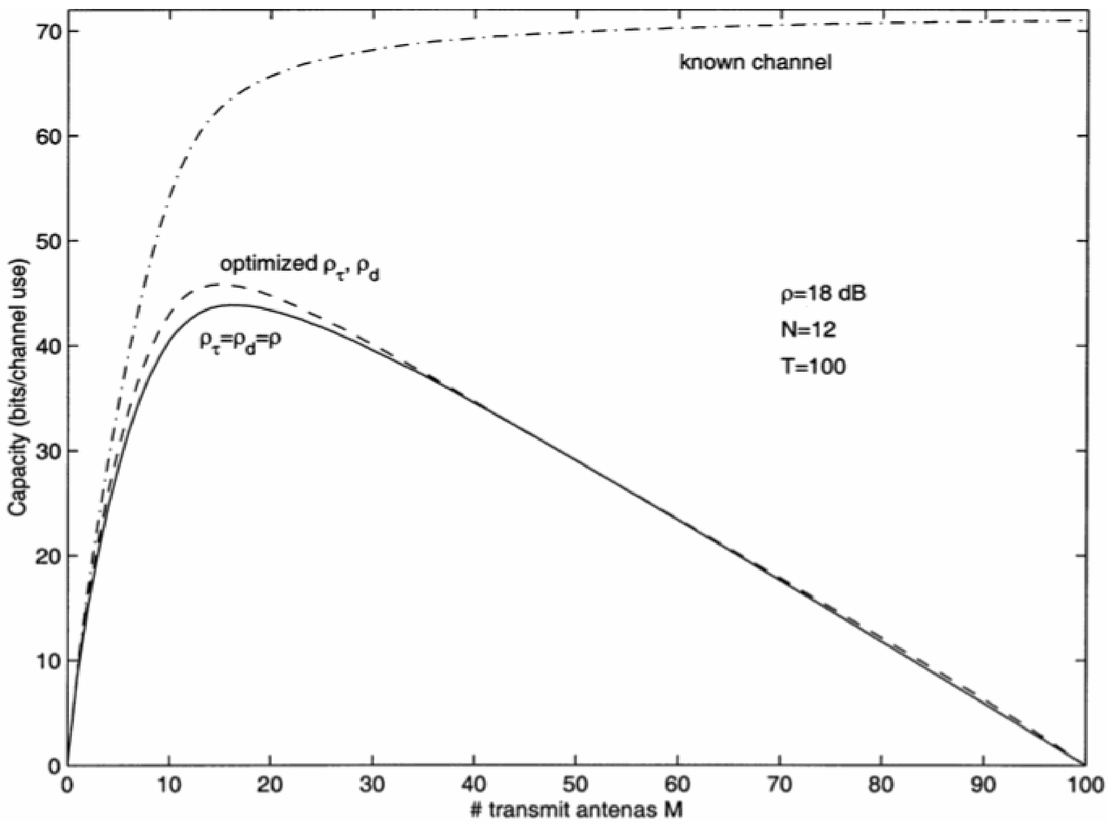


Fig. 5. Capacity as a function of number of transmit antennas M with $\rho = 18$ dB and $N = 12$ receive antennas. The solid line is optimized over T_τ for $\rho_\tau = \rho_d = \rho$ (see (40)), and the dashed line is optimized over the power allocation with $T_\tau = M$ (Theorem 3). The dash-dotted line is the capacity when the receiver knows the channel perfectly. The maximum throughput is attained at $M \approx 15$.

Figure 5: Capacity as a function of N_t with SNR = 18 dB and $N_r = 12$. For a given N_r, SNR (γ), and coherence time (T), there is an optimum N_t [13].

BER Performance with Estimated Channel Matrix [11]

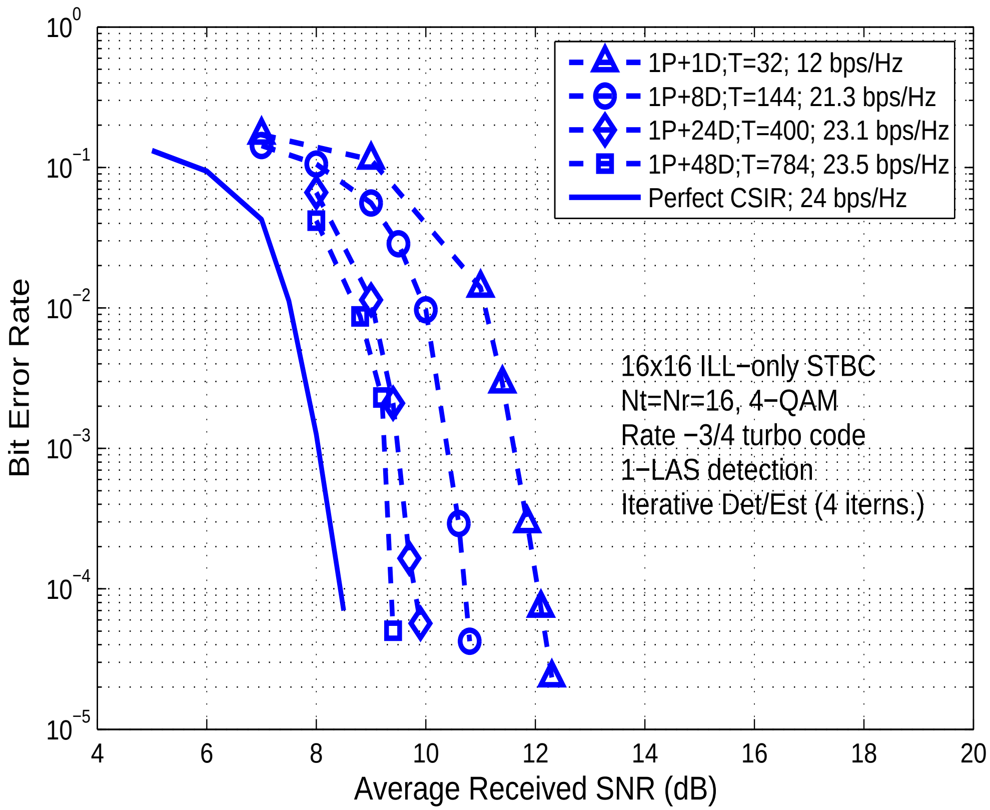


Figure 6: Turbo coded BER performance of LAS detection and channel estimation as a function of coherence time, $T = 32, 144, 400, 784$ ($N_d = 1, 8, 24, 48$), for a given $N_t = N_r = 16$. 16×16 ILL-only STBC, 4-QAM, rate-3/4 turbo code. Spectral efficiency and BER performance with estimated CSIR approaches to those with perfect CSIR in slow fading (i.e., large T).

Other Promising Large-MIMO Detection Algorithms

- Reactive Tabu Search [14]
- Probabilistic Data Association [15]
- Belief Propagation [16],[17]
- These algorithms exhibit large-dimension behavior; i.e., their bit error performance improves with increasing N_t.

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[14] N. Srinidhi, S. K. Mohammed, A. Chockalingam, and B. S. Rajan, Low-Complexity Near-ML Decoding of Large Non-Orthogonal STBCs using Reactive Tabu Search, *IEEE ISIT'2009*, Seoul, June 2009.

[15] S. K. Mohammed, A. Chockalingam, B. S. Rajan, Low-complexity near-MAP decoding of large non-orthogonal STBCs using PDA, *IEEE ISIT'2009*, Seoul, June 2009.

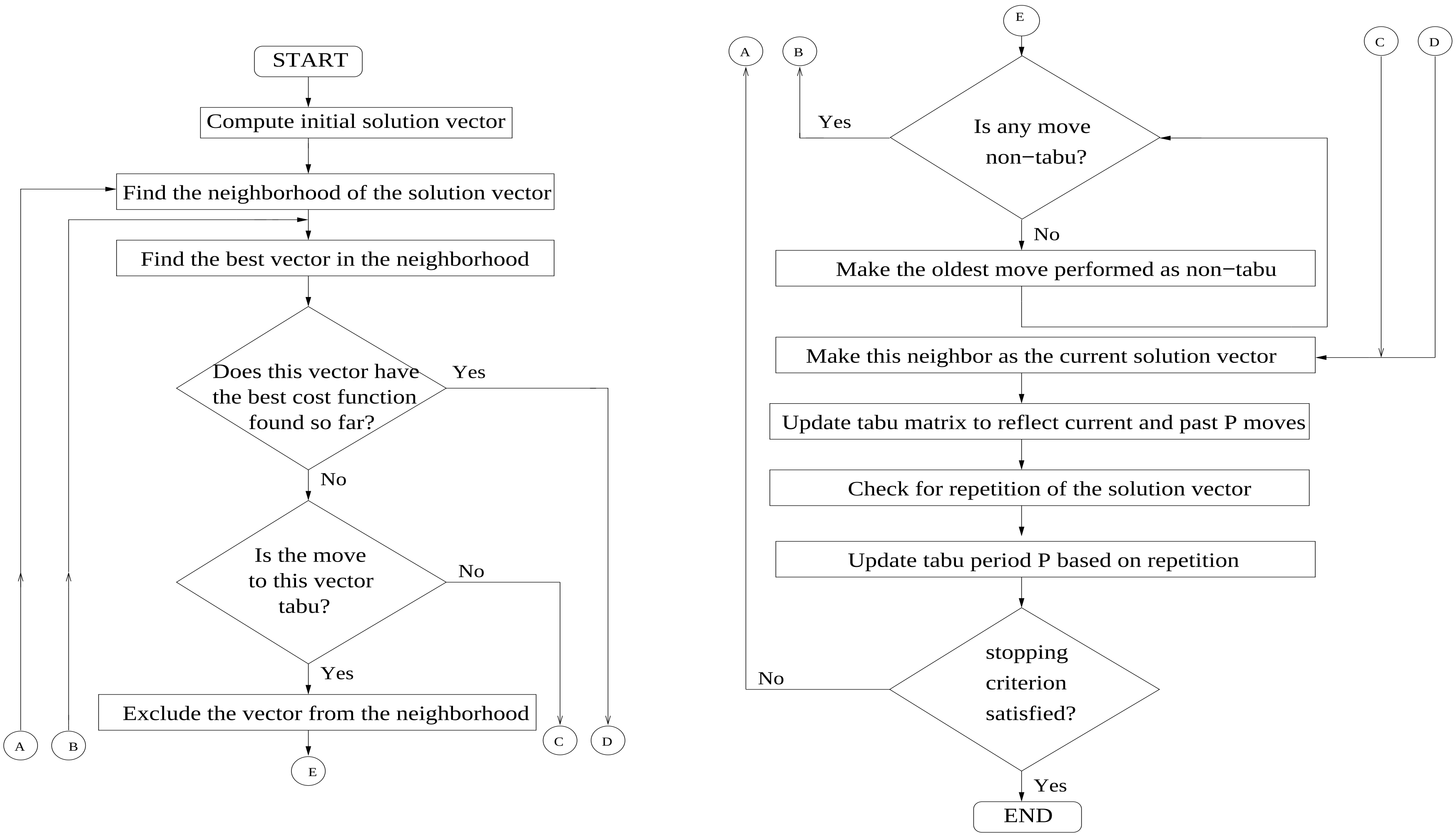
[16] S. Madhekar, P. Som, A. Chockalingam, B. S. Rajan, Belief Propagation Based Decoding of Large Non-Orthogonal STBCs, *IEEE ISIT'2009*, Seoul, June 2009.

[17] P. Som, T. Datta, A. Chockalingam, B. S. Rajan, Improved Large-MIMO Detection using Damped Belief Propagation, *IEEE ITW'2010*, Cairo, January 2010.

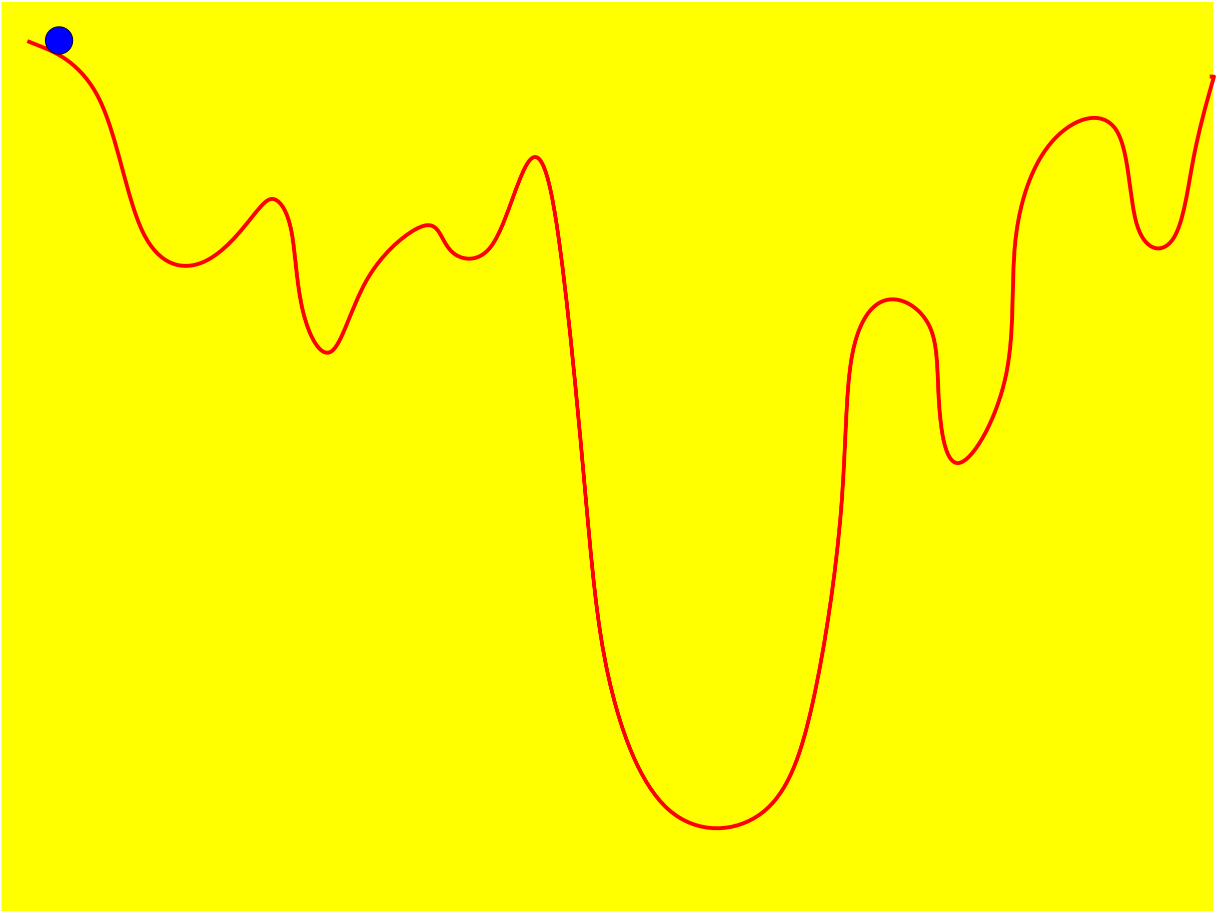
Reactive Tabu Search

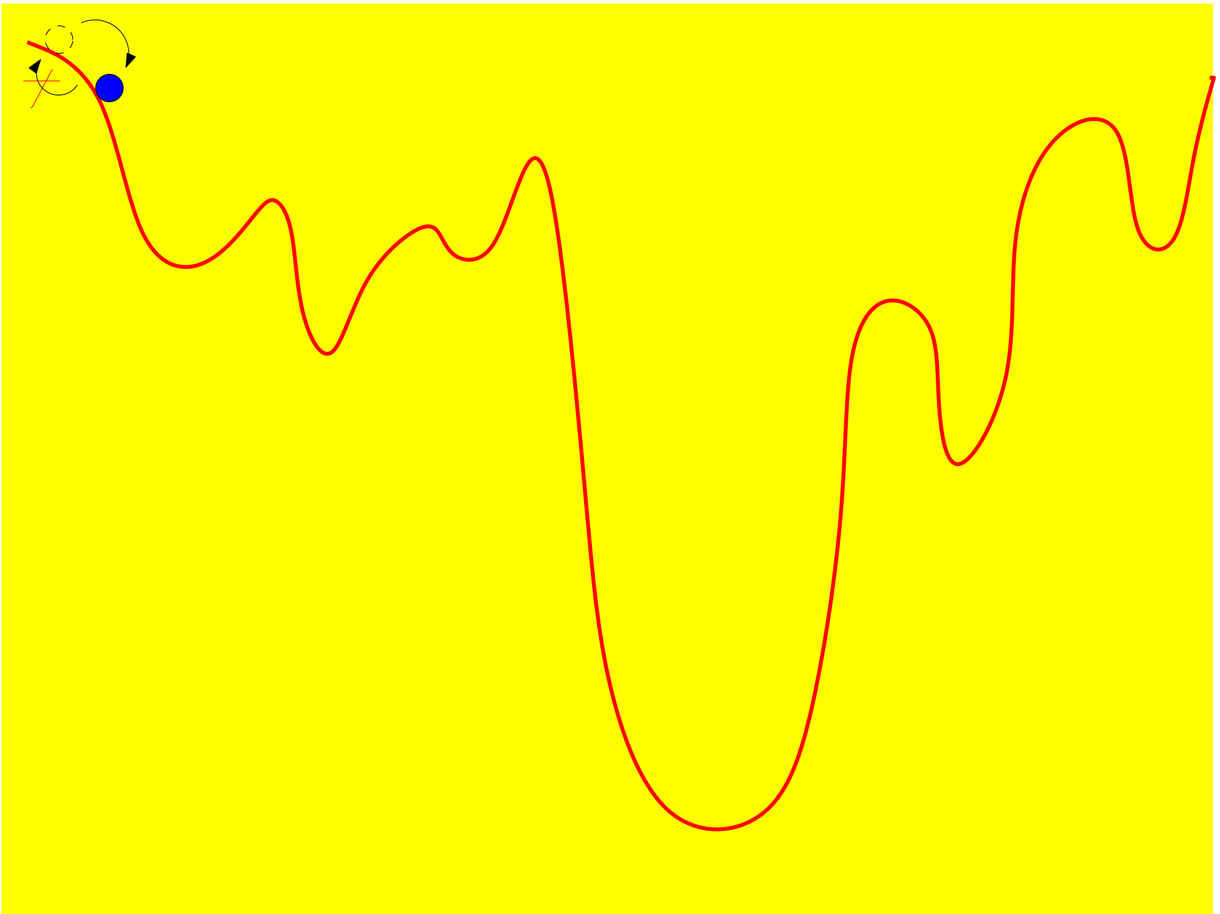
- Another iterative local search algorithm
  - A metaheuristics algorithm
  - cannot guarantee optimal solution, but generally gives near optimal solution
- Uses 'tabu' mechanism to escape from local minima or cycles
  - Certain vectors are prohibited (made tabu) from becoming solution vectors for certain number of iterations (called *tabu period*) depending on the search path
  - This is meant to ensure efficient exploration of the search space
- The reactive part adapts the tabu period

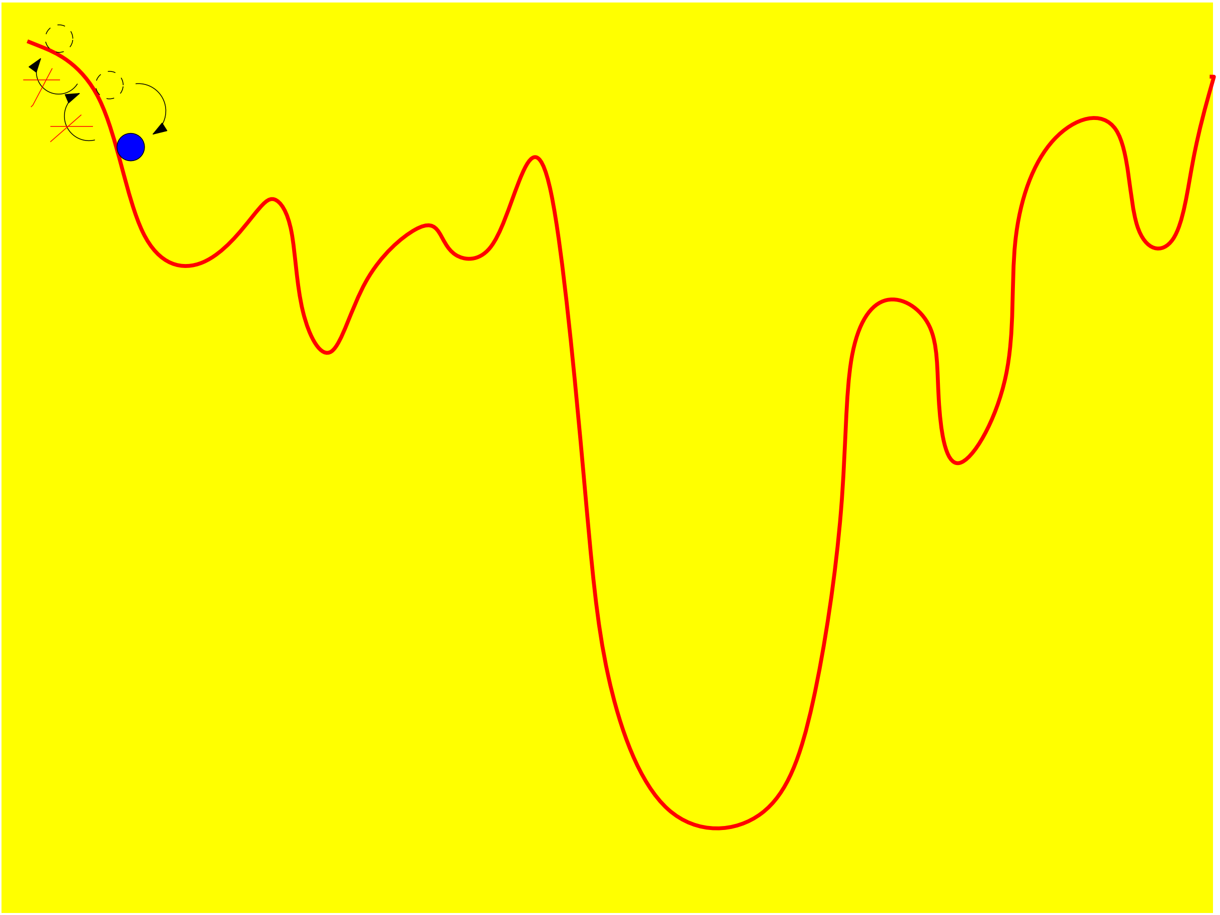
RTS Algorithm [14]

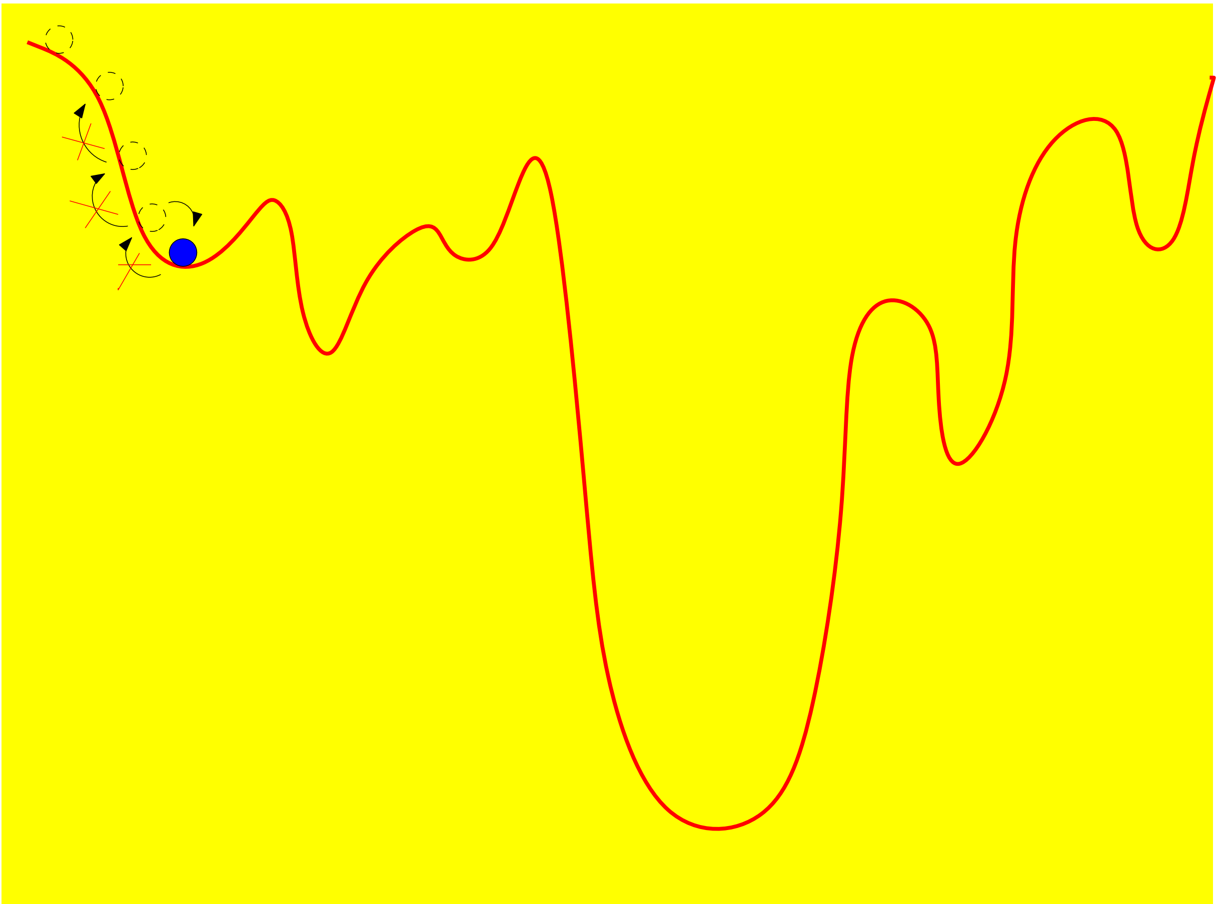


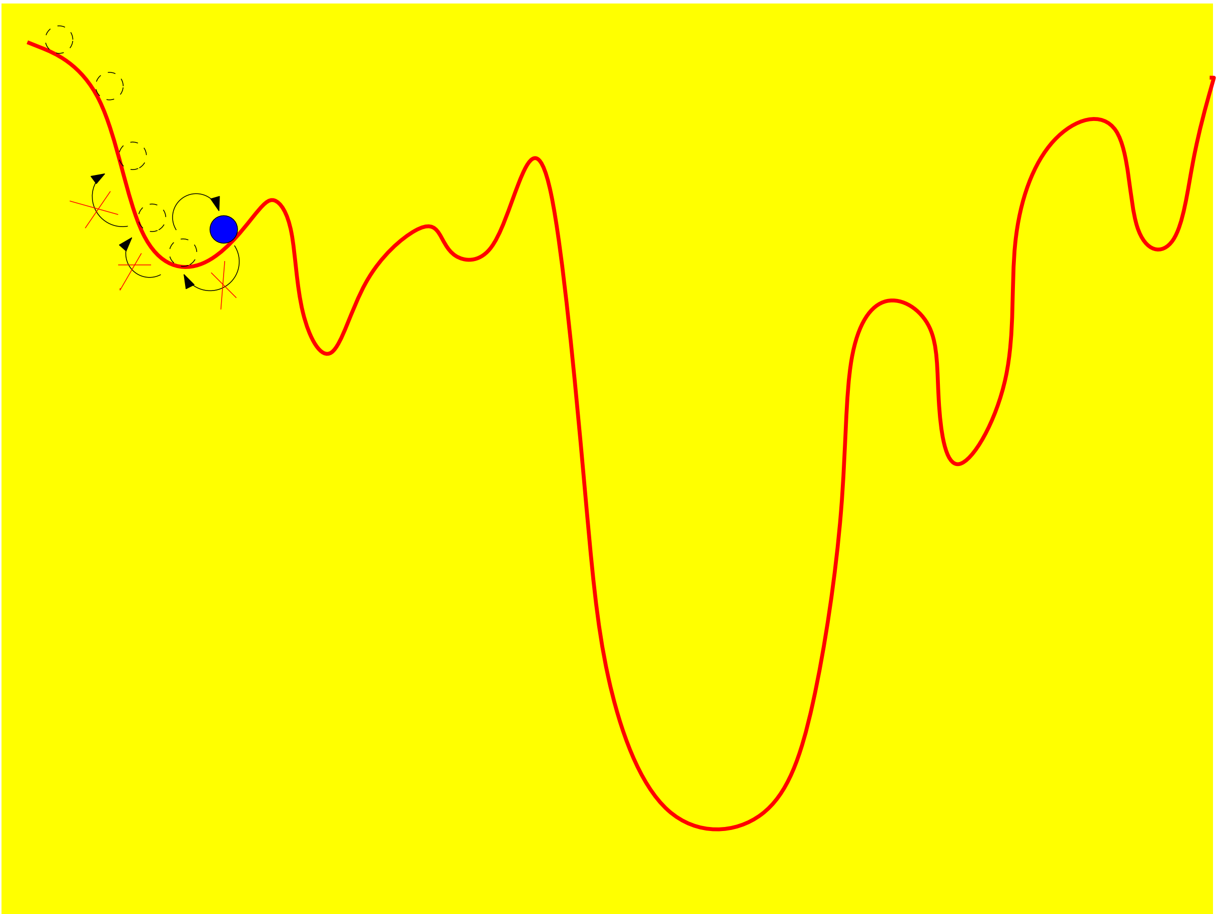
An Illustration of RTS Search Path

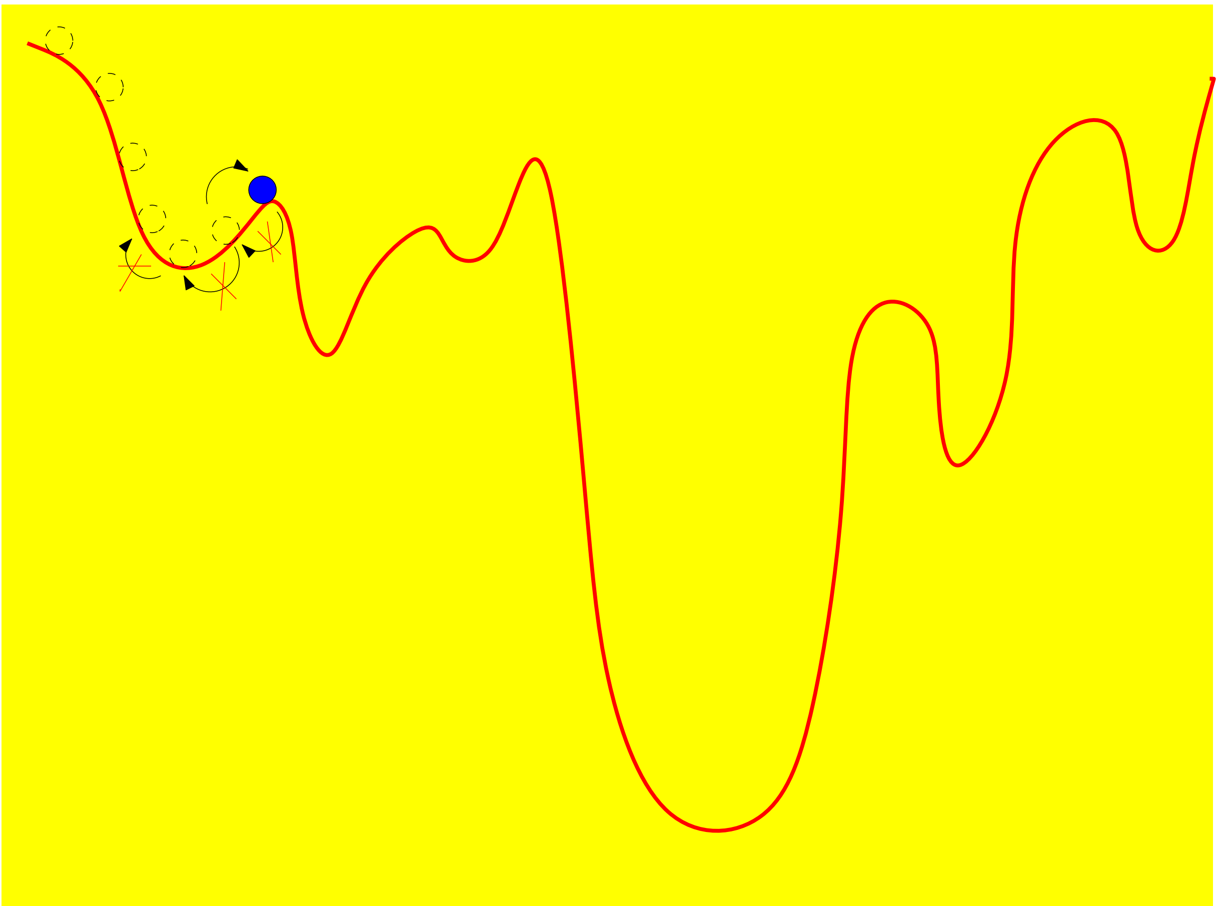


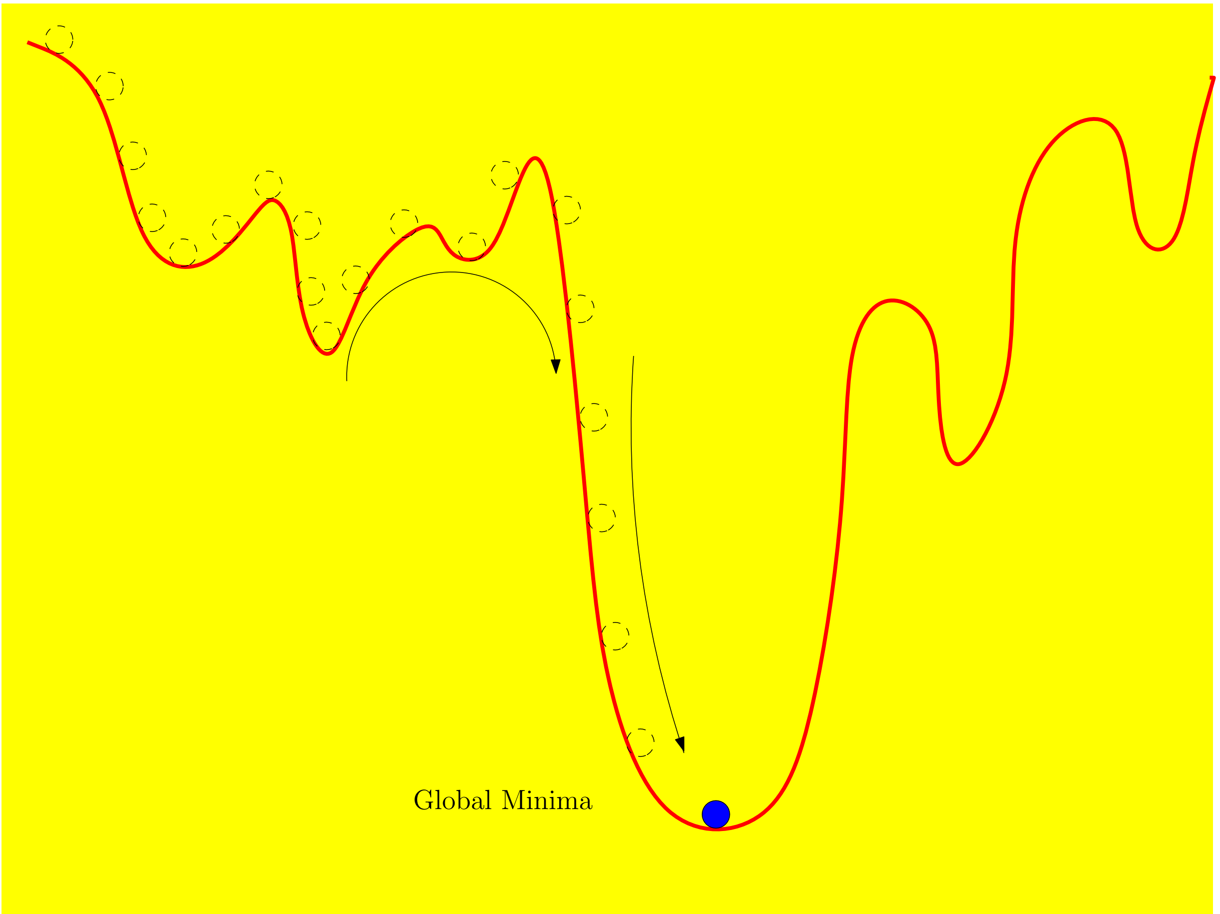




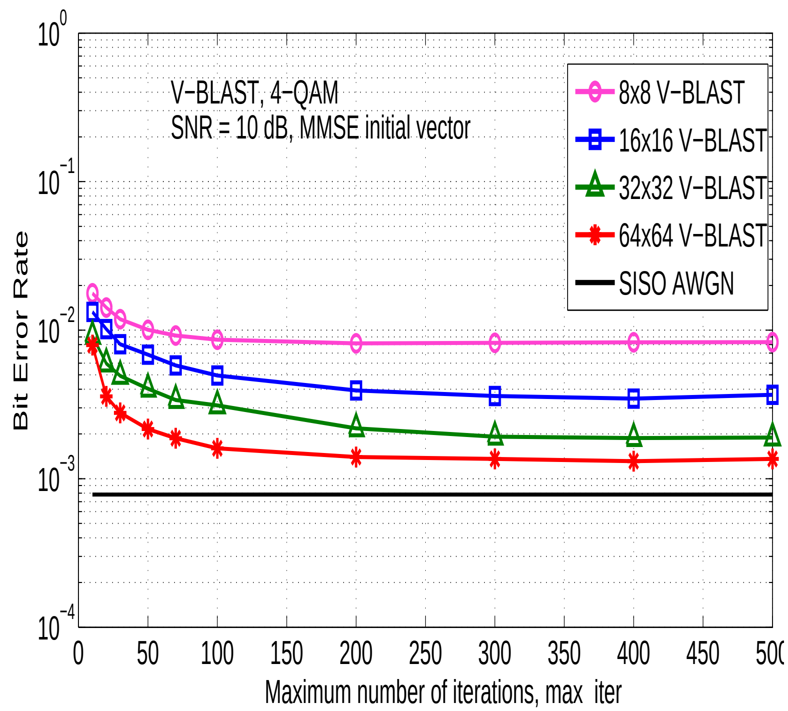




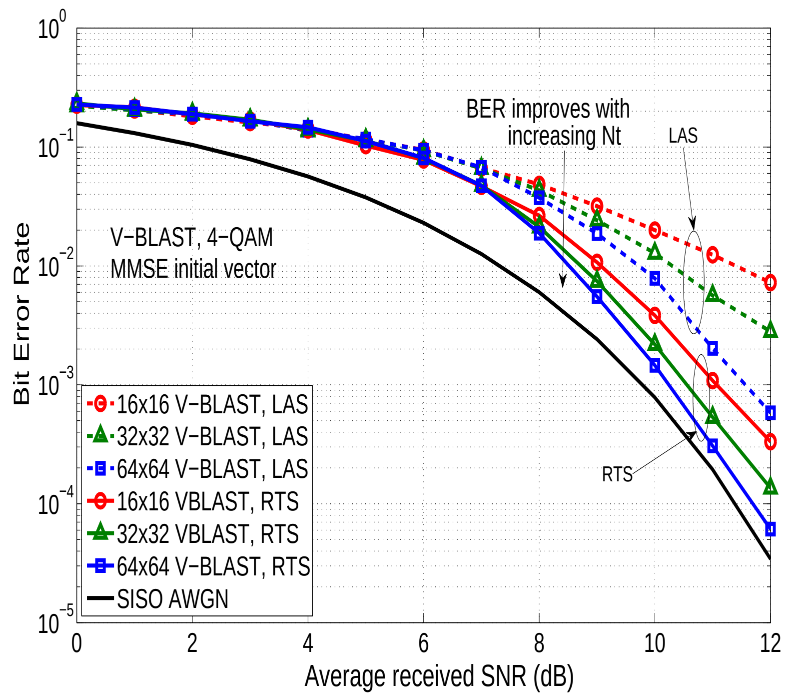



Global Minima

Performance of RTS in V-BLAST



(a) Convergence of RTS

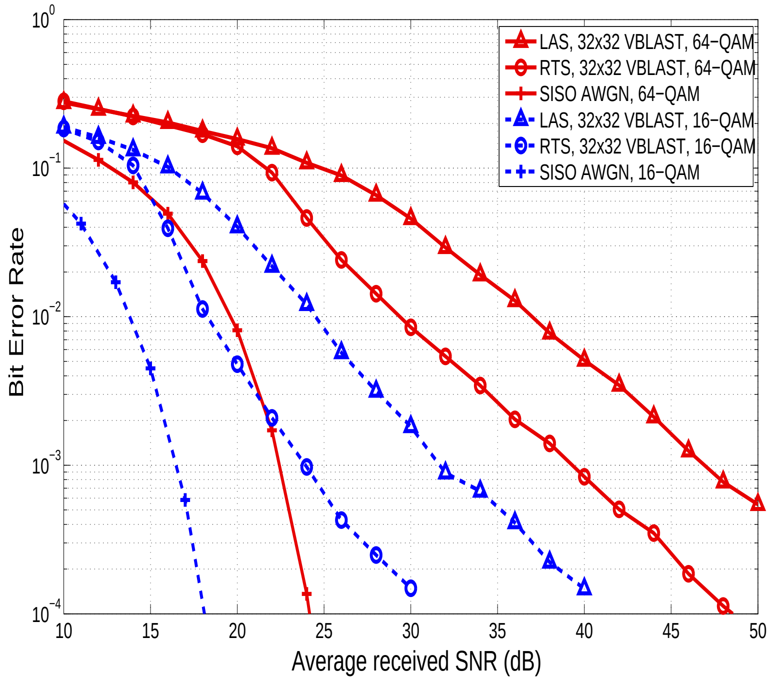


(b) RTS versus LAS

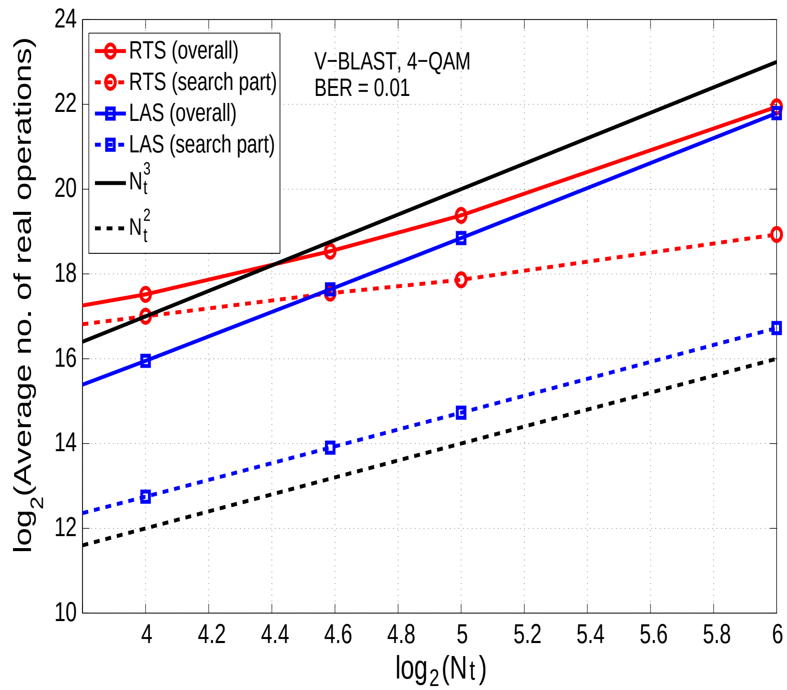
[18] N. Srinidhi, S. K. Mohammed, A. Chockalingam, B. S. Rajan, *Near-ML Signal Detection in Large-Dimension Linear Vector Channels Using Reactive Tabu Search*, Online arXiv:0911.4640v1 [cs.IT] 24 Nov 2009.

Performance/Complexity of RTS in V-BLAST

* RTS performs better than LAS at the same order of LAS complexity



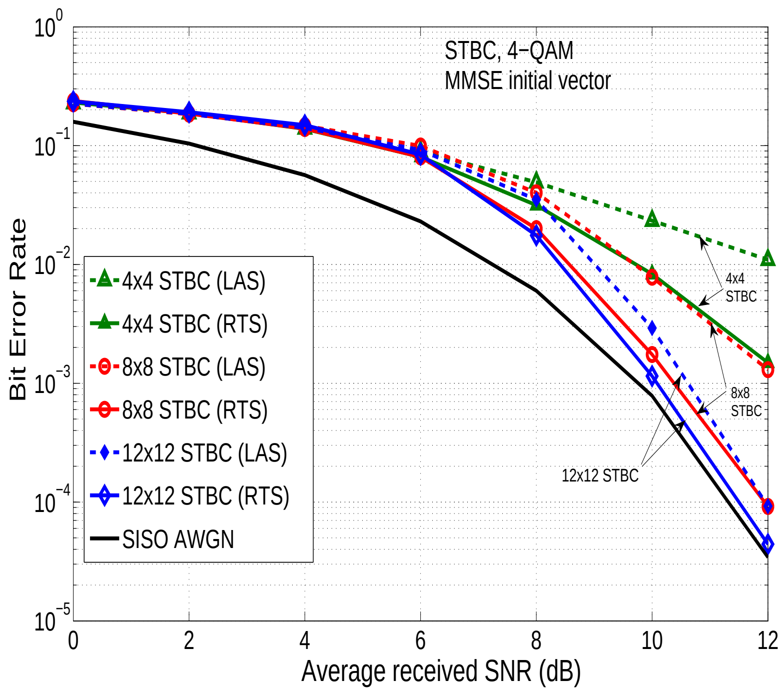
(c) Performance in 32×32 V-BLAST, M-QAM



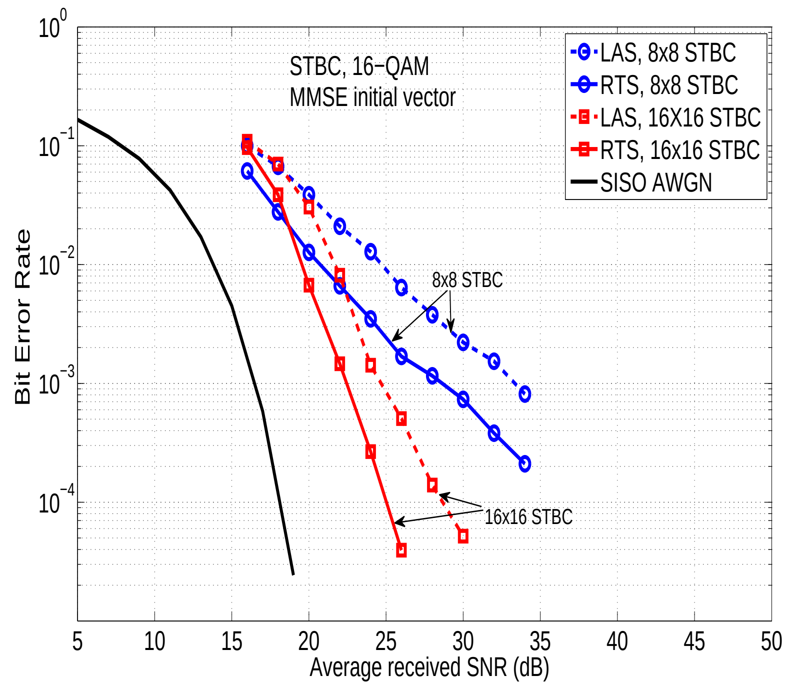
(d) Complexity

Performance of RTS in NO-STBC [14]

* RTS performs better than LAS



(e) 4-QAM



(f) 16-QAM

Probabilistic Data Association

- Originally developed for target tracking
- Used in digital communications recently
- PDA
  - A reduced complexity alternative to a posteriori probability (APP) detector/decoder/equalizer.
  - Has been applied in
    - Multiuser detection in CDMA (Luo *et al* 2001, Huang and Zhang 2004, Tan and Rasmussen 2006)
    - Turbo equalization (Yin *et al* 2004)

PDA Based Large-MIMO Detection [15]

- Iterative algorithm
  - In each iteration, $2qk$ statistic updates (one for each bit) are performed
- Likelihood ratio of bit $b_i^{(j)}$ in an iteration is

$$\Lambda_i^{(j)} \triangleq \underbrace{\frac{P\big(\mathbf{y}|b_i^{(j)}=+1\big)}{P\big(\mathbf{y}|b_i^{(j)}=-1\big)}}_{\triangleq\, \beta_i^{(j)}} \underbrace{\frac{P\big(b_i^{(j)}=+1\big)}{P\big(b_i^{(j)}=-1\big)}}_{\triangleq\, \alpha_i^{(j)}}$$

- Received signal vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{h}_{qi+j}\, b_i^{(j)} + \underbrace{\sum_{l=0}^{2k-1} \sum_{\substack{m=0 \\ m\neq q(i-l)+j}}^{q-1} \mathbf{h}_{ql+m}\, b_l^{(m)} + \mathbf{n}}_{\triangleq\, \tilde{\mathbf{n}}\ (interference+noise\ vector)}$$

$\mathbf{h}_t$: tth column of **H**

PDA Based Large-MIMO Detection [15]

- Define $p_i^{j+} \triangleq P(b_i^{(j)} = +1)$ and $p_i^{j-} \triangleq P(b_i^{(j)} = -1)$
- To compute $\beta_i^{(j)}$, approximate the distribution of $\widetilde{\mathbf{n}}$ to be Gaussian
- Mean of $\mathbf{y}$

$$\boldsymbol{\mu}_i^{j+} \triangleq \mathbb{E}(\mathbf{y}|b_i^{(j)} = +1) = \mathbf{h}_{qi+j} + \sum_{l=0}^{2k-1} \sum_{\substack{m=0 \\ m \neq q(i-l)+j}}^{q-1} \mathbf{h}_{ql+m}(2p_l^{m+} - 1)$$

$$\boldsymbol{\mu}_i^{j-} \triangleq \mathbb{E}(\mathbf{y}|b_i^{(j)} = -1) = \boldsymbol{\mu}_i^{j+} - 2\mathbf{h}_{qi+j}$$

- Covariance of $\mathbf{y}$

$$\mathbf{C}_i^j = \sigma^2 \mathbf{I}_{2N_rp} + \sum_{l=0}^{2k-1} \sum_{\substack{m=0 \\ m \neq q(i-l)+j}}^{q-1} \mathbf{h}_{ql+m}\, \mathbf{h}_{ql+m}^T\, 4p_l^{m+}(1 - p_l^{m+})$$

PDA Based Large-MIMO Detection [15]

- Using $\boldsymbol{\mu}_i^{j\pm}$ and $\mathbf{C}_i^j$, $P(\mathbf{y}|b_i^{(j)} = \pm 1)$ can be written as

$$P(\mathbf{y}|b_i^{(j)} = \pm 1) \;=\; \frac{e^{-(\mathbf{y}-\boldsymbol{\mu}_i^{j\pm})^T(\mathbf{C}_i^j)^{-1}(\mathbf{y}-\boldsymbol{\mu}_i^{j\pm})}}{(2\pi)^{N_r p}|\mathbf{C}_i^j|^{\frac{1}{2}}}$$

- Using (5), β_i^j can be written as

$$\beta_i^j\beta_i^j \;=\; e^{-\left((\mathbf{y}-\boldsymbol{\mu}_i^{j+})^T(\mathbf{C}_i^j)^{-1}(\mathbf{y}-\boldsymbol{\mu}_i^{j+})-(\mathbf{y}-\boldsymbol{\mu}_i^{j-})^T(\mathbf{C}_i^j)^{-1}(\mathbf{y}-\boldsymbol{\mu}_i^{j-})\right)}$$

- Compute $\Lambda_i^{(j)}$ using $\alpha_i^{(j)}$ and $\beta_i^{(j)}$
- Update the statistics of $b_i^{(j)}$ as

$$P(b_i^{(j)} = +1|\mathbf{y}) \;=\; \frac{\Lambda_i^{(j)}}{1+\Lambda_i^{(j)}}, \quad P(b_i^{(j)} = -1|\mathbf{y}) \;=\; \frac{1}{1+\Lambda_i^{(j)}}$$

- This completes one iteration of the algorithm

PDA Based Large-MIMO Detection [15]

- Updated values of $P(b_i^{(j)} = +1|\mathbf{y})$ and $P(b_i^{(j)} = -1|\mathbf{y})$ for all i, j are fed back as a priori probabilities to the next iteration
- Algorithm terminates after a certain number of iterations
- At the end of the last iteration,
  - decide $\widehat{b}_i^{(j)}$ as $+1$ if $\Lambda_i^{(j)} \geq 1$, and -1 otherwise
- In coded systems
  - feed $\Lambda_i^{(j)}$'s as soft inputs to the decoder

Performance of PDA in V-BLAST

* PDA algorithm also exhibits large-dimension behavior

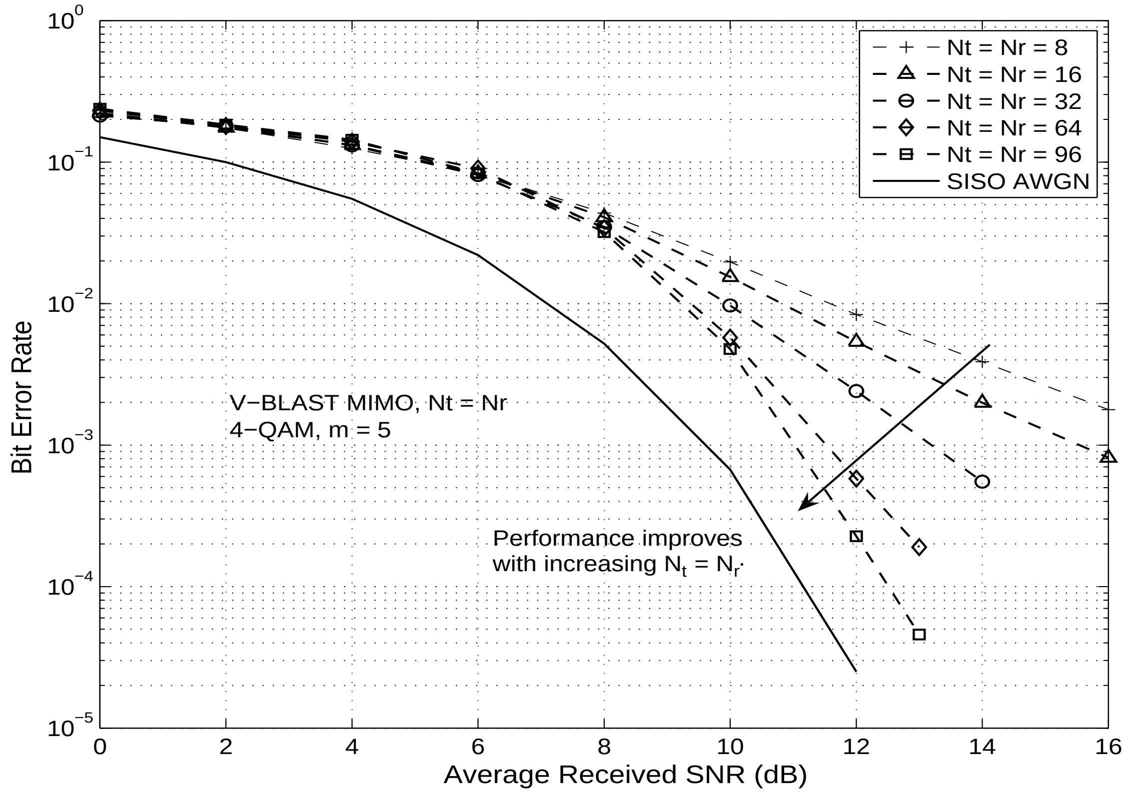
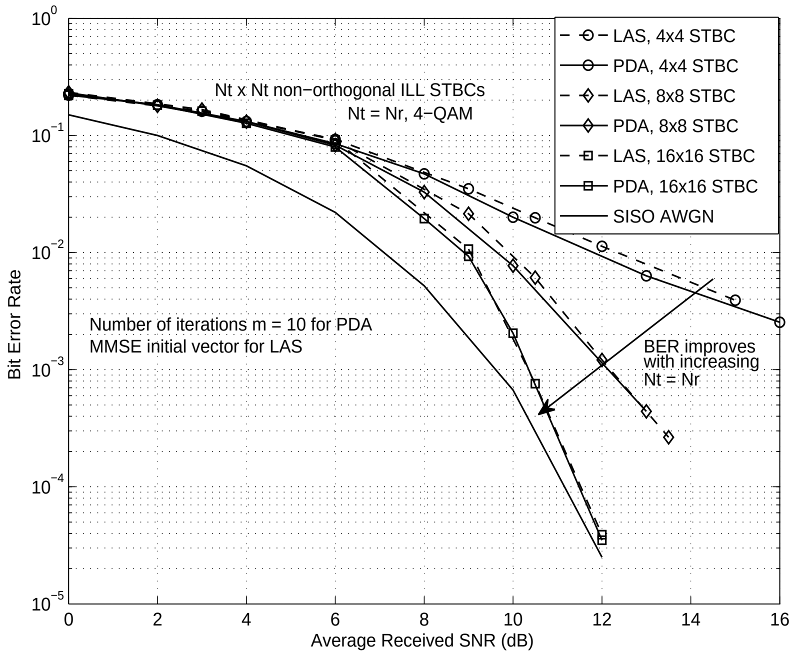
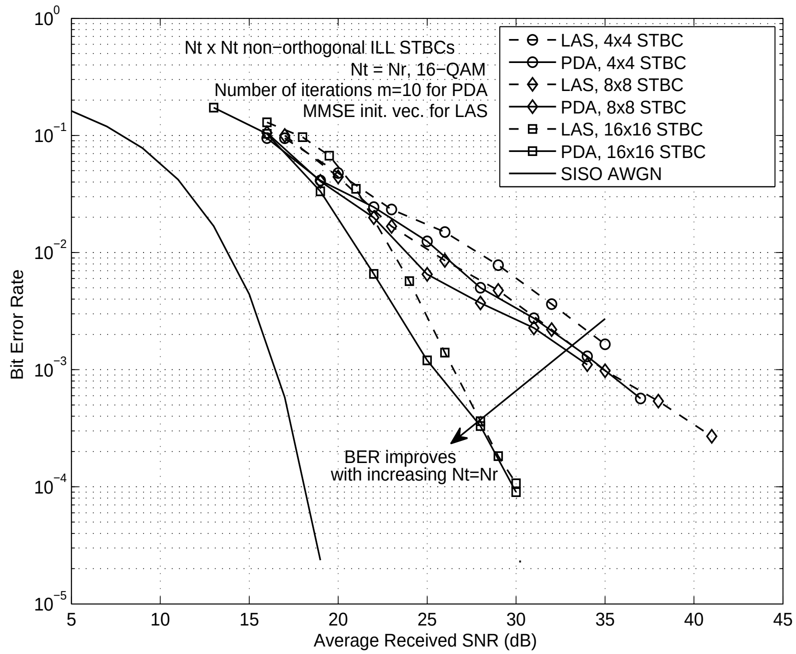


Figure 7: BER performance of PDA based detection of V-BLAST MIMO for $N_t = N_r = 8, 16, 32, 64, 96$, 4-QAM, and $m = 5$ iterations.

Performance of PDA in NO-STBC [15]



(a) 4-QAM



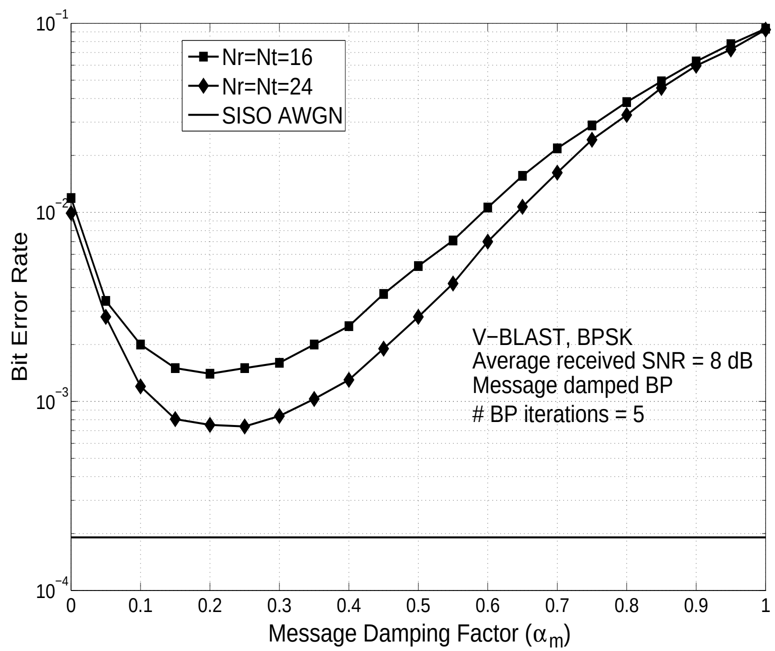
(b) 16-QAM

Large-MIMO Detection Based on Graphical Models

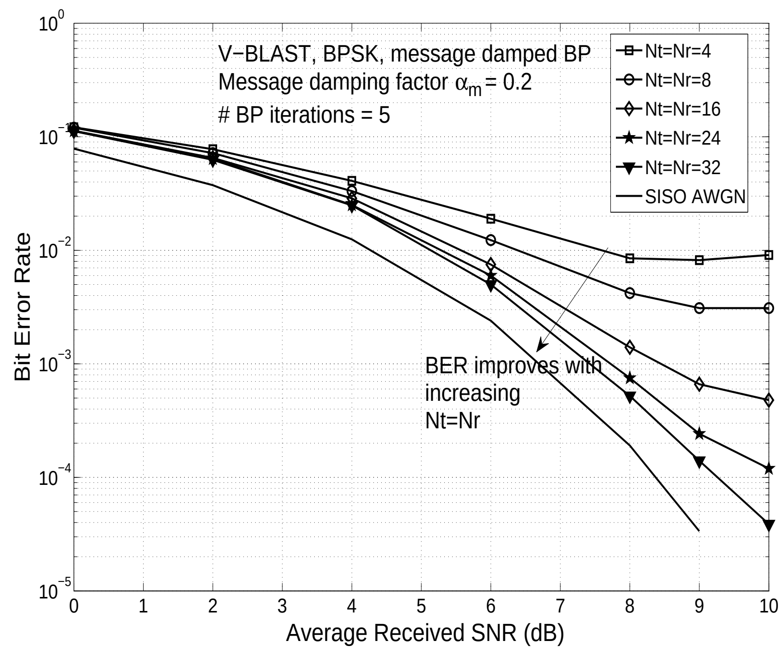
- Belief propagation (BP) is proven to work in cycle-free graphs
- BP is often successful in graphs with cycles as well
- MIMO graphical models are fully/densely connected
- Graphical models with certain simplifications/assumptions work successfully in large-MIMO detection

1. Use of Pairwise Markov Random Field (MRF) based graphical model in conjunction with message/belief damping [16],[17]
2. Use of Factor Graph (FG) based graphical model with Gaussian Approximation of Interference (GAI) [17]

Performance of Damped BP on Pairwise MRF [17]



(c) Performance as a function of damping factor



(d) BP on pairwise MRF exhibits large-dimension behavior

- Damping significantly improves performance
- Order of per-symbol complexity: $O(N_t^2)$

Large-MIMO Detection using BP on FGs [17]

- Each entry of the vector $\mathbf{y}$ is treated as a function node (observation node)
- Each symbol, $x_i \in \{\pm 1\}$, is treated as a variable node
- Key ingredient: Gaussian approximation of the interference

$$y_i \;=\; h_{ik}x_k + \underbrace{\overbrace{\sum_{j=1, j\neq k}^{2N_t} h_{ij}x_j}^{interference} + n_i}_{\triangleq\, z_{ik}},$$

is modeled as $\mathbb{C}\mathcal{N}(\mu_{z_{ik}}, \sigma^2_{z_{ik}})$ with $\mu_{z_{ik}} = \sum_{j=1,j\neq k}^{N_t} h_{ij}\mathbb{E}(x_j)$, and $\sigma^2_{z_{ik}} = \sum_{j=1,j\neq k}^{2N_t} |h_{ij}|^2 \,\mathrm{Var}(x_j) + \frac{\sigma^2}{2}$, where h_{ij} is the (i,j)th element in $\mathbf{H}$

Large-MIMO Detection using BP on FGs [17]

- With x_i's $\in \{\pm 1\}$, the log-likelihood ratio (LLR) of x_k at observation node i, denoted by Λ_i^k, is

$$\Lambda_i^k \;=\; \log \frac{p(y_i|\mathbf{H}, x_k = 1)}{p(y_i|\mathbf{H}, x_k = -1)} \;=\; \frac{2}{\sigma^2_{z_{ik}}} \Re\left(h^*_{ik}(y_i - \mu_{z_{ik}})\right)$$

- LLR values computed at observation nodes are passed to variable nodes.
- Using these LLRs, variable nodes compute the probabilities

$$p_i^{k+} \;\stackrel{\triangle}{=}\; p_i(x_k = +1|\mathbf{y}) \;=\; \frac{\exp(\sum_{l \neq i} \Lambda_l^k)}{1 + \exp(\sum_{l \neq i} \Lambda_l^k)}$$

  and pass them back to the observation nodes.

- This message passing is carried out for a certain number of iterations.
- At the end, x_k is detected as

$$\widehat{x}_k \;=\; \text{sgn}\left(\sum_{i=1}^{2N_r} \Lambda_i^k\right)$$

Message Passing on Factor Graphs [17]

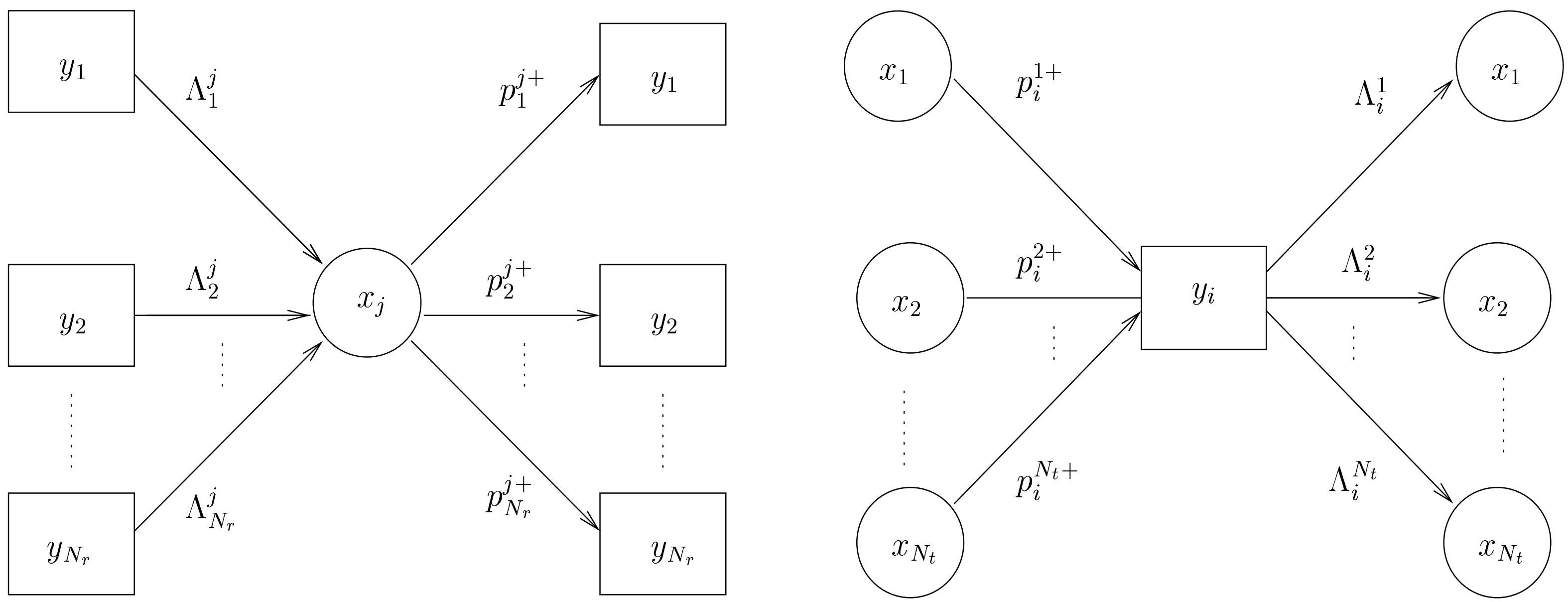
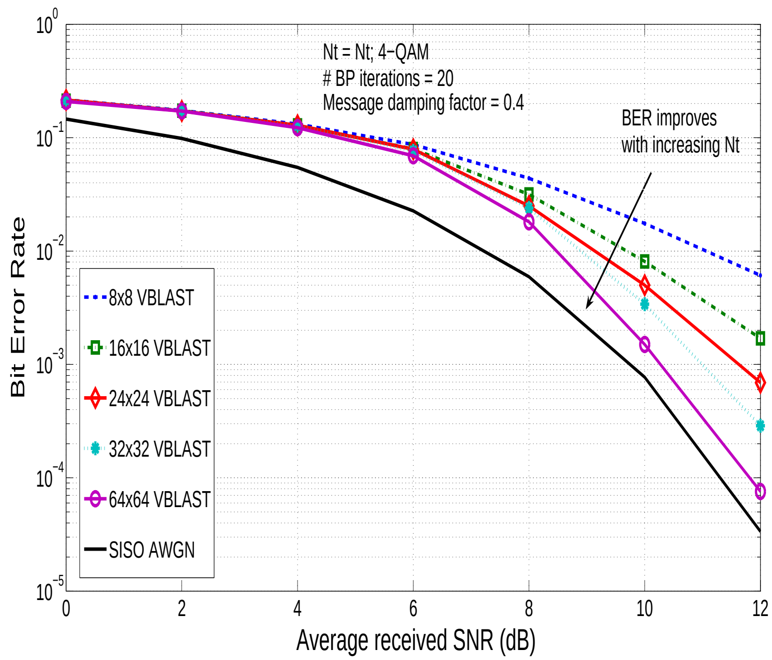
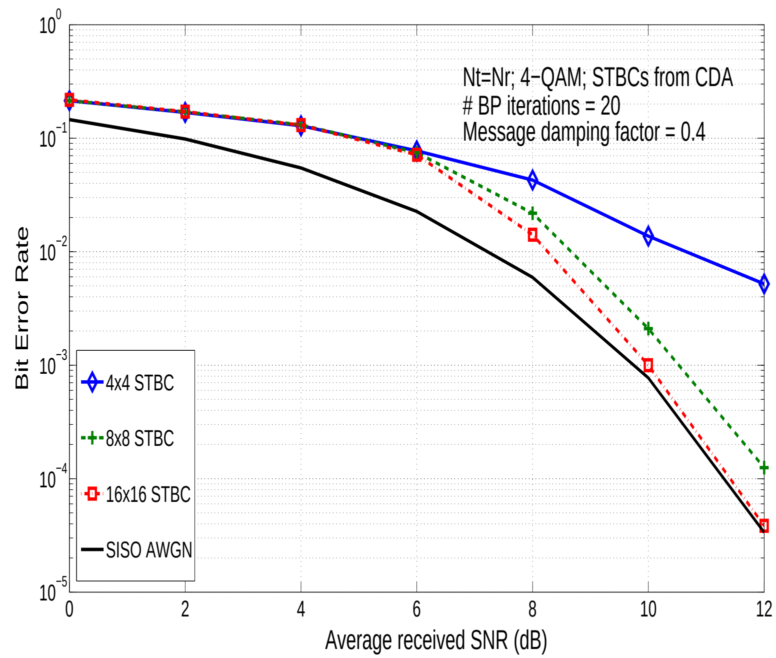


Figure 8: Message passing between variable nodes and observation nodes.

Performance of BP on FGs with GAI [17]



(a) V-BLAST



(b) Non-Orthogonal STBC

- BP with GAI achieves near-optimal performance for increasing $N_t = N_r$ with $O(N_t)$ per-symbol complexity

Large-MIMO Applications/Standardization

- Potential Applications
  - Fixed Wireless IPTV/HDTV distribution (e.g., in 5 GHz band)
    - potentially big markets in India
  - High-speed back haul connectivity between BSs/BSCs using high data rate large-MIMO links (e.g., in 5 GHz band)
  - Wireless mesh networks
- Large-MIMO in Wireless Standards?
  - Multi-Gigabit Rate LAN/PAN (e.g., in 5GHz / 60 GHz band)
    - Evolution of WiFi standards (IEEE 802.11ac and 802.11ad)
  - LTE-Advanced, WiMax (IEEE 802.16m)
  - Can consider 12×12, 16×16, 24×24, 32×32 MIMO systems

Concluding Remarks

- Low-complexity detection
  - critical enabling technology for large-MIMO
  - no more a bottleneck
- Large-MIMO systems can be implemented
- Large-MIMO approach scores high on spectral efficiency and operating SNR compared to other approaches (e.g., increasing QAM size)
- Standardization efforts can consider reaping the benefits of large-MIMO in their evolution

Thank You