A Low-Complexity Detector for Large MIMO Systems and Multicarrier CDMA Systems

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***Abstract*— We consider large MIMO systems, where by '*large*' we mean number of transmit and receive antennas of the order of tens to hundreds. Such large MIMO systems will be of immense interest because of the very high spectral efficiencies possible in such systems. We present a low-complexity detector which achieves uncoded *near-exponential diversity* performance for hundreds of antennas (i.e., achieves near SISO AWGN performance in a large MIMO fading environment) with an average per-bit complexity of just $O(N_tN_r)$, where N_t and N_r denote the number of transmit and receive antennas, respectively. With an outer turbo code, the proposed detector achieves good coded bit error performance as well. For example, in a 600 transmit and 600 receive antennas V-BLAST system with a high spectral efficiency of 450 bps/Hz (using BPSK and rate-3/4 turbo code), our simulation results show that the proposed detector performs to within about 7 dB from capacity. This practical feasibility of the proposed high-performance, low-complexity detector could potentially trigger wide interest in the theory and implementation of large MIMO systems. We also illustrate the applicability of the proposed detector in the low-complexity detection of high-rate, non-orthogonal space-time block codes and large multicarrier CDMA (MC-CDMA) systems. In large MC-CDMA systems with hundreds of users, the proposed detector is shown to achieve near single-user performance at an average per-bit complexity linear in number of users, which is quite appealing for its use in practical CDMA systems.**

***Index Terms*— Large MIMO systems, V-BLAST, low-complexity detection, near-exponential diversity, high spectral efficiency, space-time block codes, multicarrier CDMA.**

I. INTRODUCTION

MULTIPLE-input multiple-output (MIMO) systems with multiple antennas at both transmitter and receiver sides have become very popular owing to the several advantages they promise to offer, including transmit diversity and high data rates [1]-[3]. It is known that the MIMO channels have a capacity that grows linearly with the minimum of the number of antennas on the transmitter and receiver sides [4]-[6]. A key component of a MIMO system is the MIMO detector at the receiver, whose job is to recover the symbols that are transmitted simultaneously from multiple transmitting antennas. In practical applications, the MIMO detector is often the bottleneck for the overall performance and complexity.

MIMO detectors including sphere decoder and several of its variants [7]-[14] achieve near-maximum likelihood (ML) performance at the cost of high complexity. Other well known detectors including ZF (zero forcing), MMSE (minimum mean square error), and ZF-SIC (ZF with successive interference cancellation) detectors [3],[15] are attractive from a complexity view point, but achieve relatively poor performance. For example, the ZF-SIC detector (i.e., the well known V-BLAST detector with ordering [16],[17]) does not achieve the full diversity in the system. The MMSE-SIC detector has been shown to achieve optimal performance [3]. However, the order of per-bit complexity involved in MMSE-SIC and ZF-SIC detectors is cubic in number of antennas. Even reduced complexity detectors (e.g., [18]) are prohibitively complex for large number of antennas of the order of tens to hundreds. With small number of antennas, the full potential of MIMO (in terms of achieving high capacities using large number of antennas) is not achieved. A main issue with using large number of antennas, however, is the high detection complexities involved.

Our focus in this paper is on large MIMO systems, where by '*large*' we mean number of transmit and receive antennas of the order of tens to hundreds. Such large MIMO systems will be of immense interest because of the very high spectral efficiencies possible in such systems. For example, in a V-BLAST system, increased number of transmit antennas means increased data rate without bandwidth increase. However, major bottlenecks in realizing such large MIMO systems include i) physical placement of such a large number of antennas in communication terminals; for small terminal sizes, this would require a high carrier frequency operation, i.e., small carrier wavelengths for $\lambda/2$ separation to ensure independent fade coefficients[1], ii) lack of practical low-complexity detectors for such large systems, and iii) associated channel estimation issues. In this paper, we primarily address the second problem (i.e., low-complexity large MIMO detection). Specifically, we present a low-complexity detector for large MIMO systems that employ spatial multiplexing (i.e., V-BLAST), and evaluate its performance without and with channel estimation errors.

The proposed low-complexity detector has its roots in past work on Hopfield neural network (HNN) based algorithms for image restoration [19]-[22], which are meant to handle large

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[1]We, however, point out that there can be several large MIMO applications where antenna placing need not be a major issue. An example of such an scenario is to provide high-speed backbone connectivity in wireless mesh networks using large MIMO links, where large number of antennas can be placed at the backbone base stations. Also, tens of antennas can be placed in moderately sized communication terminals (e.g., laptops, set top boxes).

digital images (e.g., 512×512 image with 2^{18} pixels). In [23],[24], Sun applied his HNN based image restoration algorithms in [20]-[22] to multiuser detection (MUD) in CDMA systems on AWGN channels. This detector, referred to as the likelihood ascent search (LAS) detector, essentially searches out a sequence of bit vectors with monotonic likelihood ascent and converges to a fixed point in finite number of steps [23],[24]. The power of the LAS detector for CDMA lies in $i)$ its linear average per-bit complexity in number of users, and $ii)$ its ability to perform very close to ML detector for large number of users [23],[24]. Taking the cue from LAS detector's complexity and performance superiority in *large systems*, we, in this paper, successfully adopt the LAS detector for large MIMO systems that employ spatial multiplexing and report interesting results.

We refer to the proposed detector as MF/ZF/MMSE-LAS[2] detector depending on the initial vector used in the algorithm; MF-LAS detector uses the matched filter output as the initial vector, and ZF-LAS and MMSE-LAS detectors employ ZF and MMSE outputs, respectively, as the initial vector. Our adoption of HNN algorithms to MIMO detection in this paper, we believe, is a powerful development in large MIMO detection research. Our major findings in this paper are summarized as follows:

- In an uncoded V-BLAST system with BPSK, the proposed detector achieves '*near-exponential diversity*' performance for hundreds of antennas (i.e., achieves near SISO AWGN performance). For example, the proposed detector nearly renders a 200×200 MIMO fading channel into 200 parallel, non-interfering SISO AWGN channels.
- The detector achieves this near-exponential diversity performance with an average per-bit complexity of just $O(N_tN_r)$, where N_t and N_r denote the number of transmit and receive antennas, respectively.
- The near-exponential diversity achieving feature for hundreds of antennas at practically affordable complexity is a remarkable attribute of the proposed detector. For example, the average per-bit complexity of the well known ZF-SIC detector [16] is $O(N_t^2N_r)$, and it does not achieve full diversity[3].
- For large number of antennas, the MF-LAS, ZF-LAS, and MMSE-LAS detectors achieve almost the same performance. Hence, in such cases, the MF-LAS is preferred because of its lesser complexity than ZF/MMSE-LAS; MF-LAS does not need the matrix inversion operation needed in ZF-LAS and MMSE-LAS.
- With an outer turbo code, the proposed detector achieves good coded bit error performance as well. For example, in a 600 transmit and 600 receive antennas V-BLAST system with a high spectral efficiency of 450 bps/Hz (using BPSK and rate-3/4 turbo code), our simulation results show that the proposed detector performs to within about 7 dB from capacity.
- In terms of the effect of channel estimation errors on performance, our simulation results show that in a 200×200 V-BLAST system with BPSK and rate-1/2 turbo code, the degradation in coded BER performance of the proposed MMSE-LAS detector due to channel estimation errors is only about 0.2 dB and 0.6 dB for estimation error variances of 0.01 and 0.05, respectively, compared to perfect channel estimation.
- We also illustrate the applicability of the proposed detector in the low-complexity detection of high-rate, non-orthogonal space-time block codes (STBC) and large multicarrier CDMA (MC-CDMA) systems. In large MC-CDMA systems (with hundreds and thousands of users), the proposed detector is shown to achieve near single-user performance, at an average per-bit complexity linear in number of users, which is quite appealing for its use in practical CDMA systems.

The rest of the paper is organized as follows. In Sec. II, we present the proposed LAS detector for V-BLAST systems and its complexity. Detailed uncoded and coded bit error performance results for V-BLAST and high-rate, non-orthogonal STBC are presented in Sec. III. The LAS detector for MC-CDMA and the corresponding performance results are presented in Sec. IV. Conclusions and possible future research are presented in Sec. V.

II. Proposed Detector for V-BLAST

In this section, we present the proposed LAS detector for V-BLAST and its complexity. Consider a V-BLAST system with N_t transmit antennas and N_r receive antennas, $N_t \leq N_r$, where N_t symbols are transmitted from N_t transmit antennas simultaneously. Let $b_j \in \{+1, -1\}$ be the symbol[4] transmitted by the jth transmit antenna. Each transmitted symbol goes through the wireless channel to arrive at each of N_r receive antennas. Denote the path gain from transmit antenna j to receive antenna k by h_{kj}. Considering a baseband discrete-time model for a flat fading MIMO channel, the signal received at antenna k, denoted by y_k, is given by

$$y_k \;=\; \sum_{j=1}^{N_t} h_{kj} b_j + n_k. \tag{1}$$

The $\{h_{kj}\}$, $\forall k \in \{1, 2, \cdots, N_r\}$, $\forall j \in \{1, 2, \cdots, N_t\}$, are assumed to be i.i.d. complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E\big[\big(h_{kj}^I\big)^2\big] = E\big[\big(h_{kj}^Q\big)^2\big] = 0.5$, where h_{kj}^I and h_{kj}^Q are the real and imaginary parts of h_{kj}. The noise sample at the kth receive antenna, n_k, is assumed to be complex Gaussian with zero mean, and $\{n_k\}$, $k = 1, 2, \cdots, N_r$, are assumed to be independent with $E[n_k^2] = N_0 = \frac{N_t E_s}{\gamma}$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna [2]. Collecting the

[2]Throughout the paper, whenever we write MF/ZF/MMSE-LAS, we mean MF-LAS, ZF-LAS, and MMSE-LAS.

[3]In V-BLAST systems, the ZF, MMSE, and ZF-SIC (without ordering) detectors achieve only $N_r - N_t + 1$ order diversity but have varying SNR loss [1], i.e., for $N_t = N_r$, these detectors achieve only first order diversity. The ZF-SIC (with ordering) can have more than $N_r - N_t + 1$ order diversity because of the ordering (i.e., selection) but it still does not achieve full diversity. The proposed detector, however, nearly achieves N_t order diversity for large N_t (i.e., near-exponential diversity for large N_t).

[4]Although we present the detector assuming BPSK here, it is applicable to M-QAM and M-PAM as well. Accordingly, we present simulation results for 16-PAM and 16-QAM also in Sec. III-B.

received signals from all receive antennas, we write[5]

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = [\, y_1 \; y_2 \; \cdots \; y_{N_r} \,]^T$ is the N_r-length received signal vector, $\mathbf{b} = [\, b_1 \; b_2 \; \cdots \; b_{N_t} \,]^T$ is the N_t-length transmitted bit vector, $\mathbf{H}$ denotes the $N_r \times N_t$ channel matrix with channel coefficients $\{h_{kj}\}$, and $\mathbf{n} = [\, n_1 \; n_2 \; \cdots \; n_{N_r} \,]^T$ is the N_r-length noise vector. $\mathbf{H}$ is assumed to be known perfectly at the receiver[6], but not at the transmitter.

A. Proposed LAS Algorithm

The proposed LAS algorithm essentially searches out a sequence of bit vectors until a fixed point is reached; this sequence is decided based on an update rule. In the V-BLAST system considered, for ML detection [15], the most likely $\mathbf{b}$ is taken as that $\mathbf{b}$ which maximizes

$$\Lambda(\mathbf{b}) = \mathbf{b}^T\mathbf{H}^H\mathbf{y} + \mathbf{b}^T\left(\mathbf{H}^H\mathbf{y}\right)^* - \mathbf{b}^T\mathbf{H}^H\mathbf{H}\mathbf{b}. \tag{3}$$

The likelihood function in (3) can be written as

$$\Lambda(\mathbf{b}) = \mathbf{b}^T\mathbf{y}_{eff} - \mathbf{b}^T\mathbf{H}_{eff}\mathbf{b}, \tag{4}$$

where

$$\mathbf{y}_{eff} = \mathbf{H}^H\mathbf{y} + \left(\mathbf{H}^H\mathbf{y}\right)^*, \tag{5}$$

$$\mathbf{H}_{eff} = \mathbf{H}^H\mathbf{H}. \tag{6}$$

Update Criterion in the Search Procedure: Let $\mathbf{b}(n)$ denote the bit vector tested by the LAS algorithm in the search step n. The starting vector $\mathbf{b}(0)$ can be the output vector from any known detector. When the output vector of the MF detector is taken as the $\mathbf{b}(0)$, we call the resulting LAS detector as the MF-LAS detector. We define ZF-LAS and MMSE-LAS detectors likewise. Given $\mathbf{b}(n)$, the algorithm obtains $\mathbf{b}(n+1)$ through an update rule until a fixed point is reached. The update is made in such a way that the change in likelihood from step n to $n+1$, denoted by $\Delta\Lambda\left(\mathbf{b}(n)\right)$, is positive, i.e.,

$$\Delta\Lambda\left(\mathbf{b}(n)\right) \stackrel{\triangle}{=} \Lambda\left(\mathbf{b}(n+1)\right) - \Lambda\left(\mathbf{b}(n)\right) \geq 0. \tag{7}$$

An expression for the above change in likelihood can be obtained in terms of the gradient of the likelihood function as follows. Let $\mathbf{g}(n)$ denote the gradient of the likelihood function evaluated at $\mathbf{b}(n)$, i.e.,

$$\mathbf{g}(n) \stackrel{\triangle}{=} \frac{\partial\left(\Lambda(\mathbf{b}(n))\right)}{\partial\left(\mathbf{b}(n)\right)} = \mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n), \tag{8}$$

where

$$\mathbf{H}_{real} = \mathbf{H}_{eff} + \left(\mathbf{H}_{eff}\right)^* = 2\,\Re\left(\mathbf{H}_{eff}\right). \tag{9}$$

[5]We adopt the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^*$, and $[.]^H$ denote transpose, conjugate, and conjugate transpose operations, respectively. $\Re(.)$ and $\Im(.)$ denote the real and imaginary parts of the complex argument.

[6]The effect of imperfect channel estimates at the receiver is illustrated in the simulation results in Sec. III.

Using (4) in (7), we can write

$$\begin{aligned}\Delta\Lambda\left(\mathbf{b}(n)\right) &= \mathbf{b}^T(n+1)\mathbf{y}_{eff} - \mathbf{b}^T(n+1)\mathbf{H}_{eff}\mathbf{b}(n+1)\\ &\quad - \left(\mathbf{b}^T(n)\mathbf{y}_{eff} - \mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n)\right)\\ &= \left(\mathbf{b}^T(n+1) - \mathbf{b}^T(n)\right)\left(\mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n)\right)\\ &\quad - \left(\mathbf{b}^T(n+1) - \mathbf{b}^T(n)\right)\left(-\mathbf{H}_{real}\mathbf{b}(n)\right)\\ &\quad - \mathbf{b}^T(n+1)\mathbf{H}_{eff}\mathbf{b}(n+1) + \mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n).\end{aligned} \tag{10}$$

Now, defining

$$\Delta\mathbf{b}(n) \stackrel{\triangle}{=} \mathbf{b}(n+1) - \mathbf{b}(n), \tag{11}$$

and $i)$ observing that $\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n) = 2\mathbf{b}^T(n)\mathbf{H}_{eff}\mathbf{b}(n)$, $ii)$ adding & subtracting the term $\frac{1}{2}\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n+1)$ to the RHS of (10), and $iii)$ further observing that $\mathbf{b}^T(n)\mathbf{H}_{real}\mathbf{b}(n+1) = \mathbf{b}^T(n+1)\mathbf{H}_{real}\mathbf{b}(n)$, we can simplify (10) as

$$\begin{aligned}\Delta\Lambda\left(\mathbf{b}(n)\right) &= \Delta\mathbf{b}^T(n)\left(\mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n)\right)\\ &\quad - \frac{1}{2}\Delta\mathbf{b}^T(n)\mathbf{H}_{real}\Delta\mathbf{b}(n)\\ &= \Delta\mathbf{b}^T(n)\Big(\mathbf{g}(n) + \frac{1}{2}\mathbf{z}(n)\Big),\end{aligned} \tag{12}$$

where

$$\mathbf{z}(n) = -\mathbf{H}_{real}\Delta\mathbf{b}(n). \tag{13}$$

Now, given $\mathbf{y}_{eff}$, $\mathbf{H}_{eff}$, and $\mathbf{b}(n)$, the objective is to obtain $\mathbf{b}(n+1)$ from $\mathbf{b}(n)$ such that $\Delta\Lambda(\mathbf{b}(n))$ in (12) is positive. Potentially any one or several bits in $\mathbf{b}(n)$ can be flipped (i.e., changed from +1 to -1 or vice versa) to get $\mathbf{b}(n+1)$. We refer to the set of bits to be checked for possible flip in a step as a *check candidate set*. Let $L(n) \subseteq \{1, 2, \cdots, N_t\}$ denote the check candidate set at step n. With the above definitions, it can be seen that the likelihood change at step n, given by (12), can be written as

$$\Delta\Lambda(\mathbf{b}(n)) = \sum_{j \in L(n)} \Delta b_j(n)\Big[g_j(n) + \frac{1}{2}z_j(n)\Big], \tag{14}$$

where $b_j(n)$, $g_j(n)$, and $z_j(n)$ are the jth elements of the vectors $\mathbf{b}(n)$, $\mathbf{g}(n)$, and $\mathbf{z}(n)$, respectively. As shown in [23],[24] for synchronous CDMA on AWGN, the following update rule can be easily shown to achieve monotonic likelihood ascent (i.e., $\Delta\Lambda(\mathbf{b}(n)) > 0$ if there is at least one bit flip) in the V-BLAST system as well.

LAS Update Algorithm: Given $L(n) \subseteq \{1, 2, \cdots, N_t\}, \forall n \geq 0$ and an initial bit vector $\mathbf{b}(0) \in \{-1, +1\}^{N_t}$, bits in $\mathbf{b}(n)$ are updated as per the following update rule:

$$b_j(n+1) = \begin{cases} +1, & \text{if } j \in L(n),\ b_j(n) = -1 \\ & \text{and } g_j(n) > t_j(n), \\ -1, & \text{if } j \in L(n),\ b_j(n) = +1 \\ & \text{and } g_j(n) < -t_j(n), \\ b_j(n), & \text{otherwise}, \end{cases} \tag{15}$$

where $t_j(n)$ is a threshold for the jth bit in the nth step, which, similar to the threshold in [23],[24], is taken to be

$$t_j(n) = \sum_{i \in L(n)} \left|(\mathbf{H}_{real})_{j,i}\right|, \ \forall j \in L(n), \tag{16}$$

where $(\mathbf{H}_{real})_{j,i}$ is the element in the jth row and ith column of the matrix $\mathbf{H}_{real}$. It can be shown, as in [23],[24], that

$t_j(n)$ in (16) is the minimum threshold that ensures monotonic likelihood ascent.

It is noted that different choices can be made to specify the sequence of $L(n), \forall n \geq 0$. One of the simplest sequences correspond to checking one bit in each step for a possible flip, which is termed as a sequential LAS (SLAS) algorithm with constant threshold, $t_j = \left| (\mathbf{H}_{real})_{j,j} \right|$. The sequence of $L(n)$ in SLAS can be such that the indices of bits checked in successive steps are chosen circularly or randomly. Checking of multiple bits for possible flip is also possible. Let $L_f(n) \subseteq L(n)$ denote the set of indices of the bits flipped according to the update rule in (15) at step n. Then the updated bit vector $\mathbf{b}(n+1)$ can be written as

$$\mathbf{b}(n+1) \;=\; \mathbf{b}(n) - 2 \sum_{i \in L_f(n)} b_i(n)\mathbf{e}_i, \qquad (17)$$

where $\mathbf{e}_i$ is the ith coordinate vector. Using (17) in (8), the gradient vector for the next step can be obtained as

$$\begin{aligned} \mathbf{g}(n+1) &= \mathbf{y}_{eff} - \mathbf{H}_{real}\mathbf{b}(n+1) \\ &= \mathbf{g}(n) + 2 \sum_{i \in L_f(n)} b_i(n) \left(\mathbf{H}_{real}\right)_i \;, \end{aligned} \qquad (18)$$

where $(\mathbf{H}_{real})_i$ denotes the ith column of the matrix $\mathbf{H}_{real}$. The LAS algorithm keeps updating the bits in each step based on the update rule given in (15) until $\mathbf{b}(n) = \mathbf{b}_{fp}, \forall n \geq n_{fp}$ for some $n_{fp} \geq 0$, in which case $\mathbf{b}_{fp}$ is a fixed point, and it is taken as the detected bit vector and the algorithm terminates.

B. Complexity of the Proposed Detector for V-BLAST

The complexity of the proposed detector is analyzed as follows. Given an initial vector, the LAS algorithm part alone has an average per-bit complexity of $O(N_tN_r)$. This can be explained as follows. The complexity involved in the LAS algorithm is due to three components: $i)$ initial computation of $\mathbf{g}(0)$ in (8), $ii)$ update of $\mathbf{g}(n)$ in each step as per (18), and $iii)$ the average number of steps required to reach a fixed point. Computation of $\mathbf{g}(0)$ requires the computation of $\mathbf{H}^H\mathbf{H}$ for each MIMO fading channel realization (see Eqns. (8), (9), and (6)), which requires a per-bit complexity of order $O(N_tN_r)$. Update of $\mathbf{g}(n)$ in the nth step as per (18) using sequential LAS requires a complexity of $O(N_t)$, and hence a constant per-bit complexity. Regarding the complexity component $iii)$, we obtained the average number of steps required to reach a fixed point for sequential LAS through simulations. We observed that the average number of steps required is linear in N_t, i.e., constant per-bit complexity where the constant depends on SNR, N_t, N_r, and the initial vector (see Fig. 5). Putting the above complexity components $i)$, $ii)$, and $iii)$ together, we see that the average per-bit complexity of the LAS algorithm alone is $O(N_tN_r)$.

In addition to the above complexity, the initial vector generation also contributes to the overall complexity. The average per-bit complexity of generating initial vectors using MF, ZF, and MMSE are $O(N_r)$, $O(N_tN_r)$, and $O(N_tN_r)$, respectively. The higher complexity of ZF and MMSE compared to MF is because of the need to perform matrix inversion operation in ZF/MMSE. Again, putting the complexities of the LAS part and the initial vector generation part together, we see that the overall average per-bit complexity of the proposed MF/ZF/MMSE-LAS detector for V-BLAST is $O(N_tN_r)$.

This is in contrast with the well known ZF-SIC detector with ordering[7], whose per-bit complexity is $O(N_t^2N_r)$. Thus, proposed MF/ZF/MMSE-LAS enjoys a clear complexity advantage over ZF-SIC by an order of N_t. Thus, while the ZF-SIC becomes prohibitively complex for large number of antennas of the order of hundreds, the low-complexity attribute of MF/ZF/MMSE-LAS makes it practically viable. This advantage is well illustrated in the next section, where we show the simulation points up to 10^{-5} uncoded BER with 400 antennas – these BER results for ZF-LAS with 400 antennas were obtained within few hours of simulation run time, whereas simulation points for ZF-SIC for such large number of antennas were found to require several days of simulation run time (so we do not give ZF-SIC performance for 400 antennas).

We further point out that several detectors other than ZF-SIC, including detectors based on sphere decoding and its low-complexity variants [8],[9],[10], Markov Chain Monte Carlo (MCMC) techniques [11],[12], lattice reduction techniques [13], QR decomposition based methods [14], too have higher complexity than $O(N_tN_r)$, and hence have prohibitively high complexities for hundreds of antennas. Many of these detectors have been shown to achieve near-ML performance, but only for number of antennas typically less than ten. However, because of their higher complexities, we note that simulating these detectors is prohibitively time-consuming for hundreds of antennas. So, we could not present the BER performance of these detectors for comparison with that of the proposed detector. As we will show in the next section, a remarkable advantage of the proposed detector is that, at a much lesser and practical complexity, it achieves near-ML performance for hundreds of antennas. Alternatively put, in a large system setting, the proposed detector wins over the above detectors in complexity while matching them in near-ML performance, whereas it wins over ZF-SIC in both complexity as well as performance.

III. Results and Discussions for V-BLAST

In this section, we present the BER performance of the proposed LAS detector for V-BLAST obtained through extensive simulations, and compare with those of other detectors including ZF-SIC, MF, ZF, and MMSE. The LAS algorithm used is the sequential LAS with circular checking of bits starting from the first antenna bit. We present the performance results under two headings, namely, $i)$ uncoded BER performance, and $ii)$ turbo coded BER performance. We also quantify how far is the proposed detector's turbo coded performance away from the theoretical capacity. The SNRs in all the BER performance figures are the average received SNR per received antenna, γ, defined in Sec. II.

A. Uncoded BER Performance

MF/ZF-LAS performs increasingly better than ZF-SIC for increasing $N_t = N_r$: In Fig. 1, we plot the uncoded BER

[7]Henceforth, we use the term 'ZF-SIC' to always refer 'ZF-SIC with ordering'.

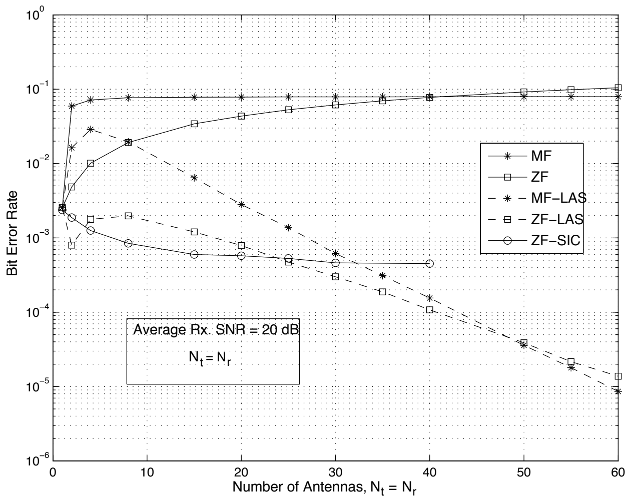

Fig. 1. Uncoded BER performance of MF/ZF-LAS detectors as a function of number of transmit/receive antennas ($N_t = N_r$) for V-BLAST at an average received SNR = 20 dB. BPSK, N_t bps/Hz spectral efficiency.

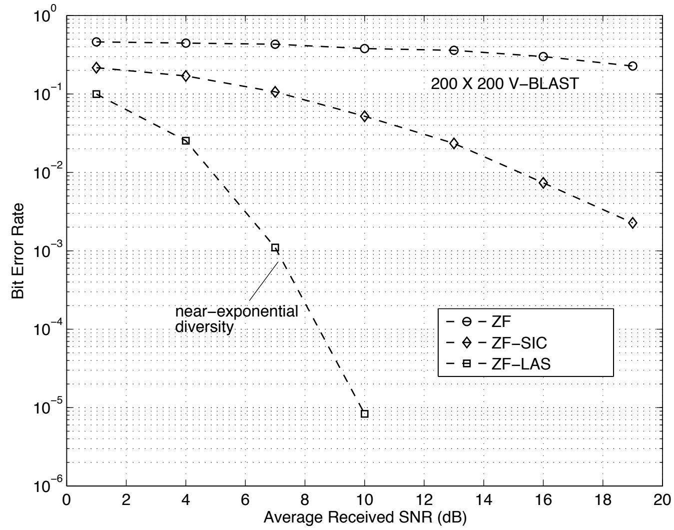

Fig. 2. Uncoded BER performance of ZF-LAS versus ZF-SIC as a function of average received SNR for a 200×200 V-BLAST system. BPSK, 200 bps/Hz spectral efficiency. ZF-LAS achieves higher order diversity (near-exponential diversity) than ZF-SIC at a much lesser complexity.

performance of the MF-LAS, ZF-LAS and ZF-SIC detectors for V-BLAST as a function of $N_t = N_r$ at an average received SNR of 20 dB with BPSK. The performance of the MF and ZF detectors are also plotted for comparison. From Fig. 1, the following observations can be made:

- The BER at $N_t = N_r = 1$ is nothing but the SISO flat Rayleigh fading BER for BPSK, given by $\frac{1}{2}\left[1 - \sqrt{\frac{\gamma}{1+\gamma}}\right]$ which is equal to 2.5×10^{-3} for $\gamma = 20$ dB [25]. While the performance of MF and ZF degrade as $N_t = N_r$ is increased, the performance of ZF-SIC improves for antennas up to $N_t = N_r = 15$, beyond which a flooring effect occurs. This improvement is likely due to the potential diversity in the ordering (selection) in ZF-SIC, whereas the flooring for $N_t > 15$ is likely due to interference being large beyond the cancellation ability of the ZF-SIC.
- The behavior of MF-LAS and ZF-LAS for increasing $N_t = N_r$ are interesting. Starting with the MF output as the initial vector, the MF-LAS always achieves better performance than MF. More interestingly, this improved performance of MF-LAS compared to that of MF increases remarkably as $N_t = N_r$ increases. For example, for $N_t = N_r = 15$, the performance improves by an order in BER (i.e., 7.5×10^{-2} BER for MF versus 7×10^{-3} BER for MF-LAS), whereas for $N_t = N_r = 60$ the performance improves by four-orders in BER (i.e., 8×10^{-2} BER for MF versus 9×10^{-6} BER for MF-LAS). This is due to the large system effect in the LAS algorithm which is able to successfully pick up much of the diversity possible in the system. This large system performance superiority of the LAS is in line with the observations/results reported in [23],[24] for a large CDMA system (large number of antennas in our case, whereas it was large number of users in [23],[24]).
- While the ZF-LAS performs slightly better than ZF-SIC for antennas less than 4, ZF-SIC performs better than ZF-LAS for antennas in the range 4 to 24. This is likely because, for antennas less than 4, the BER of ZF is small enough for the LAS to clean up the ZF initial vector into an output vector better than the ZF-SIC output vector. However, for antennas in the range of 4 to 24, the BER of ZF gets high to an extent that the ZF-LAS is less effective in cleaning the initial vector beyond the diversity performance achieved by the ZF-SIC. A more interesting observation, however, is that for antennas greater than 25, the large system effect of ZF-LAS kicks in. So, in the large system setting (e.g., antennas more than 25 in Fig. 1), the ZF-LAS performs increasingly better than ZF-SIC for increasing $N_t = N_r$. We found the number of antennas where the large system effect kicks in (i.e., the number of antennas at which the cross-over between ZF-SIC and ZF-LAS occurs) to be different for different SNRs.
- Another observation in Fig. 1 is that for antennas greater than 50, MF-LAS performs better than ZF-LAS. This behavior can be explained by observing the performance comparison between MF and ZF detectors given in the same figure. For more than 50 antennas, MF performs slightly better than ZF. It is known that ZF detector can perform worse than MF detector under high noise/interference conditions [15] (here high interference due to large N_t). Hence, starting with a better initial vector, MF-LAS performs better than ZF-LAS.

ZF-LAS outperforms ZF-SIC in large V-BLAST systems both in complexity & diversity: In Fig. 2, we present an interesting comparison of the uncoded BER performance between ZF, ZF-LAS and ZF-SIC, as a function of average SNR for a 200×200 V-BLAST system. This system being a large system, the ZF-LAS has a huge complexity advantage over ZF-SIC as pointed out before in Sec. II-B. In fact, although we have taken the effort to show the performance of ZF-SIC at such a large number of antennas like 200, we had to obtain these simulation points for ZF-SIC over days of simulation time, whereas the same simulation points for ZF-LAS were obtained in just few

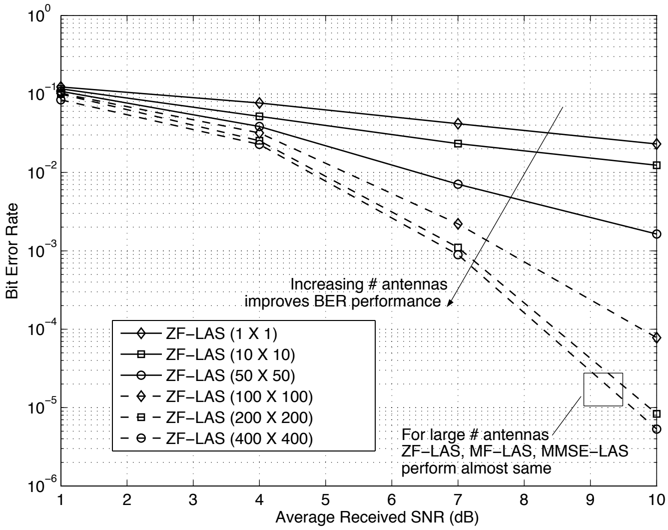

Fig. 3. Uncoded BER performance of ZF-LAS for V-BLAST as a function of average received SNR for increasing values of $N_t = N_r$. BPSK, N_t bps/Hz spectral efficiency. For large number of antennas (e.g., $N_t = N_r = 200, 400$), the performance of ZF-LAS, MF-LAS, and MMSE-LAS are almost the same.

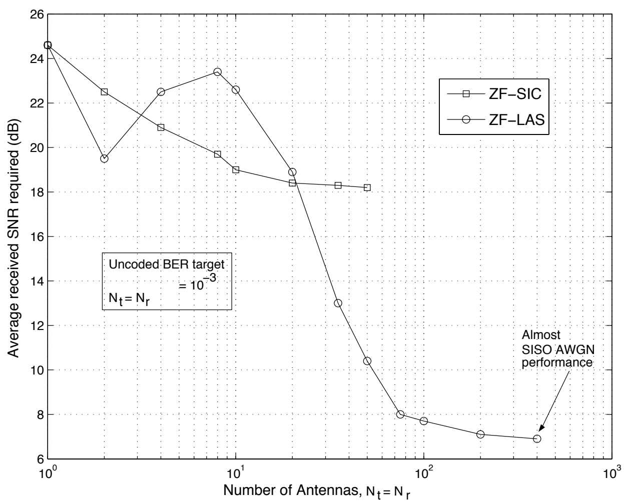

Fig. 4. Average received SNR required to achieve a target uncoded BER of 10^{-3} in V-BLAST for increasing values of $N_t = N_r$. BPSK. ZF-LAS versus ZF-SIC. ZF-LAS achieves near SISO AWGN performance.

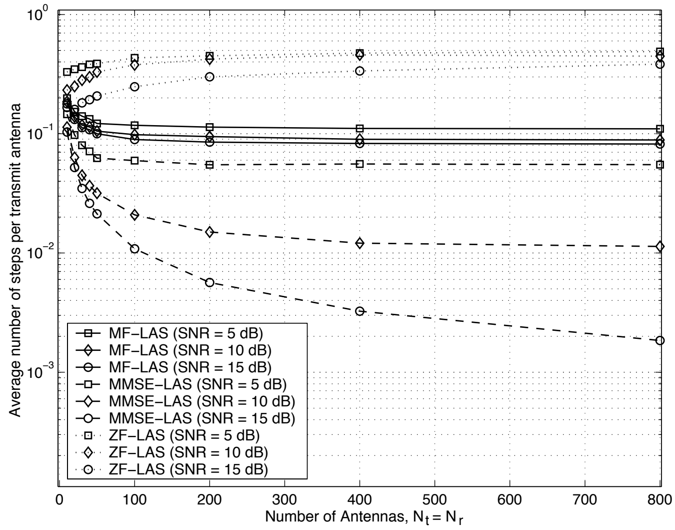

Fig. 5. Complexity of the LAS algorithm in terms of average number of steps per transmit antenna till fixed point is reached in V-BLAST as a function of $N_t = N_r$ for different SNRs and initial vectors (MF, ZF, MMSE). BPSK. Results obtained from simulations.

hours. This is due to the $O(N_t^2 N_r)$ complexity of ZF-SIC versus $O(N_t N_r)$ complexity of ZF-LAS, as pointed out in Sec. II-B. More interestingly, in addition to this significant complexity advantage, ZF-LAS is able to achieve a much higher order of diversity (in fact, near-exponential diversity) in BER performance compared to ZF-SIC (which achieves only a little better than first order diversity). This is clearly evident from the slopes of the BER curves of ZF-LAS and ZF-SIC. *Note that the BER curve for ZF-LAS is almost the same as the uncoded BER curve for BPSK on a SISO AWGN channel, given by $Q(\sqrt{\gamma})$ [25]. This means that the proposed detector nearly renders a 200×200 MIMO fading channel into 200 parallel, non-interfering SISO AWGN channels.*

LAS Detector's performance with hundreds of antennas: As pointed earlier, obtaining ZF-SIC results for more than even 50 antennas requires very long simulation run times, which is not the case with ZF-LAS. In fact, we could easily generate BER results for up to 400 antennas for ZF-LAS, which are plotted in Fig. 3. The key observations in Fig. 3 are that i) the average SNR required to achieve a certain BER performance keeps reducing for increasing number of antennas for ZF-LAS, and ii) increasing the number of antennas results in increased orders of diversity achieved (close to SISO AWGN performance for 200 and 400 antennas). We have also observed from our simulations that for large number of antennas, the LAS algorithm converges to almost the same near-ML performance regardless of the initial vector chosen. For example, for the case of 200 and 400 antennas in Fig. 3, the BER performance achieved by ZF-LAS, MF-LAS, and MMSE-LAS are almost the same (although we have not explicitly plotted the BER curves for MF-LAS and MMSE-LAS in Fig. 3). So, in such large MIMO systems setting, MF-LAS may be preferred over ZF/MMSE-LAS since ZF/MMSE-LAS require matrix inverse operation whereas MF-LAS does not.

Observation i) in the above paragraph is explicitly brought out in Fig. 4, where we have plotted the average received SNR required to achieve a target uncoded BER of 10^{-3} as a function of $N_t = N_r$ for ZF-LAS and ZF-SIC. It can be seen that the SNR required to achieve 10^{-3} with ZF-LAS significantly reduces for increasingly large $N_t = N_r$. For example, the required SNR reduces from about 25 dB for a SISO Rayleigh fading channel to about 7 dB for a 400×400 V-BLAST system using ZF-LAS. As we pointed out in Fig. 3, this 400×400 system performance is almost the same as that of a SISO AWGN channel where the SNR required to achieve 10^{-3} BER is also close to 7 dB [25], i.e., $20 \log \left(Q^{-1}(10^{-3})\right) \approx 7$ dB.

B. Turbo Coded BER Performance

In this subsection, we present the turbo coded BER performance of the proposed LAS detector. We also quantify how far is the proposed detector's performance away from the theoretical capacity. For a $N_t \times N_r$ MIMO system model

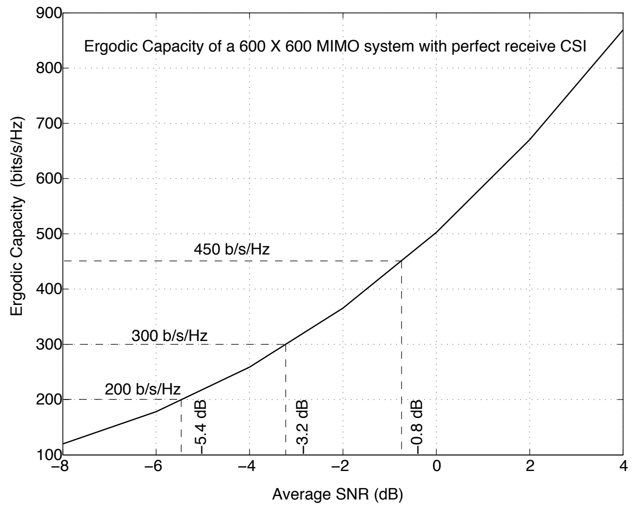

Fig. 6. Ergodic capacity for 600×600 MIMO system with receive CSI.

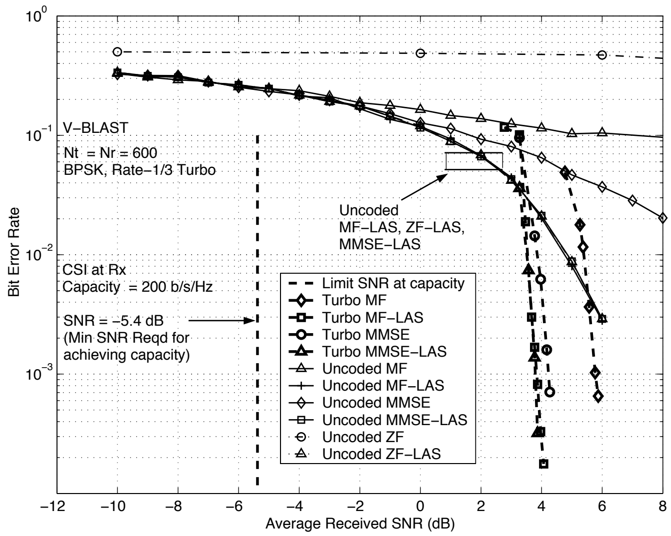

Fig. 7. BER performance of various detectors for rate-1/3 turbo-encoded data using BPSK symbols in a 600×600 V-BLAST system. 200 bps/Hz spectral efficiency. Proposed MF/ZF/MMSE-LAS detectors' performance is away from capacity by 9.4 dB.

in Sec. II with perfect channel state information (CSI) at the receiver, the ergodic capacity is given by [5]

$$C \;=\; E\left[\log\det\left(\mathbf{I}_{N_r} + (\gamma/N_t)\,\mathbf{H}\mathbf{H}^H\right)\right], \qquad (19)$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix and γ is the average SNR per receive antenna. We have evaluated the capacity in (19) for a 600×600 MIMO system through Monte-Carlo simulations and plotted it as a function of average SNR in Fig. 6. Figure 7 shows the simulated BER performance of the proposed LAS detector for a 600×600 MIMO system with BPSK and rate-1/3 turbo code (i.e., spectral efficiency = $600 \times 1 \times \frac{1}{3} = 200$ bps/Hz). Figure 8 shows similar performance plots for rate-3/4 turbo code at a spectral efficiency of $600 \times 1 \times \frac{3}{4} = 450$ bps/Hz. From the capacity curve in Fig. 6, the minimum SNRs required at 200 bps/Hz and 450 bps/Hz spectral efficiencies are -5.4 dB and -0.8 dB, respectively. The following interesting observations can be made from Figs. 7 and 8:

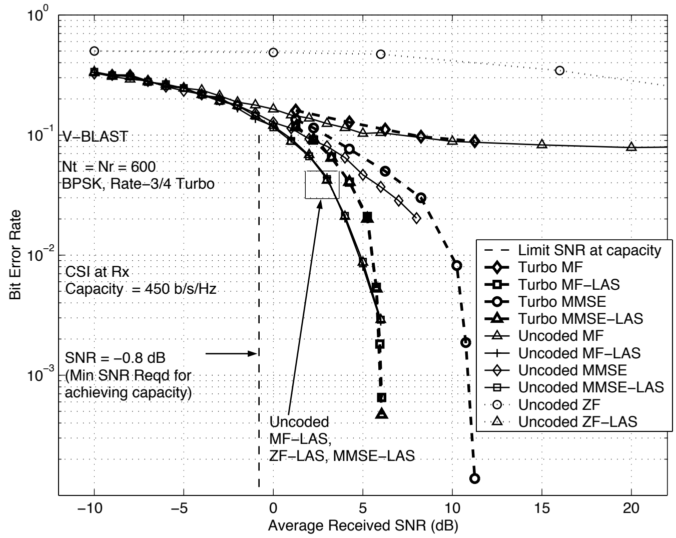

Fig. 8. BER performance of various detectors for rate-3/4 turbo-encoded data using BPSK symbols in a 600×600 V-BLAST system. 450 bps/Hz spectral efficiency. Proposed MF/ZF/MMSE-LAS detectors' performance is away from capacity by 7 dB.

- In terms of uncoded BER, the performance of MF, ZF, and MMSE are different, with ZF and MMSE performing the worst and best, respectively. But the performance of MF-LAS, ZF-LAS, and MMSE-LAS are almost the same (near-exponential diversity performance) with the number of antennas being large ($N_t = N_r = 600$).
- With a rate-1/3 turbo code (Fig. 7), all the LAS detectors considered (i.e., MF-LAS, ZF-LAS, MMSE-LAS) achieve almost the same performance, which is about 9.4 dB away from capacity (i.e., near-vertical fall of coded BER occurs at about 4 dB). Turbo coded MF/MMSE without LAS also achieve good performance in this case (i.e., less than only 2 dB away from turbo coded MF/ZF/MMSE-LAS performance). This is because the uncoded BER of MF and MMSE at around 4 to 6 dB SNR are small enough for the turbo code to be effective. However, this is not the case with turbo coded ZF without LAS. As can be seen, in the range of SNRs shown, the uncoded BER of ZF without LAS is so high (close to 0.5) that the vertical fall of coded BER can happen only at very high SNRs, because of which we have not shown the performance of turbo coded ZF without LAS.
- A clear superiority of the proposed MF/ZF/MMSE-LAS over MF/MMSE without LAS in terms of coded BER comes about when high-rate turbo codes are used. For example, when a rate-3/4 turbo code is used (Fig. 8), the MF/ZF/MMSE-LAS performs to within about 7 dB from capacity[8] (i.e., minimum required SNR at capacity is -0.8 dB and the vertical fall of the coded BER occurs at 6 dB). Whereas the performance of rate-3/4 turbo coded MF/MMSE without LAS are much farther away from capacity. Vertical fall for turbo coded MMSE without LAS occurs only at 11.5 dB (i.e., 5 to 6 dB away from turbo coded MF/ZF/MMSE-LAS). The performance

[8] We point out that we have fed the ± 1 valued output vector from LAS to the turbo decoder. However, if soft values are generated and used instead, the performance is expected to move further towards the capacity.

TABLE I
NEARNESS TO CAPACITY OF VARIOUS DETECTORS FOR 600×600 V-BLAST WITH BPSK AND VARIOUS TURBO CODE RATES. PROPOSED LAS DETECTOR PERFORMS TO WITHIN ABOUT 9.4 dB, 7.7 dB, 6.8 dB FROM CAPACITY FOR 200, 300, AND 450 BPS/HZ SPECTRAL EFFICIENCIES, RESPECTIVELY.

Code Rate, Spectral Eff.	Min. SNR at capacity	Vertical fall of coded BER occurs at			
		LAS	ZF	MF	MMSE
Rate-1/3 turbo, 200 bps/Hz	-5.4 dB	4 dB	high	6 dB	4.5 dB
Rate-1/2 turbo, 300 bps/Hz	-3.2 dB	4.5 dB	high	high	6 dB
Rate-3/4 turbo, 450 bps/Hz	-0.8 dB	6 dB	high	high	high

of turbo coded MF without LAS is unacceptably poor (there is no vertical fall in the SNRs shown). This clear coded performance advantage of the proposed detector is primarily due to the near-exponential fall of uncoded BER achieved using LAS.

- Another interesting way to interpret the results by comparing the plots in Figs. 7 and 8 is that a 2 dB increase in received SNR γ (from 4 dB to 6 dB) achieves an increase of 250 bps/Hz spectral efficiency (from 200 to 450 bps/Hz).

In Table I, we summarize the performance of various detectors in terms of their nearness to capacity in a 600×600 V-BLAST system using BPSK, and rate-1/3, rate-1/2 and rate-3/4 turbo codes.

Effect of Channel Estimation Errors: As pointed out in Sec. I, apart from the detector, a major bottleneck in large MIMO systems is channel estimation [28],[29]. We have evaluated the effect of channel estimation errors on the performance of the proposed detector. We consider a channel estimation error model where the estimated channel matrix, $\widehat{\mathbf{H}}$, is taken to be

$$\widehat{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H}, \tag{20}$$

where $\Delta\mathbf{H}$ is the estimation error matrix, the entries of which are assumed to be i.i.d. complex Gaussian with zero mean and variance σ_e^2. In Fig. 9, we have plotted the uncoded as well as rate-1/2 turbo coded BER performance of the proposed MMSE-LAS detector with BPSK for different values of $\sigma_e^2 = 0, 0.01, 0.05, 0.1$. Note that the $\sigma_e^2 = 0$ plot corresponds to the case of perfect channel estimation. As expected, the performance degrades as σ_e^2 is increased. It can be observed however that, compared to the case of perfect channel estimation, the performance degradation in achieving an uncoded BER of 10^{-2} is only about 0.25 dB and 1 dB for estimation error variances of 0.01 and 0.05, respectively. Even at a large error variance of 0.1, the proposed MMSE-LAS achieves a much higher diversity than MMSE without LAS with perfect channel estimation. In terms of coded BER, the performance degradation compared to perfect channel estimation is only about 0.2 dB and 0.6 dB for error variances of 0.01 and 0.05, respectively. Thus, the sensitivity of the proposed detector's performance to estimation errors shown in Fig. 9 illustrates that channel estimation schemes that render estimation error variances to be about 0.01 to 0.05 are

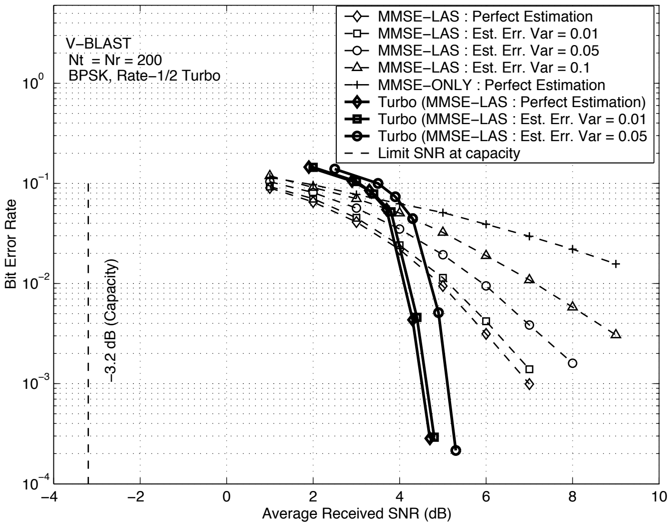

Fig. 9. Effect of channel estimation errors on uncoded and coded BER performance of the proposed MMSE-LAS detector in a 200×200 V-BLAST system with BPSK and rate-1/2 turbo code. Channel estimation error variance, $\sigma_e^2 = 0, 0.01, 0.05, 0.1$.

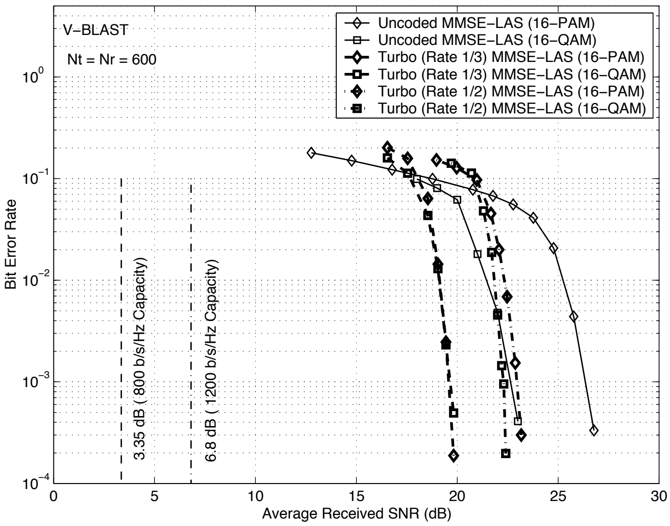

Fig. 10. Uncoded and coded BER performance of MMSE-LAS in a 600×600 V-BLAST system for 16-PAM and 16-QAM with rate-1/2 and rate-1/3 turbo codes.

good to keep the performance loss small. The investigation of estimation algorithms and efficient pilot schemes for accurate channel estimation in large MIMO systems as such would be important topics for future work.

Performance of M-PAM/M-QAM: Although the LAS algorithm in Sec. II is presented assuming BPSK, it can be adopted for M-ary modulation including M-PAM and M-QAM. In the case of BPSK, the elements of the data vector take values from $\{\pm 1\}$. M-PAM symbols take discrete values from $\{A_m, 1 \leq m \leq M\}$ where $A_m = (2m - 1 - M)$, $m = 1, 2, \cdots, M$, and M-QAM is nothing but quadrature PAM. Similar to the search over binary vectors for BPSK, likelihood ascent search can be performed over M-ary symbol vectors. We have adopted the LAS algorithm for M-PAM/M-QAM and evaluated the performance of the LAS detector for 4-PAM/4-QAM and 16-PAM/16-QAM without and with coding. In M-PAM/M-QAM also, we have observed large

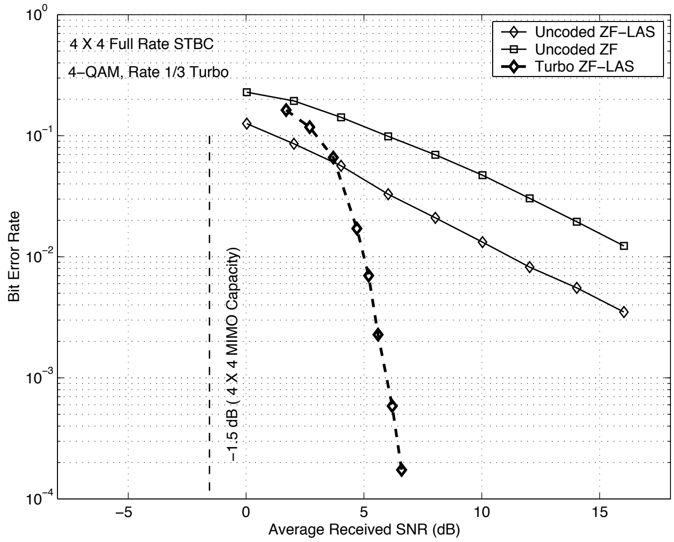

Fig. 11. Uncoded and coded BER performance of ZF-LAS for a 4×4 full-rate (i.e., rate-4) non-orthogonal STBC from Division Algebras given in (21). 4-QAM, rate-1/3 turbo code. 8/3 bps/Hz spectral efficiency.

system behavior of the proposed detector similar to those presented for BPSK. As an example, in Fig. 10, we present the uncoded and coded performance of the MMSE-LAS detector in a 600×600 V-BLAST system for 16-PAM/16-QAM with rate-1/2 and rate-1/3 turbo codes at spectral efficiencies of $600 \times 4 \times \frac{1}{2} = 1200$ bps/Hz and $600 \times 4 \times \frac{1}{3} = 800$ bps/Hz, respectively.

C. Proposed LAS Detector for High-Rate STBCs

Since the placement of several antennas can be an issue in small-sized communication terminals, high-rate space time codes can be used instead of pure V-BLAST; the advantages of the space-time codes approach being $i)$ less number of antennas, and $ii)$ transmit diversity. Since multiple time slots are involved in the space-time approach, additional decoding delay would be involved compared to V-BLAST. Low-complexity decoding of high-rate, non-orthogonal STBCs is a challenge. Here, we show that high-rate, non-orthogonal STBCs can be easily decoded using the proposed LAS detector while achieving good performance.

Explicit construction of high-rate, full-diversity, non-orthogonal STBCs from Division Algebras have been discussed in detail in [26]. An $n \times n$ STBC is said to be of full-rate, if there are n^2 variables in it, or, equivalently, the rate of the code is n complex symbols per channel use. An example of a full-rate, full-diversity, non-orthogonal STBC from Division Algebras for 4 transmit antennas is shown below [26]:

$$\mathbf{S} = \frac{1}{2}\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}, \tag{21}$$

where the $\{s_{ij}\}$, $i, j \in \{1, 2, 3, 4\}$ are defined as follows. Let $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_{16}]$ denote the data symbol vector. Then, the symbols $\{s_{ij}\}$, $i, j \in \{1, 2, 3, 4\}$ in the space-time block code $\mathbf{S}$ in (21) are given by

$$\begin{aligned}
s_{11} &= x_1 + x_2 e^{\frac{j2\pi}{16}} + x_3 e^{\frac{j4\pi}{16}} + x_4 e^{\frac{j6\pi}{16}}, \\
s_{21} &= x_5 + x_6 e^{\frac{j2\pi}{16}} + x_7 e^{\frac{j4\pi}{16}} + x_8 e^{\frac{j6\pi}{16}}, \\
s_{31} &= x_9 + x_{10} e^{\frac{j2\pi}{16}} + x_{11} e^{\frac{j4\pi}{16}} + x_{12} e^{\frac{j6\pi}{16}}, \\
s_{41} &= x_{13} + x_{14} e^{\frac{j2\pi}{16}} + x_{15} e^{\frac{j4\pi}{16}} + x_{16} e^{\frac{j6\pi}{16}}, \\
s_{12} &= e^{\frac{j}{2}}\big(x_{13} + jx_{14} e^{\frac{j2\pi}{16}} - x_{15} e^{\frac{j4\pi}{16}} - jx_{16} e^{\frac{j6\pi}{16}}\big), \\
s_{22} &= x_1 + jx_2 e^{\frac{j2\pi}{16}} - x_3 e^{\frac{j4\pi}{16}} - jx_4 e^{\frac{j6\pi}{16}}, \\
s_{32} &= x_5 + jx_6 e^{\frac{j2\pi}{16}} - x_7 e^{\frac{j4\pi}{16}} - jx_8 e^{\frac{j6\pi}{16}}, \\
s_{42} &= x_9 + jx_{10} e^{\frac{j2\pi}{16}} - x_{11} e^{\frac{j4\pi}{16}} - jx_{12} e^{\frac{j6\pi}{16}}, \\
s_{13} &= e^{\frac{j}{2}}\big(x_9 - x_{10} e^{\frac{j2\pi}{16}} + x_{11} e^{\frac{j4\pi}{16}} - x_{12} e^{\frac{j6\pi}{16}}\big), \\
s_{23} &= e^{\frac{j}{2}}\big(x_{13} - x_{14} e^{\frac{j2\pi}{16}} + x_{15} e^{\frac{j4\pi}{16}} - x_{16} e^{\frac{j6\pi}{16}}\big), \\
s_{33} &= x_1 - x_2 e^{\frac{j2\pi}{16}} + x_3 e^{\frac{j4\pi}{16}} - x_4 e^{\frac{j6\pi}{16}}, \\
s_{43} &= x_5 - x_6 e^{\frac{j2\pi}{16}} + x_7 e^{\frac{j4\pi}{16}} - x_8 e^{\frac{j6\pi}{16}}, \\
s_{14} &= e^{\frac{j}{2}}\big(x_5 - jx_6 e^{\frac{j2\pi}{16}} - x_7 e^{\frac{j4\pi}{16}} + jx_8 e^{\frac{j6\pi}{16}}\big), \\
s_{24} &= e^{\frac{j}{2}}\big(x_9 - jx_{10} e^{\frac{j2\pi}{16}} - x_{11} e^{\frac{j4\pi}{16}} + jx_{12} e^{\frac{j6\pi}{16}}\big), \\
s_{34} &= e^{\frac{j}{2}}\big(x_{13} - jx_{14} e^{\frac{j2\pi}{16}} - x_{15} e^{\frac{j4\pi}{16}} + jx_{16} e^{\frac{j6\pi}{16}}\big), \\
s_{44} &= x_1 - jx_2 e^{\frac{j2\pi}{16}} - x_3 e^{\frac{j4\pi}{16}} + jx_4 e^{\frac{j6\pi}{16}}.
\end{aligned}$$

The STBC in (21) sends 16 symbols in 4 time slots using 4 transmit antennas. A detailed discussion on such codes for arbitrary number of antennas can be seen in [26]. Well known decoding algorithms which extract the full-diversity property of these codes are the sphere-decoding and MCMC algorithms [11]-[13], which are prohibitively complex when the number of antennas exceed ten. With the proposed MF/ZF/MMSE-LAS detector, however, such high-rate, non-orthogonal STBCs can be decoded for large number of antennas/time slots at low-complexity while achieving good performance as well. The STBC received signal model can be written in an equivalent V-BLAST form [27], and hence the proposed detector in Sec. II can be used in the STBC decoding. As an example, in Fig. 11, we present the uncoded and turbo coded performance of the proposed ZF-LAS detector for the rate-4 STBC shown in (21) using 4-QAM and rate-1/3 turbo code (i.e., a spectral efficiency of 8/3 bps/Hz). We observe that the proposed ZF-LAS detector performs to within about 8 dB from the 4×4 MIMO capacity.

IV. LAS Detector for Multicarrier CDMA

In this section, we present the proposed LAS detector for multicarrier CDMA, its performance and complexity. Consider a K-user synchronous multicarrier DS-CDMA system with M subcarriers. Let $b_k \in \{+1, -1\}$ denote the binary data symbol of the kth user, which is sent in parallel on M subcarriers [30],[31]. Let N denote the number of chips-per-bit in the signature waveforms. It is assumed that the channel is frequency non-selective on each subcarrier and the fading is slow (assumed constant over one bit interval) and independent from one subcarrier to the other.

Let $\mathbf{y}^{(i)} = \Big[y_1^{(i)} \ y_2^{(i)} \ \cdots \ y_K^{(i)}\Big]^T$ denote the K-length received signal vector on the ith subcarrier; i.e., $y_k^{(i)}$ is the output of the kth user's matched filter on the ith subcarrier. Assuming that the inter-carrier interference is negligible, the K-length received signal vector on the ith subcarrier $\mathbf{y}^{(i)}$ can be written in the form

$$\mathbf{y}^{(i)} = \mathbf{R}^{(i)}\mathbf{H}^{(i)}\mathbf{A}\mathbf{b} + \mathbf{n}^{(i)}, \tag{22}$$

where $\mathbf{R}^{(i)}$ is the $K \times K$ cross-correlation matrix on the ith subcarrier, with its entries $\rho_{lj}^{(i)}$'s denoting the normalized cross correlation coefficient between the signature waveforms of the lth and jth users on the ith subcarrier. $\mathbf{H}^{(i)}$ represents the $K \times K$ channel matrix, given by

$$\mathbf{H}^{(i)} = diag\left\{h_1^{(i)}, h_2^{(i)}, \cdots, h_K^{(i)}\right\}, \quad (23)$$

where the channel coefficients $h_k^{(i)}$, $i = 1, 2, \cdots, M$, are assumed to be i.i.d. complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E\left[\left(h_{kI}^{(i)}\right)^2\right] = E\left[\left(h_{kQ}^{(i)}\right)^2\right] = 0.5$, where $h_{kI}^{(i)}$ and $h_{kQ}^{(i)}$ are the real and imaginary parts of $h_k^{(i)}$. The K-length data vector $\mathbf{b}$ is given by

$$\mathbf{b} = \left[\begin{array}{cccc} b_1 & b_2 & \cdots & b_K \end{array}\right]^T, \quad (24)$$

and the $K \times K$ diagonal amplitude matrix $\mathbf{A}$ is given by

$$\mathbf{A} = diag\left\{A_1, A_2, \cdots, A_K\right\}, \quad (25)$$

where A_k denotes the transmit amplitude of the kth user. The K-length noise vector $\mathbf{n}^{(i)}$ is given by

$$\mathbf{n}^{(i)} = \left[\begin{array}{cccc} n_1^{(i)} & n_2^{(i)} & \cdots & n_K^{(i)} \end{array}\right]^T, \quad (26)$$

where $n_k^{(i)}$ denotes the additive noise component of the kth user on the ith subcarrier, which is assumed to be complex Gaussian with zero mean with $E[n_k^{(i)}\left(n_j^{(i)}\right)^*] = \sigma^2$ when $j = k$ and $E[n_k^{(i)}\left(n_j^{(i)}\right)^*] = \sigma^2\rho_{kj}^{(i)}$ when $j \neq k$. We assume that all the channel coefficients are perfectly known at the receiver.

A. LAS Algorithm for MC-CDMA

We note that once the likelihood function for the MC-CDMA system in the above is obtained, it is straightforward to adopt the LAS algorithm for MC-CDMA. Accordingly, in the multicarrier system considered, the most likely $\mathbf{b}$ is taken as that $\mathbf{b}$ which maximizes

$$\begin{aligned}\Lambda(\mathbf{b}) &= \sum_{i=1}^{M}\left(\mathbf{b}^T\mathbf{A}(\mathbf{H}^{(i)})^*\mathbf{y}^{(i)} + \mathbf{b}^T\mathbf{A}\mathbf{H}^{(i)}(\mathbf{y}^{(i)})^*\right) \\ &\quad -\mathbf{b}^T\left(\sum_{i=1}^{M}\mathbf{A}\mathbf{H}^{(i)}\mathbf{R}^{(i)}(\mathbf{H}^{(i)})^*\mathbf{A}\right)\mathbf{b}. \quad (27)\end{aligned}$$

The likelihood function in (27) can be written in a form similar to Eqn. (4.11) in [15] as

$$\Lambda(\mathbf{b}) = \mathbf{b}^T\mathbf{A}\mathbf{y}_{ceff} - \mathbf{b}^T\mathbf{H}_{ceff}\mathbf{b}, \quad (28)$$

where

$$\mathbf{y}_{ceff} = \sum_{i=1}^{M}\left(\left(\mathbf{H}^{(i)}\right)^*\mathbf{y}^{(i)} + \mathbf{H}^{(i)}\left(\mathbf{y}^{(i)}\right)^*\right), \quad (29)$$

$$\mathbf{H}_{ceff} = \sum_{i=1}^{M}\mathbf{A}\mathbf{H}^{(i)}\mathbf{R}^{(i)}\left(\mathbf{H}^{(i)}\right)^*\mathbf{A}. \quad (30)$$

Now observing the similarity between (28) and (4) in Sec. II-A, the LAS algorithm for MC-CDMA can be arrived at, along the same lines as that of V-BLAST in the previous section, with $\mathbf{y}_{eff}$, $\mathbf{H}_{eff}$ and $\mathbf{H}_{real}$ replaced by $\mathbf{y}_{ceff}$, $\mathbf{H}_{ceff}$, and $\mathbf{H}_{creal}$, respectively, with all other notations, definitions, and procedures in the algorithm remaining the same.

B. Complexity of the Proposed Detector for MC-CDMA

The complexity of the proposed detector for MC-CDMA can be analyzed in a similar manner as done for V-BLAST in Sec. II. First, given an initial vector, the LAS operation part alone in MC-CDMA has an average per-bit complexity of $O(MK)$, which is due to $i)$ initial computation of $\mathbf{g}(0)$ in (8), which requires $O(MK)$ complexity per bit, $ii)$ update of $\mathbf{g}(n)$ in each step as per (18), which requires $O(K)$ complexity for sequential LAS, and hence constant per-bit complexity, and $iii)$ the average number of steps required to reach a fixed point, which, through simulations, is found to have a constant per-bit complexity. Next, the initial vector generation using MMSE or ZF has a per-bit complexity of $O(K^2)$ for $K > M$. Finally, combining the above complexities involved in the LAS part and the initial vector generation part, the overall average per-bit complexity of the MMSE/ZF-LAS detector for MC-CDMA is $O(K^2)$. The initial vector generation using MF has a per-bit complexity of only $O(M)$. Hence, if the MF output is used as the initial vector, then the overall average per-bit complexity of the MF-LAS is the same as that of the LAS alone, which is $O(MK)$. For large K, the performance of MF-LAS, ZF-LAS, and MMSE-LAS are almost the same (see Fig. 13), and hence MF-LAS is preferred because of its linear complexity in number of users, K, for a given M.

C. Results and Discussions for MC-CDMA

We evaluated the BER performance of the proposed LAS detector for MC-CDMA through extensive simulations. We evaluate the uncoded BER performance of the proposed LAS detector as a function of average SNR, number of users (K), number of subcarriers (M), and number of chips per bit (N). We also evaluate the BER performance as a function of *loading factor*, α, where, as done in the CDMA literature [15], we define $\alpha \stackrel{\triangle}{=} \frac{K}{MN}$. We call the system as underloaded when $\alpha < 1$, fully loaded when $\alpha = 1$, and overloaded when $\alpha > 1$. Random binary sequences of length N are used as the spreading sequences on each subcarrier. In order to make a fair comparison between the performance of MC-CDMA systems with different number of subcarriers, we keep the system bandwidth the same by keeping MN constant. Also, in that case we keep the total transmit power to be the same irrespective of the number of subcarriers used. In the simulation plots we show in this section, we have assumed that all users transmit with equal amplitude[9]. The LAS algorithm used is the SLAS with circular checking of bits starting from the first user's bit.

First, in Fig. 12, we present the BER performance of MF/ZF-LAS detectors as a function of average SNR in a single carrier (i.e., $M = 1$) *underloaded system*, where we consider $\alpha = 2/3$ by taking $K = 200$ users and $N = 300$ chips per bit. For comparison purposes, we also plot the performance of MF and ZF without LAS. Single user (SU) performance, which corresponds to the case of no multiuser interference (i.e., $K = 1$), is also shown as a lower bound on the achievable multiuser performance. From Fig. 12, we can observe that the

[9]We note that we have simulated the MF/ZF-LAS performance in near-far conditions as well. Even with near-far effect, the MF/ZF-LAS detectors have been observed to achieve near single-user performance.

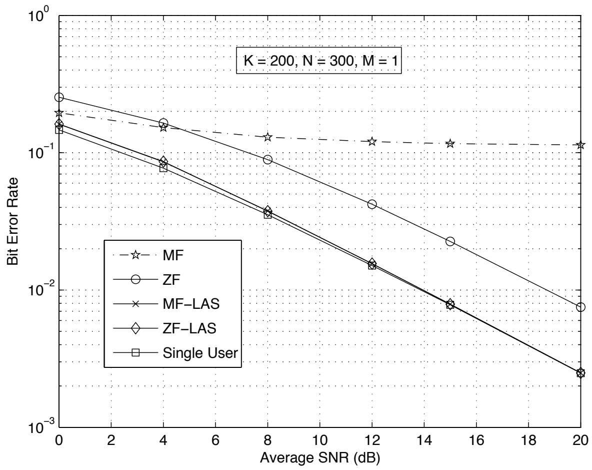

Fig. 12. BER performance of ZF-LAS and MF-LAS detectors as a function of average SNR for single carrier CDMA in Rayleigh fading. $M = 1$, $K = 200$, $N = 300$, i.e., $\alpha = 2/3$.

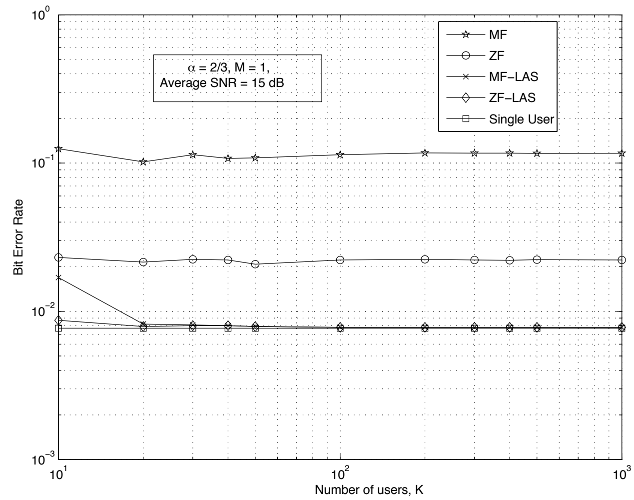

Fig. 13. BER performance of ZF-LAS and MF-LAS detectors as a function of number of users, K, for single carrier CDMA ($M = 1$) in Rayleigh fading for a fixed $\alpha = 2/3$ and average SNR = 15 dB. N varied from 15 to 1500.

performance of MF and ZF detectors are far away from the SU performance. Whereas, the ZF-LAS as well as MF-LAS detectors almost achieve the SU performance. We point out that, like ZF detector, other suboptimum detectors including MMSE, SIC, and PIC detectors [15] also do not achieve near SU performance for the considered loading factor of 2/3, whereas the MF-LAS detector achieves near SU performance, that too at a lesser complexity than these other suboptimum detectors.

Next, in Fig. 13, we show the BER performance of the MF/ZF-LAS detectors for $M = 1$ as a function of number of users, K, for a fixed value of $\alpha = 2/3$ at an average SNR of 15 dB. We varied K from 10 to 1000 users. SU performance is also shown (as the bottom most horizontal line) for comparison. It can be seen that, for the fixed value of $\alpha = 2/3$, both the MF-LAS as well as the ZF-LAS achieve near SU performance (even in the presence of 1000 users), whereas the ZF and MF detectors do not achieve the SU performance.

In Fig. 14, we show the BER performance of the MF/ZF-LAS detectors as a function of average SNR for different number of subcarriers, namely, $M = 1, 2, 4$, keeping a constant $MN = 100$, for a *fully loaded system* (i.e., $\alpha = 1$) with $K = 100$. Keeping $\alpha = 1$ and $K = 100$ for all cases means that $i)$ $N = 100$ for $M = 1$, $ii)$ $N = 50$ for $M = 2$, and $iii)$ $N = 25$ for $M = 4$. The SU performance for $M = 1$ (1st order diversity), $M = 2$ (2nd order diversity), and $M = 4$ (4th order diversity) are also plotted for comparison. These diversities are essentially due to the frequency diversity effect resulting from multicarrier combining of signals from M subcarriers. It is interesting to see that even in a fully loaded system, the MF/ZF-LAS detectors achieve all the frequency diversity possible in the system (i.e., MF/ZF-LAS detectors achieve SU performance with 1st, 2nd and 4th order diversities for $M = 1, 2$ and 4, respectively). On the other hand, ZF detector is unable to achieve the frequency diversity in the fully loaded system, and its performance is very poor compared to MF/ZF-LAS detectors.

Next, in Fig. 15, we present the BER performance of ZF/MF-LAS detectors in a MC-CDMA system with $M = 4$ as a function of loading factor, α, where we vary α from 0.025 to 1.5. We realize this variation in α by fixing $K = 30$, $M = 4$, and varying N from 300 to 5. The average SNR considered is 8 dB. From Fig. 15, it can be observed that as α increases all detectors loose performance, but the MF/ZF-LAS detectors can offer relatively good performance even at *overloaded conditions* of $\alpha > 1$. Another observation is that at $\alpha > 1$, MF-LAS performs slightly better than ZF-LAS. This is because $\alpha > 1$ corresponds to a high interference condition, and it is known in MUD literature [15] that ZF can perform worse than MF at low SNRs and high interference. In such cases, starting with a better performing MF output as the initial vector, MF-LAS performs better.

As pointed out earlier, the average per-bit complexity of the MF-LAS for MC-CDMA is $O(MK)$, and for a given M it is linear in number of users, K. Other detectors in the literature have higher complexities; for example, the per-bit complexity is $O(K^2)$ for PIC and for detectors based on semi-definite programming [32], and $O(K^{2.5})$ for detectors based on semi-definite relaxation [33]. Hence these detectors are prohibitively complex for large K. SIC without ordering has a linear per-bit complexity in K, but it does not achieve near-ML performance for large K. MF-LAS, on the other hand, achieves both linear per-bit complexity and near-ML performance for large K.

Further to our present work on the application of MF/ZF-LAS detectors for MC-CDMA, several extensions are possible on the practical application of these detectors in CDMA systems. Two such useful extensions are $i)$ MF/ZF/MMSE-LAS for frequency selective CDMA channels with RAKE combining; we point out that a similar approach and system model adopted here for MC-CDMA is applicable, by taking a view of equivalence between frequency diversity through MC combining and multipath diversity through RAKE combining, and $ii)$ MF/ZF/MMSE-LAS for asynchronous CDMA systems, which can be carried out once the system model is appropriately written in a form similar to (22). These two extensions can allow MF/ZF-LAS detectors to be practical in CDMA systems (e.g., 2G and 3G CDMA systems), with

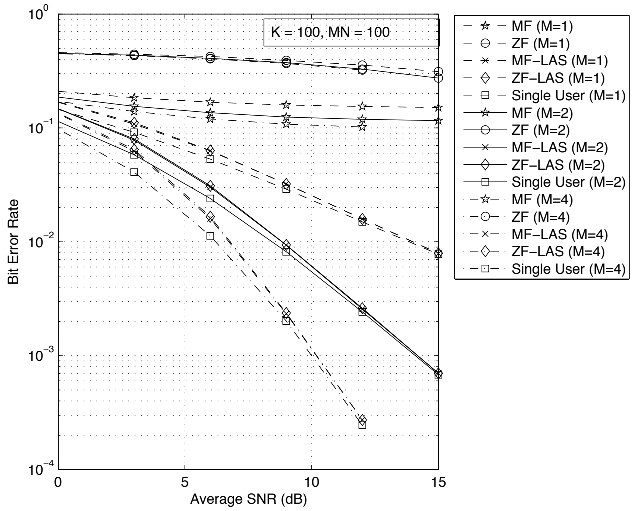

Fig. 14. BER performance of ZF-LAS and MF-LAS detectors as a function of average SNR for multicarrier CDMA in Rayleigh fading. $M = 1, 2, 4$, $\alpha = 1$, $K = 100$, $MN = 100$.

potential for significant gains in system capacity. Current approaches to MUD considered in practical CDMA systems are mainly PIC and SIC. However, the illustrated fact that MF-LAS can easily outperform PIC/SIC detectors in performance and complexity for large K suggests that MF-LAS can be a powerful MUD approach in practical CDMA systems.

V. Conclusions

We presented a high-performance (near-exponential diversity achieving), low-complexity $\big($average per-bit complexity of just $O(N_t N_r)\big)$ detector for large MIMO systems that employ spatial multiplexing using hundreds of antennas (achieving high spectral efficiencies of the order of hundreds of bps/Hz). We also illustrated the effect of channel estimation errors on the performance of the proposed detector. We conclude this paper by pointing to the following remark made by the author of [2] in its preface in 2005: *"It was just a few years ago, when I started working at AT&T Labs – Research, that many would ask 'who would use more than one antenna in a real system?' Today, such skepticism is gone."* Extending this sentiment, we believe large MIMO systems would be practical in the future, and the practical feasibility of low-complexity detectors like the LAS detector presented in this paper could be a potential trigger to create wide interest in the theory and implementation of large MIMO systems. For example, antenna and RF technologies and channel estimation for large MIMO systems could open up as new focus areas in large MIMO research. As mentioned in Sec. I, a likely practical application of large MIMO is to provide high-speed backbone connectivity in wireless mesh networks using large MIMO links, where large number of antennas can be placed at the backbone base stations (several other potential large MIMO applications can be thought of as well). Finally, with its superiority in performance and complexity for large number of users, the proposed LAS detector for MC-CDMA can be a powerful approach to MUD implementations in practical multicarrier and CDMA systems.

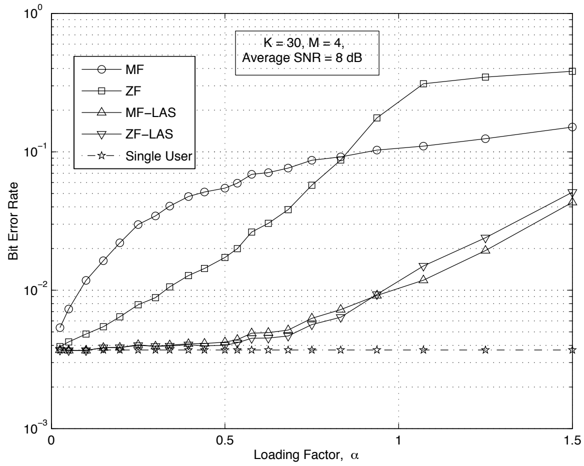

Fig. 15. BER performance of ZF-LAS and MF-LAS detectors as a function of loading factor, α, for multicarrier CDMA in Rayleigh fading. $M = 4$, $K = 30$, N varied from 300 to 5, average SNR = 8 dB.

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