Low-Complexity Detection and Performance in Multi-Gigabit High Spectral Efficiency Wireless Systems

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***Abstract*—Recently, we reported a low-complexity likelihood ascent search (LAS) detection algorithm for large MIMO systems with several tens of antennas that can achieve high spectral efficiencies of the order of tens to hundreds of bps/Hz. Through *simulations*, we showed that this algorithm achieves increasingly near SISO AWGN performance for increasing number of antennas in *i.i.d.* Rayleigh fading. However, no bit error performance analysis of the algorithm was reported. In this paper, we extend our work on this low-complexity large MIMO detector in two directions: $i)$ We report an asymptotic bit error probability analysis of the LAS algorithm in the large system limit, where $N_t, N_r \rightarrow \infty$ keeping $N_t = N_r$, where N_t and N_r are the number of transmit and receive antennas, respectively. Specifically, we prove that the error performance of the LAS detector for V-BLAST with 4-QAM in i.i.d. Rayleigh fading converges to that of the maximum-likelihood (ML) detector as $N_t, N_r \rightarrow \infty$ keeping $N_t = N_r$. $ii)$ We present simulated BER and nearness to capacity results for V-BLAST as well as high-rate non-orthogonal STBC from Division Algebras (DA), in a more realistic spatially correlated MIMO channel model. Our simulation results show that $a)$ at an uncoded BER of 10^{-3}, the performance of the LAS detector in decoding 16×16 STBC from DA with $N_t = N_r = 16$ and 16-QAM degrades in spatially correlated fading by about 7 dB compared to that in i.i.d. fading, and $b)$ with a rate-3/4 outer turbo code and 48 bps/Hz spectral efficiency, the performance degrades by about 6 dB at a coded BER of 10^{-4}. Our results further show that providing asymmetry in number of antennas such that $N_r > N_t$ keeping the total receiver array length same as that for $N_r = N_t$, the detector is able to pick up the extra receive diversity thereby significantly improving the BER performance.**

I. Introduction

Multi-gigabit wireless transmissions at high spectral efficiencies can be achieved using *large MIMO systems* that employ tens of antennas at the transmitter and receiver sides [1]. Major challenges in practically realizing such large MIMO systems include $i)$ physical placement of large number of antennas in communication terminals[1], $ii)$ lack of practical low-complexity detectors for such large systems, and $iii)$ channel estimation issues. Low-complexity detection of MIMO signals is a challenging problem [2]-[3]. In a key recent development in *low-complexity large MIMO detection*, we, in [4],[5], have reported a detection algorithm, termed as likelihood ascent search (LAS) algorithm, and showed that large MIMO signals originating from several tens/hundreds of antennas can be detected at practically affordable complexities, and that too achieving near capacity performance. For example, in a 600 transmit and 600 receive antennas V-BLAST system with a high spectral efficiency of 200 bps/Hz (using BPSK and rate-1/3 turbo code), our simulation results showed that the LAS detector achieves an uncoded BER performance that is almost the same as the SISO AWGN performance, and a turbo coded BER performance that is close to within about 4.6 dB from theoretical MIMO capacity, which is a significant result both from a low-complexity detection viewpoint in a large MIMO system context as well as from a nearness to capacity performance viewpoint. We had also adopted the proposed detector for the low-complexity decoding of high-rate non-orthogonal space-time block codes (STBC) from division algebras (DA) [6], and showed that the detector performs close to within about 5.5 dB of the theoretical capacity in decoding a 16×16 full-rate non-orthogonal STBC from DA (which has 256 complex data symbols in one STBC matrix) using 4-QAM and rate-3/4 turbo code at 24 bps/Hz.

Our new contribution in this paper is that we extend our work in [4],[5] in two directions: $i)$ we present an analytical proof[2] that the error performance of the LAS detector for V-BLAST with 4-QAM in i.i.d. Rayleigh fading converges to that of the ML detector in the large system limit where $N_t, N_r \rightarrow \infty$, keeping $N_t = N_r$, where N_t and N_r denote the number of transmit and receive antennas, respectively, and $ii)$ instead of the i.i.d. fading model we considered earlier (which is inadequate a model when practical MIMO propagation conditions are considered), here we consider a more realistic MIMO channel model proposed by Gesbert *et al* in [7], which is characterized by carrier frequency, scattering radii at the transmitter and receiver, distance between transmit and receive linear arrays, angular spread at the transmitter and receiver, number of physical scatterers, and inter-antenna spacing at the transmitter and receiver, and report interesting simulated uncoded/turbo coded BER results that illustrate the effect of the scattering environment and spatial correlation in the MIMO channel on the LAS detector's bit error performance.

Although we present simulation results for a 5 GHz system in an outdoor environment in this paper, the proposed low-complexity detection approach using non-orthogonal STBC signaling at the transmitter can be relevant in multi-gigabit 60 GHz WPAN/VHT systems [11]-[15] under non-LOS (NLOS) conditions. This approach has the advantages of high spectral efficiencies and increased robustness to blockages (e.g., obstruction due to moving persons/objects which are likely in indoor environments).

II. System Model

Consider a V-BLAST system with N_t transmit antennas and N_r receive antennas, $N_t \leq N_r$. Let $\mathbf{x}_c \in \mathbb{C}^{N_t \times 1}$ be the symbol vector transmitted, and $\mathbf{H}_c \in \mathbb{C}^{N_r \times N_t}$ denote the channel matrix, such that its (i,j)th entry $h_{i,j}$ is the complex channel gain from the jth transmit antenna to the ith receive antenna. Assuming rich scattering, we model the entries of $\mathbf{H}_c$

[1]For small terminal sizes, this would require a high carrier frequency operation, i.e., small carrier wavelengths for $\lambda/2$ separation to ensure independent fade coefficients. Also, tens of antennas can be placed in moderately sized communication terminals (e.g., laptops, set top boxes) that can enable interesting spectrally efficient high data rate applications like wireless IPTV.

[2]Only simulated BER performance of the LAS detector showing increasingly near SISO AWGN performance for increasing number of antennas was reported in [4],[5], and no bit error performance analysis was reported.

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as i.i.d $\mathcal{CN}(0,1)$. Let $\mathbf{y}_c \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{n}_c \in \mathbb{C}^{N_r \times 1}$ denote the received signal vector and the noise vector, respectively, at the receiver, where the entries of $\mathbf{n}_c$ are modeled as i.i.d $\mathcal{CN}(0,\sigma^2)$. The received signal vector can then be written as

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c. \tag{1}$$

Let $\mathbf{y}_c$, $\mathbf{H}_c$, $\mathbf{x}_c$, and $\mathbf{n}_c$ be decomposed into real and imaginary parts as follows:

$$\begin{aligned}\mathbf{y}_c &= \mathbf{y}_I + j\mathbf{y}_Q, \quad \mathbf{x}_c = \mathbf{x}_I + j\mathbf{x}_Q, \\ \mathbf{n}_c &= \mathbf{n}_I + j\mathbf{n}_Q, \quad \mathbf{H}_c = \mathbf{H}_I + j\mathbf{H}_Q.\end{aligned} \tag{2}$$

Further, we define $\mathbf{H}_r \in \mathbb{R}^{2N_r \times 2N_t}$, $\mathbf{y}_r \in \mathbb{R}^{2N_r \times 1}$, $\mathbf{x}_r \in \mathbb{R}^{2N_t \times 1}$, and $\mathbf{n}_r \in \mathbb{R}^{2N_r \times 1}$ as

$$\mathbf{H}_r = \begin{pmatrix} \mathbf{H}_I & -\mathbf{H}_Q \\ \mathbf{H}_Q & \mathbf{H}_I \end{pmatrix},$$

$$\mathbf{y}_r = [\mathbf{y}_I^T \ \mathbf{y}_Q^T]^T, \quad \mathbf{x}_r = [\mathbf{x}_I^T \ \mathbf{x}_Q^T]^T, \quad \mathbf{n}_r = [\mathbf{n}_I^T \ \mathbf{n}_Q^T]^T. \tag{3}$$

Now, (1) can be written as

$$\mathbf{y}_r = \mathbf{H}_r\mathbf{x}_r + \mathbf{n}_r. \tag{4}$$

Henceforth, we shall work with the real-valued system in (4). For notational simplicity, we drop subscripts r in (4) and write

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{H} = \mathbf{H}_r \in \mathbb{R}^{2N_r \times 2N_t}$, $\mathbf{y} = \mathbf{y}_r \in \mathbb{R}^{2N_r \times 1}$, $\mathbf{x} = \mathbf{x}_r \in \mathbb{R}^{2N_t \times 1}$, $\mathbf{n} = \mathbf{n}_r \in \mathbb{R}^{2N_r \times 1}$. In this real-valued system model, the real-part of the complex data symbols will be mapped to $[x_1, \cdots, x_{N_t}]$ and the imaginary-part of these symbols will be mapped to $[x_{N_t+1}, \cdots, x_{2N_t}]$. For M-QAM, $[x_1, \cdots, x_{N_t}]$ can be viewed to be from an underlying M-PAM signal set and so is $[x_{N_t+1}, \cdots, x_{2N_t}]$. Let $\mathbb{A}_i$ denote the M-PAM signal set from which x_i takes values, $i = 1, 2, \cdots, 2N_t$; e.g., for 4-QAM, $\mathbb{A}_i = \{1, -1\}$ for $i = 1, 2, \cdots, 2N_t$. Now, define a $2N_t$-dimensional signal space $\mathbb{S}$ to be the Cartesian product of $\mathbb{A}_1$ to $\mathbb{A}_{2N_t}$. The ML solution vector, $\mathbf{d}_{ML}$, is given by

$$\mathbf{d}_{ML} = \underset{\mathbf{d} \in \mathbb{S}}{\arg\min} \|\mathbf{y} - \mathbf{H}\mathbf{d}\|^2 = \underset{\mathbf{d} \in \mathbb{S}}{\arg\min} \left(\mathbf{d}^T\mathbf{H}^T\mathbf{H}\mathbf{d} - 2\mathbf{y}^T\mathbf{H}\mathbf{d}\right). \tag{6}$$

In the following, we present the low-complexity LAS detection algorithm we reported in [4],[5] for M-QAM.

A. LAS Algorithm for Large MIMO Detection

The LAS algorithm starts with an initial solution $\mathbf{d}^{(0)}$, given by $\mathbf{d}^{(0)} = \mathbf{B}\mathbf{y}$, where $\mathbf{B}$ is the initial solution filter, which can be a matched filter (MF) or zero-forcing (ZF) filter or MMSE filter. The index m in $\mathbf{d}^{(m)}$ denotes the iteration number in a given search stage. The ML cost function after the kth iteration in a given search stage is given by

$$C^{(k)} = {\mathbf{d}^{(k)}}^T\mathbf{H}^T\mathbf{H}\mathbf{d}^{(k)} - 2\mathbf{y}^T\mathbf{H}\mathbf{d}^{(k)}. \tag{7}$$

The $\mathbf{d}$ vector is updated from kth to $(k+1)$th iteration by updating one symbol, say, the pth symbol, as

$$\mathbf{d}^{(k+1)} = \mathbf{d}^{(k)} + \lambda_p^{(k)}\mathbf{e}_p, \tag{8}$$

where $\mathbf{e}_p$ denotes the unit vector with its pth entry only as one, and all other entries as zero. Since $\mathbf{d}^{(k)}$ and $\mathbf{d}^{(k+1)}$ should belong to $\mathbb{S}$, and therefore $\lambda_p^{(k)}$ can take only certain integer values. For example, for 16-QAM, $\mathbb{A}_p = \{-3, -1, 1, 3\}$), and $\lambda_p^{(k)}$ can take values only from $\{-6, -4, -2, 0, 2, 4, 6\}$. Using (7) and (8), and defining a matrix $\mathbf{G}$ as

$$\mathbf{G} \ \stackrel{\triangle}{=} \ \mathbf{H}^T\mathbf{H}, \tag{9}$$

we can write the cost difference $C^{(k+1)} - C^{(k)}$ as

$$\mathcal{F}(l_p^{(k)}) \ \stackrel{\triangle}{=} \ C^{(k+1)} - C^{(k)} \ = \ {l_p^{(k)}}^2 a_p - 2l_p^{(k)}|z_p^{(k)}|,$$

where $z_p^{(k)}$ is the pth entry of the $\mathbf{z}^{(k)}$ vector given by $\mathbf{z}^{(k)} = \mathbf{H}^T(\mathbf{y} - \mathbf{H}\mathbf{d}^{(k)})$, $a_p \stackrel{\triangle}{=} (\mathbf{G})_{p,p}$ is the (p,p)th entry of the $\mathbf{G}$ matrix, and $l_p^{(k)} = |\lambda_p^{(k)}|$. The value of $l_p^{(k)}$ which gives the largest descent in the cost function from the kth to the $(k+1)$th iteration (when symbol p is updated) is obtained as

$$l_{p,opt}^{(k)} \ = \ 2\left\lfloor \frac{|z_p^{(k)}|}{2a_p} \right\rceil, \tag{10}$$

where $\lfloor . \rceil$ denotes the rounding operation. If $d_p^{(k)}$ were updated using $l_{p,opt}^{(k)}$, it is possible that the updated value does not belong to $\mathbb{A}_p$. To avoid this, we adjust $l_{p,opt}^{(k)}$ so that the updated value of $d_p^{(k)}$ belongs to $\mathbb{A}_p$. Let

$$s \ = \ \underset{p}{\arg\min} \ \mathcal{F}(l_{p,opt}^{(k)}). \tag{11}$$

If $\mathcal{F}(l_{s,opt}^{(k)}) < 0$, the update for the $(k+1)$th iteration is

$$\mathbf{d}^{(k+1)} \ = \ \mathbf{d}^{(k)} + l_{s,opt}^{(k)}\,\mathrm{sgn}(z_s^{(k)})\mathbf{e}_s, \tag{12}$$

$$\mathbf{z}^{(k+1)} \ = \ \mathbf{z}^{(k)} - l_{s,opt}^{(k)}\,\mathrm{sgn}(z_s^{(k)})\mathbf{g}_s. \tag{13}$$

where $\mathbf{g}_s$ is the sth column of $\mathbf{G}$. If $\mathcal{F}(l_{s,opt}^{(k)}) \geq 0$, then the search terminates, and $\mathbf{d}^{(k)}$ is declared as the detected vector.

III. Asymptotic Analysis of LAS Algorithm

In this section, we prove the asymptotic convergence of the error probability of the LAS detector to that of the ML detector for $N_t, N_r \to \infty$ with $N_t = N_r$ in V-BLAST. Consider 4-QAM, i.e., $\mathbb{S} \in \{+1, -1\}^{2N_t}$, and let $N_t = N_r$. An n-symbol update on a data vector $\mathbf{d} \in \mathbb{S}$ transforms $\mathbf{d}$ to $(\mathbf{d} - \Delta\mathbf{d}_n)$ such that $(\mathbf{d} - \Delta\mathbf{d}_n) \in \mathbb{S}$. Further, $(\mathbf{d} - \Delta\mathbf{d}_n)$ is obtained by changing n symbols in $\mathbf{d}$ at distinct indices given by the n-tuple $\mathbf{u}_n \stackrel{\triangle}{=} (i_1, i_2, \cdots, i_n)$, $1 \leq i_j \leq 2N_t, \forall j = 1, \cdots, n$. Therefore, we can write $\Delta\mathbf{d}_n$ as

$$\Delta\mathbf{d}_n = \sum_{k=1}^{n} 2d_{i_k}\mathbf{e}_{i_k}, \tag{14}$$

where d_{i_k} is the i_kth element of $\mathbf{d}$. Let $\mathbb{L}_n \subseteq \mathbb{S}$ denote the set of data vectors such that for any $\mathbf{d} \in \mathbb{L}_n$, if a n-symbol update is performed on $\mathbf{d}$ resulting in a vector $(\mathbf{d} - \Delta\mathbf{d}_n)$, then $||\mathbf{y} - \mathbf{H}(\mathbf{d} - \Delta\mathbf{d}_n)|| \geq ||\mathbf{y} - \mathbf{H}\mathbf{d}||$. Our main result in this section is Theorem 2. To prove Theorem 2, we need the following Lemmas 1 to 5, Slutsky's theorem [8], and Theorem 1. Here, we state the Lemmas and Theorem 1 without proof due to page limit[3], and provide the proof for Theorem 2.

Lemma 1: Let $\mathbf{d} \in \mathbb{S}$. Then, $\mathbf{d} \in \mathbb{L}_n$ if and only if, for any n-update on $\mathbf{d}$, $n \in [1, 2 \cdots, 2N_t]$,

$$\left(\mathbf{y} - \mathbf{H}\mathbf{d} + \frac{1}{2}\mathbf{H}\Delta\mathbf{d}_n\right)^T\left(\mathbf{H}\Delta\mathbf{d}_n\right) \ \geq \ 0. \tag{15}$$

If $\mathbf{d} \in \mathbb{L}_1$, then using Lemma 1 and (5), we can write

$$\left(\mathbf{n} + \mathbf{H}(\mathbf{x} - \mathbf{d}) + \mathbf{h}_p d_p\right)^T\left(\mathbf{h}_p d_p\right) \geq 0, \ \forall p = 1, \cdots, 2N_t. \tag{16}$$

[3]Proofs of Lemmas 1 to 5 and Theorem 1 are available online arXiv:0806.2533v1 [cs.IT] 16 Jun 2008 [9].

Lemma 2: Assuming uniqueness of the ML vector, a symbol vector $\mathbf{d} \in \mathbb{S}$ is the ML vector if and only if the noise vector $\mathbf{n}$ satisfies the following set of equations

$$\Big(\mathbf{n} + \mathbf{H}(\mathbf{x} - \mathbf{d}) + \Big(\sum_{j=1}^{n} \mathbf{h}_{i_j} d_{i_j}\Big)\Big)^T \Big(\sum_{j=1}^{n} \mathbf{h}_{i_j} d_{i_j}\Big) \geq 0, \quad (17)$$

$\forall n = 1, \cdots, 2N_t$, and for all possible n-tuples $\{i_1, \cdots, i_n\}$ for each n.

Definition: For each $\mathbf{d} \in \mathbb{S}$ and for each integer m, $1 \leq m \leq 2N_t$, we associate a region $\mathbb{R}_{\mathbf{d}^m} \in \mathbb{R}^{2N_t}$, such that a noise vector $\mathbf{n} \in \mathbb{R}_{\mathbf{d}^m}$ if and only if $\mathbf{n}$ satisfies the set of equations in (17) $\forall n = 1, \cdots, m$, and for all possible n-tuples $\{i_1, \cdots, i_n\}$ for each n. Then define $\mathbb{R}_{\mathbf{d}} \triangleq \mathbb{R}_{\mathbf{d}^{2N_t}}$.

Lemma 3: If $\mathbf{n} \in \mathbb{R}_{\mathbf{d}}$, then $\mathbf{d}$ is the ML vector. Let $\mathbf{d}_i, \mathbf{d}_j \in \mathbb{S}$ and $\mathbf{d}_i \neq \mathbf{d}_j$. Then $\mathbb{R}_{\mathbf{d}_i}$ and $\mathbb{R}_{\mathbf{d}_j}$ are disjoint.

Lemma 4: Let $\mathbf{h} \in \mathbb{R}^{2N_t}$ be a random vector with i.i.d entries distributed as $\mathcal{N}(0, 0.5)$. Let $\{\mathbf{h}_i\}, i = 1, 2, \cdots, m$ be a set of vectors, with each $\mathbf{h}_i \in \mathbb{R}^{2N_t}$ and having i.i.d entries distributed as $\mathcal{N}(0, 0.5)$, $\mathbb{E}[\mathbf{h}_i \mathbf{h}_j^T] = 0$ for $i \neq j$, and $\mathbb{E}[\mathbf{h}\mathbf{h}_j^T] = 0$ for $j = 1, \cdots, m$. Then

$$\lim_{N_t \to \infty} \frac{\sum_{k=1}^{m} \mathbf{h}^T \mathbf{h}_k}{m N_t} = 0. \quad (18)$$

Lemma 5: Let $i_j \in \mathbf{u}_n$, $j = 1, \cdots, n$. Define a r.v $z_{\mathbf{u}_n}$ as

$$z_{\mathbf{u}_n} \triangleq \frac{\sum_{k=1}^{n} \sum_{j=k+1}^{n} \mathbf{h}_{i_j}^T \mathbf{h}_{i_k} d_{i_j} d_{i_k}}{\sum_{j=1}^{n} \|\mathbf{h}_{i_j}\|^2}. \quad (19)$$

For any $\mathbf{d} \in \mathbb{S}$, $z_{\mathbf{u}_n}$ converges to zero in probability as $N_t \to \infty$, i.e., $z_{\mathbf{u}_n} \xrightarrow{p} 0$ as $N_t \to \infty$ $\forall n = 2, 3, \cdots, 2N_t$.

Theorem 1: Let $\mathbf{d} \in \mathbb{S}$ and $\mathbf{n} \in \mathbb{R}_{\mathbf{d}^1}$. Then $\mathbf{n} \in \mathbb{R}_{\mathbf{d}}$ in probability as $N_t \to \infty$, i.e., for any δ, $0 \leq \delta \leq 1$, there exists an integer $N(\delta)$ such that for $N_t > N(\delta)$, $p(\mathbf{n} \in \mathbb{R}_{\mathbf{d}}) > 1 - \delta$.

Theorem 2: The data vector/bit error probability of the LAS detector converges to that of the ML detector as $N_t, N_r \to \infty$ keeping $N_t = N_r$.

Proof: Let $\mathbf{d}_{LAS}$ be the final output symbol vector of the LAS algorithm given $\mathbf{x}$, $\mathbf{H}$ and $\mathbf{n}$. The algorithm terminates if and only if no 1-update results in any further decrease of the cost function. This implies that for the given $\mathbf{x}$, $\mathbf{H}$ and $\mathbf{n}$, $\mathbf{d}_{LAS} \in \mathbb{L}_1$, and therefore it must be true that $\mathbf{n}$ satisfies (16) with $\mathbf{d}$ replaced by $\mathbf{d}_{LAS}$. These set of equations are the same which define the region $\mathbb{R}_{\mathbf{d}^1}$. Therefore, replacing $\mathbf{d}$ by $\mathbf{d}_{LAS}$, we can equivalently claim that $\mathbf{n} \in \mathbb{R}_{\mathbf{d}^1_{LAS}}$. Using Theorem 1, we can further claim that asymptotically as $N_t \to \infty$, $\mathbf{n} \in \mathbb{R}_{\mathbf{d}_{LAS}}$ in probability. From Lemma 3, we know that if $\mathbf{n} \in \mathbb{R}_{d_{LAS}}$, then $\mathbf{d}_{LAS}$ is indeed the ML vector for the given $\mathbf{x}$, $\mathbf{H}$ and $\mathbf{n}$. Therefore, we can state that asymptotically as $N_t \to \infty$, $\mathbf{d}_{LAS}$ is indeed the ML vector in probability. That is, for any δ, $0 \leq \delta \leq 1$, there exists an integer $N(\delta)$ such that for $N_t \geq N(\delta)$

$$P(\mathbf{d}_{LAS} \text{ is the ML vector}) > (1 - \delta). \quad (20)$$

Therefore, we can write that for $N_t \geq N(\delta)$

$$\begin{aligned} P_{LAS}(err) &= P(\mathbf{d}_{LAS} \neq \mathbf{x}) \quad (21) \\ &= P(\mathbf{d}_{LAS} \neq \mathbf{x} \,|\, \mathbf{d}_{LAS} = \text{ML vector}) P(\mathbf{d}_{LAS} = \text{ML vector}) \\ &+ P(\mathbf{d}_{LAS} \neq \mathbf{x} \,|\, \mathbf{d}_{LAS} \neq \text{ML vector}) P(\mathbf{d}_{LAS} \neq \text{ML vector}). \end{aligned}$$

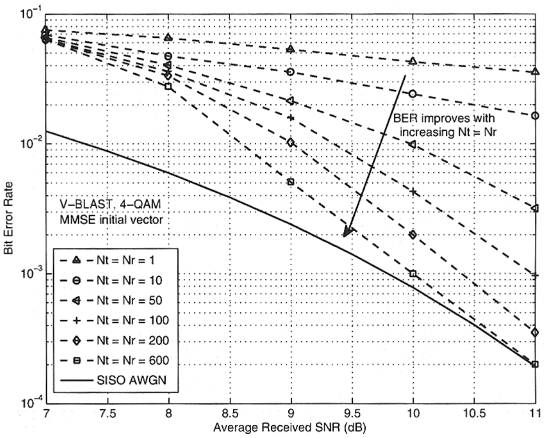

Fig. 1. Simulated BER performance of the LAS detector for V-BLAST as a function of average received SNR for increasing values of $N_t = N_r$. MMSE initial vector, 4-QAM. LAS detector achieves near SISO AWGN performance at high SNRs for large $N_t = N_r$.

From (20), we have $P(\mathbf{d}_{LAS} \neq \text{ML vector}) \leq \delta$. Also, $P(\mathbf{d}_{LAS} \neq \mathbf{x} \,|\, \mathbf{d}_{LAS} = \text{ML vector})$ is the probability of error for the ML detector, which we denote by $P_{ML}(err)$. Using these, we can bound the probability of error for the LAS detector as

$$\begin{aligned} P_{LAS}(err) &\leq P_{ML}(err) + \delta \, P(\mathbf{d}_{LAS} \neq \mathbf{x} \,|\, \mathbf{d}_{LAS} \neq \text{ML vector}) \\ &\leq \mathbf{P}_{ML}(err) + \delta. \quad (22) \end{aligned}$$

Since δ can be arbitrarily small, we can conclude from (22) that indeed as $N_t \to \infty$, the symbol vector error probability of the LAS detector converges to that of the ML detector. This proof can be adapted to show that apart from the symbol vector error probability, the bit error probability of the LAS detector also converges to that of the ML detector. The proof for the bit error probability convergence is along the same lines as (21) and (22), except that instead of defining the error event as $\mathbf{d}_{LAS} \neq \mathbf{x}$, we define error events for each bit; e.g., for the pth bit, the error event is defined as $d_{p_{LAS}} \neq x_p$. □

Simulation Results and Discussions: In Fig. 1 we present the simulated BER performance of the LAS detector for V-BLAST with 4-QAM and MMSE initial vector for increasing $N_t = N_r$. Since analytical expressions for ML performance in the large MIMO system limit for different signal sets are not available, we have plotted the SISO AWGN performance for comparison. It can be seen that for large $N_t = N_r$, the BER performance of the LAS detector approaches the SISO AWGN performance at high SNRs. Since large MIMO systems can be viable in practice due to the availability of low-complexity detectors like the LAS detector, analytical BER expressions for the ML performance in the large MIMO system limit would be quite useful as a benchmark for comparing the performance of practical detectors in large MIMO systems. The statistical mechanics approach employed in [10] for large CDMA system BER analysis can be investigated for such an analysis.

IV. Performance of LAS Detector in MIMO Spatial Fading Correlation

In [4],[5], we focused on the low-complexity and nearness to capacity performance aspects of the LAS detector, assuming an i.i.d. fading channel model (where the entries of the channel matrix are modeled as independent complex Gaussian r.v's). However, MIMO propagation conditions witnessed in practice often render the i.i.d. fading model as inadequate, and several more realistic MIMO channel models that take into account the scattering environment, spatial correlation, etc., have been investigated in the literature [7]. For example, spatial correlation at the transmit and/or receive side can affect the rank structure of the MIMO channel resulting in degraded MIMO capacity. The structure of scattering in the propagation environment can also affect the capacity [7]. Hence, it is of interest to investigate the performance of the LAS detector in more realistic MIMO channel models. In our present study, we use the NLOS MIMO channel model proposed by Gesbert *et al* in [7], which is briefly described below. This model can be appropriate in application scenarios like high data rate HDTV/wireless IPTV distribution using high spectral efficiency large MIMO links, where large N_t and N_r can be placed at the base station (BS) and customer premises equipment (CPE), respectively.

A. MIMO Channel Model in [7]

The propagation scenario for the MIMO channel model considered is shown in Fig. 2, where linear arrays of N_t omnidirectional transmit antennas with spacing d_t, and N_r omnidirectional receive antennas with spacing d_r are considered [7]. The propagation path between the transmit and receive arrays is obstructed on both sides of the link by a number of significant near-field scatterers (e.g., large objects) referred to as transmit and receive scatterers, which are modeled as omnidirectional ideal scatterers. The maximum range of the scatterers from the horizontal axis at the transmit and receive sides are denoted by D_t and D_r, respectively. When omnidirectional antennas are used, D_t and D_r correspond to the transmit and receive scattering radii, respectively. On the receive side, the signal reflected by the scatterers onto the antennas impinge on the array with an angular spread θ_r, which is a function of the distance between the array and the scatterers. Similarly, angular spread θ_t is defined on the transmit side. The range between the local scatterers at the transmit and receive sides is denoted by R. It is assumed that the scatterers are located adequately far from the antennas so that the plane-wave assumption holds. Further, local scattering condition is assumed, i.e., $D_t << R$ and $D_r << R$. The number of scatterers on each side, S, is considered to be large enough (typically > 10) for random fading to occur. The complex channel gain matrix as per this model is given by [7]

$$\mathbf{H}_c = \frac{1}{\sqrt{S}}\mathbf{R}^{1/2}_{\theta_r,d_r}\mathbf{G}_r\mathbf{R}^{1/2}_{\theta_S,2D_r/S}\mathbf{G}_t\mathbf{R}^{1/2}_{\theta_t,d_t}. \qquad (23)$$

where $\mathbf{G}_t = [\mathbf{g}_1\mathbf{g}_2\cdots\mathbf{g}_{N_t}]$ is an $S\times N_t$ i.i.d. Rayleigh fading matrix, $\mathbf{g}_n \sim \mathcal{CN}(\mathbf{0},\mathbf{I}_S)$, $\mathbf{G}_r = [\mathbf{g}_1\mathbf{g}_2\cdots\mathbf{g}_{N_r}]$, $\mathbf{R}^{1/2}_{\theta_t,d_t}$ and $\mathbf{R}^{1/2}_{\theta_r,d_r}$ are the $N_t\times N_t$ and $N_r\times N_r$ matrices controlling the transmit and receive antenna correlations, respectively, whose expressions are given in [7].

B. Simulation Results and Discussions

We have evaluated the uncoded as well as the turbo coded BER performance of the LAS detector under various parameter settings in the above MIMO channel model through simulations. We consider the following parameters in the simulations: $f_c = 5$ GHz, $R = 500$ m, $S = 30$, $D_t = D_r = 20$ m, $\theta_t = \theta_r = 90^\circ$, and $d_t = d_r = 2\lambda/3$. For the considered carrier frequency of 5 GHz, $\lambda = 6$ cm and $d_t = d_r = 4$ cm. We have evaluated the performance of both V-BLAST as well as full-rate non-orthogonal STBCs from DA in [6] which offer high spectral efficiencies. Large MIMO systems using full-rate non-orthogonal STBCs can be suitable for practical implementation when the quasi-static fading assumption is valid; for e.g., with static transmitter/receivers in residential wireless IPTV distribution application scenarios, the fading would be slow meeting the quasi-static fading assumption. Accordingly, here, we present the simulation results for large MIMO systems employing 16×16 and 12×12 STBCs from DA.

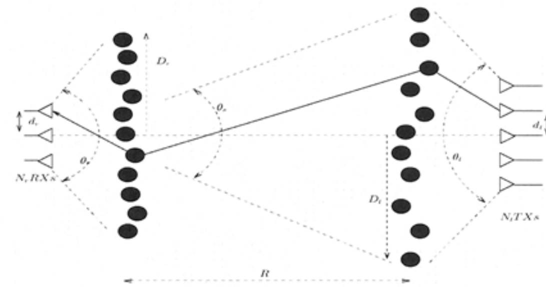

Fig. 2. Propagation scenario for the MIMO fading channel model.

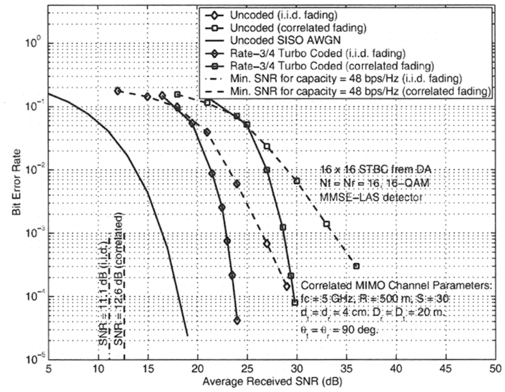

Fig. 3. Uncoded/coded BER performance of the proposed LAS detector in i.i.d. fading as well as correlated MIMO fading in [7] with parameters $f_c = 5$ GHz, $R = 500$ m, $S = 30, D_t = D_r = 20$ m, $\theta_t = \theta_r = 90^\circ$, and $d_t = d_r = 2\lambda/3 = 4$ cm. 16×16 STBC from DA, $N_t = N_r = 16$, 16-QAM, rate-3/4 turbo code, **48 bps/Hz spectral efficiency**.

Spatial Correlation Degrades BER Performance: In Fig. 3, we plot the BER performance of the LAS detector in decoding 16×16 full-rate non-orthogonal STBC from DA with $N_t = N_r = 16$ and 16-QAM. In all the simulations, MMSE output vector is used as the initial vector in the LAS algorithm. Perfect knowledge of the channel matrix is assumed at the receiver. Uncoded BER as well as rate-3/4 turbo coded BER (48 bps/Hz spectral efficiency) with i.i.d. fading and correlated fading are shown. Uncoded SISO AWGN performance is also plotted for comparison. In addition, from the MIMO capacity formula in [1], we evaluated the theoretical minimum SNR required to achieve a capacity of 48 bps/Hz and shown in Fig. 3 for i.i.d. as well as correlated fading. It is seen that the minimum SNR required to achieve a certain capacity gets increased for correlated fading compared to i.i.d. fading. From the BER plots in Fig. 3, it can be observed that at an uncoded BER of 10^{-3}, the performance in correlated fading degrades by about 7 dB compared that in i.i.d. fading. Likewise, at a rate-3/4 turbo coded BER of 10^{-4}, a performance loss of about 6 dB is observed in correlated fading compared to that in i.i.d. fading. In terms of nearness to capacity, the vertical fall of the coded BER for i.i.d. fading occurs at about 24 dB SNR, which is about 13 dB away from theoretical minimum required SNR of 11.1 dB. With correlated fading, the detector is observed to perform close to capacity within about 18.5 dB. One way to alleviate such degradation in performance due to spatial correlation and scattering environment can be by providing more number of dimensions at the receive side, as we will see in Fig. 4 and 5.

Asymmetry in Number of Antennas with $N_r > N_t$: Figure 4 illustrates that the LAS detector can achieve substantial improvement in uncoded as well as coded BER performance compared to $N_r = N_t$ by increasing N_r beyond N_t for 16-QAM in i.i.d. fading. For example, by comparing the LAS detector performance in decoding 12×12 STBC from DA with $(N_t = N_r = 12)$ versus $(N_t = 12, N_r = 18)$ we observe that the uncoded BER performance with $(N_t = 12, N_r = 18)$ improves by about 14 dB compared to $(N_t = N_r = 12)$ at 10^{-3} BER. Even the uncoded BER performance with $(N_t = 12, N_r = 18)$ is significantly better than the coded BER performance with $(N_t = N_r = 12)$. This improvement is essentially due to the ability of the LAS detector in effectively picking up the additional diversity orders provided by the increased number of receive antennas. With a rate-3/4 turbo code (36 bps/Hz spectral efficiency), at a coded BER of 10^{-4}, the LAS detector achieves a significant performance improvement of about 12.5 dB with $(N_t = 12, N_r = 18)$ compared to that with $(N_t = N_r = 12)$. In fact, with $(N_t = 12, N_r = 18)$ the vertical fall of coded BER is such that it is only about 7 dB from the theoretical minimum SNR to achieve capacity.

A similar advantage of exploiting antenna asymmetry with $N_r > N_t$ and 16-QAM in correlated fading is illustrated in Fig. 5, where we have maintained $N_r d_r = 72$ cm and $d_t = d_r$ in both the cases of symmetry (i.e., $N_t = N_r = 12$) as well as asymmetry (i.e., $N_t = 12, N_r = 18$). From Fig. 5, it can be observed that with asymmetry and rate-3/4 turbo code (i.e., 36 bps/Hz), the LAS detector achieves near-capacity performance to within just about 8 dB, which is a significant result which points to the potential of realizing practical high spectral efficiency multi-gigabit large MIMO systems that can achieve near-capacity performance even in the presence of spatial correlations.

V. Conclusion

While the spectral efficiencies achieved in current MIMO wireless standards (e.g., IEEE 802.11n and 802.16e) are only about 10 bps/Hz or less, the practical feasibility of the proposed LAS detector and its ability to perform well at much higher spectral efficiencies through the use of 16×16 and 12×12 non-orthogonal STBCs can enable practical implementation of multi-gigabit MIMO wireless systems with spectral efficiencies in excess of 10 bps/Hz. This approach can also be relevant in multi-gigabit 60 GHz WPAN/VHT systems under NLOS conditions; the advantages being higher spectral efficiencies and robustness to blockages (e.g., obstruction due to moving persons/objects) in indoor environments.

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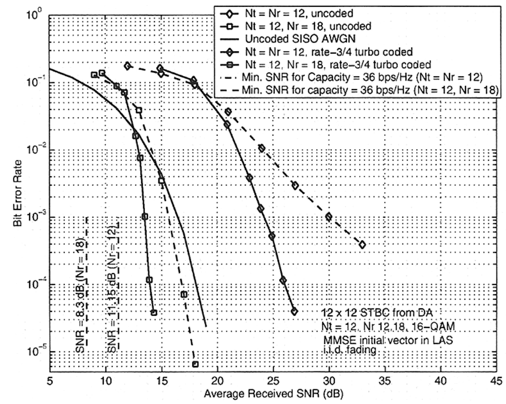

Fig. 4. Effect of asymmetric large MIMO with $N_r > N_t$ in i.i.d. fading. 12×12 STBC from DA, $N_t = 12$, $N_r = 12, 18$, 16-QAM, rate-3/4 turbo code, **36 bps/Hz spectral efficiency**. MMSE initial vector in LAS detection.

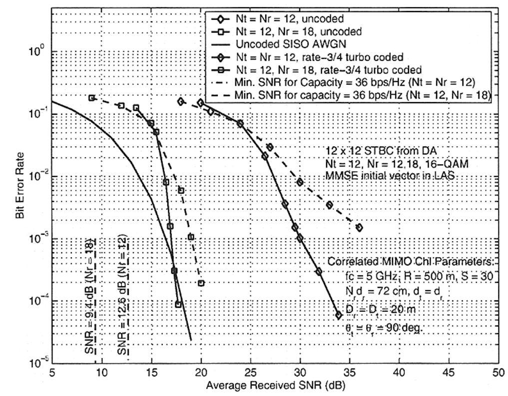

Fig. 5. Effect of asymmetric large MIMO with $N_r > N_t$ in correlated MIMO fading in [7] keeping $N_r d_r$ constant and $d_t = d_r$. $N_r d_r = 72$ cm, $f_c = 5$ GHz, $R = 500$ m, $S = 30$, $D_t = D_r = 20$ m, $\theta_t = \theta_r = 90^\circ$, 12×12 STBC from DA, $N_t = 12$, $N_r = 12, 18$, 16-QAM, rate-3/4 turbo code, **36 bps/Hz spectral efficiency**. MMSE initial vector in LAS detection.

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