

# Increasing System Capacity in Uplink Multiuser MIMO Using Sparsity Exploiting Receiver

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**Abstract**— In multiuser communication on the uplink, all subscribed users may not be active simultaneously. This leads to sparsity in the activity pattern in the users' transmissions, which can be exploited in the multiuser MIMO receiver at the base station (BS). Because of no transmissions from inactive users, joint detection at the BS has to consider an augmented signal set that includes zero. In this paper, we propose a receiver that exploits this inactivity-induced sparsity and considers the zero-augmented signal set. The proposed receiver is based on Markov Chain Monte Carlo techniques. Near-optimal performance and increased system capacity (in terms of number of users in the system) are demonstrated. For example, a multiuser MIMO system with  $N = 32$  receive antennas at the BS and an user activity factor of 0.2 supports 51 uplink users meeting a QoS of  $10^{-3}$  coded bit error rate.

**Keywords** – Sparse signal detection, uplink multiuser MIMO, user activity, MCMC techniques, system capacity.

## I. INTRODUCTION

Sparsity in naturally occurring signals is key to compressive sensing/sampling idea which asserts that sparse signals can be reconstructed from fewer samples [1], [2], [3]. The idea of compressed sensing has become very popular due to its theoretical elegance and application in various fields including communication. For example, in wireless channels with large delay spreads, the channel impulse response generally shows sparse behavior (i.e., only a few multipath taps have strong received powers) [4]. In case of sensor networks, where an unmanned aerial vehicle collects data from a set of deployed sensors, the number of active sensors can be smaller than the total number of sensors to save sensing resources [5]. In on-off random access channels, detection of active users has been shown to be mathematically equivalent to sparsity detection in compressed sensing [6].

In multiuser communication on the uplink, all subscribed users may not be active simultaneously, as the typical activity factor (the probability that each user is active) is low. This inactivity of users creates sparsity in the total signal. In this paper, we are interested in exploiting such sparsity that occurs in multiuser MIMO systems on the uplink. This is of particular interest given that there is growing interest in large MIMO systems with tens to hundreds of antennas at the base station (BS) [7]. The BS detects the users signals based on their spatial signatures. This puts a limit on the number of users in the system (system capacity) for a given number of receive antennas at the BS. Exploitation of the inactivity-induced sparsity can increase the system capacity. Alternately, to achieve a certain system capacity, the number of BS receive antennas can be reduced.

The problem of signal detection in sparse multiuser MIMO systems becomes a special case of the compressive sensing problem with finite alphabet constraint. Low complexity receivers for multiuser detection where the number of active users is unknown has been studied in [8] using the optimal strategy of jointly detecting the set of active users and their data. Detection techniques exploiting sparsity under finite alphabet constraint has been studied in [5], [9], [10], [11]. A sphere decoder (SD) based receiver is implemented in [9] for DS-CDMA. In [5], the authors have shown results with SD and semi-definite relaxation (SDR) algorithms modified for sparse detection. Multiuser detection for sparse CDMA systems has also been recently investigated in [11] using Lasso (least-absolute shrinkage and selection operator) based algorithms. Another low-complexity QR decomposition based algorithm named decision directed detector (DDD) has also been proposed in [11] which takes a serial interference cancellation approach for detecting transmitted symbols.

In this paper, our focus is on low complexity sparsity exploiting detection in large uplink multiuser MIMO systems with tens to hundreds of antennas at the BS and similar number of users with one or more antennas. The symbol alphabet is augmented by zero to capture the user inactivity. The computational complexity of the detection problem with augmented alphabet becomes more when users transmit from complex alphabets (e.g. QAM, PSK) compared to when real alphabets (e.g., PAM) are used. Augmenting zero into a complex alphabet makes it a non-standard alphabet, and hence the corresponding complex system model can not be converted into an equivalent real system model. Therefore, we work with the complex system model and modify the optimum detection criterion to exploit the user inactivity induced sparsity by adding scaled zeroth norm of the wanted symbol vector as a penalty term. This optimum detection being exponential in complexity (and hence prohibitive for large systems), it becomes necessary to devise low complexity techniques that can render near optimality in performance.

In this paper, we demonstrate that Markov chain Monte Carlo (MCMC) simulation approach – a class of low complexity sampling based algorithms, shown to be effective in communication problems like CDMA multiuser detection and MIMO detection [12], [13] – can be successful in achieving almost optimal performance in the sparse multiuser MIMO environment. This, however, needs modifications to the sampling and stopping strategies compared to conventional MCMC algorithms. Sparsity exploitation by the proposed MCMC based receiver is shown to increase the number of uplink users sup-

ported in a multiuser MIMO system meeting QoS requirement in terms of coded bit error rate (BER) performance.

## II. SYSTEM MODEL

Consider the uplink in a multiuser MIMO system with  $K$  synchronized users, each user having one transmit antenna and BS having  $N$  receive antennas (Fig. 1). Extension of the system model to have users with multiple antennas is straightforward. Transmission is organized into frames, each frame consisting of  $\tau$  channel uses. Each user is assumed to be active in a frame (i.e., transmit in that frame) with probability  $\alpha$ , known as the activity factor. That is, at the start of each frame, each user decides to transmit with probability  $\alpha$ , independent of its transmission in other frames and transmissions from other users. Let the users transmit their symbols from the complex alphabet  $\mathbb{S}$ , where  $|\mathbb{S}| = M = 2^m$  and each complex symbol contains  $m$  bits. Let the set of active users in one frame be denoted by  $\mathbb{A}$ . Let the transmitted symbol vector from  $k$ th user be denoted by  $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^\tau]$ . If  $k \in \mathbb{A}$ , then  $x_k^t \in \mathbb{S}$ , else  $x_k^t = 0$ ,  $\forall t = 1, 2, \dots, \tau$  and  $\forall k = 1, 2, \dots, K$ . The symbols transmitted by each user are formed from the coded bits which are output from a convolution encoder of rate  $r$ . Information bit vector  $\mathbf{b}_k = [b_k^1, b_k^2, \dots, b_k^B]$  of length  $B$  is the input to the convolution encoder, where  $B/r = \tau m$ . Gray mapping from coded bits to modulation symbols is assumed.

The transmitted symbol vectors from all users can be written in a matrix form as  $\mathbf{X} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_K^T]^T$ . Let  $\mathbf{H} \in \mathbb{C}^{N \times K}$ , given by  $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K]$ , denote the channel gain matrix, where  $\mathbf{h}_k = [h_{1k}, h_{2k}, \dots, h_{Nk}]^T$  is the channel gain vector from user  $k$  to the BS, and  $h_{jk}$  denotes the channel gain from  $k$ th user to  $j$ th receive antenna at the BS. Assuming rich scattering and adequate spatial separation between users and BS antenna elements,  $h_{jk}, \forall j, k$  are assumed to be independent Gaussian with zero mean and  $\sigma_k^2$  variance such that  $\sum_k \sigma_k^2 = K$ .  $\sigma_k^2$  models the imbalance in received powers from different users, and  $\sigma_k^2 = 1$  corresponds to the perfect power control scenario. We assume that BS knows all the channel entries, and that the channel gains remain constant over one frame. Now, the received signal matrix at the BS in a frame, denoted by  $\mathbf{Y} \in \mathbb{C}^{N \times \tau}$ , can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}, \quad (1)$$

where  $\mathbf{W}$  is the noise matrix of size  $N \times \tau$  whose entries are modeled as i.i.d.  $\mathcal{CN}(0, \sigma^2)$ . The average received SNR in dB scale is defined as  $10 \log_{10} \left( \frac{\alpha K E_s}{\sigma^2} \right)$ , where  $E_s$  is the average symbol energy and  $10 \log_{10} \left( \frac{E_s}{\sigma^2} \right)$  is the transmit power to noise ratio for each user.

## III. RECEIVER STRUCTURE

The receiver structure at the BS consists of three parts, namely,

- sparse randomized-MCMC algorithm for vector-wise data detection,
- decoder pre-processing of detected symbols, and

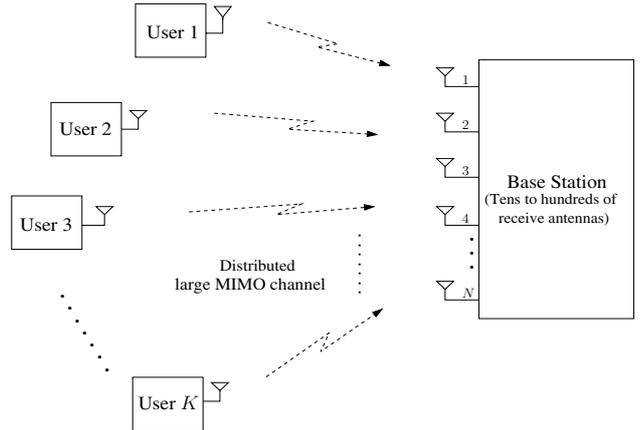


Fig. 1. Large multiuser MIMO system on the uplink.

- convolution decoding.

These functions are presented in the following subsections.

### A. Sparse Randomized MCMC

In the detection phase, the receiver takes the received vector in each channel use and detects the symbols independent of other channel uses. In  $t$ th channel use,  $t = 1, 2, \dots, \tau$ , the BS receives a  $N \times 1$  vector  $\mathbf{y}^t$ , the  $t$ th column of  $\mathbf{Y}$  which can be written as

$$\mathbf{y}^t = \mathbf{H}\mathbf{x}^{(t)} + \mathbf{w}^t, \quad (2)$$

where  $\mathbf{w}^t$  is the  $t$ th column vector of  $\mathbf{W}$ , and  $\mathbf{x}^{(t)}$  is the  $t$ th column vector of  $\mathbf{X}$  which denotes the assembled symbol vector transmitted from all users in the  $t$ th channel use. Note that the symbol from an inactive user in a channel use is zero. Hence, the symbols of  $\mathbf{x}^{(t)}$  come from an augmented alphabet  $\mathbb{S}_0 = \mathbb{S} \cup 0$ . The optimum detection rule then becomes

$$\mathbf{x}_{opt} = \arg \max_{\hat{\mathbf{x}} \in \mathbb{S}_0^K} p(\hat{\mathbf{x}} | \mathbf{y}, \mathbf{H}), \quad (3)$$

where

$$\begin{aligned} p(\hat{\mathbf{x}} | \mathbf{y}, \mathbf{H}) &\propto p(\mathbf{y} | \hat{\mathbf{x}}, \mathbf{H}) p(\hat{\mathbf{x}}) \\ &= \exp \left( - \frac{\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2}{\sigma^2} \right) \left( \frac{\alpha}{M} \right)^{\|\hat{\mathbf{x}}\|_0} (1 - \alpha)^{K - \|\hat{\mathbf{x}}\|_0} \\ &\propto \exp \left( - \frac{\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2}{\sigma^2} \right) \left( \frac{\alpha}{1 - \alpha} \right)^{\|\hat{\mathbf{x}}\|_0}. \end{aligned} \quad (4)$$

So the optimum detection rule can be simplified as

$$\mathbf{x}_{opt} = \arg \min_{\hat{\mathbf{x}} \in \mathbb{S}_0^K} (\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2 + c \|\hat{\mathbf{x}}\|_0), \quad (5)$$

where  $c = \sigma^2 \log \left( \frac{1 - \alpha}{\alpha / M} \right)$ . The index  $t$  is dropped for notational simplicity from now on. It can be observed that, due to the zero augmented nature of the transmit alphabet, the detection has to be done in complex domain itself. The detection problem in (5) can be solved by using MCMC simulations.

1) *Conventional MCMC*: In conventional MCMC or Gibbs sampling based detection, the algorithm starts with an initial symbol vector, denoted by  $\mathbf{x}^{(i=0)}$ . In each iteration of the algorithm, an updated symbol vector is obtained by sampling each symbol from its probability distribution keeping others fixed. For  $k$ th coordinate (i.e.  $k$ th user) the update is done as

$$x_k^{(i+1)} \sim p\left(x_k | x_1^{(i+1)}, x_2^{(i+1)}, \dots, x_{k-1}^{(i+1)}, x_{k+1}^{(i)}, \dots, x_K^{(i)}, \mathbf{y}, \mathbf{H}\right). \quad (6)$$

The detected symbol vector in a given iteration is chosen to be that symbol vector which has the least maximum-likelihood (ML) cost in all the iterations up to that iteration.

2) *Randomized MCMC*: In conventional MCMC, stalling problem occurs due to the MCMC iterations getting trapped in poor local solutions, beyond which the ML cost does not improve with increasing iterations for a long time. A simple, yet effective, idea to avoid such traps is a randomization strategy in the sampling [14]. In each iteration, instead of updating  $x_i^{(t)}$ 's as per the Gibbs sampling rule in (6) with probability 1 (as done in conventional MCMC),  $x_i^{(t)}$ 's are updated as in (6) with probability  $(1 - q_i)$  and with probability  $q_i$  they are sampled from uniform distribution. This randomized strategy (randomized MCMC) alleviates the stalling problem in the case of 4-QAM alphabet. A random restart strategy, where randomized MCMC algorithm is run multiple times each time with a different random initial vector and choose the best solution among multiple restarts, achieves near-optimal performance for higher-order QAM. Reduced complexity is achieved using a stopping criterion based on a standardized ML cost [14].

3) *Sparse R-MCMC*: Due to the sparse nature of the transmitted vector in the considered system model, the probabilities in (6) are calculated using the modified ML metric (5). The computation of standardized ML cost required in the calculation of stopping criterion and restart criterion in [14] is done as follows:

$$\phi(\hat{\mathbf{x}}) = \frac{\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2 - N\sigma^2 - cK\alpha}{\sqrt{N}\sigma^2}. \quad (7)$$

Let the detected symbol vector at time  $t$  be denoted as  $\tilde{\mathbf{x}}^{(t)}$ . The uncoded symbol error rate is obtained by taking the average over the number of mismatches among  $\mathbf{x}^{(t)}$  by Monte Carlo simulations. The per-symbol complexity of sparse R-MCMC is quadratic in  $K$  which is suited for large multiuser MIMO systems.

### B. Decoder pre-processing of detected symbols

$\tilde{\mathbf{x}}^{(t)}$ 's,  $t = 1, 2, \dots, \tau$ , are stacked together to form  $\tilde{\mathbf{X}} = [\tilde{\mathbf{x}}^{(1)}, \tilde{\mathbf{x}}^{(2)}, \dots, \tilde{\mathbf{x}}^{(\tau)}]$ . From this matrix, the receiver has to extract soft values of the bits that will be fed to the convolution decoder. This process is done in the following two steps:

1) *Estimating the active set  $\mathbb{A}$  using optimal threshold*: Let  $\hat{\mathbf{x}}_k$  denote the  $k$ th row of  $\tilde{\mathbf{X}}$ . Hence  $\hat{\mathbf{x}}_k$  contains all the detected symbols from user  $k$ . Since any user has either transmitted in all channel uses within a frame or has transmitted in

none, the receiver compares the number of non-zeros entries in  $\hat{\mathbf{x}}_k$  to a threshold  $\theta$  to decide whether the user is active or not. Let the set of active users estimated from  $\tilde{\mathbf{X}}$  be  $\tilde{\mathbb{A}}$ . Hence,

$$k \in \tilde{\mathbb{A}} \quad \text{iff} \quad \|\hat{\mathbf{x}}_k\|_0 \geq \theta. \quad (8)$$

The optimal choice of  $\theta$  is done by minimizing the support recovery error rate  $P_s$ , defined by  $\mathcal{E}\left(\frac{|\mathbb{A} \cap \tilde{\mathbb{A}}^C| + |\mathbb{A}^C \cap \tilde{\mathbb{A}}|}{K}\right)$ , where  $\mathbb{A}^C$  denotes the complementary set of  $\mathbb{A}$ .

Let  $p_{10}$  be the probability of a non-zero symbol being detected as zero and  $p_{01}$  be the probability of zero being detected as a non-zero symbol. Under the assumption that all error events in vector-wise detection using sparse R-MCMC are independent, the support recovery error rate can be written as function of  $\theta$  as

$$P_s(\theta) = \alpha \sum_{n=0}^{\theta-1} \binom{\tau}{n} (p_{10})^{\tau-n} (1-p_{10})^n + (1-\alpha) \sum_{n=\theta}^{\tau} \binom{\tau}{n} (p_{01})^n (1-p_{01})^{\tau-n}, \quad (9)$$

where  $\binom{\tau}{n} = \frac{\tau!}{n!(\tau-n)!}$ . Hence the optimal choice of  $\theta$  is

$$\theta_{opt} = \arg \min_{0 \leq \theta \leq \tau} P_s(\theta). \quad (10)$$

Clearly,  $\theta_{opt}$  is the minimum integer value of  $n$  such that

$$\frac{\alpha \binom{\tau}{n} (p_{10})^{\tau-n} (1-p_{10})^n}{(1-\alpha) \binom{\tau}{n} (p_{01})^n (1-p_{01})^{\tau-n}} > 1. \text{ Simplifying, we get}$$

$$\theta_{opt} = \left\lceil \frac{\log\left(\frac{\alpha}{1-\alpha}\right) + \tau \log\left(\frac{p_{10}}{1-p_{01}}\right)}{\log\left(\frac{p_{10}p_{01}}{(1-p_{10})(1-p_{01})}\right)} \right\rceil. \quad (11)$$

In low to medium SNRs, due to large value to  $c$  in (5),  $p_{10} > p_{01}$  and thus  $\theta_{opt} < \tau/2$ . If  $c = 0$ , which happens in high SNR and/or  $\alpha = \frac{M}{M+1}$ , then  $\theta_{opt} \approx \tau/2$ .

2) *Generating soft values from sparse R-MCMC detected outputs*: Now consider a matrix  $\tilde{\mathbf{X}}$  of size  $|\tilde{\mathbb{A}}| \times \tau$  which is obtained by deleting the rows in  $\tilde{\mathbf{X}}$  corresponding to inactive users. Hence, the  $a$ th row in  $\tilde{\mathbf{X}}$  corresponds to detected symbols from the  $a$ th active user, where  $a = 1, 2, \dots, |\tilde{\mathbb{A}}|$ . Note that these detected symbols are hard values from the augmented alphabet  $\mathbb{S}_0$ . We obtain the reduced channel matrix  $\tilde{\mathbf{H}}$  of size  $N \times |\tilde{\mathbb{A}}|$  which contains only the channel gains from active users ( $a$ th column of  $\tilde{\mathbf{H}}$  is the channel gains from  $a$ th active user to the BS antennas). Knowing  $\mathbf{Y}$ ,  $\tilde{\mathbf{H}}$  and  $\tilde{\mathbf{X}}_0$ , the receiver needs to estimate the soft values of every transmitted bit. As in Sec. III-A, the operation is done for each channel use  $t$ ,  $t = 1, 2, \dots, \tau$ . Let the  $t$ th column of  $\tilde{\mathbf{X}}$  be denoted as  $\tilde{\mathbf{x}}^{(t)}$ , whose  $a$ th entry corresponding to  $a$ th active user is denoted as  $\tilde{x}_a^{(t)}$ .

Let the set  $\mathbb{S}$  be partitioned into  $\mathbb{S}_i^+$  and  $\mathbb{S}_i^-$  for each  $i$ ,  $i = 1, 2, \dots, m$ , where  $\mathbb{S}_i^+$  is the set of all the symbols in which  $i$ th bit is +1, and  $\mathbb{S}_i^-$  is the set of all the symbols in which  $i$ th bit is -1. Let the soft value of the  $i$ th bit of the  $a$ th active user in  $t$ th channel use be denoted as  $L_{a,i}^t$ . Let  $\tilde{\mathbf{x}}_{-a}^{(t)}$  denote the

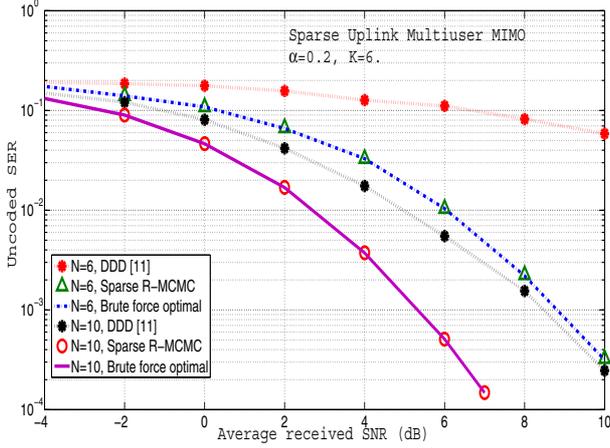


Fig. 2. Uncoded SER comparison between sparse R-MCMC, DDD, and brute force optimal algorithms in uplink multiuser MIMO. ( $K = 6, N = 6$ ) and ( $K = 6, N = 10$ ), 4-QAM,  $\alpha = 0.2$ .

vector containing all elements in  $\bar{\mathbf{x}}^{(t)}$  other than  $\bar{x}_a^{(t)}$ . Now  $L_{a,i}^t$  can be written as

$$L_{a,i}^t = \log \left( \frac{\sum_{s \in \mathcal{S}_i^+} p(\bar{x}_a^{(t)} = s | \bar{\mathbf{x}}_{-a}^{(t)}, \mathbf{y}^t, \bar{\mathbf{H}})}{\sum_{s \in \mathcal{S}_i^-} p(\bar{x}_a^{(t)} = s | \bar{\mathbf{x}}_{-a}^{(t)}, \mathbf{y}^t, \bar{\mathbf{H}})} \right). \quad (12)$$

Thus the receiver generates  $\tau m$  soft values for each active user, which are fed to the convolution decoder.

For convolution decoding, we use trellis based BCJR decoder [15]. The decoder outputs  $B$  decoded bits for each user. We calculate the coded error rate performance for the active users who are detected correctly as the active ones.

#### IV. RESULTS AND DISCUSSIONS

The uncoded symbol error rate (SER) as well as coded BER performance of the sparse R-MCMC algorithm are evaluated through simulations. We have used the rate-1/2 convolution code given by the generator  $(1 + D^2, 1 + D + D^2)$ . Figures 2 to 7 and I are for the case of perfect power control scenario whereas in 8 and II we consider imperfect power control case.

In Fig. 2, we compare the uncoded SER performance of sparse R-MCMC algorithm with those of the decision-directed decoder (DDD) algorithm in [11] as well as brute force optimal detector for ( $K = 6, N = 6$ ) and ( $K = 6, N = 10$ ) systems with  $\sigma_k^2 = 1, \forall k$ . It is seen that sparse R-MCMC algorithm gives almost optimal performance, whereas the performance of DDD algorithm is far from optimal (due to error propagation in serial interference cancellation approach).

Figure 3 shows a uncoded SER performance comparison between sparse R-MCMC and DDD algorithms for  $N = 32$  and  $K = 20, 32, 50$  for  $\alpha = 0.2$ . It is seen that sparse R-MCMC outperforms DDD by a significant margin. The performance of DDD for under-determined system of  $K = 50, N = 32$  is not shown as that would involve exhaustive search over  $K - N + 1$  symbols. Also, the performance of brute force optimal detector is not possible for these large systems.

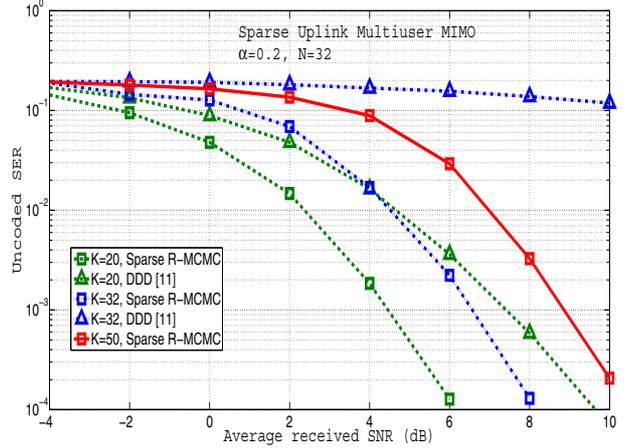


Fig. 3. Uncoded SER performance comparison between sparse R-MCMC and DDD algorithms for  $K = 20, 32, 50, N = 32$ , 4-QAM,  $\alpha = 0.2$ .

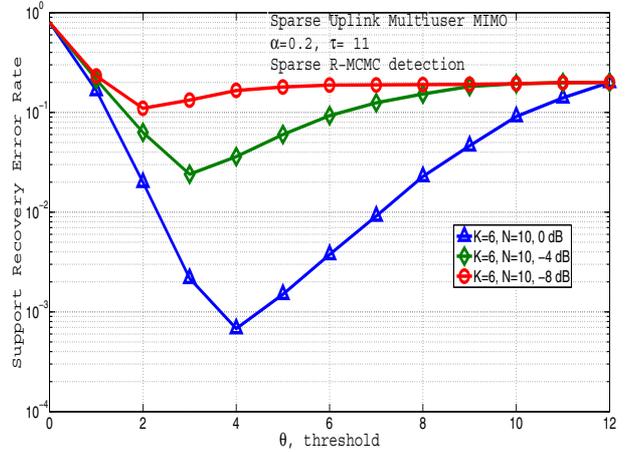


Fig. 4. Variation of support recovery error rate with  $\theta$  for  $K = 6, N = 10, \alpha = 0.2, \tau = 11$ , 4-QAM.

Figure 4 shows the variation of support recovery error rate with  $\theta$  for a  $K = 6, N = 10, \alpha = 0.2, \tau = 11$  system with sparse R-MCMC detection and post detector processing for average receive SNRs of -8 dB, -4 dB, and 0 dB. The figure shows that performance is optimal for  $\theta = 2, 3$ , and 4 for SNR = -8 dB, -4 dB, 0 dB, respectively. The calculated values of  $\theta_{opt}$  from (11), respectively, are also 2, 3, and 4. Figure 5 shows support recovery error rate comparison between sparse R-MCMC and DDD algorithms for  $N = 32, K = 20, 32, 50, \alpha = 0.2$  and  $\tau = 51$ . The figure shows that sparse R-MCMC performs much better than DDD for both  $K = 20, N = 32$  and  $K = 32, N = 32$ . Figure 5 shows support recovery error rate comparison between sparse R-MCMC and DDD algorithms for  $N = 32, K = 20, 32, 50, \alpha = 0.2$  and  $\tau = 51$ . The figure shows that sparse R-MCMC performs much better than DDD in terms of support recovery as well.

The coded BER comparison between sparse R-MCMC and DDD algorithms for  $K = 20, 32, 50, N = 32, \alpha = 0.2, \tau = 51$  is shown in Fig. 6. The convolution coded sparse R-MCMC receiver is able to achieve  $2 \times 10^{-4}$  BER at av-

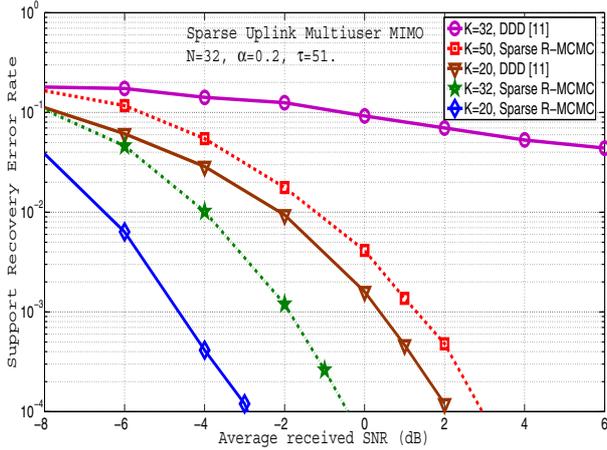


Fig. 5. Support recovery error rate comparison between sparse R-MCMC and DDD for  $K = 20, 32, 50, N = 32, \alpha = 0.2, \tau = 51$ , and 4-QAM.

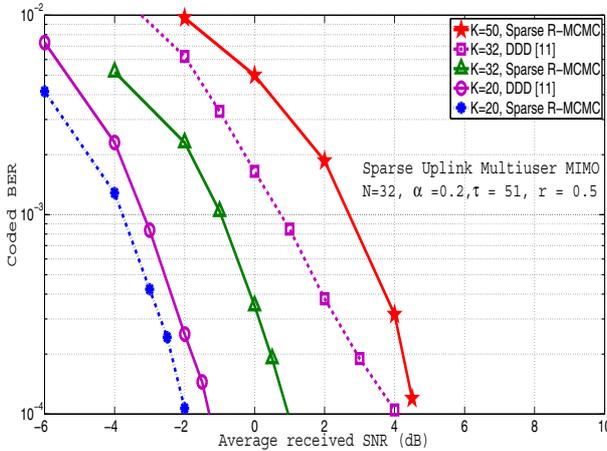


Fig. 6. Coded BER performance comparison between sparse R-MCMC and DDD algorithms for  $K = 20, 32, 50, N = 32, \alpha = 0.2, \tau = 51$ , 4-QAM, rate-1/2 convolution code.

verage receive SNR of -2.3 dB, 0.5 dB and 4.3 dB for  $(K = 20, N = 32)$ ,  $(K = 32, N = 32)$ ,  $(K = 50, N = 32)$  systems, respectively.

Figure 7 and Table-I illustrate the potential increase in the system capacity (number of users supported in the system) due to sparsity exploitation. Figure 7 shows that variation in coded BER performance with increasing number of users  $K$  for various values of activity factors ( $\alpha = 0.1, 0.2, 0.3, 0.4, 1$ ) for  $N = 32$  and sparse R-MCMC detection. For this experiment, we have kept each user's transmission power to noise ratio at -7 dB. It is seen that, with  $N = 32$  BS receive antennas and  $\alpha = 0.2$  activity factor, 51 users can be supported meeting an coded BER of  $1 \times 10^{-3}$ .

Figure 8 and Table-II illustrate the potential increase in the system capacity (number of users supported in the system) due to sparsity exploitation in the absence of perfect power control in the BS. 22 different users. The power imbalance is simulated by choosing different  $\sigma_k^2$  for different users, with  $\sigma_k^2$  being uniformly distributed between -3 dB to 3

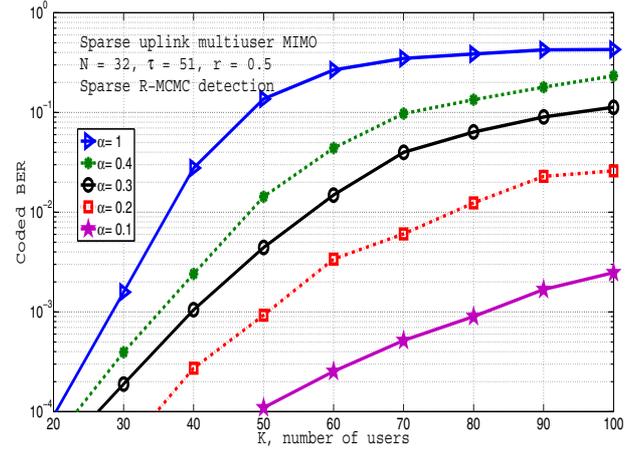


Fig. 7. Coded BER performance of sparse R-MCMC as a function of number of users  $K$  for  $N = 32$  receive antennas at the BS and for different activity factors  $\alpha = 0.2, 0.3, 0.4, 1$  and 4-QAM.

Activity factor $\alpha$	No. of uplink users supported Perfect power control case at $10^{-3}$ coded BER
1	28
0.4	35
0.3	40
0.2	51
0.1	82

TABLE I

NUMBER OF USERS THAT CAN BE SUPPORTED USING  $N = 32$  RECEIVE ANTENNAS AT THE BS FOR DIFFERENT ACTIVITY FACTORS USING SPARSE R-MCMC DETECTION (OBTAINED FROM FIG. 7).

dB. Figure 8 shows that variation in coded BER performance with increasing number of users  $K$  for various values of activity factors ( $\alpha = 0.1, 0.2, 0.3, 0.4, 1$ ) for  $N = 32$  and sparse R-MCMC detection. For this experiment, we have kept each user's transmission power to noise ratio at -7 dB as in 7. It is seen that, with  $N = 32$  BS receive antennas and  $\alpha = 0.2$  activity factor, 32 users can be supported meeting an coded BER of  $1 \times 10^{-3}$ , which is more than that of  $\alpha = 1$  for this power imbalance scenario also.

## V. CONCLUSION

We considered the problem of sparse signal detection in multiuser MIMO systems on the uplink. Sparsity is natural in this system due to inactivity of some fraction of the subscribed users in the system. The inactivity of the users was modeled by zero-augmenting of the transmission alphabet. We proposed a MCMC based receiver that exploited this sparsity in jointly detecting the users' data at the BS receiver. Simulation results showed that the proposed receiver achieved near-optimal performance. Also, it was shown that by exploiting the inactivity-induced sparsity in the signal detection, the system capacity (in terms of number of users supported in the system) can be increased beyond the number of receive an-

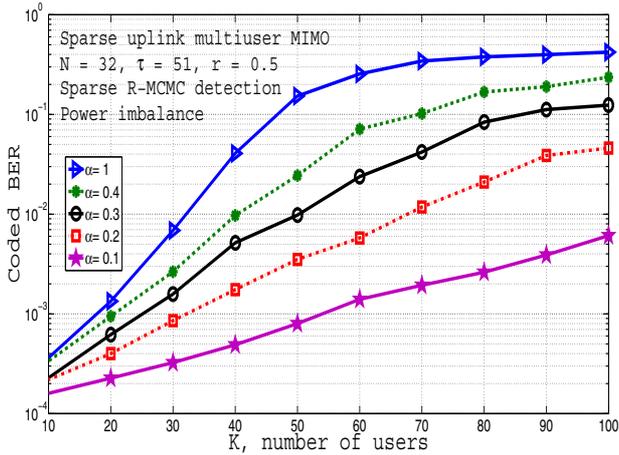


Fig. 8. Coded BER performance of sparse R-MCMC as a function of number of users  $K$  for  $N = 32$  receive antennas at the BS and for different activity factors  $\alpha = 0.2, 0.3, 0.4, 1$  and 4-QAM for  $-3$  dB to  $3$  dB power imbalance scenario.

Activity factor $\alpha$	No. of uplink users supported Power imbalance case at $10^{-3}$ coded BER
1	18
0.4	21
0.3	25
0.2	32
0.1	54

TABLE II

NUMBER OF USERS THAT CAN BE SUPPORTED USING  $N = 32$  RECEIVE ANTENNAS AT THE BS FOR DIFFERENT ACTIVITY FACTORS USING SPARSE R-MCMC DETECTION FOR  $-3$  DB TO  $3$  DB POWER IMBALANCE SCENARIO (OBTAINED FROM FIG. 8).

tennas at the BS while meeting QoS requirement in terms of coded BER performance. Future extension to this work can include capacity analysis in a multi-cell environment.

## REFERENCES

- [1] E. J. Candes and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, pp. 21-30, Mar. 2008.
- [2] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.
- [3] E. J. Candes and T. Tao, "Near-optimal signal recovery from random projections: universal encoding strategies?," *IEEE Trans. Inf. Theory*, vol. 52, pp. 5406-5425, Dec. 2006.
- [4] W. Dongming, H. Bing, Z. Junhui, G. Xiqi, and Y. Xiaohu, "Channel estimation algorithms for broadband MIMO-OFDM sparse channel," *Proc. IEEE PIMRC'2003*, Beijing, Sep. 2003.
- [5] Z. Tian, G. Leus, and V. Lottici, "Detection of sparse signals under finite-alphabet constraints," *Proc. IEEE ICASSP'2009*, pp. 2349-2352, Taipei, Apr. 2009.
- [6] A. K. Fletcher, S. Rangan, and V. K. Goyal, "A sparsity detection framework for on-off random access channels," *Proc. IEEE ISIT'2009*, pp. 169-173, Seoul, Jun. 2009.
- [7] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO: How many antennas do we need?," online arXiv:1107.1709v1 [cs.IT] 8 Jul 2011.
- [8] E. Biglieri and M. Lops, "Multiuser detection in a dynamic environment—Part I: user identification and data detection," *IEEE Trans. Inf. Theory*, vol. 53, no. 9, pp. 3158-3170, Sep. 2007.
- [9] D. Angelosante, E. Biglieri, and M. Lops, "Low-complexity receivers for multiuser detection with an unknown number of active users," *Proc. IEEE PIMRC'2008*, Cannes, Sep. 2008.
- [10] A. T. Campo, A. G. Fabregas, and E. Biglieri, "Large-system analysis of multiuser detection with an unknown number of users: a high-SNR approach," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3416-3428, Jun. 2011.
- [11] H. Zhu and G. B. Giannakis, "Exploiting sparse user activity in multiuser detection," *IEEE Trans. Commun.*, vol. 59, no. 2, pp. 454-465, Feb. 2011.
- [12] B. Farhang-Boroujeny, H. Zhu, and Z. Shi, "Markov chain Monte Carlo algorithms for CDMA and MIMO communication systems," *IEEE Trans. Signal Process.*, vol. 54, no. 5, pp. 1896-1909, May 2006.
- [13] R-R. Chen, R. Peng, and B. Farhang-Boroujeny, "Markov chain Monte Carlo: applications to MIMO detection and channel equalization," *Proc. IEEE ITA'09*, San Diego, Feb. 2009.
- [14] T. Datta, N. A. Kumar, A. Chockalingam, and B. S. Rajan, "A novel MCMC algorithm for near-optimal detection in large-scale uplink multiuser MIMO systems," *Proc. IEEE ITA'2012*, pp. 69-77, San Diego, Feb. 2012.
- [15] F. Jelinek, L. R. Bahl, J. Cocke, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, vol. IT-20, pp. 284-287, Mar. 1974.