Capacity Analysis of Time-Indexed Media-based Modulation

Bharath Shamasundar[†], Lakshmi Narasimhan Theagarajan[††], and A. Chockalingam[†]
[†] Department of ECE, Indian Institute of Science, Bangalore 560012
[††] Department of EE, Indian Institute of Technology, Palakkad, Kanjikode 678623

***Abstract*—Time-indexed media-based modulation (TI-MBM) is an index modulation scheme where time slots in a transmission frame are indexed to convey additional information bits in media-based modulation (MBM). It was shown in the literature that, in frequency selective fading channels with inter-symbol interference, TI-MBM with cyclic-prefixed single-carrier (CPSC) scheme can achieve better transmission rates, bit error performance, and multipath diversity gain compared to conventional MBM. In this paper, we analyze the capacity of TI-MBM with CPSC, which has not been reported before. We show that, for a given channel matrix, selecting the index of the activated time slots and RF mirrors according to uniform distribution is suboptimal. We derive the probabilities with which time slots and RF mirrors in TI-MBM can be activated such that the achievable transmission rate is maximized. Further, we prove that the transmit symbols from a Gaussian mixture distribution, whose mixture weights are chosen to be the product of these probabilities, can achieve capacity.**

***Index Terms*—Media-based modulation, index modulation, time-slot indexing, capacity analysis, Gaussian mixture distribution.**

I. Introduction

Conventionally, a communication system conveys information bits using symbols from a modulation alphabet $\mathbb{A}$ (say, QAM or PSK) that are transmitted through a wireless fading channel. A recently proposed modulation technique called as the *media-based modulation* (MBM) [1]-[7] takes a different approach in which channel gains are used to form a channel alphabet which is used to convey additional information bits along with the bits conveyed by the symbols from the conventional modulation alphabet $\mathbb{A}$.

An MBM transmitter consists of one transmit antenna and m_{rf} passive elements called 'radio frequency (RF) mirrors' placed near the transmit antenna (see Fig. 1). Each of these RF mirrors can be in one of the two states, viz., ON state or OFF state, depending on an information bit controlling it. A mirror reflects the RF signal when it is in the ON state and allows the signal to pass through when it is in the OFF state. Since there are m_{rf} RF mirrors, there are $2^{m_{rf}}$ possible different ON and OFF combinations of the RF mirrors. Each combination is called a 'mirror activation pattern (MAP)'. It is known that even a small perturbation in the near field of the transmit antenna can lead to multiple random reflections resulting in a different end-to-end channel between the transmit and receive antennas. The $2^{m_{rf}}$ different MAPs create $2^{m_{rf}}$ different near fields, and this results in $2^{m_{rf}}$ independent end-to-end channels. This set of $2^{m_{rf}}$ channels form the 'channel alphabet' in MBM. In a given channel use, one of the MAPs is selected based on the m_{rf} information bits, which is equivalent to selecting one of the channel symbols from the channel alphabet. Also, a source symbol from a conventional modulation alphabet $\mathbb{A}$ is transmitted from the antenna using $\log_2|\mathbb{A}|$ information bits. Therefore, the achieved rate in MBM is given by $\eta_{\text{MBM}} = m_{rf} + \log_2|\mathbb{A}|$ bits per channel use (bpcu). It has been shown that MBM can achieve superior bit error performance at lesser hardware complexity compared to conventional SIMO and MIMO systems [1]-[4].

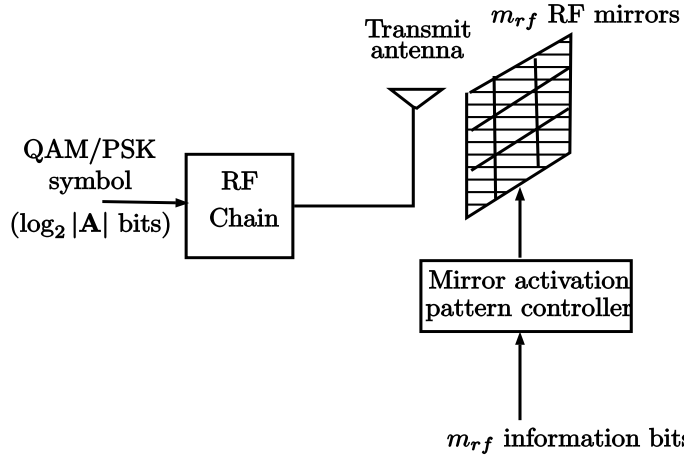

Fig. 1: Schematic diagram of an MBM transmitter.

To improve the performance of MBM in frequency-selective fading channels, the idea of 'time-slot indexing' in MBM was proposed [8], [9]. Time-indexed MBM (TI-MBM) is a block transmission scheme in which time is divided into frames. Each frame consists of N channel uses (time slots). Out of the N time slots, K are used for transmitting MBM signal vectors and the remaining $N-K$ are left unused. The K active time slots are chosen based on information bits. Thus, along with the bits conveyed by the conventional MBM signals, the choice of the active time slots also conveys additional bits. It has been shown that TI-MBM can achieve significantly better bit error performance compared to conventional MBM in frequency-selective fading channels. Recently, diversity analysis of TI-MBM has been carried out in [10], where it is shown that TI-MBM can

This work was supported in part by the Visvesvaraya PhD Scheme of the Ministry of Electronics & IT, Government of India, J. C. Bose National Fellowship, Department of Science and Technology, Government of India, and the Intel India Faculty Excellence Program.

achieve full diversity under certain conditions.

In this paper, we present an information theoretic analysis of the TI-MBM scheme to characterize its capacity, which has not been reported before. We analytically derive a capacity expression for TI-MBM in the high signal-to-noise (SNR) regime. We also prove that, for a given channel instance, capacity can be achieved in TI-MBM by activating the time slots and RF mirrors according to certain probabilities. We also prove that Gaussian mixture input distribution can achieve the capacity of TI-MBM.

The rest of the paper is organized as follows. The system model of TI-MBM is introduced in Sec. II. The capacity analysis of TI-MBM is presented in Sec. III. Numerical results are presented in Sec. IV. Conclusions are presented in Sec. V.

II. TI-MBM SYSTEM MODEL

We consider an MBM system with one transmit antenna and m_{rf} RF mirrors placed near it. Let n_r be the number of receive antennas. We consider a frequency-selective fading channel with L channel taps and an exponential power delay profile. We consider cyclic-prefixed single-carrier (CPSC) scheme to tackle inter-symbol interference (ISI). In TI-MBM with CPSC, the transmitted data is divided into frames. Each frame consists of $N+L-1$ channel uses (time slots), where N is the number of channel uses for the data part and $L-1$ is the number channel uses for the cyclic prefix (CP) part. Out of the N time slots in the data part in a frame, only K time slots are activated and the remaining $N-K$ time slots are left unused (inactive time slots). The choice of the K active time slots in a frame is made using $\lfloor \log_2 \binom{N}{K} \rfloor$ information bits. This choice is represented using an N-length vector which is referred to as the 'time-slot activation pattern (TAP)' [8]. The TAP indicates the active and inactive status of the N time slots in a frame. Also, each of the m_{rf} RF mirrors is made ON or OFF using one information bit. So, m_{rf} information bits control the ON/OFF status of the m_{rf} mirrors, leading to $N_m \triangleq 2^{m_{rf}}$ possible ON/OFF combinations of the m_{rf} RF mirrors. Each ON/OFF combination of m_{rf} mirrors is called a 'mirror activation pattern (MAP)' [8]. In a given active time slot, one of the N_m possible MAPs is chosen based on m_{rf} information bits. Further, a symbol from a modulation alphabet $\mathbb{A}$ (e.g., QAM/PSK) is transmitted in each active time slot to convey $\log_2 |\mathbb{A}|$ information bits. Therefore, the achieved transmission rate of TI-MBM system with CPSC is given by

$$\eta = \frac{1}{N+L-1}\left\{ \left\lfloor \log_2 \binom{N}{K} \right\rfloor + K(m_{rf} + \log_2 |\mathbb{A}|) \right\} \quad \text{bpcu.} \tag{1}$$

In the following subsections, we present the TI-MBM signal set and the system model.

A. TI-MBM signal set

Let $\mathbb{A}_0 \triangleq \mathbb{A} \cup 0$. The MBM signal set ($\mathbb{S}_{\text{MBM}}$) is defined as follows:

$$\mathbb{S}_{\text{MBM}} = \Big\{ \mathbf{s}_{k,p} \in \mathbb{A}_0^{N_m};\ k = 1, \cdots, N_m;\ p = 1, \cdots, |\mathbb{A}| \Big\}$$
$$\text{s.t}\ \ \mathbf{s}_{k,p} = \big[0 \cdots 0 \underbrace{s_p}_{k\text{th entry}} 0 \cdots 0\big]^T,\ s_p \in \mathbb{A}, \tag{2}$$

where k is the index of the MAP used. In TI-MBM, an MBM vector from $\mathbb{S}_{\text{MBM}}$ is transmitted in an active time slot and the inactive time slots are left vacant, which is equivalent to transmitting an $N_m \times 1$ zero vector. The TI-MBM signal set is a set of $NN_m \times 1$-sized signal vectors given by

$$\mathbb{S}_{\text{TI-MBM}} = \Big\{ \mathbf{x} = \big[\mathbf{x}_0^T\ \mathbf{x}_1^T\ \cdots\ \mathbf{x}_{N-1}^T\big]^T : \mathbf{x}_i \in \mathbb{S}_{\text{MBM}} \cup \mathbf{0},$$
$$\|\mathbf{x}\|_0 = K,\ \mathbf{t}(\mathbf{x}) \in \mathbb{T} \Big\}, \tag{3}$$

where $\mathbb{T}$ denotes the set of all valid TAPs and $\mathbf{t}(\mathbf{x})$ denotes the N-length TAP corresponding to the transmitted vector $\mathbf{x}$ [8].

To facilitate the derivation of the capacity of TI-MBM, we define 'time-slot activation matrix' and 'mirror activation matrix' as follows. The time-slot activation matrix (TAM) matrix is of the form

$$\mathbf{T} = \begin{bmatrix} | & | & & | \\ \mathbf{t}_1 & \mathbf{t}_2 & \cdots & \mathbf{t}_K \\ | & | & & | \end{bmatrix}, \tag{4}$$

where $\mathbf{t}_i,\ i \in \{1, \cdots, K\}$, is the $N \times 1$ vector corresponding to the ith active time slot and $\mathbf{T} \in \{0,1\}^{N \times K}$. The vector $\mathbf{t}_i$ corresponds to the ith active time slot and has a 1 in kth position if kth time slot is the ith active time slot of the frame. The remaining $N-1$ entries of $\mathbf{t}_i$ are zeros. For example, if $\mathbf{t}_1 = [0\ 1\ 0\ 0]^T$, then the second time slot is the first active time slot of the frame. There are $N_T \triangleq 2^{\lfloor \log_2 \binom{N}{K} \rfloor}$ possible TAPs, and hence there are N_T number of TAMs. We denote these N_T matrices by $\mathbf{T}_j, j = 1, \cdots, N_T$. Next, the mirror activation matrix (MAM) is an $NN_m \times N$ matrix of the form

$$\mathbf{M} = \text{blkdiag}[\mathbf{m}_{i_1}^{(1)}, \mathbf{m}_{i_2}^{(2)}, \cdots, \mathbf{m}_{i_N}^{(N)}]^T, \tag{5}$$

where $\mathbf{m}_{i_k}^{(k)}$ is an $N_m \times 1$ vector corresponding to the kth time slot of the frame and blkdiag() denotes the block diagonal operator. If the kth time slot is active, then $\mathbf{m}_{i_k}^{(k)}$ has 1 in the i_kth ($i_k = 1, \cdots, N_m$) position with the rest of the entries as zero. This indicates that the i_kth MAP is used in the kth time slot. Whereas, if the kth time slot is inactive, then $\mathbf{m}_{i_k}^{(k)}$ is an N_m-length zero vector. For example, if $\mathbf{m}_{i_1}^{(1)} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$ and $\mathbf{m}_{i_2}^{(2)} = [0\ 0\ 0\ 0\ 0\ 1\ 0\ 0]^T$, then the first time slot is inactive, whereas the second time slot is active and uses the sixth MAP (i.e., $i_2 = 6$). There are N_m different MAPs possible in each active time slot, and hence there are $N_M \triangleq N_m^K = 2^{Km_{rf}}$ different possible MAMs for a given TAM, which we denote by $\mathbf{M}_i, i = 1, \cdots, N_M$.

B. TI-MBM received signal

We assume that the channel is invariant for one TI-MBM frame duration. At the receiver, the $Nn_r \times 1$ TI-MBM received signal vector, after removing the CP, is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{6}$$

where $\mathbf{y} = [\mathbf{y}_1^T \ldots \mathbf{y}_N^T]^T$, $\mathbf{y}_j$ is the MBM signal received at the jth channel use, $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{Nn_r})$ is the $Nn_r \times 1$ noise vector, and $\mathbf{H}$ is the $Nn_r \times NN_m$ equivalent block circulant channel matrix given by

$$\mathbf{H} = \text{circ}[\mathbf{H}_0, \mathbf{H}_1, \cdots, \mathbf{H}_{L-1}, \mathbf{0}, \cdots, \mathbf{0}], \tag{7}$$

where circ$[\cdot]$ denotes the circulant operator and $\mathbf{H}_l$, $l \in \{0, \cdots, L-1\}$, is the $n_r \times N_m$ channel matrix corresponding to the lth multipath whose (i,k)th entry is $h_{i,k}^{(l)}$, where $h_{i,k}^{(l)}$ denotes the channel gain from the transmit antenna to the ith receive antenna for the lth path when the kth MAP is used, $i \in \{1, \cdots, n_r\}$, $k \in \{1, \cdots, N_m\}$, $l \in \{0, \cdots, L-1\}$. The maximum likelihood (ML) detection rule for TI-MBM signal detection is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}_{\text{TI-MBM}}}{\text{argmin}} \ \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \tag{8}$$

III. TI-MBM CAPACITY ANALYSIS

In order to obtain an expression for the capacity of TI-MBM, we rewrite the received signal in (6) as

$$\mathbf{y} = \mathbf{H}\mathbf{M}\mathbf{T}\mathbf{s} + \mathbf{n}, \tag{9}$$

where $\mathbf{s} \in \mathbb{A}^K$ is the vector containing the K symbols from the modulation alphabet $\mathbb{A}$ that is transmitted in the K active time slots, $\mathbf{T} \in \{0,1\}^{N \times K}$ is the TAM, and $\mathbf{M} \in \{0,1\}^{NN_m \times N}$ is the MAM. The channel is assumed to be known at both the transmitter and the receiver. Now, we give the following upper bound on the capacity of the TI-MBM channel.

Theorem III.1. *The capacity of TI-MBM for a given channel matrix* $\mathbf{H}$ *can be upper bounded as*

$$C \leq \max_{\{\alpha_i\},\{\beta_j\},\{\mathbf{Q}_{ij}\}} \frac{\sum_{i=1}^{N_M} \sum_{j=1}^{N_T} \alpha_i \beta_j \left(\log_2 \det(\mathbf{D}_{ij}) - \log_2 \alpha_i \beta_j \right)}{N+L-1}, \tag{10}$$

where $\mathbf{D}_{ij} = \mathbf{I}_{Nn_r} + \gamma \mathbf{H}\mathbf{M}_i\mathbf{T}_j\mathbf{Q}_{ij}\mathbf{T}_j^H\mathbf{M}_i^H\mathbf{H}^H$, $\alpha_i = p(\mathbf{M}_i)$, $\beta_j = p(\mathbf{T}_j)$, $\mathbf{Q}_{ij}$ *is the covariance matrix of* $\mathbf{s}$*, and* γ *is the SNR.*

Proof. The capacity of TI-MBM can be written as [11]

$$\begin{aligned} C &= \max_{f(\mathbf{x})} \frac{1}{N+L-1} \left(I(\mathbf{x}; \mathbf{y}|\mathbf{H}) \right) \\ &= \max_{f(\mathbf{x})} \frac{1}{N+L-1} \left(H(\mathbf{y}) - H(\mathbf{y}|\mathbf{H}, \mathbf{x}), \right) \\ &= \max_{f(\mathbf{x})} \frac{1}{N+L-1} \left(H(\mathbf{y}) - H(\mathbf{n}) \right), \end{aligned} \tag{11}$$

where $I(\cdot)$ denotes the mutual information, $H(\cdot)$ denotes the entropy of a random variable, and $f(\mathbf{x})$ denotes the probability density function (PDF) of the transmit TI-MBM signal vector $\mathbf{x}$. It can be seen that the mutual information is maximized by maximizing the entropy of the received signal $\mathbf{y}$. From (9), the mutual information between $\mathbf{x}$ and $\mathbf{y}$ can be written as

$$I(\mathbf{x}; \mathbf{y}|\mathbf{H}) = I(\mathbf{M}_i, \mathbf{T}_j, \mathbf{s}; \mathbf{y}|\mathbf{H}). \tag{12}$$

Using the chain rule, we rewrite (12) as

$$I(\mathbf{x}; \mathbf{y}|\mathbf{H}) = I(\mathbf{s}; \mathbf{y}|\mathbf{H}) + I(\mathbf{T}_j; \mathbf{y}|\mathbf{H}, \mathbf{s}) + I(\mathbf{M}_i; \mathbf{y}|\mathbf{H}, \mathbf{T}_j, \mathbf{s}). \tag{13}$$

Let $p(\mathbf{M}_i) = \alpha_i$, $i \in \{1, \cdots, N_M\}$ and $p(\mathbf{T}_j) = \beta_j$, $j \in \{1, \cdots, N_T\}$ be the probability mass functions (PMF) for the MAMs and TAMs, respectively, such that $\sum_{i=1}^{N_M} \alpha_i = 1$ and $\sum_{j=1}^{N_T} \beta_j = 1$. Now, the first term in (13) can be written as

$$\begin{aligned} I(\mathbf{s}; \mathbf{y}|\mathbf{H}) &= \sum_{j=1}^{N_T} \beta_j I(\mathbf{s}; \mathbf{y}|\mathbf{H}, \mathbf{T}_j) \\ &= \sum_{i=1}^{N_M} \sum_{j=1}^{N_T} \alpha_i \beta_j I(\mathbf{s}; \mathbf{y}|\mathbf{H}, \mathbf{M}_i, \mathbf{T}_j). \end{aligned} \tag{14}$$

Since $\mathbf{s}$ and $(\mathbf{M}_i, \mathbf{T}_j)$ are independent, we know that $I(\mathbf{s}; \mathbf{y}|\mathbf{H})$ is maximum when $f(\mathbf{s})$ is complex Gaussian. However, this distribution may not maximize the second and third terms of (13), and therefore we bound them as follows:

$$I(\mathbf{T}_j; \mathbf{y}|\mathbf{H}, \mathbf{s}) \leq H(\mathbf{T}_j) = -\sum_{j=1}^{N_T} \beta_j \log_2 \beta_j. \tag{15}$$

$$I(\mathbf{M}_i; \mathbf{y}|\mathbf{H}, \mathbf{T}_j, \mathbf{s}) \leq H(\mathbf{M}_i) = -\sum_{i=1}^{N_M} \alpha_i \log_2 \alpha_i. \tag{16}$$

Substituting (14), (15), and (16) in (13), an upper bound on the capacity of TI-MBM can be obtained as

$$\begin{aligned} C \leq C_u \triangleq \max_{\{\alpha_i\},\{\beta_j\},f(\mathbf{s})} \frac{1}{N+L-1} \Bigg[&-\sum_{i=1}^{N_M} \alpha_i \log_2 \alpha_i \\ &- \sum_{j=1}^{N_T} \beta_j \log_2 \beta_j + \sum_{i=1}^{N_M} \sum_{j=1}^{N_T} \alpha_i \beta_j I(\mathbf{s}; \mathbf{y}|\mathbf{H}, \mathbf{M}_i, \mathbf{T}_j) \Bigg]. \end{aligned} \tag{17}$$

Note that only the last term in the above equation depends on $f(\mathbf{s})$. Thus, when $f(\mathbf{s})$ is complex Gaussian, the mutual information in the last term is maximized. Now, the upper bound in (17) can be written as [14]

$$\begin{aligned} C_u = \max_{\{\alpha_i\},\{\beta_j\},\{\mathbf{Q}_{ij}\}} \frac{1}{N+L-1} \Bigg[\sum_{i=1}^{N_M} \sum_{j=1}^{N_T} \alpha_i \beta_j \big(&\log_2 \det(\mathbf{D}_{ij}) \\ &- \log_2 \alpha_i \beta_j \big) \Bigg], \end{aligned} \tag{18}$$

where $\mathbf{D}_{ij} = \mathbf{I}_{Nn_r} + \gamma \mathbf{H}\mathbf{M}_i\mathbf{T}_j\mathbf{Q}_{ij}\mathbf{T}_j^H\mathbf{M}_i^H\mathbf{H}^H$, $\mathbf{Q}_{ij}$ is the covariance matrix of $\mathbf{s}$, and γ is the SNR. □

Next, using this upper bound, we find an expression for the TI-MBM capacity in the high SNR regime. Let $\{\mathbf{Q}_{ij}^*\}$ be the covariance matrix, and $\{\alpha_i^*\}$ and $\{\beta_j^*\}$ be the PMFs of MAMs and TAMs, respectively, that maximize the mutual information in (18). Now, we have the following result.

Theorem III.2. *The capacity of TI-MBM in the high SNR regime for a given channel matrix $\mathbf{H}$ is given by*

$$\lim_{\gamma\to\infty} C = \frac{\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i^*\beta_j^*\Big[\log_2\det(\mathbf{D}_{ij}^*) - \log_2\alpha_i^*\beta_j^*\Big] + 2Nn_r}{N+L-1}, \tag{19}$$

where $\mathbf{D}_{ij}^* = \mathbf{I}_{Nn_r} + \mathbf{H}\mathbf{M}_i\mathbf{T}_j\mathbf{Q}_{ij}^*\mathbf{T}_j^H\mathbf{M}_i^H\mathbf{H}^H$.

Proof. Let $F(.)$ be the mutual information defined as follows:

$$\begin{aligned} F(\alpha_i,\beta_j,\mathbf{Q}_{ij}) &\triangleq I(\mathbf{x};\mathbf{y}|\mathbf{H}) \\ &= h(\mathbf{y}) - h(\mathbf{y}|\mathbf{H},\mathbf{x}) \\ &= -\int_{\mathbb{C}^{Nn_r}} f(\mathbf{y})\log_2 f(\mathbf{y}) - Nn_r\log_2(\pi e). \end{aligned} \tag{20}$$

The PDFs required for the computation of $F(.)$ are:

$$f(\mathbf{y}) \triangleq \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j f_{ij}(\mathbf{y}), \tag{21}$$

$$f_{ij}(\mathbf{y}) \triangleq \frac{1}{\pi^{Nn_r}\det(\mathbf{D}_{ij})}\exp(-\mathbf{y}^H\mathbf{D}_{ij}^{-1}\mathbf{y}). \tag{22}$$

From Theorem III.1, the PDF of $\mathbf{x}$ that maximizes the mutual information is given by

$$\begin{aligned} f(\mathbf{x}) &= \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j f_{\mathbf{M}_i\mathbf{T}_j\mathbf{s}}(\mathbf{M}_i\mathbf{T}_j\mathbf{s}) \\ &= \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j\frac{1}{\pi^{NN_m}\det(\mathbf{M}_i\mathbf{T}_j\mathbf{Q}_{ij}\mathbf{T}_j^H\mathbf{M}_i^H)} \\ &\qquad \exp\left[-\mathbf{x}^H(\mathbf{M}_i\mathbf{T}_j\mathbf{Q}_{ij}\mathbf{T}_j^H\mathbf{M}_i^H)^{-1}\mathbf{x}\right]. \end{aligned} \tag{23}$$

From (23), we can see that $f(\mathbf{x})$ is a Gaussian mixture with the mixture coefficients $\{\alpha_i\beta_j\}$. The asymptotic ($\gamma\to\infty$) mutual information at $\{\alpha_i^*\}$, $\{\beta_j^*\}$, and $\{\mathbf{Q}_{ij}^*\}$ can be obtained [12],[13] as follows:

$$\begin{aligned} &\lim_{\gamma\to\infty} F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*) = \\ &\quad -\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i^*\beta_j^*\log_2\left(\sum_{m=1}^{N_M}\sum_{n=1}^{N_T}\frac{\alpha_m^*\beta_n^*}{\det(\mathbf{D}_{ij}^*+\mathbf{D}_{mn}^*)}\right) + Nn_r, \\ &\quad = -\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i^*\beta_j^*\log_2\left(\frac{\alpha_i^*\beta_j^*}{\det(2\mathbf{D}_{ij}^*)} + \right. \\ &\qquad \left.\sum_{\substack{m=1\\(m,n)\neq(i,j)}}^{N_M}\sum_{n=1}^{N_T}\frac{\alpha_m^*\beta_n^*}{\det(\mathbf{D}_{ij}^*+\mathbf{D}_{mn}^*)}\right) + Nn_r. \end{aligned} \tag{24}$$

Since $\det(\mathbf{D}_{ij}^*+\mathbf{D}_{mn}^*) \propto \gamma^{Nn_r-K}$ [12] and $Nn_r > K$, we have the following:

$$\lim_{\gamma\to\infty}\frac{\alpha_m^*\beta_n^*}{\det(\mathbf{D}_{ij}^*+\mathbf{D}_{mn}^*)} = 0, \tag{25}$$

$$\begin{aligned} \lim_{\gamma\to\infty} F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*) = \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i^*\beta_j^*\Big[\log_2\det(\mathbf{D}_{ij}) - \\ \log_2\alpha_i^*\beta_j^*\Big] + 2Nn_r. \end{aligned} \tag{26}$$

Since $\{\alpha_i^*\}$, $\{\beta_j^*\}$, and $\{\mathbf{Q}_{ij}^*\}$ are the solutions to the optimization problem in (18), we have

$$\lim_{\gamma\to\infty} C \le \lim_{\gamma\to\infty}\frac{F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*)}{N+L-1}. \tag{27}$$

However, $F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*)$ is the mutual information between $\mathbf{x}$ and $\mathbf{y}$. Hence, we have

$$\lim_{\gamma\to\infty}\frac{F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*)}{N+L-1} \le \lim_{\gamma\to\infty} C. \tag{28}$$

Combining (27) and (28), we get $\lim_{\gamma\to\infty} C = \lim_{\gamma\to\infty}\frac{F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*)}{N+L-1}$. □

Using Theorem III.2, the average capacity of TI-MBM can be given by $\lim_{\gamma\to\infty} C = \mathbb{E}_{\mathbf{H}}\lim_{\gamma\to\infty}\frac{F(\alpha_i^*,\beta_j^*,\mathbf{Q}_{ij}^*)}{N+L-1}$. Further, the above theorem suggests that the optimal PMFs of MAMs and TAMs that achieve capacity need not be uniform distributions. In the next theorem, we find the optimal mixture coefficients $\{\alpha_i^*\beta_j^*\}$ and the optimal covariance matrices $\{\mathbf{Q}_{ij}^*\}$ that achieve the capacity of TI-MBM.

Theorem III.3. *The optimal $\{\mathbf{Q}_{ij}^*\}$ and $\{\alpha_i^*\beta_j^*\}$ that achieve the TI-MBM capacity is given by $\mathbf{Q}_{ij}^* = \mathbf{V}_{ij}\mathbf{\Sigma}_{ij}^*\mathbf{\Sigma}_{ij}^{*H}\mathbf{V}_{ij}^H$ and $\alpha_i^*\beta_j^* = \frac{\det(\mathbf{D}_{ij}^*)}{\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\det(\mathbf{D}_{ij}^*)}$, where*

$$\mathbf{\Sigma}_{ij}^* = diag\left(\sqrt{\sigma_{ij}^{(1)*}},\cdots,\sqrt{\sigma_{ij}^{(n_c)*}}\right), \tag{29}$$

$$\sigma_{ij}^{(p)*} = \left(\frac{1}{\theta_{ij}\ln 2} - \frac{1}{\gamma\lambda_{ij}^{(p)2}}\right)^+, \quad p = 1,\cdots,n_c, \tag{30}$$

and $n_c = rank(\mathbf{H}\mathbf{M}_i\mathbf{T}_j)$.

Proof. First, using the Lagrange multipliers, we solve the mutual information maximization problem as follows:

$$\max_{\{\alpha_i\},\{\beta_j\},\{\mathbf{Q}_{ij}\}} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T} \alpha_i\beta_j \left[\log_2 \det(\mathbf{D}_{ij}) - \log_2 \alpha_i\beta_j\right]$$
$$\text{s.t} \sum_{i=1}^{N_M}\alpha_i = 1, \quad \sum_{j=1}^{N_T}\beta_j = 1, \text{ and } \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j \mathrm{Tr}(\mathbf{Q}_{ij}) = 1. \tag{31}$$

Since we are interested in the mixture coefficients $\{\alpha_i\beta_j\}$, we modify the above optimization problem as

$$\max_{\{\alpha_i\},\{\beta_j\},\{\mathbf{Q}_{ij}\}} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T} \alpha_i\beta_j \left[\log_2 \det(\mathbf{D}_{ij}) - \log_2 \alpha_i\beta_j\right]$$
$$\text{s.t} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j = 1 \text{ and } \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j \mathrm{Tr}(\mathbf{Q}_{ij}) = 1. \tag{32}$$

Note that there are $N_M N_T$ mixture coefficients $\{\alpha_i\beta_j\}$. For convenience, we denote these mixture coefficients by δ_l, $l \in \{1, \cdots, N_M N_T\}$. For any feasible $\mathbf{Q}_{ij}$, the Lagrange function for the optimal mixture coefficients δ_l^* is given by

$$J(\delta_l, \mu) = \sum_{l=1}^{N_M N_T} \delta_l \log_2 \det(\mathbf{D}_{ij}) - \sum_{l=1}^{N_M N_T} \delta_l \log_2 \delta_l - \mu\Big(\sum_{i=1}^{N_M N_T} \delta_l - 1\Big). \tag{33}$$

Setting the derivatives of $J(\delta_l, \mu)$ w.r.t. δ_l to zero and using the constraint $\sum_{l=1}^{N_m N_T} \delta_l = 1$ to find μ, we get the optimal mixture coefficients to be

$$\delta_l^* = \alpha_i^*\beta_j^* = \frac{\det(\mathbf{D}_{ij})}{\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\det(\mathbf{D}_{ij})}. \tag{34}$$

Next, we find the optimal $\{\mathbf{Q}_{ij}\}$. Let $n_c = \mathrm{rank}(\mathbf{H}\mathbf{M}_i\mathbf{T}_j)$. Now, the covariance matrix $\mathbf{Q}_{ij}$ can be decomposed as $\mathbf{Q}_{ij} = \mathbf{V}_{ij}\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\mathbf{V}_{ij}^H$, where $\mathbf{V}_{ij} \in \mathbb{C}^{K\times n_c}$ is the right singular matrix of $\mathbf{H}\mathbf{M}_i\mathbf{T}_j$, $\boldsymbol{\Sigma}_{ij} = \mathrm{diag}\big(\sqrt{\sigma_{ij}^1}, \cdots, \sqrt{\sigma_{ij}^{n_c}}\big)$ and $\sqrt{\sigma_{ij}^l}$ is the power allocated to the lth mode of the channel matrix $\mathbf{H}\mathbf{M}_i\mathbf{T}_j$. Let $\boldsymbol{\Lambda}_{ij} = \mathrm{diag}(\lambda_{ij}^{(1)}, \ldots, \lambda_{ij}^{(n_c)}) \in \mathbb{C}^{n_c\times n_c}$ be the diagonal matrix containing the non-zero singular values of $\mathbf{H}\mathbf{M}_i\mathbf{T}_j$. Now, the optimization problem in (32) becomes

$$\max_{\{\alpha_i,\{\beta_j\},\{\boldsymbol{\Sigma}_{ij}\}} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j\Big[\log_2\det(\mathbf{I}_{N n_r} + \gamma\boldsymbol{\Lambda}_{ij}\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\boldsymbol{\Lambda}_{ij}^H) - \log_2\alpha_i\beta_j\Big]$$
$$\text{s.t} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j\mathrm{Tr}(\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H) = 1 \text{ and } \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j = 1. \tag{35}$$

Here, we use the following approximation [13]:

$$\log_2\det(\mathbf{I}_{N n_r} + \gamma\boldsymbol{\Lambda}_{ij}\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\boldsymbol{\Lambda}_{ij}^H) \approx n_c\log_2\mathrm{Tr}\left(\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\right) + r_{ij}, \tag{36}$$

where r_{ij} is a quantity that is independent of $\boldsymbol{\Sigma}_{ij}$ [13],[14]. Let $G_{ij} \triangleq \mathrm{Tr}(\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H)$. From (36), for any feasible $\{\alpha_i\beta_j\}$, the optimization problem in (35) becomes

$$\max_{\{G_{ij}\}} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j\Big[n_c\log_2 G_{ij} + r_{ij} - \log_2\alpha_i\beta_j\Big]$$
$$\text{s.t} \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j G_{ij} = 1 \text{ and } \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j = 1. \tag{37}$$

The Lagrangian function to solve this optimization problem is

$$J(G_{ij},\zeta) = \sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\Big[\alpha_i\beta_j\Big(n_c\log_2 G_{ij} + r_{ij} - \log_2\alpha_i\beta_j\Big)\Big] - \zeta\Big(\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j G_{ij} - 1\Big). \tag{38}$$

Differentiating this Lagrangian function w.r.t. G_{ij} and equating the derivative to zero gives

$$G_{ij} = \frac{n_c}{\zeta}, \quad \forall\ (i,j) \in \{1,\cdots,N_M\}\times\{1,\cdots,N_T\}. \tag{39}$$

Further, using the constraint $\sum_{i=1}^{N_M}\sum_{j=1}^{N_T}\alpha_i\beta_j G_{ij} = 1$ to solve for ζ and substituting this value of ζ in (39) gives us $G_{ij} = 1$ for all (i,j). Now, to find the optimal $\mathbf{Q}_{ij}$, the optimization problem in (35) reduces to

$$\max_{\boldsymbol{\Sigma}_{ij}} \ \log_2\det\left(\mathbf{I}_{N n_r} + \gamma\boldsymbol{\Lambda}_{ij}\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\boldsymbol{\Lambda}_{ij}^H\right) \ \text{s.t}\ \mathrm{Tr}\left(\boldsymbol{\Sigma}_{ij}\boldsymbol{\Sigma}_{ij}^H\right) = 1. \tag{40}$$

It can be noted that the optimization problem (40) is the well known power allocation problem encountered in parallel MIMO channel. Thus, the optimal solution $\mathbf{Q}_{ij}^*$ is given by [15]

$$\mathbf{Q}_{ij}^* = \mathbf{V}_{ij}\boldsymbol{\Sigma}_{ij}^*\boldsymbol{\Sigma}_{ij}^{*H}\mathbf{V}_{ij}^H, \tag{41}$$

where $\mathbf{V}_{ij}$ is as described before,

$$\boldsymbol{\Sigma}_{ij}^* = \mathrm{diag}\Big(\sqrt{\sigma_{ij}^{(1)*}}, \cdots, \sqrt{\sigma_{ij}^{(n_c)*}}\Big)$$

is the diagonal water-filling power allocation matrix which satisfies $\mathrm{Tr}(\mathbf{Q}_{ij}) = 1$, and

$$\sigma_{ij}^{(p)*} = \left(\frac{1}{\theta_{ij}\ln 2} - \frac{1}{\gamma\lambda_{ij}^{(p)2}}\right)^+, \quad p = 1,\cdots,n_c, \tag{42}$$

where θ_{ij} satisfies

$$\sum_{p=1}^{n_c}\left(\frac{1}{\theta_{ij}\ln 2} - \frac{1}{\gamma\lambda_{ij}^{(p)2}}\right)^+ = 1. \tag{43}$$

□

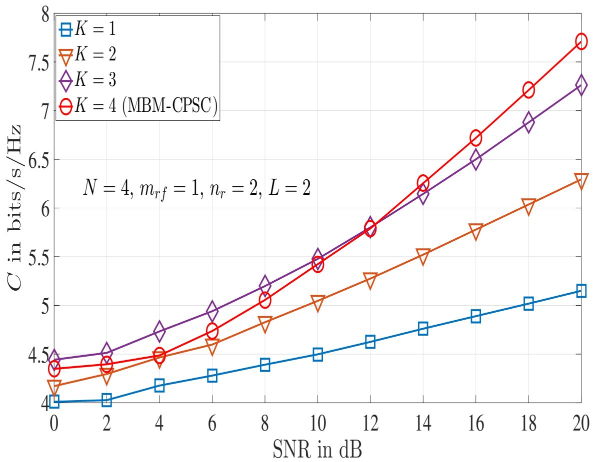

Fig. 2: Capacity of TI-MBM with $N = 4$, $m_{rf} = 1$, $n_r = 2$, $L = 2$, and $K = 1, 2, 3, 4$.

IV. Numerical Results

In this section, we present numerical results on the capacity of TI-MBM. We numerically simulated the capacity of TI-MBM with CPSC using the derived theoretical expressions. The capacity of the TI-MBM system with $N = 4$, $m_{rf} = 1$, $n_r = 2$, and $L = 2$ is plotted in Fig. 2 for $K = 1, 2, 3, 4$. The channel matrix is an $Nn_r \times NN_m = 8 \times 8$ random complex Gaussian matrix generated using the channel model described in (7). It can be seen that the capacity increases with the increase in the number of the active time slots. Note that $K = 4$ corresponds to conventional MBM-CPSC scheme (with no time-slot indexing). For this case of $K = 4$, the capacity falls below that of the TI-MBM system with $K = 3$ up to an SNR value of 12 dB. This shows that TI-MBM with an appropriate choice of the value of K can provide higher capacity than conventional MBM-CPSC systems.

It was shown in [8] that TI-MBM system achieves a better bit-error rate (BER) performance compared to conventional MBM-CPSC system. Further, it was shown in [10] that while MBM-CPSC can not extract the channel multipath diversity, TI-MBM with suitable choice of K can extract the full channel multipath diversity. Our results suggests that, with a suitable choice of the number of active time slots K, TI-MBM can achieve not only better BER performance and higher diversity, but also higher capacity compared to conventional MBM-CPSC.

V. Conclusions

We carried out an information theoretic analysis of TI-MBM with CPSC. Specifically, we derived the capacity of the TI-MBM system under the assumption that the channel state information is available at both transmitter and receiver. Our analysis showed that the capacity can be achieved when the input TI-MBM signal has Gaussian mixture distribution with the mixture weights being the product of the probabilities of time activation matrices and mirror activation matrices. Further, we showed that, with a suitable number of active time slots, TI-MBM can achieve higher capacity compared to conventional MBM-CPSC.

References

[1] A. K. Khandani, "Media-based modulation: a new approach to wireless transmission," *Proc. IEEE ISIT'2013*, pp. 3050-3054, Jul. 2013.

[2] A. K. Khandani, "Media-based modulation: converting static Rayleigh to AWGN," *Proc. IEEE ISIT'2014*, Jun. 2014.

[3] E. Seifi, M. Atamanesh, A. K. Khandani, "Media-based MIMO: outperforming known limits in wireless," *Proc. IEEE ICC'2016*, May 2016.

[4] Y. Naresh and A. Chockalingam, "On media-based modulation using RF mirrors," *IEEE Trans. Veh. Tech.*, vol. 66, no. 6, pp. 4967-4983, Jun. 2017.

[5] B. Shamasundar, S. Jacob, T. Lakshmi Narasimhan, A. Chockalingam, "Media-based modulation for the uplink in massive MIMO systems," *IEEE Trans. Veh. Tech.*, vol. 67, no. 9, pp. 8169-8183, Sep. 2018.

[6] E. Basar and I. Altunbas, "Space-time channel modulation," *IEEE Trans. Veh. Tech.*, vol. 66, no. 8, pp. 7609-7614, Aug. 2017.

[7] I. Yildirim, E. Basar, and I. Altunbas, "Quadrature channel modulation," *IEEE Wireless Commun. Lett.*, vol. 6, no. 6, pp. 790-793, Dec. 2017.

[8] B. Shamasundar, S. Jacob, and A. Chockalingam, "Time-indexed media-based modulation," *Proc. IEEE VTC'2017 Spring,* Jun. 2017.

[9] B. Shamasundar, S. Bhat, S. Jacob, and A. Chockalingam, "Multidimensional index modulation in wireless communications," *IEEE Access,* vol. 6, pp. 589-604, Nov. 2017.

[10] R. Gupta, B. Shamasundar, and A. Chockalingam, "Diversity analysis of time-indexed media-based modulation," *Proc. IEEE VTC'2019 Fall*, Sep. 2019.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory,* 2nd ed., Wiley Publishers, Jul. 2006.

[12] L. He, J. Wang, and J. Song, "Spatial modulation for more spatial multiplexing: RF-chain-limited generalized spatial modulation aided mm-Wave MIMO with hybrid precoding," *IEEE Trans. Commun.*, vol. 66, no. 3, pp. 986-998, Mar. 2018.

[13] S. Guo, H. Zhang, and M. S. Alouini, "MIMO capacity with reduced RF chains," online: arXiv:1901.03893v3 [eess.SP] 7 Feb. 2019.

[14] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 5, pp. 684-702, Jun. 2003.

[15] D. Tse and P. Vishwanath, *Fundamentals of Wireless Communications,* Cambridge Univ. Press, May. 2005.