Effect of Mobility on Power Control and System Capacity in CDMA Cellular Wireless Networks

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ABSTRACT

We consider a CDMA cellular wireless access network that handles various classes of calls that are characterised by different bit rates and E_b/N_0 requirements. For the uplink of such a system we study the relationship between power control, admission control and user mobility. We compare two methods for power control: Constant Received Power Control (CRPC) in which the total power received at each base station (BS) is kept constant, and each Mobile Station (MS) is assigned a fraction of this received power, and Constant Signal to Interference Ratio (SIR) Power Control (CSPC) in which each MS is dynamically power controlled such that the measured SIR at the Base Station is kept constant. In steady state, for stationary mobiles, CSPC should provide a larger admission region than CRPC. We calculate the admission regions for CRPC and CSPC under various mobility conditions, and find that with high mobility CRPC outperforms the CSPC based algorithm.

I. INTRODUCTION

Wideband CDMA has been proposed as the access scheme for future multimedia cellular wireless access networks. Provided the power control and synchronization problems can be overcome, CDMA is a very attractive multiple access technique for wireless networks. The advantages of CDMA over other multiple access schemes include higher spectral reuse efficiency, greater immunity to multipath fading, gradual overload capability, simple exploitation of voice inactivity, and the ability to make use of soft handoff procedures. CDMA capacity is limited only by the total level of interference from all the connected users.

CDMA utilizes statistical multiplexing without the complex radio channel allocation and reallocation that is required in frequency-division multiple access (FDMA) and time-division multiple access (TDMA) systems. When a resource is shared statistically, however, in order to guarantee that each contending user obtains its desired share of the resource, and hence some guaranteed level of quality of service (QoS), it is important to implement an *admission control*. In CDMA systems the QoS required by a call is specified in terms of a minimum signal-to-inteference power (SIR) ratio, and it is adequate that this minimum SIR is violated only with a small probability.

We consider the problem of determining the admission control criterion for the uplink of a multicellular CDMA system. Owing to asynchronous transmission on the uplink, power control is an essential part of a CDMA based network as it overcomes the near-far problem. The power control algorithm employed may have a significant effect on the admission control since CDMA based networks are interference limited, and the power control algorithms keeps the interference under check. We consider two power control algorithms: Constant SIR Power Control (CSPC), which is an iterative algorithm based on SIR (signal to interference ratio) measurements, and the Constant Received Power Control (CRPC) algorithm, which aims at keeping the total uplink power received by a base station (BS) constant. CSPC offers the prospect of more economical operations because each mobile (MS) transmits just enough power to attain the required SIR. However, CSPC introduces a coupling among the power control loops of all the mobiles present, as the power transmitted by a mobile depends on the transmit levels of all other mobiles in the network. Owing to this coupling, it has been observed ([2]) that, even after a call is admitted, during the evolution of this algorithm, the transmitter power fluctuates erratically and the signal to interference ratio falls below the desired level. Thus under high mobility conditions, due to frequent power updates, the communication quality may significantly deteriorate. Although the CRPC algorithm is not optimum in terms of efficient resource utilization, as the received power is maintained at a fixed level irrespective of the interference observed, it may work better under high user mobility conditions as the power updates are not coupled with one another.

In this paper we analyze a model of a multicellular multiclass CDMA system operating under CSPC or CRPC, and thus compare the system capacity obtained with these two approaches for various degrees of mobility.

In [2], Bambos discusses the problems associated with power control when the channel gains are time varying. Rosberg [1] proposes to allocate an additional cost in terms of SIR to tackle user mobility in a channelized system. In this paper, we analyse the effect of mobility on the two power control algorithms, CRPC and CSPS, and compare their call admission regions.

The paper is organised as follows. In Section II we briefly discuss the system model under consideration, and introduce some notation. In Section III we obtain the admission region for multiclass calls under CSPC and CRPC based power control algorithms. Section IV compares the numerical values of admission thresholds for the two algorithms.

II. SYSTEM MODEL

We consider the uplink of a multiservice direct sequence code division multiple access (DS-CDMA) cellular system with standard uniform hexagonal cells. The forward and reverse links use disjoint frequency bands. We consider only the reverse link as it is generally accepted to be the limiting factor in capacity calculations. The system supports different classes of calls. The classes are differentiated by their QOS requirements, i.e., the bit rate, and the minimum target probability of bit error, or equivalently, E_b/N_0. The propagation model consists of distance losses and shadow fading whose effects are captured by the channel gains. We assume that the Rayleigh fading component is averaged out at the time scale under consideration. The shadow fading term being spatial depends only upon the relative positions of transmitter and receiver. This term can be represented as composed of two components, one due to the scatterers around the MS and the other due to scatterers around the BS.

All analyses are done with respect to a *reference* cell, indexed 0. There are $M-1$ other cells. The number of active MSs in cell $j, 0 \leq j \leq M-1$, is denoted by N_j. The MSs in cell j are indexed by $k_j, 1 \leq k_j \leq N_j$. The path attenuation at epoch t, from MS k_j to BS i is denoted by $G_{ik_j}(t)$.

We assume C classes of calls, distinguished from one another in terms of their QoS requirements, i.e., class i has a target minimum bit energy to noise power spectral density ratio γ_i, and bit rate of R_i. We assume that a transmission can tolerate an outage probability of P_{out}. Let W be the total uplink bandwidth available. Then the QoS requirement for the i^{th} mobile can be encapsulated into a minimum signal to interference power ratio (SIR) requirement

$$\Gamma_i = \frac{\gamma_i R_i}{W}$$

Define

$$\alpha_i = \frac{\Gamma_i}{\Gamma_i + 1}$$

III. ADMISSION CONTROL ANALYSIS

A. Constant Received Power Control (CRPC)

The objective of CRPC is that the total power received at a BS from all the active mobiles of its cell is kept constant irrespective of the other cell interference observed. The total signal power received at a BS is assumed to be bounded above by S_T. As in [8], S_T could be determined from transmitter power constraints, or a physical power receipt constraint at the BS. Since every cell uses the same policy, we will see that the consequence of the CRPC approach is to assign to each new admitted call a fixed fraction of S_T, and the MS is then power controlled so that the power received from the MS is maintained at this fraction of S_T. It follows that as the MS moves around within its cell the signal to interference ratio (SIR) at the BS receiver will vary. Then the objective of admission control is to ensure that each admitted call receives its desired QoS (minimum SIR) if it remains in the cell throughout its life time, i.e., intra cell mobility.

Consider the reference cell. At time t, the BS in this cell receives signal power $P_i(t)$ from MS $i, 1 \leq i \leq N_0$. Denote by $I(t)$ the total other cell average interference power received at the reference cell BS at time t. It is easily seen that the SIR requirement for each MS in the reference cell is satified if for all i, and for all t,

$$P_i(t) \geq \alpha_i\left(\sum_{j=1}^{N_0} P_j(t) + I(t) + \eta_0\right)$$

where η_0 is the receiver noise power. According to the CRPC objective, suppose we ensure that, for all t, $\sum_{j=1}^{N_0} P_j(t) = S_T$. It follows that the receive power allotted to MS i, $P_i(t)$, should be proportional to α_i. Let $P_i(t)$ be some fraction f_i of S_T. Once a call is admitted and the fraction f_i is assigned to it, then power control of MS i is achieved simply by the MS adjusting its transmit power in order to maintain the target received power at the BS. Since the fading under consideration is frequency independent, we assume reciprocity in the attenuations in each direction between the BS and the MS, the required adjustment in transmit power at the MS can be estimated at the MS by making measurements on the pilot signal from the BS.

This receive power allocation approach is feasible, if and only if, for all t

$$S_T \geq (S_T + I(t) + \eta_0) \sum_{i=1}^{N_0} \alpha_i$$

Since $I(t)$ is a random quantity due to the mobility of the users in other cells and shadow fading, we relax the above requirement by allowing the inequality to be violated with a very small probability, the probability of outage P_{out}. Thus we require that

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \mathbf{1}_{\left\{I(y)+\eta_0 \; > \; \frac{1-\sum_{i=1}^{N_0} \alpha_i}{\sum_{i=1}^{N_0} \alpha_i} S_T\right\}} dy \leq P_{out} \quad (1)$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function of $\{\cdot\}$. Since the term $\sum_{i=1}^{N_0} \alpha_i$ appears, Equation 1 expresses the admission control condition for CRPC. We will now use other cell interference analysis to bring Equation 1 into a more usable form. We neglect η_0 since the ratio $\frac{\eta_0}{S_T}$ is negligibly small.

Define

$$Z(t) \;=\; \frac{I(t)}{S_T}$$

Recalling that an MS is allocated a fraction of the total received power at each BS, we see that

$$Z(t) \;=\; \sum_{j=1}^{M} \sum_{k_j=1}^{N_j} \frac{G_{0k_j}(t)}{G_{jk_j}(t)}\; f_{k_j} \quad (2)$$

We assume the gain process of each MS to be stationary and egodic, and the gain processes to be independent from MS to MS. Let Z denote the marginal random variable of the $Z(t)$ process. The requirement in Equation 1 can then be written as

$$Prob\left(Z > \frac{1-\sum_{i=1}^{N_0} \alpha_i}{\sum_{i=1}^{N_0} \alpha_i}\right) \leq P_{out} \quad (3)$$

Z is the sum of a large number of independent random variables. Hence invoking a central limit theorem, the random variable Z is approximately normal with some mean μ_z and variance σ_z^2. Using this normal approximation in Equation 3, we find that the admission region for the reference cell is given by,

$$\sum_{i=1}^{N_0} \alpha_i \;\leq\; \frac{1}{1+\mu_z + Q^{-1}(P_{out})\,\sigma_z} \quad (4)$$

where $Q(\cdot)$ is the tail of the normal distribution; i.e.,

$$Q(x) \;=\; \int_x^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{y^2}{2}}\, dy.$$

We have considered arbitrary mobility while obtaining this admission rule since, with the CRPC based algorithm, low or high mobility does not affect the performance of the power control. We note that this admission control region is similar to the one obtained in [7], where it was shown that for a single cell with multiclass users the admission criterion is

$$\sum_{i=1}^{N_0} \frac{\Gamma_i}{1+\Gamma_i} = \sum_{i=1}^{N_0} \alpha_i \ \leq \ 1$$

The parameter μ_z is the mean other cell interference, and σ_z is the variability that occurs owing to other cell users' mobility, and shadow fading. We do not have a contribution from the same cell users' mobility since the total received power at the BS from the same cell users is always kept constant.

Given the values of $f_{k_j}, 1 \leq j \leq M, 1 \leq k_j \leq N_j$, the parameters μ_z and σ_z can be determined from Equation 2. Power law path loss and log-normal shadowing are assumed for carrying out the computation. The computation can also include the effect of hard handoff or soft handoff (the ratio $\frac{G_{0k_j}}{G_{jk_j}}$ will be calculated accordingly). The parameters μ_z and σ_z are then evaluated numerically (see [5]). It is observed that only the MSs in the first two tiers of interfering cells contribute significantly towards other cell interference. Given μ_z and σ_z the admission control approach for the reference cell is then fully specified via Equation 4.

A.1 Admission Control with CRPC

Note that admission control into the reference cell requires knowledge of the parameters μ_z and σ_z; these depend on the allocation fractions f_{k_j}, and these fractions in turn depend on other cell interference observed in cell j, which includes interference from calls in the reference cell. As a new call of class i arrives to the reference cell, it specifies its α_i requirement. The admission controller can execute the following algorithm to decide whether the call is admissible or not.

Let B_j denote the right hand-side of Equation 4 at cell j. B_j is known to each cell either by a knowledge of all the f_{k_j} values, or by measurement and statistical computation. When a call arrives in cell 0, it specifies its value of α, and the condition Equation 4 is checked using the known value of B_0. If the condition is violated then the call is rejected. However, even if the condition for cell 0 is not violated, admission of this call could lead to changes in $B_j, j \neq 0$, such that the existing configuration of calls in some cell $j, j \neq 0$, becomes infeasible. Thus with the new call assumed to be admitted, the values of $B_j, j \neq 0$, need to be recalculated. If the condition in Equation 4 is still not violated in any cell, then the values of f_{k_j} are updated for all k_j, and the process is repeated until the f_{k_j} values converge, or Equation 4 gets violated in some cell. The new call is admissible only in the former case. Note that even though this admission control process is complex, and requires network state to be available at some computation point, once the procedure admits a call the SIR requirements are met for all calls no matter how long they stay or how rapidly they move.

B. Constant SIR Power Control (CSPC)

The objective of CSPC is that power received from an MS (at its home BS) should be just enough so that the target SIR requirement for that MS is achieved. To achieve this, the BS measures the interference power relative to each MS, and the attenuation from the MS through bit error measurements, and then commands the MS to appropriately set its transmit power so that the desired SIR is achieved. We view these adjustments as taking place at periodic epochs separated by *measurement intervals*. We normalise time to the measurement interval. Denote by $Q_{k_j}(t)$ the power transmitted by MS k_j in the interval starting at epoch $t, t \in \{0, 1, 2, \ldots\}$. Let $G_{ik_j}(t)$ denote the average attenuation from MS k_j to BS i during the measurement interval starting at epoch t. It follows that the total interference plus noise power, with respect to the signal received from MS i in cell 0, measured at the reference BS 0 during the interval $(t, t+1)$ is given by

$$\sum_{j_0=1; j_0 \neq i}^{N_0} Q_{j_0}(t)\, G_{0j_0}(t) + \sum_{j=1}^{M} \sum_{k_j=1}^{N_j} Q_{k_j}(t)\, G_{0k_j}(t) \ + \ \eta_0$$

CSPC then commands MS i in the reference cell to set its transmit power in the interval $(t+1, t+2)$, i.e., $Q_i(t+1)$, as follows

$$\begin{aligned} Q_i(t+1)\, G_{0i}(t) \ &= \\ &\Gamma_i \left(\sum_{j_0=1; j_0 \neq i}^{N_0} Q_{j_0}(t)\, G_{0j_0}(t) \right. \\ &\left. + \sum_{j=1}^{M} \sum_{k_j=1}^{N_j} Q_{k_j}(t)\, G_{0k_j}(t) \ + \ \eta_0 \right) \end{aligned} \tag{5}$$

Assuming that the changes in transmit power at each iteration are small, we can approximately rewrite the above equation as

$$\begin{aligned} Q_i(t+1)\, G_{0i}(t) \ &= \\ &\alpha_i \left(\sum_{j_0=1}^{N_0} Q_{j_0}(t)\, G_{0j_0}(t) \right. \\ &\left. + \sum_{j=1}^{M} \sum_{k_j=1}^{N_j} Q_{k_j}(t)\, G_{0k_j}(t) \ + \ \eta_0 \right) \end{aligned} \tag{6}$$

where α_i is as defined earlier.

Denote the average value of the attenuation ratio $\frac{G_{0k_j}(t)}{G_{jk_j}(t)}$ over all the MSs in cell j by $\beta_{0j}(t)$. Then, defining $P_j(t)$ as the total signal power received at BS j from MSs in cell j, we can write approximately

$$\sum_{k_j=1}^{N_j} Q_{k_j}(t)\, G_{0k_j}(t) \ \approx \ P_j(t)\, \beta_{0j}(t)$$

Assuming that the changes in the attenuation $G_{jk_j}(t)$ over each update interval are small, we can then sum Equation 6 over all the MSs in the reference BS ($1 \leq i \leq N_0$), to obtain

$$P_0(t+1) = \left(\sum_{i=1}^{N_0} \alpha_i \right) \left(P_0(t) + \sum_{j=1}^{M} P_j(t) \beta_{0j}(t) + \eta_0 \right) \tag{7}$$

Notice that, given $P_0(t+1)$, the value of $Q_i(t+1)$ can be calculated as (see Equation 6)

$$Q_i(t+1) = \left(\frac{\alpha_i}{\sum_{i=1}^{N_0} \alpha_i} \right) \frac{P_0(t+1)}{G_{0i}(t)}$$

Writing the total uplink received power equation for each BS $j, 1 \leq j \leq M-1$, and putting these equations together into a compact matrix format, we get

$$\underline{P}\,(t+1) \ = \ \underline{\underline{\mathcal{A}}}\,(\ \underline{\underline{B}}(t)\, \underline{P}\,(t) \ + \underline{\eta}) \tag{8}$$

where $\underline{P}$ is an M dimensional column vector representing the total received power at each of the M BSs, and $\underline{\underline{\mathcal{A}}}$ is an $M \times M$ diagonal matrix with diagonal entries $\mathcal{A}_{jj} \ = \ \sum_{k_j=1}^{N_j} \ \alpha_{k_j}$;

$\underline{\underline{B}}(t)$ is an $M \times M$ matrix whose diagonal entries are all ones, and the off-diagonal elements $B_{ij}(t) = \beta_{ij}(t), 0 \leq i,j \leq (M-1)$. Finally, $\underline{\eta}$ is the vector of thermal receiver noise powers, taken to be constants.

The matrix $\underline{\underline{B}}(t)$ has random entries because of the random variations in the channel gains. Thus Equation 8 is a *random coefficient autoregressive (RCA) process*. $\{\underline{P}(t)\}$ is a vector stochastic process, and we need to understand the conditions for its convergence to a stationary regime. Let the process converge (in distribution) to a random (power) vector P^*. The condition for such convergence should give us a necessary condition on the maximum number of admissible users in any of the cells. In the full generality of the problem, such a condition is hard to derive as the random matrices $\underline{\underline{B}}(t)$ are correlated.

In order to obtain a tractable analysis, and to study the effect of mobility, we consider the following model. The users move in such a manner that starting with a change in the matrix $\underline{\underline{B}}$ at t, for the next k measurement intervals $\underline{\underline{B}}$ does not change (the mobiles are fixed during this period). Then at $t+k$ an independent and identically distributed sample of $\underline{\underline{B}}$ is taken. With this model, we can analytically study two extreme situations, $k=1$ and $k=\infty$. In the former case (i.e., $\underline{\underline{B}}(t)$ are independent and indentically distributed matrices) we can apply the theory in the Appendix to obtain a condition for admission control. In the latter case, $\underline{\underline{B}}$ is sampled at $t=0$ and then stays fixed; the power vector converges if the Peron-Frobenius eigenvalue of the matrix $\underline{\underline{A}} \times \underline{\underline{B}}$ is less than one; requiring that the probability of this not happening is small gives another condition for admission control.

This approach can provide a loose constraint for admission control. We will show via simulations that even if this constraint is satisfied, the target SIR is not met with the desired outage probability. This happens because every MS updates its power level without any knowledge of the action taken by other MSs and thus due to coupling of power control updates, the SIR tend to often fall below the target.

As suggested in [1], in order to satisfy the QoS requirement under various mobility conditions, we have to inflate the target SIR by a factor so that the desired QoS is met. For each degree of mobility, the amount of inflation of SIR required depends upon the actual SIR that is achieved (say $\alpha^{'}$), with the desired outage probability, when the target SIR (α) is *not* inflated. The inflated value of target SIR ($\alpha^{''}$) can then be evaluated as,

$$\alpha^{''} = \frac{\alpha}{\alpha^{'}}\,\alpha$$

With this inflated target SIR, the RCA stability condition deflates the number of users that are admissible. In the next section we will show by simulation that this procedure succeeds in providing the desired QoS, but with a reduced number of admissible users.

IV. NUMERICAL RESULTS

We consider two classes of calls. The target minimum bit energy to noise power spectral density ratio $\gamma = 5\ dB$ and the total bandwidth available $W = 5\ MHz$. The bit rate requirement $R_1 = 10Kbps, R_2 = 20Kbps$. Thus $\alpha_1 = .0063$ and $\alpha_2 = .0125$. The number of cells $M = 19$. We assume that the standard deviation of the shadow fading component lies between $6-8\ dB$. The tolerable probability of outage is $P_{out} = 10^{-3}$.

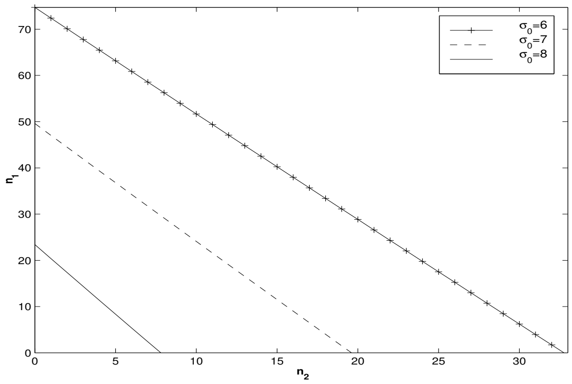

Fig. 1. CRPC: Admission region for two call classes, and three values of σ.

Let n_1 and n_2 be the number of class 1 and 2 calls in the reference cell. We assume that the cells are uniformly loaded, i.e, in every cell there are n_1 and n_2 calls of class 1 and 2, and the MSs are only capable of hard handoff at the cell boundary.

Figure 1 shows a sample admission region for the CRPC based algorithm. Results are shown for three values of the shadow fading standard deviation σ. For the CSPC based algorithm, we first obtain a contraint on the admission region using the eigenvalue approach discussed in the previous section (see also the Appendix). For $\sigma_0 = 8dB$, this yields

$$n_1\alpha_1 + n_2\alpha_2 \leq 0.164$$

which is slightly looser than the corresponding admission region for CRPC; see Figure 1. If calls are admitted in this region, then it can be expected that the power vector will converge, but we have found through simulation that the desired SIR outage QoS is not met. In order to satisfy the QoS requirement we have to use a tighter admission region.

The simulation setup is as follows. We consider only class 1 users. The above admission constraint for the CSPC algorithm gives $n_1 \leq 26 (\approx 0.164/0.0063)$. With these many users in the each cell, we start with an arbitrary initial power vector. We sample the matrix entries β_{ij} independently, and keep these fixed for the next k iterations. The CSPC algorithm is used to update the MS powers at each update epoch. After k update epochs the gain matrix entries are again sampled. This is continued for a large number of iterations and the actual SIR distribution, of the average MS, over the simulation is obtained. Figure 2 shows the cumulative distribution function (CDF) of the received SIR with k as a parameter. Recall that the higher the value of k the lower is the mobility. It is observed that for more than 60% of the time the SIR falls below the target α_1. To attempt to satisfy the desired QoS, we inflate the target SIR (α_1) as suggested in previous section. The received SIR CDFs are shown in Figure 3; notice that the actual target SIR α_1 is now met with the desired outage probability. With the inflated target SIR, however, we cannot accommodate 26 users as the stability condition gets violated. We compute the acceptable number of users from the stability conditions for different values of parameter k. Figure 4 shows the comparison for calls of type 1. The number of acceptable calls is plotted versus the mobility parameter k. Since CRPC is unaffected by mobility, the corresponding curve is just a horizontal line. Notice that CRPC works better than the CSPC algorithm when the mobility is high, but under very low mobility conditions (large k) CSPC performs better, as expected.

V. CONCLUSION

In the context of a CDMA cellular system carrying multimedia traffic, we have studied the effect of power control and mobility on the admission region. We compared Constant Received Power Control (CRPC) with Constant SIR Power Control (CSPC), and showed that under high mobility conditions CRPC can permit the admission of more calls than CSPC. Analytical determination of the CSPC admission region with user mobility is an important problem for future research.

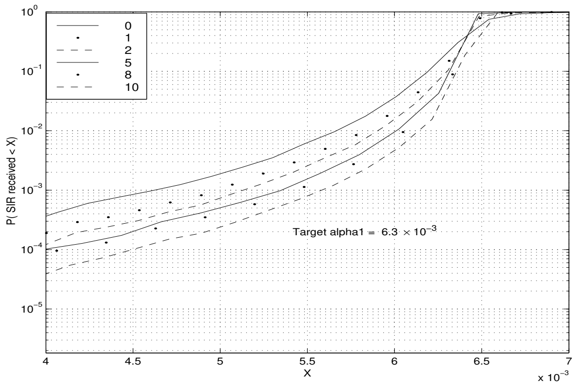

Fig. 2. CSPC: CDF of SIR using nominal target SIR in updates; $N = 26$.

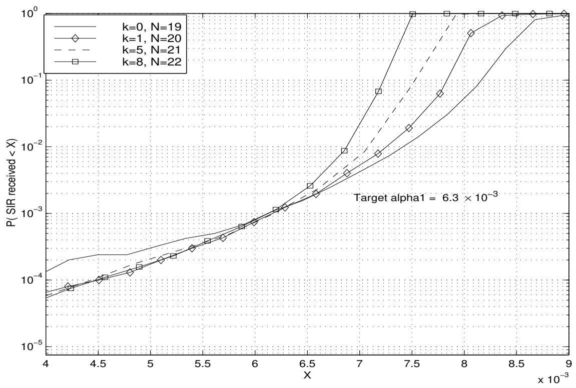

Fig. 3. CSPC:CDF of SIR using inflated target SIR in updates.

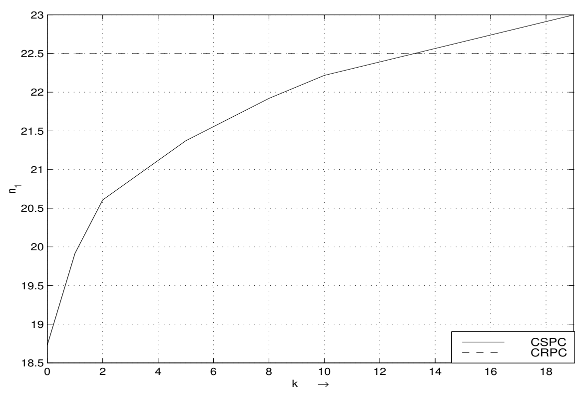

Fig. 4. Admissible number of users under CSPC and CRPC for different mobility conditions.

APPENDIX

Motivated by Equation 8 for CSPC, we consider the general problem of the existence of a limit distribution of the solution Y_n of a general random difference equation of the form

$$Y_n = M_n Y_{n-1} + Q \qquad n \geq 1 \tag{9}$$

where $M_1, M_2, \cdots$ is an infinite sequence of i.i.d. random real $m \times m$ matrices and Q is a nonrandom, nonzero vector in $\Re^m$. The RCA process of Equation 8 corresponds to the same problem if we assume that the elements of the gain matrix are i.i.d., and the gain matrix is sampled at each update epoch (i.e., $k = 1$). Given Y_0, Y_n has same distribution as

$$Y_n = \sum_{k=1}^{n} M_1 M_2 \cdots M_{k-1} Q + M_1 M_2 \cdots M_n Y_0$$

We state the following results from existing literature on random difference equations.

Result 1: Kesten [6] showed that if

$$E(\log^+ \| M_1 \|) < \infty$$

Then $\alpha = \lim_{n\to\infty} \frac{1}{n} \log \| M_1 M_2 \cdots M_n \|$ exists and is finite w.p. 1, where, for $x > 0$, $\log^+(x) = (\log(x))^+$ and $\| \cdot \|$ is the Euclidean norm of the matrix.

Result 2: Cohen et al. [3] considers the hypothesis that $\{M_{1,(ij)};\ i,j = 1, \cdots, m\}$ have finite mean and variance. Defining C to be the positive semidefinite matrix $E(M_1^T M_1)$,i.e.,

$$C_{ij} = \sum_{k=1}^{m} E(M_{1,(ki)}\ M_{1,(kj)})$$

it is shown that

$$E(\log^+ \| M_1 \|) < \infty$$

and

$$\alpha \leq \log(\ [r(C)]^{1/2})$$

where $r(C)$ is the spectral radius (max eigenvalue) of C.

Result 3: Kesten [6] further showed that if $\alpha < 0$ then the series

$$R = \sum_{n=1}^{\infty} M_1 M_2 \cdots M_{n-1} Q$$

converges w.p. 1 and the distribution of the solution Y_n of Equation 9 converges to that of R, independently of Y_0.

It follows that a sufficient condition for the existence of a limit distribution of Y_n is that $r(C) < 1$.

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