Optimal Resource Allocation Policies for a Multiaccess Fading Channel with a Quality of Service Constraint

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I. INTRODUCTION

Future wireless networks will support traffic with diverse quality of service (QoS) requirements. Wireless resources, e.g., the bandwidth and the power at the mobiles, being scarce, it is important to use them judiciously and thus the traditional problem of sharing the resources efficiently among the contending users is important. At the system level, the problem that we address is similar to the problem in Capone and Stavrakakis [1]. Considering a TDMA based system they have determined whether a given QoS vector is achievable, and the efficient resource allocation policy that delivers it. They considered a multiaccess link with T slots per frame, shared among M variable bit rate (VBR) unbuffered sources. Because of strict delay requirements the packets that cannot be served in the frame following their arrival are assumed to be lost. Thus the QoS measure was the rate of dropping packets. They determined the admissible region and a simple scheduling policy that delivers the achievable QoS.

In [1] it was assumed that the physical layer is transparent,i.e., the channel delivers the data error-free. Now consider the case where a user transmits a packet in its allocated slot in the frame and the channel experienced is bad (e.g. a fading wireless channel). Though the packet is not dropped in the transmitter buffer, it eventually gets dropped at the receiver end. If, however, some power control mechanism is available then one can increase the transmitter power so that the packet can be received error free at the receiver, but this may violate the average transmitter power constraint. Thus while evaluating the achievable regions one must take into account the physical layer constraints and the time varying nature of the wireless channel. In this paper we consider the uplink of a wireless system with M users communicating with a base station. All users can transmit simultaneously unlike a TDMA based system considered in [1]. We will first consider a fluid model and subsequently specialize to a packetized model. The resource to be shared is the transmission rate and the power. We consider the problem addressed in [1] from an information theoretic perspective. We incorporate the effect of temporal variations in the channel. The information theory based analysis helps in obtaining the limit of what could possibly be achieved using an efficient channel coding-decoding scheme and also provides insight into good rate and power control policies. The QoS vector under consideration would be the acceptable number of bits dropped per slot on an average, or the probability of dropping packets. We will obtain a criterion for determining whether a given QoS vector is admissible and if it is, we provide a scheduling policy so that the available resources are utilized optimally while satisfying the QoS requirements.

II. SYSTEM MODEL

Consider M variable bit rate (VBR) sources with strict delay requirements sharing a multipoint to point communication link. We assume a slotted system; the bits generated over a slot are accumulated and a corresponding request for allocation is made towards the end of slot. The channel gain (fading) process is assumed to be slowly varying and stays constant over a slot. We assume that the Base Station (BS) has a good estimate of the channel in the next slot. Each user has a long term average transmission power constraint. The BS acts as a scheduler, which depending upon the channel conditions of the users and their respective requirements, allocates the bit rates and the transmission powers to different users. The rescheduling of the resources takes place at the slot boundaries. The bits which cannot be transmitted in the slot following their arrival are assumed to be lost. This may be too strict a requirement but realistic enough to be considered as a first step towards the more general problem of buffered service at each node. The bits to be transmitted can be chosen at random from the lot so that contiguous loss of data can be avoided. Depending upon the channel bandwidth B and the slot length T, by Nyquist theorem, let N be the possible channel uses in a slot. Considering the problem at the physical layer, if a rate of r bits per channel use is allocated to a user then a bit stream of length $r * N$ arrives at the channel encoder from the higher layer which is encoded into a codeword of length N symbols. We assume that the codebooks corresponding to various allowable rates are available at the transmitter. This could be a reasonable assumption from an information theoretic viewpoint as the codeword lengths are fixed to N symbols. We further assume negligible encoding time. We denote by I the set $\{1, 2, \cdots, M\}$ of users and from now on, the index $i \in I$ unless otherwise stated. We assume that the decoder at the BS has successive decoding capability.

A. Notation and Problem Formulation

Let the maximum acceptable number of bits lost per slot on an average for user i be l_i. Since the requirement is on the fraction of bits lost on a long run, it can be computed as a ratio of l_i and the expected number of bits that arrive per slot. So we consider l_i as our QoS parameter. Let, in the k^{th} slot, $s_i(k)$ be the rate requested for and $h_i(k)$ be the channel gain of the i^{th} user. The rate $s_i(k)$ is such that the value $s_i(k) * N$ is the number of bits that arrive in the previous slot. Define $h(k) = [h_1(k), h_2(k), \cdots, h_M(k)]$. Let $\{S_i(k)\}$and $\{H(k)\}$ be the random processes representing $s_i(k)$ and $h(k)$ respectively. The channel gain process $\{H(k)\}$ is assumed to be jointly stationary and ergodic. The process $\{S_i(k)\}$ is assumed to be stationary and ergodic for each $i \in I$. Also $\{S_i(k)\}$ and $\{S_j(k)\}$ are independent for $j \neq i$. If in slot k, the user i transmits a signal $x_i(k)$, then the receiver gets $y(k) = \sum_{i=1}^{M} h_i(k)x_i(k) + \eta(k)$, where $\eta(k)$ represents the N dimensional additive white Gaussian noise (AWGN) vector in slot k. The transmitter i has an average transmit power constraint of $\bar{P}_i$; define $\bar{P} = [\bar{P}_1, \bar{P}_2, \cdots, \bar{P}_M]$. Let, in the k^{th} slot, the BS scheduler allocate the rate $r_i(k)$ to user i and define $r(k) = [r_1(k), r_2(k), \cdots, r_M(k)]$. The cost of allocating a rate vector $r(k)$ is the transmitter power vector $p(k)$ which depend upon $h(k)$ through the Shannon's formula for the ergodic capacity for the AWGN multiple access channels. Let $\{R(k)\}$ and $\{P(k)\}$ be the vector valued random processes representing $r(k)$ and $p(k)$ respectively. Since the codeword lengths are finite, there will be a small error probability, i.e., the probability of decoding a codeword incorrectly which depends on the codeword length N and decreases exponentially with the codeword length. The random coding bound (refer to the Appendix) gives a bound on the achievable probability of decoding error for codewords of length N. Thus given any channel state, to achieve an error probability P_e, there is a corresponding rate penalty of ϵ or equivalently a higher power cost for a given rate vector. Thus given a power vector P, channel gain vector h and ϵ, the rate vector should lie in the region (capacity region),

$$C(h, P) = \left\{ R : R(\mathcal{J}) \leq \frac{1}{2} \log \left(1 + \frac{\sum_{j \in \mathcal{J}} h_j P_j}{\sigma^2} \right) - |\mathcal{J}|\epsilon \qquad \text{for every } \mathcal{J} \subset \mathrm{I} \right\}$$

where σ^2 denotes the ambient noise power, $R(\mathcal{J}) = \sum_{i \in \mathcal{J}} R_i$ and $|\mathcal{J}|$ denotes the cardinality of the set $\mathcal{J}$. The decoding error probability will correspond to bits lost at the receiver. Define an indicator function representing error event for user i in slot k by $I_{err,i}(k)$ where $E(I_{err}) = P_e$. The number of bits lost (not transmitted or arriving in error at the B.S) for user i in slot k is $N((S_i(k) - R_i(k))^+ + I_{err,i}(k)R_i(k))$. In order to satisfy the QoS requirement of each user we should have,

$$\limsup_{K \to \infty} \frac{1}{K} [\sum_{k=1}^{K} \{((S_i(k) - R_i(k))^+ + I_{err,i}(k)R_i(k))N\}] \leq l_i, \text{ for } i \in I.$$

Also the power constraint should be satisfied, i.e.,

$$\limsup_{K \to \infty} \frac{1}{K} [\sum_{k=1}^{K} P(k)] \leq \bar{P}.$$

Our objective is to find the boundary of the achievable QoS region and the scheduling policy that achieves any given QoS requirement within this region subject to the users' transmitter power constraints. Thus we minimize a weighted sum of the expected number of bits lost per slot, subject to the power constraints. By varying the weights, we can determine the QoS region boundary. If l lies in the achievable QoS region, we determine the rate and power allocation policies. We restrict ourselves to stationary Markovian policies (rules), i.e., in the k^{th} slot, the allocated rate vector $r(k)$ and the power vector $p(k)$ are functions of the $(s(k), h(k))$ pair only.

III. Analysis

Denote the space of all stationary Markovian policies by $\mathcal{M}$. A policy $\pi \in \mathcal{M}$ is a pair of mappings

$$r: \mathcal{R}_+^M \times (0,1]^M \rightarrow \mathcal{R}_+^M; \quad p: \mathcal{R}_+^M \times (0,1]^M \rightarrow \mathcal{R}_+^M$$

where $r(s,h)$ is the allocated rate vector and $p(s,h)$ is the allocated power vector. Let $\mathcal{M}' \subset \mathcal{M}$ be a subset of policies for which the information theoretic constraints are satisfied, i.e., for all $(s,h), r(s,h) \in C(h, p(h,s))$. Define the cost functions J^π and K^π as the expected number of bits dropped per slot and the average transmitter power under the policy $\pi \in \mathcal{M}'$ respectively. Since the process $(S(k), H(k))$ is stationary and ergodic, we have

$$J^\pi = E^\pi_{(S,H)}[w \cdot N((S - r(S,H))^+ + I_{err}(S,H) r(S,H))], \quad K^\pi = E^\pi_{(S,H)}[p(S,H)]$$

where the weights w_i are nonnegative and $x \cdot y = \sum x_i y_i$ for vectors x, y. Let $\mathcal{M}_{\bar{P}}$ be the subset of policies which satisfy the power constraint, i.e.,

$$\mathcal{M}_{\bar{P}} = \{\pi \in \mathcal{M}' : K^\pi \leq \bar{P}\}.$$

The problem is,

$$\min\{J^\pi \; : \; \pi \; \in \; \mathcal{M}_{\bar{P}}\}. \tag{1}$$

Since the functional J^π is convex and the sets $\{\pi : K^\pi \leq \bar{P}\}$ and $\{\pi : r(s,h) \in C(h, p(s,h)), \; \forall \; (s,h) \text{ pairs}\}$ are convex in the space of all policies, there exists a Lagrange multiplier $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_M)$ (Refer to Section 2.6 of Aubin [6]) such that the above optimization problem is equivalent to

$$\min\{J^\pi + \lambda \cdot K^\pi : \pi \in \mathcal{M}'\}.$$

where λ is determined by the power constraint $\bar{P}$. For fixed λ, define

$$V^\pi = E^\pi_{(S,H)}[w \cdot N((S - r(S,H))^+ + P_e r(S,H)) + \lambda \cdot p(S,H)].$$

Thus the objective is,

$$\min\{V^\pi : \pi \in \mathcal{M} \text{ and } r(s,h) \in C(h, p(s,h)) \text{ for all } (s,h) \text{ pairs}\}.$$

Since the multiplier λ is fixed, the problem gets decoupled over the (s,h) pairs and is the same as solving the following problem for all fixed (s,h) pairs,

$$\min_{(r,p)}\{w \cdot N((s-r)^+ + P_e r) + \lambda \cdot p\} \qquad \text{subject to} \qquad r \in C(h,p). \tag{2}$$

Let q be the received power vector. Define, for a given rate vector r and an (s,h) pair,

$$Q(h,r) = \{q : \exists \, p \text{ s.t. } q_i = h_i p_i, \; r \in C(h,p)\}.$$

The region $Q(h,r)$ is a contra-polymatroid [2]. The optimization problem (2) can be restated as,

$$\min_{(r,q)} \{w \cdot N((s-r)^+ + P_e r) + \frac{\lambda}{h} \cdot q\} \text{ subject to } q \in Q(h,r)$$

where, by an abuse of notation, $\frac{\lambda}{h}$ denotes the row vector $(\frac{\lambda_1}{h_1}, \cdots, \frac{\lambda_M}{h_M})$. Without loss of generality, let us assume that $\frac{\lambda_1}{h_1} \geq \frac{\lambda_2}{h_2} \geq \cdots \geq \frac{\lambda_M}{h_M}$. For any given rate vector r, the minimum value of $\frac{\lambda}{h} \cdot q$ subject to above said contra-polymatroid constraint is given by (see Lemma 3.3 of [2]),

$$\sum_{i=1}^{M} \frac{\lambda_i}{h_i} \{ f(\sum_{k=1}^{i}(r_k+\epsilon)) - f(\sum_{k=1}^{i-1}(r_k+\epsilon)) \},$$

where $f(x) := \sigma^2(e^{2\beta x} - 1)$, $\beta = \ln(2)$ and $f(0) := 0$. Rewriting the above problem, we get a convex objective function and the problem is

$$\min_r \Big\{ \sum_{i=1}^{M} w_i N((s_i - r_i)^+ + P_e r_i) + \sum_{i=1}^{M} \frac{\lambda_i}{h_i} \{ f(\sum_{k=1}^{i}(r_k+\epsilon)) - f(\sum_{k=1}^{i-1}(r_k+\epsilon)) \} \Big\}.$$

In order to remove $(\cdot)^+$ from the above problem we use an extra constraint $r \leq s$. There exists a Lagrange multiplier $\mu \geq 0$ such that the above problem is equivalent to,

$$\min_r \sum_{i=1}^{M} \Big[w_i N(s_i - r_i + P_e r_i) + \frac{\lambda_i}{h_i} \{ f(\sum_{k=1}^{i}(r_k+\epsilon)) - f(\sum_{k=1}^{i-1}(r_k+\epsilon)) \} + \mu_i r_i \Big].$$

The solution to the above problem that satisfies the Kuhn-Tucker conditions gives us the allocated rate vector r. It can be obtained as follows:
Let $(u_1, u_2, \cdots, u_n)$ solve the following system of equations,

$$\sum_{k=i}^{M} \left(\frac{\lambda'_k}{h_k} - \frac{\lambda'_{k+1}}{h_{k+1}} \right) 2^{2\sum_{m=1}^{k}(u_m+\epsilon)} = v_i \ \text{ for } \ i = \{1, \cdots, M\} \tag{3}$$

where $\frac{\lambda'_{M+1}}{h_{M+1}}$ and v_{M+1} are both zero, $v_i = w_i(1-P_e)N$ and $\lambda'_i = 2\beta\sigma^2\lambda_i$ for all $i \in I$.
Let $\mathcal{J} = \{j : s_j \leq u_j, j \in I\}$. Now in the system of equations (3), set $u_i = s_i$ for $i \in \mathcal{J}$ and again solve the resulting subset of equations (3) for $u_i, i \in I \setminus \mathcal{J}$, yielding the solution u'. The rate allocation policy is $r_i = \min(s_i, u'_i)$ for all $i \in I$. The power allocation policy is $p_i = \frac{1}{h_i}\{f(\sum_{k=1}^{i}(r_k+\epsilon)) - f(\sum_{k=1}^{i-1}(r_k+\epsilon)\}$. When $s_i \leq u'_i, \forall i \in I$, define the system to be underloaded and overloaded otherwise. The Lagrange multiplier λ is obtained from the average power constraint.

Theorem III.1: Given $\bar{P}, s, w, \lambda \in \mathcal{R}_+^M$ and $h \in (0,1]^M$ satisfying $\frac{\lambda_1}{h_1} \geq \frac{\lambda_2}{h_2} \geq \cdots \geq \frac{\lambda_M}{h_M}$, the pair (r,p) obtained as above is the optimal rate and power allocation policy. The boundary of the QoS region is $E_{(S,H)}[w \cdot N((S - r(S,H))^+ + P_e r(S,H))]$, a function of w.

Remark 1: The rate allocation region is convex and in the overload region, the rate r_i is larger for the user with larger value of w_i and/or larger value of the channel gain h_i.

Remark 2: If l lies in the QoS region, then we say that the users are admissible. If admissible, the weight w is any element of the set $\{w : E_{(S,H)}\{w \cdot ((S - r(S,H))^+ + P_e r(S,H))N\} \leq l\}$. We illustrate the theorem by the following examples.

A. Examples

1) **Single User Channel** ($M = 1$): Given the channel gain h, if $s \leq \frac{1}{2}\log(\frac{h(1-P_e)N}{2\ln(2)\lambda\sigma^2}) - \epsilon$, then $r = s$; else $r = \frac{1}{2}\log(\frac{h(1-P_e)N}{2\ln(2)\lambda\sigma^2}) - \epsilon$. The corresponding power allocation is $p = \min(\frac{\sigma^2}{h}(2^{2s}-1), (c - \frac{\sigma^2}{h})^+)$, where c is a constant. Notice that for the case $r = \frac{1}{2}\log(\frac{h(1-P_e)N}{2\ln(2)\lambda\sigma^2}) - \epsilon$, the power allocation policy has the familiar water filling form.

2) **Two User Scheduling Region** ($M = 2$): Assume that the QoS constraints are admissible. Let w be the weights such that $w_1 > w_2$. Define $u_1 = \frac{1}{2}\log\left(\frac{h_1 h_2(v_1 - v_2)}{\lambda'_1 h_2 - \lambda'_2 h_1}\right) - \epsilon$ and $u_2 = \frac{1}{2}\log\left(\frac{(\lambda'_1 h_2 - \lambda'_2 h_1)v_2}{\lambda'_2(v_1 - v_2)h_1}\right) - \epsilon$. Given h, the rate

allocation policy is as follows:
If $s \le u$ then $r = s$; If $s_1 \le u_1$, then $r_1 = s_1$ and $r_2 = \min\{s_2, \frac{1}{2}\log(\frac{v_2 h_2}{\lambda_2^{'}}) - s_1 - 2\epsilon\}$; If $s_2 \le u_2$, then $r_1 = \min\{s_1, \frac{1}{2}\log\left(\frac{v_1 h_1 h_2}{\lambda_1^{'} h_2 - \lambda_2^{'} h_1 + \lambda_2^{'} h_1 e^{2\beta(s_2+\epsilon)}}\right) - \epsilon\}$ and $r_2 = s_2$; Else $r = u$.

The rate region is shown in Figure III-A. The values of λ_1 and λ_2 are obtained from the average power constraints. The averaging has to be carried out over the distributions of S and H. The computation of these multipliers can be done numerically using an iterative algorithm as suggested in [2]. □

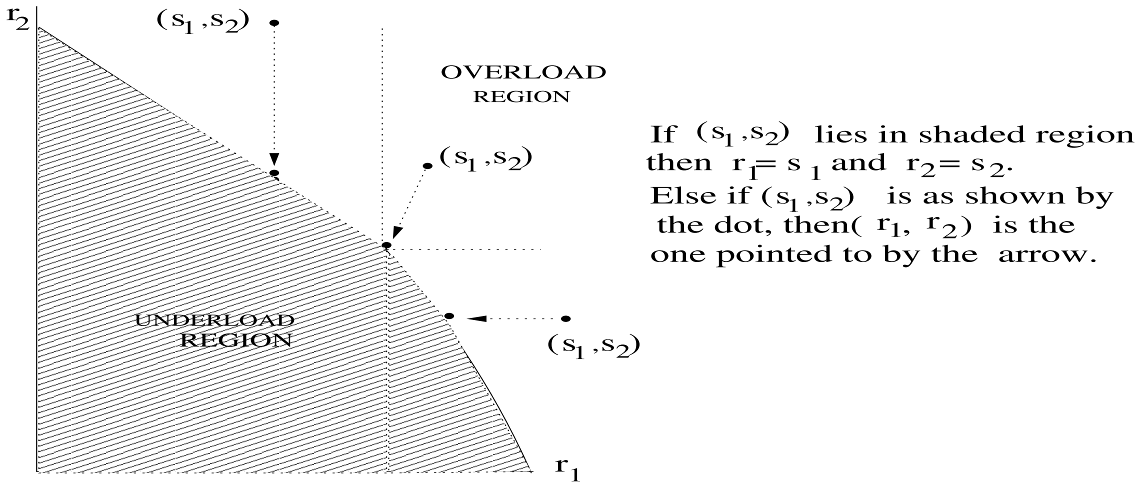

Fig. 1. Two user rate allocation region for a fixed channel gain h

IV. Packet Level View

We specialize the above problem to the case where multiple packets of fixed length N arrive in a slot and N is chosen same as the allowed number of channel uses per slot for simplification. If packet size is different, the following analysis goes through with minor modification. The scheduling policy has to choose some of the packets to service in the slot while dropping the rest of the packets. The QoS measure is the packet dropping probability. Since dropping probability is related to the rate of dropping packets through the packet arrival rate, we consider rate of dropping packets as our quality criterion. Since the packet lengths are fixed to N, the number of packets per slot $s \in {Z_+}^M$ where Z_+ is the set of nonnegative integers. We assume the packet arrival process to be stationary and ergodic. If we use the same rate allocation policy as obtained in Section III , then for any h, the allocated rate vector r can take values in $\mathcal{R}^M$ but we cannot serve a fraction of a packet. Thus the search of policies should be over a class of Markovian policies which allocates integral rates to each user. Let, corresponding to s and channel gain vector h, the policy given by the result in Section III be (r, p). Consider a policy which allocates a rate $\lfloor r \rfloor$. This policy under-utilizes the power available at the transmitter. Thus the QoS region is unnecessarily restricted. We give an improved policy which is a randomization between two stationary Markov policies (refer [4]).

Consider, for example, the single user case. Given h, the rate allocation policy obtained in Section III is $r = \min(s, z)$ where $z = \frac{1}{2}\log(\frac{h(1-P_e)N}{\lambda^{'}}) - \epsilon$. Let the corresponding average power cost be $\overline{P}(h)$ where averaging is done with respect to S, the packet arrival process. Define two new policies $\overline{\Pi}$ and $\underline{\Pi}$ which allocate the rates according to $r = \min(s, \lceil z \rceil)$ and $r = \min(s, \lfloor z \rfloor)$ respectively. Let the corresponding power costs be $K^{\overline{\Pi}}(h)$ and $K^{\underline{\Pi}}(h)$. Clearly, $K(\underline{\Pi}) \le \overline{P}(h) \le K(\overline{\Pi})$. Thus given that the channel state is h, a randomized policy is to choose the policy $\overline{\Pi}$ with probability $\pi(h) = \frac{\overline{P}(h) - K(\underline{\Pi})}{K(\overline{\Pi}) - K(\underline{\Pi})}$ and $\underline{\Pi}$ with $1 - \pi(h)$. We can verify that the average power constraint $\bar{P}$ is satisfied with equality.

Consider the multiuser case. Given (s, h), let $u_i^{'}$ be as computed in Section III. Define two new policies as $r_i = \min(s_i, g(u_i^{'}))$ where the function $g(\cdot)$ for the policy $\overline{\Pi}$ is $g(x) = \lceil x \rceil$ and for $\underline{\Pi}$ is $g(x) = \lfloor x \rfloor$. Let $\pi_i(h)$ be the probability of choosing the policy $\overline{\Pi}$ for i^{th} user. Define $\pi(h) = [\pi_1(h), \pi_2(h), \cdots, \pi_M(h)]$. Let $\bar{P}(h)$ be the average power cost, where the averaging is done with respect to S, when the policy is such that the function $g(x) = x$. The probability vectors $\pi(h)$ can be computed iteratively using Newton method so that the average power used under the randomized policy (randomized between $\overline{\Pi}$ and $\underline{\Pi}$) is equal to $\bar{P}(h)$. The existence of such a vector is guaranteed since the power requirement vector is continuous in $\pi(h)$ and for $\pi(h) = 0$, it assumes a value less than$\bar{P}(h)$ and for $\pi(h) = 1$, its value is strictly greater than $\bar{P}(h)$.

Consider rather an extreme situation where a single packet of variable size arrives in each slot and the packet is either served or dropped. There is no fragmentation allowed. The QoS measure is the expected number of packets dropped. We consider only the symmetric system, i.e., the average power constraint is same for each user and the arrival and channel gain process is symmetric. The policy $\Pi(s, h)$ is a mapping $\Pi : (s, h) \to \{0, 1\}^M$, where $\Pi_i(s, h) = 1$ indicates that the i^{th} user packet is served and $\Pi_i(s, h) = 0$ otherwise. Thus a policy gives a rate vector $r^\pi(s, h)$. Let $C^{pi}(s, h)$ counts the number of packets dropped under policy π when in state (s, h). Let $P^\pi(s, h)$ be the power requirement under the policy π when in state (s, h). Using the Shannon formula for information theoretic capcaity of multiaccess fading channels, we have a constraint that $r^\pi(s, h) \in C(h, P^\pi(s, h))$. Let $\bar{P}$ be the average power constraint. Let $\mathcal{M}_{\bar{P}}$ be the subset of policies,

$$\mathcal{M}_{\bar{P}} = \{\pi \in \Pi : E^\pi_(s, h)(P^\pi(s, h)) \le \bar{P} \text{ and } r^\pi(s, h) \in C(h, P^\pi(s, h))\}.$$

Thus the optimization problem is $\min_{\pi \in \mathcal{M}_{\bar{P}}} E_{(s,h)}(C^\pi(s, h))$. Using the Lagrangian argument. Let λ be the Lagrange multiplier such that the problem is equivalent to,

$$\min_{\pi \in \Pi} E_{(s,h)}(C^\pi(s, h) + \lambda \cdot P^\pi(s, h)) \text{ s.t. } r^\pi(s, h) \in C(h, P^\pi(s, h))\}.$$

Due to symmetry of the system, we can take the multiplier λ to be a scalar. Now using the decoupling argument as in Section III, the objective is,

$$\min_{\pi \in \Pi}(C^\pi + \lambda \sum_{i=1}^{M} P_i^\pi) \text{ s.t. } r^\pi \in C(h, P^\pi)\},$$

for each (s, h) pair, Define an ordering t as $t(i) > t(j)$ if $h_i > h_j$. As in Section III, given a policy π, the minimum value of $\sum_{i=1}^{M} P_i$ is given by,

$$\sum_{i=1}^{M} \frac{1}{h_{t(i)}} \{f(\sum_{k=1}^{i}(s_{t(k)}\pi_{t(k)})) - f(\sum_{k=1}^{i-1}(s_{t(k)}\pi_{t(k)}))\}.$$

The function $f(\cdot)$ is same as in Section III. Define

$$V(\pi) = C^\pi + \lambda \sum_{i=1}^{M} \frac{1}{h_{t(i)}} \{f(\sum_{k=1}^{i}(s_{t(k)}\pi_{t(k)})) - f(\sum_{k=1}^{i-1}(s_{t(k)}\pi_{t(k)}))\}$$

The objective is $\min_{\pi \in \Pi} V(\pi)$.

For the single user case, it turns out that a threshold policy is optimal. If$\frac{1}{h}(f(s) - 1) \le \frac{1}{\lambda}$ then serve the packet, else drop it. The constant λ satisfies the average power constraint.

V. Conclusion

We have considered the uplink of an M user cellular wireless system. Transmission is in slots, in each of which data arrives at each user. Some or all of data at each transmitter in each slot is transmitted and the rest is discarded. The QoS measure is the fraction of data lost. In this paper, we have provided a result that provides the achievable region in the M dimensional QoS space and the optimal rate and power allocation policies on the boundary of the region. The main result is for fluid arrival and fluid service. We also provide some results for the packet model. In ongoing work we are developing similar results for the case when data can be buffered from slot to slot. There are some QoS constraints.

APPENDIX

Now we show a result (refer [5]) used in the paper. Given the channel realization h, the probability of decoding any user incorrectly while using codewords of length N and maximum likelihood decoding, is given by $p(N) \leq \sum_{\mathcal{J}} p(\mathcal{J}, N)$, where $\mathcal{J} \subset I$. Using the random coding bound [5],

$$p(\mathcal{J}, N) \leq exp\Big(- N\rho\Big[- R(\mathcal{J}) + \frac{1}{2}\log\Big(1 + \frac{\sum_{i\in\mathcal{J}} \mathcal{P}_i(h)\ h_i}{\sigma^2(1+\rho)}\Big)\Big]\Big)$$

where $R(\mathcal{J}) = \sum_{i\in\mathcal{J}} R(i)$. Assuming that $\exists\ \epsilon > 0$ such that for all $\mathcal{J} \subset I$,

$$R(\mathcal{J}) \leq \frac{1}{2}\log\Big(1 + \frac{\sum_{i\in\mathcal{J}} \mathcal{P}_i(h)\ h_i}{\sigma^2}\Big) - |\mathcal{J}|\epsilon.$$

Thus the bound on the probability of decoding any user incorrectly is, $P(error) = exp(\ \frac{\rho N}{2}\log(1+\rho)) \sum_{\mathcal{J}\subset I} exp(-N\rho|\mathcal{J}|\epsilon)$. Thus we can compute the value of ϵ as we need $p(error) \leq P_e$ where as ρ is parameter taking values in $[0, 1]$.

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