Effect of Soft Handoffs on the Downlink Admission Region for CDMA Multimedia Cellular Systems

Munish Goyal, Anurag Kumar
Dept. of Electrical Communication Engg,
Indian Institute of Science, Bangalore 560012
e-mail: `munish, anurag@ece.iisc.ernet.in`

ABSTRACT

We consider a CDMA based cellular system and analyse the performance of the downlink under soft handoffs. We consider traffic with some desired quality of service requirements expressed as bandwidth guarantees and a target bit energy to noise spectral density ratio. We analytically study the effect of handoff threshold on the soft handoff region and hence on the admission control region. The analysis involves level crossing analysis of an Ornstein-Uhlenbeck process across a handoff threshold profile. We compare the admission regions for hard handoff and soft handoff with and without diversity combining.

I. INTRODUCTION

At present, the majority of the traffic in cellular networks is voice and the traffic volumes on the uplink and the downlink are similar. Unlike voice communication, there are many applications such as Internet access and streaming media transfer from media servers in which the utilization of radio resources is strongly biased towards the downlink. There is a need to study the downlink in order to obtain an admission control region for given quality of service (QoS) requirements. The downlink performs significantly better than the uplink because of no intracell interference as the orthogonal variable spreading factor (OVSF) codes, used in third generation wireless systems, maintain orthogonality.

We assume that all the base stations (BSs) transmit at the same total average power level. This is reasonable since the BS transmitter is limited by some maximum power. As a mobile moves within a cell, the amount of power allocated varies according to its relative need. The power allocation needs to be updated every few seconds because of the change in the channel gains for different mobiles. This update is termed as *power allocation* from the BS rather than power control, as in the uplink. An MS near the boundary of the cell may be in soft handoff, in which case it can receive signal power from two or more BS and the signals received are coherently combined at the MS rake receiver. Thus a BS needs to transmit reduced power to the MS, since the power required by it in the soft hand off region is divided among two or more BS's, which helps in interference reduction, but at the expense of the cell dwell time [10]. Soft handoff is normally initiated when the ratio of the signal strengths from two nearby BS's observed at the mobile crosses some predetermined threshold. The width of soft handoff region is also determined by the handoff threshold. Thus the effective cell boundary may be much larger than the actual boundary. This extended coverage by a BS may lead to the reduction of the allowable number of users in the system for quality of service guarantees. Hence the effect of the width of the soft handoff region and user mobility must be considered while evaluating the admission region for guaranteeing a minimum service quality.

In [6], an admission control algorithm for serving multiple classes which require different SIR and transmission rates is proposed. But most of the available work including [6] and [10] does not consider the effect of the soft handoff region on the admission thresholds. The analysis is carried out under the common assumption that the mobiles are uniformly distributed within the cell and the BS has to provide the desired power to all the MSs within the cell. The cell boundary normally expands for a cellular system using soft handoffs. We propose to look at how the soft handoff effects the distribution of the mobiles and hence the admission thresholds. Our work is closely related to the work of De Hoz and Cordier [2]. Given the fraction of mobiles in soft handover within each cell, they obtained a theoretical limit of the downlink capacity for voice calls. They carried out a deterministic analysis based on the mean profile of fading process.

We propose to derive the downlink capacity for multimedia traffic with soft handoff and diversity combining.

Our main contribution is to characterize the soft handoff region by a level crossing analysis of the Ornstein-Uhlenbeck process across a handoff threshold profile. This yields an effective distribution of MS locations in terms of the BS power required to serve them. This distribution is then used in standard analysis to obtain the admission control regions. The analysis extends the result in [9] for multimedia traffic with soft handoffs.

This paper is organized as follows. In Section II, we model the system under consideration and obtain the admission control region. This admission region is shown to depend upon the distribution of a random variable Z. Given the position of the mobile r, we obtain the distribution of Z in Section III. Using level crossing analysis, we show how handoff region is defined in terms of handoff thresholds and obtain the distribution of r under various thresholds in Section IV. In Section V, we give numerical results for the admission region.

II. Problem Formulation

Let there be 'C' classes of calls, distinguished from one another in terms of their diverse Quality of Service (QOS) requirements, i.e., target minimum bit energy to noise power spectral density ratio of η_i and the bit rate of ρ_i for the i^{th} class. Let W be the total bandwidth available, then the QOS requirement for the i^{th} mobile can be encapsulated into a minimum signal to interference power ratio (SIR) requirement as $\Gamma_i = \frac{\eta_i \rho_i}{W}$.

The model under consideration is that of a traditional cellular system. The reference cell is indexed as 0. Let the distance between the two adjacent cell BS's be d, and the distance of the k^{th} mobile in the reference cell from the j^{th} cell BS be $r_{jk}(t)$ and the angle it makes with its BS, with respect to a reference line, be $\theta_{jk}(t)$. Assuming that the time scale under consideration is greater than the correlation time scale for the Rayleigh component of the fading process, the Rayleigh component is averaged out. The shadow fading term is spatial and depends only upon the relative positions of the transmitter and the receiver. This term can be represented as two components, one due to the scatterers around the MS and the other due to scatterers around the BS written respectively as $10^{\frac{-\xi_k(r_{jk}(t),\theta_{jk}(t))}{10}}$ and $10^{\frac{-\xi_{jk}(\theta_{jk}(t))}{10}}$. The logarithms of both the components are normal $(0, \sigma_0^2/2)$. The first component depends upon both r and θ and is common to all the BSs, i.e., it is MS specific and depends on its polar coordinates, while the other one depends only upon the angle that the MS makes with the BS and is independent from one BS to another. Assume that the path loss factor is 4. The channel gain process for j^{th} transmitter and the k^{th} receiver is given by, for $\forall j, \forall k$,

$$G_{jk}(t) = (r_{jk}(t))^{-4} \; 10^{-(\xi_{jk}(\theta_{jk}(t)) + \xi_k(r_{jk}(t),\theta_{jk}(t)))/10}$$

At time t, let $P_i(t)$ be the power transmitted from the reference BS to the i^{th} MS in the cell, and $I_i(t)$ be the total other cell interference observed by this MS. Let there be N active terminals in the reference cell. The QOS for each of the MS is satisfied iff for $\forall i, \forall t$, $\frac{P_i(t)\; G_{0i}(t)}{I_i(t)} \geq \Gamma_i$. The interference caused to the i^{th} MS, due to transmissions from other BSs each of which is transmitting at S_T, is $I_i(t) = \sum_{j=1}^{\infty} S_T G_{ji}(t)$. Assuming full loading of the downlink, the QoS constraint can now be written as

$$P_i(t) \geq \Gamma_i \; S_T \; \sum_{j=1}^{\infty} \frac{G_{ji}(t)}{G_{0i}(t)} := \Gamma_i \; S_T Z_i(t) \tag{1}$$

Summing Eqn.(1) over i we require $\sum_{i=1}^{N} \Gamma_i Z_i(t) \leq 1$ for all t. Note that $Z_i(t)$ is a random process due to the random variations in the channel gains. The process $Z_i(t)$ is for mobile i and depends only on its location; we assume that the process $Z_i(t), 1 \leq i \leq N$, are independent, have identical law (under homogeneous scatterers assumption), and are stationary and ergodic. We allow the constraint in Eqn.(1) to be violated with a very small probability termed as the probability of outage P_{out}. Thus the QoS constraint is satisfied if $P(\sum_{i=1}^{N} \Gamma_i Z_i > 1) \leq P_{out}$.

Let $N = \sum_{j=1}^{C} n_j$. Applying the Chernoff bound, for all $\theta \geq 0$, the L.H.S. of the QoS constraint is,

$$\begin{aligned} P\left(\sum_{i=1}^{N} \Gamma_i Z_i \geq 1\right) &\leq e^{-\theta} \; E\left(e^{\theta \sum_{i=1}^{N} \Gamma_i Z_i}\right) \\ &\leq e^{-\theta} \; \prod_{j=1}^{C} \left(E\left(e^{\theta \; \Gamma_j \; Z_j}\right)\right)^{n_j} \\ &\leq e^{-\sup_{\theta \geq 0}\left(\theta - \sum_{j=1}^{C} n_j \; \chi_j(\theta)\right)} \end{aligned}$$

where $\chi_j(\theta) = \ln \; E\left(e^{\theta \; \Gamma_j \; Z_j}\right)$ is the effective capacity for calls of class j. Let $P_{out} = e^{-\delta}$. The admission region is then given by,

$$A = \left\{ \underline{n} : \sup_{\theta \geq 0}\left(\theta - \sum_{j=1}^{C} n_j \; \chi_j(\theta)\right) \geq \delta \right\}.$$

We approximate the boundary of the admission region via a hyperplane as in [3]. If θ^* gives the supremum for $n = n^*$, a point on the boundary, then we take $\underline{n}$ as feasible if,

$$\sum_{j=1}^{C} n_j \; \chi_j(\theta^*) \leq \theta^* - \delta. \tag{2}$$

Thus in order to compute $\chi_j(\theta^*)$, we need to evaluate the distribution of random variable Z_j.

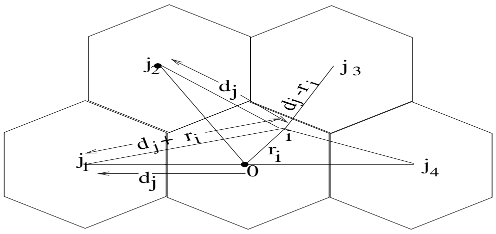

Fig. 1. Shows the approximate distance of the MS i in the reference cell from the other cell base stations.

III. Distribution of Z_i

Define $\nu_j \in \{1, -1, 0\}$. The distance from the j^{th} BS to the i^{th} mobile in the 0^{th} cell, r_{ji} is approximately equal to $d_j + \nu_j r_{0i}$ (Refer Fig 1). The random variable Z_i can be written as $Z_i = \sum_{j=1}^{\infty} 10^{(\xi_{0i} - \xi_{ji})/10}\ 10^{-\mu_{ji}/10}$ where $\mu_{ji} = 40\ \log_{10}\left(\frac{d_j + \nu_j r_i}{r_i}\right)$. Applying the Fenton-Wilkinson method followed by some algebra we show that given the distance r_i of the mobile from its BS, and that the mobile i is served only by the reference BS, the random variable Z_i has a distribution of the form e^{Ω_i} where Ω_i is normal $(m_i, v_i + \beta^2\sigma_0^2/2)$ where,

$$v_i = \ln\left((e^{\beta^2\sigma_0^2/2} - 1)\ \frac{\sum_{j=1}^{\infty}(e^{-2\ \beta\mu_{ji}})}{(\sum_{j=1}^{\infty}(e^{-\ \beta\mu_{ji}}))^2}\ + 1\right),$$
$$m_i = -\left[\frac{\beta^2\sigma_0^2 - 2v_i}{4} + \ln(\textstyle\sum_{j=1}^{\infty}(e^{-\ \beta\mu_{ji}}))\right],$$

and $\beta = \frac{\ln(10)}{10}$. Since r_i is a random variable we need to know its distribution in order to evaluate $\chi_i(\theta)$. Recall that the soft handoff has the effect of reducing the power requirement from each BS for a MS in the soft handoff region. We will capture this effect of soft handoff within our model by determining a distribution of r_i which keeps the MS within a smaller effective cell and thus reducing the power from the BS and increasing the capacity.

IV. Distribution of r_i

The random variable r_i is the distance of an MS from its home cell BS. The simplest case is the one with hard handoff. Hard handoff takes place at the cell boundary. We neglect the small hysteresis used in networks with hard handoff scheme to avoid the ping-pong effect. As d is the distance between the two adjacent cell base stations, the maximum value that the random variable r_i can take is $d/2$. We assume that the MS moves freely in any direction within the cell and r_i is distributed uniformly over $[0, d/2]$. Vijayan et al [9] model hard handoff via a level crossing analysis. We intend to extend the approach for the soft handoff case.

During soft hand off, the connection to the BS which is expected to control the mobile in the future is established before the connection with the current BS is broken. A transition to the soft hand off region is said to take place when the pilot signal power from the parent BS falls below the pilot power from any of the neighboring BS by some threshold. There will be some region around the cell boundary where the mobile is in soft hand off state. In this region, there are two or more BS's transmitting the same signal to the mobile. Since these signals are coherently combined, the power required from any one of them is significantly less than in the case when only one is transmitting. But now power has to be allocated for the MS even when it is beyond the cell boundary. Thus the distribution of r_i depends upon the handoff threshold chosen which in turn will affect the capacity region.We need to determine the PDF of r_i for various handoff mechanisms.

We formulate the hand off problem for the case of two adjacent BSs, A and B separated by distance d units, with an MS moving from A to B along the line joining A and B at a constant speed. The mobile measures the pilot power from the two BSs. When the MS is at a distance r from BS A, the measured pilot signal levels $a(r)$ and $b(r)$ received from the BS A and B respectively are proportional to $(r)^{-4}\ 10^{-\xi_a(r)/10}$ and $(d-r)^{-4} 10^{-\xi_b(r)/10}$ respectively. The random processes $\xi_a(r)$ and $\xi_b(r)$ are zero mean stationary normal processes independent of each other and the autocorrelation function is (Gudmundson [5]), $E(\xi(r_1)\xi(r_2)) = \frac{\sigma_0^2}{2}\ e^{\frac{-|r_1 - r_2|}{d_0}}$, where d_0 determines how fast the correlation decays with distance r. The process $X(r) = \frac{\xi_b(r) - \xi_a(r)}{\sigma_0}$ is a stationary zero mean Gaussian process with covariance function $e^{\frac{-|\Delta r|}{d_0}}$. According to the hand over algorithm, the mobile is said to enter the soft hand off at a distance R from BS A, if the ratio of the two signal strengths at that point goes below some threshold T_h for the first time i.e, $R = min\{r : \frac{a(r)}{b(r)} \leq T_h, r \geq 0\}$. Taking logarithms and dividing throughout by σ_0, we get $R = min\{r : X(r) \leq y(r), r \geq 0\}$, where $y(r)$ is given by, $y(r) = \frac{10\log(T_h)}{\sigma_0} + \frac{40}{\sigma_0}\log\left(\frac{r}{d-r}\right)$. Thus as the MS moves from A to B, R is the location at which the random process $X(r)$ first crosses the curve $y(r)$ from above. Since R is a random variable, we evaluate the PDF of R, the distance from the BS A where the process $X(r)$ crosses the curve $y(r)$ from above for the first time. Assume that R is proper and that the process does not cross the curve before a distance of at least $d/4$ is covered from the BS A. This assumption is reasonable from the fact that the soft hand off region lies close to cell boundary. In the region of interest, starting from $d/4$ onwards, the curve $y(r)$ can be approximated by a line $s(r) = mr + c$ (See Figure 2) where, $m = \frac{d \times 10.625}{\sigma_0}$ and $c = \frac{-10.625 \times d + 10\log(T_h)}{\sigma_0}$.

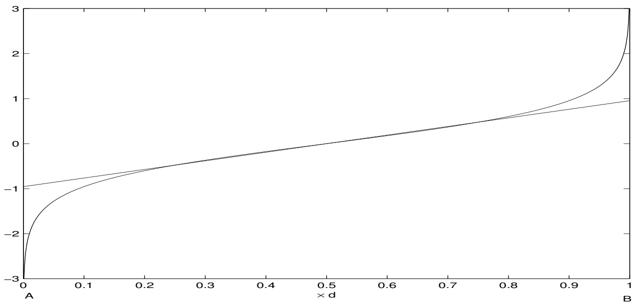

Fig. 2. The curve $y(r)$ and the $s(r)$.

We now attempt to solve the general problem of the process $x(r)$ crossing the line $s(r)$ for the first time. The argument r is the distance moved by the mobile starting from $d/4$ onwards from the BS A. The process $x(r)$ is an **Ornstein-Uhlenbeck process** [1] with the transition density say $f(x(r_1), r_1 \mid x(r_0), r_0)$. Denote by $\mathbf{g}(\mathbf{s}(\mathbf{r}), \mathbf{r} \mid \mathbf{x}(\mathbf{0}), \mathbf{0})\ \mathbf{dr}$ the first passage time probability that $x(r)$ passes a linear boundary $s(r)$ from above for the first time at a distance $(r, r + dr)$ from the BS given that the process takes a value $x(0) > s(0)$ at $r = 0$. If there exists an R such that $x(R) \leq s(R)$ with probability one, , then by "sorting" the process out as to when it first crossed the boundary, one obtains a renewal equation, for $r > 0$,

$$f(y, R \mid x, 0) = \int_0^R dr\ g(s(r), r \mid x, 0)\ f(y, R \mid s(r), r)$$

The above equation is an integral equation for $g(\cdot \mid \cdot)$.

Unfortunately, the evaluation of the first passage PDF through a varying boundary is in general a formidable task so that the available analytical results are scarce and only in few cases are the first passage length PDF known in closed form [8], [1]. The renewal equation has not yet been solved except for a constant boundary and that too only for a level zero. The evaluation of the above equation numerically is time consuming and requires the use of a large computational facility. In Ricciardi et al [7], the first passage density through a time varying boundary for a one dimensional diffusion process has been proved to satisfy a new Volterra integral equation of the second kind involving two arbitrary continuous functions. There exist efficient numerical procedures for solving such integral equations. Applying this procedure we can obtain $g(s(r), r)$ (See [4] for details).

We examine the effect of various thresholds on the depth of the soft handoff region. Define $\tau_h = \frac{10\log(T_h)}{\sigma_0}$ and $x = \frac{4r-d}{d}$. The distribution of the random variable R for various values of τ_h is as shown in the Figure 3. As

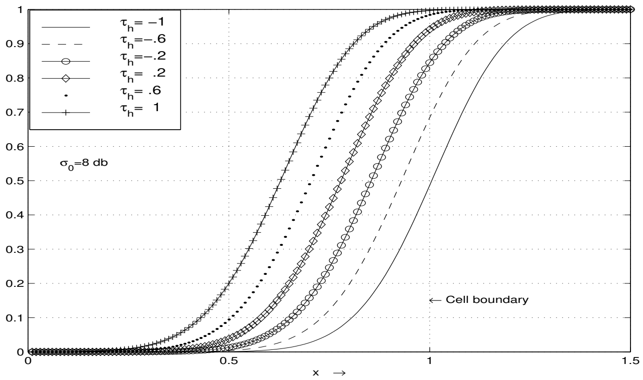

Fig. 3. Plots the probability distribution function of the distance where the ratio of the two pilot signal power crosses a threshold from above for the first time.

T_h decreases, the probability with which a handoff is initiated at a point along the line joining two BS decreases. It means that handoff initiation is delayed as expected.

Given a threshold T_h, let R_1 be the distance from the reference BS A where the MS enters the soft handoff and R_2 be the distance where it departs from this region and moves to the other cell (See fig. 4). Since R is random, we need a probabilistic condition to define R_1 and R_2. Define $R_1 = min\{r :\ P(R(T_h) < r) > P_h\}$ where P_h is a given probability. Thus, for example, for $\tau_h = 0.2$, $P_h = 0.5$, $R_1 = 0.9 \times \frac{d}{2}$ (See Figure 3). By symmetry, $R_2 = 1.1 \times \frac{d}{2}$ for same values of τ_h and P_h. If the mobile combines the two signals coherently then the power required from each BS is roughly half the power required without diversity combining. In coherent detection, the mobile at a distance $z \in [R_1, R_2 + R_1 - v]$ is same as it being at a distance $u + z - R_1$ since the power required at point z is same as that at $u + z - R_1$ (See figure 4(a)). The distribution of r_i for the two cases is shown in Figure 5. We can now evaluate $\chi_i(\theta)$ and hence the admission region.

V. Example

We use the above analysis to obtain the admission region from Eqn. 2. Let there be two classes of calls. The target minimum bit energy to noise power spectral

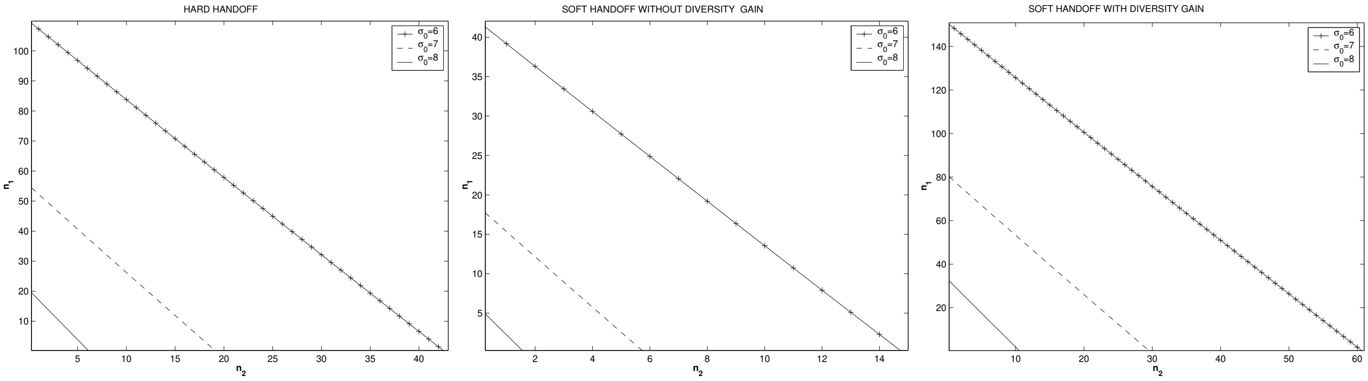

Fig. 6. Admission region for hard handoff, soft handoff without combining and soft handoff with combining.

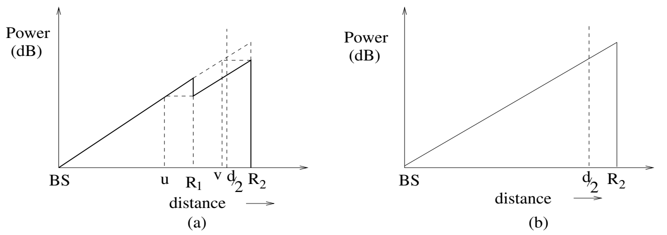

Fig. 4. Shows transmitter power profile required for a BS-MS pair as MS moves along the line joining BS A-B, (a) with diversity reception and (b) without diversity reception.

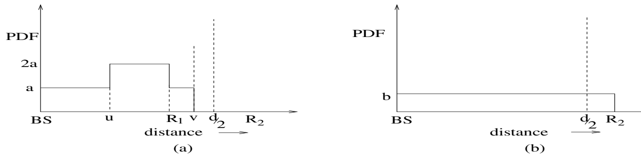

Fig. 5. Shows probability density function of r_i for the two cases .

density ratio $\eta = 5\ dB$ and the total bandwidth available $W = 5\ MHz$. The bit rate requirement $\rho_1 = 10KHz, \rho_2 = 20KHz$. Thus $\Gamma_1 = .0063$ and $\Gamma_2 = .0125$. Let the value of $\sigma_0 \in \{6, 7, 8\}$dB. Let n_1 and n_2 be the number of class 1 and 2 calls active respectively. Let the target probability of outage is $P_{out} = 10^{-3}$. Then (n_1, n_2) is feasible if it lies within the admission region. The admission region for the following cases are shown in figure 6.

Case 1: Hard handoff: The BS provides the power to only those MSs that are with in its cell boundary.

Case 2: Soft handoff without Diversity Gain: Let the threshold values $T_h = .2$ and $P_h = .5$. Then as in Section IV, R_1 and R_2 are $0.9 \times \frac{d}{2}$ and $1.1 \times \frac{d}{2}$ respectively. The soft handoff region is symmetric around the cell boundary.

Case 3: Soft handoff with Diversity Gain: With same values of R_1 and R_2 but now a BS has to provide only half the power to a MS in the soft handoff region.

VI. Conclusion

The admission region is the smallest for the case when MS does not combine the signals from two BS coherently (without diversity gain) and the largest for coherent detection. The results are quite intuitive because the interference is least in the case when MS utilizes diversity gain. Although it may become higher than in hard handoff case when soft handoff region gets wider. The main contribution of this work was in the development of an analysis for the soft handoff region.

References

[1] I.F.Blake and W.C.Lindsey,"Level-crossing problems for random processes," *IEEE Trans. on Info. Theory*, IT-19(3):295–315, May 1973.

[2] De Hoz, A.; Cordier, C.,"W-CDMA downlink performance analysis,"*IEEE VTC 1999*, Pg 968 -972, 1999

[3] J.S.Evans and D.Everitt,"Effective bandwidth-based admission control for multiservice CDMA cellular networks,"*IEEE Trans. Veh. Tech.*, 48(1):36–46, Jan 1999.

[4] Munish Goyal,"Radio Resource Management in Multimedia CDMA Cellular Systems," ME Thesis, IISc, Bangalore,2000.

[5] M.Gudmundson,"Correlation model for shadow fading in mobile radio systems,"*Electron. Lett.*, 27:2145–2146, Nov 1991.

[6] Keunyong Kim and Youngnam Han," A call admission algorithm with optimal power allocation for multiple class in CDMA system," *IEEE VTC*, Vol 6, 2666-2671, 2000.

[7] A.Buonocore, A.G.Noble, and L.M.Ricciardi," A new integral equation for the evaluation of first-passage-time probability densities,"*Adv. Appl. Prob.*, 19:784–800, 1987.

[8] Arnold J.F. Siegert," On the first passage time density problem," *Physics Review*, 81(4):617–623, Feb. 1951.

[9] R.Vijayan and J.Holtzman,"A model for analyzing handoff algorithms,"*IEEE Trans. Veh. Tech.*, 42(3):351–356, Aug. 1993.

[10] A.J.Viterbi,"CDMA-Principles of Spread Spectrum Communication,"Addison-Wesley, 1995.