Performance Analysis of the Bluetooth Physical Layer

Aditya Karnik and Anurag Kumar
Dept. of Electrical Communication Engg.
Indian Institute of Science, Bangalore, 560012, INDIA
email: {karnik, anurag}@ece.iisc.ernet.in

Abstract—**Bluetooth is an emerging, short range, indoor wireless network technology, based on FH-CDMA/TDD, which allows a collection of devices to form small overlapping networks, called piconets, in an ad hoc fashion. It is likely that the Bluetooth specification may be adopted in the IEEE 802.15 Wireless Personal Area Network standard. In this paper, we present an analysis of the Bluetooth physical layer. We consider a single room office environment (LOS propagation), with a number of piconets distributed in it. In the present work we have focussed only on the interference between piconets. Assuming a uniform distribution of Bluetooth devices in the room, we provide an analysis of the outage probability as a function of the number of overlapping piconets. We then characterise the temporal correlation in the outage process. This leads to an approximation for the mean outage duration. Combining the outage probability with the mean outage duration, we provide a 2-state Markov model for the Bluetooth channel.**

I. Introduction

Bluetooth is an emerging indoor wireless network technology which aims at replacing many proprietary cables that connect one device to another with one universal short-range radio link. Various devices such as laptops, cell-phones, PDAs and desktops can be part of a Bluetooth system. Beyond untethering devices, Bluetooth also provides a mechanism to form small, ad-hoc networks of devices called piconets. Because of the ad-hoc nature of their formation, piconets are arbitrarily distributed in space and in fact form a system of overlapping networks. Moreover, a piconet is an autonomous network with a central controller and operates using FH-CDMA. One research objective is to characterise the behaviour of the wireless channel associated with piconets in order to understand the performance of applications over Bluetooth.

In this paper we study the performance of the hopping channel of a piconet as the number of piconets increases in a given area. We consider a typical scenario for Bluetooth, namely an office environment with piconets randomly distributed in it. The performance measure we use is the outage probability with respect to a target signal-to-noise ratio at the receiver. We find that the outage probability is of the order of 10^{-2} and expect it to increase linearly with the number of piconets. We then characterise the temporal correlation in the outage process. This leads to an approximation for the mean outage duration. Combining the outage probability with the outage duration, we provide a 2-state Markov model for the Bluetooth channel. Markov models for wireless channels ([1], [2]) are of interest because of their analytical simplicity. Such models would be useful, for example, in the performance analysis of TCP controlled applications over Bluetooth (see [3]).

The paper is organised as follows. In Section II, we briefly describe the Bluetooth physical layer. In Section III, indoor propagation models are discussed. In Section IV, we analyse the statistics of the outage process and provide a 2-state Markov channel model based on it. Results are provided in Section V, and we conclude in Section VI.

To appear in IEEE International Conference on Personal Wireless Communications (ICPWC 2000), December 2000, Hyderabad, India. A part of this work was done during this author's summer internship in IBM, India Research Lab, New Delhi.

II. Bluetooth Physical Layer

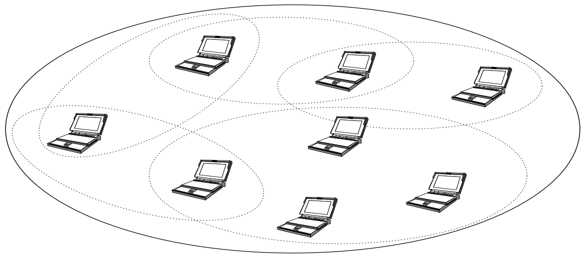

Fig. 1. In a Bluetooth network piconets overlap to form a scatternet.

Bluetooth ([4]) is a short range ($<$ 10m) radio technology which operates in the 2.4GHz ISM band. Since the ISM band is free for use, there are many sources of interference, e.g., microwave ovens, WLAN etc. In addition, Bluetooth uses low power; in the range of 1mW-100mW. To avoid interference, the physical layer is based on FH-CDMA. Bluetooth uses 79 carrier frequencies and as per the regulations, the signal bandwidth is limited to 1MHz. Nominal bit rate is 1Mbps. Sensitivity level (the input level for which the raw BER specification is met) and co-channel interference resistance are specified as -70dBm and 11-14dB respectively for BER < 0.001.

Figure 1 shows a network of overlapping Bluetooth piconets. By definition, the unit that establishes and coordinates a piconet is called the *master*. All other participants in the piconet are called *slaves*. There can be a maximum of 7 slaves in a piconet. A slow frequency hopping channel is associated with every piconet; each piconet has a unique hopping sequence determined by the master. Time is divided into slots of 625 μs. The timing is maintained by the master and slaves synchronise to the clock of the master.

Higher layer packets are fragmented into Bluetooth physical layer packets. Each packet has 126 bits of control information and a maximum payload of 2745 bits. Packet transmission takes place on one hop frequency and can take multiple slot. Full duplex communication is achieved by implementing TDD. Access control is exercised by the master. Only communication between the master and a slave is possible. The master transmits in odd numbered slot and the corresponding slave responds in the next slot. $1/3$ or $2/3$ FEC can be used for data. In addition, 1-bit acknowledgment indication ARQN is implemented. If the data is received incorrectly, it is retransmitted at the next opportunity.

III. Modelling Indoor Wireless Propagation

Propagation of radio waves inside a building is a very complicated process, and it depends significantly on the indoor environment (e.g. office, factory), and topography (LOS, obstructed). The statistics of the indoor channel varies with time due to motion of people and equipment. A survey of indoor propagation measurement and models is provided in [5].

The indoor channel is characterised by high path losses and large variations in losses. The reported values of the path loss exponent η are 1.5-1.8 for LOS and 2.4-2.8 for obstructed factory channels. In a factory environment ([6]), the overall path loss is found to be log-normal with 7.1 dB standard deviation, whereas in an office environment ([7]), the signal envelope is found to be Rician distributed (K ratio of about 2 dB) around the mean determined by the propagation losses.

Motion within the building causes signal fading which compares well with the Rician distribution with K equal to 6.8 dB and 10 dB for office and factory environments respectively ([6], [7]). If the degree of motion is small (as in an office environment), the fading is extremely slow; the Doppler bandwidth is about 4 Hz. The channel is then said to be quasi-static. The fading statistics are, however, statistically non-stationary. but wide-sense stationary characteristics are exhibited for periods up to 2s. Since many digital symbols are transmitted during this period at practical data rates, this type of fading is referred to as quasi-wide-sense stationary.

In our analysis, since we consider LOS propagation in an open office environment, we take $\eta = 2$ and a Rician signal distribution with $K = 6$ dB.

IV. SIR Analysis of Overlapping Piconets

We consider a single circular room (radius R) office environment (LOS propagation), with a number of Bluetooth devices distributed randomly in it to form M piconets. Because of the ad-hoc nature of the formation of piconets, *devices in a piconet* are spatially randomly distributed in the room leading to overlap among the piconets. Note that, in our results M will be a parameter, i.e. we are interested in analysing the performance of the Bluetooth system as the number of piconets increases in a given area.

Since Bluetooth operates in the ISM 2.45 GHz band, there are many sources of interference other than Bluetooth devices, e.g. microwave ovens and IEEE 802.11 Wavelans. We, however, focus on the interference generated within the Bluetooth system only [1]. A piconet experiences interference when a frequency hit occurs, i.e., the transmission frequency of a packet in a piconet matches with that being used in one or more other piconets for some overlapping duration. Figure 2 shows example hop sequences for 4 piconets. The *first row* shows the frequency hit regions of the *reference piconet*. T denotes the hop time. Note that, the piconets are not synchronised in time. We assume that the time offset of the i^{th} piconet $(2 \le i \le M-1)$, denoted by t_i, with respect to the reference piconet are uniformly distributed in $[0, T]$.

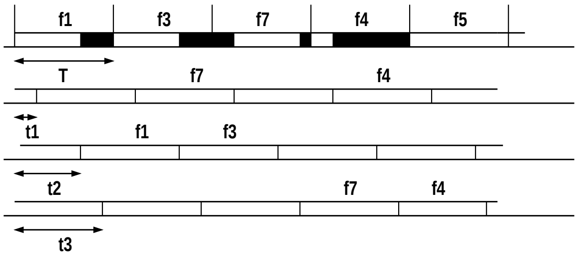

Fig. 2. Frequency hopping system. Black boxes indicate overlap of hop frequencies.

Since the devices in the piconets are spatially distributed, *the interfering piconet* is not a fixed location or a device but a random device in that piconet transmitting at a given time. We assume a physically stationary environment (little or infrequent movement) so that the channel is quasi-static and quasi-wide-sense stationary. Hence, for the *duration of a frequency hit* on the single frequency, constant interference power can be assumed to be received from the individual interferers.

We consider a reference piconet and denote by $\{\Gamma(t), t \ge 0\}$, the SIR process.

$$\Gamma(t) = \frac{P_S(t)}{\sum_{j=1}^{N(t)} P_I^j(t) + \frac{N_0 W}{2}}$$

where, at time t, $P_S(t)$ is the desired signal power, $P_I^j(t)$ is the interference power due to the j^{th} interfering piconet and $N(t)$ is the number of interferers. Recall that, in Bluetooth a device can transmit in alternate slots. In addition, a slave can transmit only when polled. Hence, the receiver is not a fixed device and the SIR process we characterise is of the reference piconet as a whole.

A. Outage Probability

Outage occurs when the bit SIR falls below the resistance ratio (denoted by ν) specified by the standard. We denote by

[1] IEEE802.11 uses more power than Bluetooth but is a slower hop system (50 hops/s vs 1600 hops/s in Bluetooth). It remains to be investigated which system will affect the other more. Also microwave ovens collocated with the work space in an office is an unlikely scenario.

Γ_b the bit SIR and seek the stationary outage probability P_{out}.

$$P_{out} := P(\frac{P_S}{\sum_{j=1}^{N_b} P_I^j + \frac{N_0 W}{2}} < \nu) \quad (1)$$

P_S is the desired signal power and P_I^j denotes the power received from the j^{th} interferer. N_b denotes the number of interferers in a bit duration. Let $P_{I,n} := \sum_{j=1}^{n} P_I^j$. A Bluetooth receiver has -70dBm sensitivity level to meet BER < 0.001. With 1 Mbps bit rate, the noise power is approximately 2.5×10^{-6}mW. Hence ignoring the thermal noise,

$$\begin{aligned} P_{out} &= P(P_S < \nu \sum_{j=1}^{N_b} P_I^j) \\ &= \sum_{n=0}^{M-1} P(P_S < \nu P_{I,n} | N_b = n) P(N_b = n) \end{aligned}$$

Recall that in the indoor environment under consideration, the mean received power is given by the inverse square law and the signal fading is Rician. Thus, given the distance of the reference transmitter $D_s = d_s$ and j^{th} interferer $D_j = d_j$ from the reference receiver, P_S and P_I^j are non-central chi-square distributed with means $(d_s/d_0)^{-2}$, $(d_j/d_0)^{-2}$ respectively. In the indoor channels $d_0 = 1$m ([8]). It then follows that $P_{I,n}$ is non-central chi-square distributed with $2n$ degrees of freedom.

The reference receiver is located randomly in the room. Let D_r denote the distance of the receiver *from the center of the room* and let $F_{D_r}(.)$ be its cdf. Denote by $\underline{D} := (D_S, D_1, D_2, \ldots, D_n)$ the vector of distances of the $n+1$ transmitters (one reference transmitter and n interferers) from the receiver. Let $F_{\underline{D}}(.|D_r)$ denote the conditional distribution of $\underline{D}$ given the location of the receiver. Then,

$$\begin{aligned} & P(P_S < \nu P_{I,n} | N_b = n) = \\ & \quad \int_0^R P(P_S < \nu P_{I,n} | D_r = \tilde{r}, N_b = n) dF_{D_r}(\tilde{r}) \\ & P(P_S < \nu P_{I,n} | D_r = \tilde{r}, N_b = n) = \qquad (2) \\ & \quad \int_A P(P_S < \nu P_{I,n} | \underline{D} = \underline{d}, D_r = \tilde{r}, N_b = n) dF_{\underline{D}}(\underline{d}|\tilde{r}) \\ & P(P_S < \nu P_{I,n} | \underline{D} = \underline{d}, D_r = \tilde{r}, N_b = n) = \\ & \quad \int_0^\infty F_{P_S}(\nu x | d_s) dF_{P_{I,n}}(x | d_1, \ldots, d_n) \end{aligned}$$

where, $A = \{\underline{d} : 0 < d_s, d_i \le R + \tilde{r}, 1 \le i \le n\}$. Note that, $F_{P_S}(.|d_s)$ and $F_{P_{I,n}}(.|d_1, \ldots, d_n)$ are the non-central chi-square distributions of P_S and $P_{I,n}$ respectively as discussed above.

B. Distribution of N_b

We model the frequency hopping pattern as a discrete time Markov chain; a state denotes a frequency. Hopping patterns of piconets are independent and identically distributed. We consider $\{X_k, k \ge 0\}$ with one-step transition probabilities given as below. This represents the ideal random hopping pattern.

$$P(X_{k+1} = f_j | X_k = f_k) = \begin{cases} \frac{1}{N_f - 1} & f_j \ne f_k \\ 0 & f_j = f_k \end{cases}$$

N_f is the total number of frequencies available for transmission. Denote by τ the bit duration and by K the number of bits in a slot(hop); thus $T = K\tau$. It can be shown that the probability of piconet j hitting a bit in the reference piconet is

$$P_{hit} = \frac{1}{N_f}[1 + \frac{1}{K}]$$

Independence among the piconets implies that

$$P(N_b = n) = \begin{pmatrix} M-1 \\ n \end{pmatrix} (P_{hit})^n (1 - P_{hit})^{M-1-n} \quad (3)$$

C. 2-State Markov Channel Model

In order to characterise the temporal correlation in the outage process, consider a bit b and $N_b = n$. If the next bit $b+1$ starts in the same hop frequency as b , then N_{b+1} consists of the interferers out of n that remain in the same hop in $b+1$, denoted by V, and the new interferers out of $M-1-n$ piconets that were using a different hop in b, denoted by Z. However, if $b+1$ starts with a new frequency (b is the last bit in the slot) then N_{b+1} equals the interferers from potential $M-1-V$ piconets. Let this be denoted by W. Let u indicate the hop change, i.e., $u = 0$ if there is a hop change and 1 otherwise. Then,

$$N_{b+1} = u(V + Z) + (1 - u)W \quad (4)$$

$V := \sum_{j=1}^{N_b} v_j$, $Z := \sum_{j=1}^{M-1-N_b} z_j$, $W := \sum_{j=1}^{M-1-V} w_j$ where ($B(p)$ denotes the Bernoulli distribution with probability of 1 being p), $u \sim B(1 - 1/K)$, $v_j \sim B(1 - 1/K)$, $z_j \sim B(1/K \times 1/N_f)$ and $w_j \sim B(P_{hit})$. Recall that, K is the number of bits per slot.

Using Equation 4, we obtain the one step joint distribution and then approximate the outage process by a Markov process. This gives us a 2-state Markov model as follows. We say that the channel is in *good* state, denoted by 'g', if $\Gamma_b \ge \nu$ or BER on the channel is below the specified value and is in *bad* state, denoted by 'b', when $\Gamma_b < \nu$, i.e., when the outage occurs. The transition probabilities have been derived in Appendix A. Let P_{ij} denote the transition probability of going from state i to state j. Then, the outage duration (in number of bits) is geometrically distributed with mean $1/(1 - P_{bb})$. Also the outage probability calculated from the Markov model, denoted by P_{out}^M, is given by $P_{gb}/(P_{gb} + P_{bg})$.

V. Results

We take $R = 5$m and assume that the devices are distributed uniformly in the room *with respect to the center of* *the room.* Then if the receiver is at the center, the cdf of D, the distance of a random device from the receiver is given by $F_D(d) = d^2/R^2, 0 \leq d \leq R$. Given $D_r = \tilde{r}$, the interferers are assumed to be independently and identically distributed in the room. Let $I_1(x) := \frac{1}{\pi} \arccos\left(\frac{(\tilde{r}-d+x)^2+\tilde{r}^2-d^2}{2\tilde{r}(\tilde{r}-d+x)}\right)$ and $I_2(x) := \frac{1}{\pi} \arccos\left(\frac{(d-\tilde{r}+x)^2+\tilde{r}^2-d^2}{2\tilde{r}(d-\tilde{r}+x)}\right)$. Then, it can be shown that if $\tilde{r} > R - \tilde{r}$,

$$F_D(d|\tilde{r}) = \begin{cases} \int_0^{2d} I_1(x) \frac{2(\tilde{r}-d+x)}{R^2} dx & 0 \leq d \leq R - \tilde{r} \\ \int_0^{R-\tilde{r}+d} I_1(x) \frac{2(\tilde{r}-d+x)}{R^2} dx & R - \tilde{r} < d \leq \tilde{r} \\ \frac{(d-\tilde{r})^2}{R^2} + \int_0^{R+\tilde{r}-d} I_2(x) \frac{2(d-\tilde{r}+x)}{R^2} dx & \tilde{r} < d \leq R + \tilde{r} \end{cases}$$

Similar expressions can be found if $\tilde{r} \leq R - \tilde{r}$. Figure 3 shows the conditional distance distribution $F_D(d|\tilde{r})$ for $\tilde{r} = 0, 1, 2, 3, 4, 5$m.

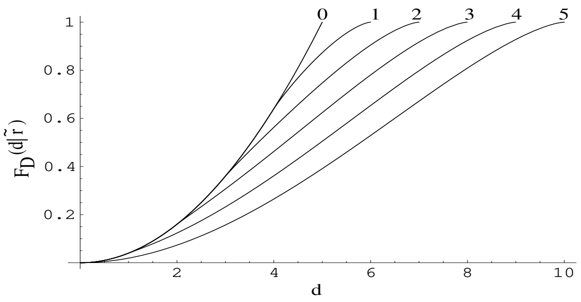

Fig. 3. Distance cdfs for receiver offset $\tilde{r} = (0, 1, 2, 3, 4, 5)$m

A. Outage Probability

Since the calculation of exact outage probability is complicated, even numerically, we resort to finding bounds. Define $g(d_s, d_1, \ldots, d_n, \tilde{r}, n) := P(P_S < \nu P_{I,n} | \underline{d}, \tilde{r}, n)$. Then, Equation 2 can be written as,

$$P(P_S < \nu P_{I,n} | D_r = \tilde{r}, N_b = n) = E_{d_s|\tilde{r}} E_{d_n|\tilde{r}} \ldots E_{d_1|\tilde{r}} [g(d_s, d_1, \ldots, d_n, \tilde{r}, n)] \quad (5)$$

Note that $g(.)$ is a decreasing function of $\{d_i, 1 \leq i \leq n\}$ and an increasing function of d_s. Also, if $\tilde{r_1} \leq \tilde{r_2}$ (see Figure 3),

$$P(D_i > t | \tilde{r_1}) \leq P(D_i > t | \tilde{r_2}) \quad (6)$$

With this stochastic dominance, it follows that, for $1 \leq i \leq n$

$$E_{d_i|\tilde{r_1}}[g(\ldots, d_i, \ldots)] \geq E_{d_i|\tilde{r_2}}[g(\ldots, d_i, \ldots)] \quad \text{if } \tilde{r_1} \leq \tilde{r_2} \quad (7)$$

$$E_{d_s|\tilde{r_1}}[g(d_s, \ldots)] \leq E_{d_s|\tilde{r_2}}[g(d_s, \ldots)] \quad \text{if } \tilde{r_1} \leq \tilde{r_2} \quad (8)$$

Using Equation 7 and 8 in Equation 5, a lower bound on P_{out} can be found.

As a simple calculation, assume that there are only two piconets and there is no fading on the channel. Then $P_S = (\frac{D_S}{d_0})^{-2}$ and $P_I = (\frac{D_I}{d_0})^{-2}$. Then,

$$P(P_S < \nu P_I | N_b = 1) = P(D_I^2 < \nu D_S^2 | N_b = 1)$$
$$= \int_0^R P(D_I < \nu^{1/2} D_S | D_r = \tilde{r}, N_b = 1) f_{D_r}(\tilde{r}) d\tilde{r}$$

$$P(D_I < \nu^{1/2} D_S | D_r = \tilde{r}, N_b = 1) \quad (9)$$
$$= \int_0^{R+\tilde{r}} P(D_I < \nu^{1/2} d_s | D_S = d_s, \tilde{r}, 1) dF_{D_S}(d_s|\tilde{r})$$
$$= E_{D_S|\tilde{r}}[F_{D_I}(\nu^{1/2} d_s | d_s, \tilde{r})]$$
$$\geq E_{D_S|\tilde{r}=0}[F_{D_I}(\nu^{1/2} d_s | d_s, \tilde{r})] \quad (10)$$
$$\geq E_{D_S|\tilde{r}=0}[F_{D_I}(\nu^{1/2} d_s | d_s, \tilde{r} = 5)] \quad (11)$$

Note that, Equation 10 follows from Equation 8 and Equation 11 follows from Equation 6. Approximating $F_D(d|\tilde{r} = 5)$ by a 4^{th} degree polynomial and taking $\nu = 25(14\text{dB})$, we get $P(P_S < \nu P_I | N_b = 1) \geq 0.939992$ and hence with $P(N_b = 1) = 1/N_f$, $P_{out} \geq 0.01189$. Table I shows lower bounds on P_{out} with signal fading.

TABLE I
LOWER BOUNDS ON OUTAGE PROBABILITY FOR $M = 2, 3$ PICONETS

M-1	Lower bound on P_{out}	
	14dB	11dB
1	0.0114	0.0103
2	0.0227	0.0204

B. Outage Duration

TABLE II
PARAMETERS OF THE 2-STATE MARKOV MODEL

M-1	P_{gb}	P_{bg}	P_{out}^M	EB	EG
1	0.00005	0.005	0.010	200	20000
2	0.000055	0.0035	0.016	285	18018

Using the derivation of transition probabilities in Appendix A, and lower bounds on the outage probability derived in Section V-A we calculate approximate values of the P_{gb} and P_{bg} for ν =14dB and 400 bits packet size. We also calculate the outage probability from this Markov model (P_{out}^M), Results are shown in Table II. Table II also shows in bits the mean outage duration (EB) and the mean duration in good state (EG).

C. Discussion

It is very complicated to calculate even the approximate values of P_{out} as the number of piconets increases. We, therefore, have results only for $M \leq 3$ piconets. However, the insight gained with these results allows us to predict the performance for larger values of M. The results indicate that when a frequency hit occurs it is difficult to maintain the resistance ratio of 14dB as specified by the standard. Hence, co-channel interference is avoided mainly through the use of large number of frequencies. For larger values of M, we expect that P_{out} will increase almost linearly, in the form of $\frac{(M-1)}{N_f}$. Though the Markov model is only an approximation (as seen from Table II), we expect that when number of piconets is small outages are infrequent (once in 20000 bits) This is because piconets are not time-synchronised and frequency hits occur with probability $\frac{1}{N_f}$. However, outages persist for approximately 200 bits which is considerable duration considering $1/3$ and $2/3$ FEC used in Bluetooth. Also in certain packet types FEC is not mandatory. As M increases, duration of good state will reduce at the cost of longer outage durations.

VI. Conclusions

We considered a typical scenario for Bluetooth piconets and found the outage probability and the mean outage duration of the SIR process. This led us to a 2-state Markov model for the Bluetooth channel. The outage probability appears to increase almost linearly with the number of piconets. Longer outage durations combined with simple FEC schemes imply that to the applications on Bluetooth, the channel will degrade rapidly as the number of piconets increases.

Appendix

I. Derivation of transition probabilities for the outage process

We derive $P_{bb} := P(\Gamma_{b+1} < \nu | \Gamma_b < \nu)$. Derivation of P_{gg} follows similarly.

$$P(\Gamma_{b+1} < \nu | \Gamma_b < \nu) = \frac{P(\Gamma_{b+1} < \nu, \Gamma_b < \nu)}{P(\Gamma_b < \nu)}$$

$$P(\Gamma_{b+1} < \nu, \Gamma_b < \nu) = P(\Gamma_b < \nu) - P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu)$$

In the following equations, when a random variable is *given*, it is understood that the value it takes is given by the corresponding small letter.

$$\begin{aligned}
&P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu) \\
&\quad = \sum_{n=0}^{M-1} P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | N_b) P(N_b = n) \\
&P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | N_b) \\
&\quad = P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | u = 1, N_b) P(u = 1) \\
&\quad + P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | u = 0, N_b) P(u = 0)
\end{aligned}$$

Note that u is independent of N_b. Let $P_{I,n} := \sum_{j=1}^{n} P_I^j$. From now on we denote $u = 1$ by u_1 and $u = 0$ by u_0.

• $u = 1 \Rightarrow$ no hop change. Recall that, the channel is assumed to be quasi-static and quasi-stationary. Then constant P_S is received for the hop time. Also, for the duration of a hit, constant interference power is received from the individual interferers.

$$\begin{aligned}
&P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | u_1, N_b) \\
&\quad = \sum_{z=1}^{M-1-n} \sum_{v=1}^{n} P(P_S < \nu P_{I,Z+V}, P_S \geq \nu P_{I,n}, Z = z, \\
&\quad V = v | u_1, N_b) \\
&P(P_S < \nu P_{I,Z+V}, P_S \geq \nu P_{I,n}, Z = z, V = v | u_1, N_b) \\
&\quad = P(P_S < \nu P_{I,z+v}, P_S \geq \nu P_{I,n} | Z, V, u_1, N_b) \\
&\quad P(Z = z, V = v | u_1, N_b) \\
&\quad = P(\nu P_{I,n} \leq P_S < \nu P_{I,z+v} | Z, V, u_1, N_b) \\
&\quad P(Z = z | N_b) P(V = v | N_b)
\end{aligned}$$

• $u = 0 \Rightarrow$ hop changes. Hence, the receiver changes. We indicate by P_S^{b+1} and P_S^b the signal powers received in bits $b+1$ and b respectively. Then,

$$\begin{aligned}
&P(\Gamma_{b+1} < \nu, \Gamma_b \geq \nu | u_0, N_b) \\
&\quad = \sum_{v=1}^{n} P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, V = v | u_0, N_b) \\
&P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, V = v | u_0, N_b) \\
&\quad = P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, | V, u_0, N_b) \\
&\quad P(V = v | N_b) \\
&P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, | V, u_0, N_b) \\
&\quad = \sum_{w=1}^{M-1-v} P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, \\
&\quad W = w | V, u_0, N_b) \\
&P(P_S^{b+1} < \nu P_{I,W}, P_S^b \geq \nu P_{I,n}, W = w | V, u_0, N_b) \\
&\quad = P(W = w | V) P(\nu P_{I,n} \leq P_S^b | W, V, u_0, N_b) \\
&\quad P(P_S^{b+1} < \nu P_{I,w} | W, V, u_0, N_b)
\end{aligned}$$

since the power distribution at different receivers is independent.

References

[1] H. S. Wang and N. Moayeri, "Finite-State Markov Channel- A Useful Model for Radio Communication Channels," *IEEE Transactions on Vehicular Technology*, vol. 44, no. 1, pp. 163–171, February 1995.

[2] M. Zorzi, R.R. Rao, and L.B. Milstein, "On the Accuracy of a First-order Markov Model for Data Transmission on Fading Channels," in *Proc. IEEE ICUPC'95*, November 1995, pp. 211–215.

[3] Anurag Kumar, "Comparative Performance Analysis of Versions of TCP in Local Network with a Lossy Link," *IEEE/ACM Transactions on Networking*, vol. 6, no. 4, August 1998.

[4] *The Bluetooth Specification*, available from http://www.bluetooth.com/.

[5] Homayoun Hashemi, "The Indoor Radio Propagation Channel," *Proceedings of The IEEE*, vol. 81, no. 7, pp. 943–968, July 1993.

[6] T. S. Rappaport and C. D. McGillem, "UHF Fading in Factories," *IEEE JSAC*, vol. 7, no. 1, pp. 40–48, January 1989.

[7] Robert J.C. Bultitude, "Measurement, Characterisation and Modelling of Indoor 800/900 MHz Radio Channels for Digital Communications," *IEEE Communications Magazine*, vol. 25, pp. 5–12, June 1987.

[8] Theodore Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 1996.