Performance Analysis of Wireless Ad-Hoc Networks*

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Abstract

With the growing importance of wireless ad hoc networks (particularly, in applications such as wireless local area networks, and ad hoc sensor networks) it is important to develop an understanding of their performance. Performance analysis of wireless ad hoc networks is a challenging task owing to the fact that such analysis must take into account the interactions between the wireless physical layer, radio propagation, multiple access, random topology, routing, and the characteristics of the application that generates the traffic carried by the network. In this paper we discuss the major issues in the performance of wireless ad hoc networks in the light of recent research in this area. Among the topics surveyed are capacity scaling results, stochastic capacity, Bluetooth performance, and the performance of TCP.

1 Introduction

For the purpose of performance analysis, one needs to abstract out the essential (performance governing) aspects of a system so as to build a mathematical model. Wireless ad hoc networks (henceforth abbreviated as WANETs) are communication networks in which even the network

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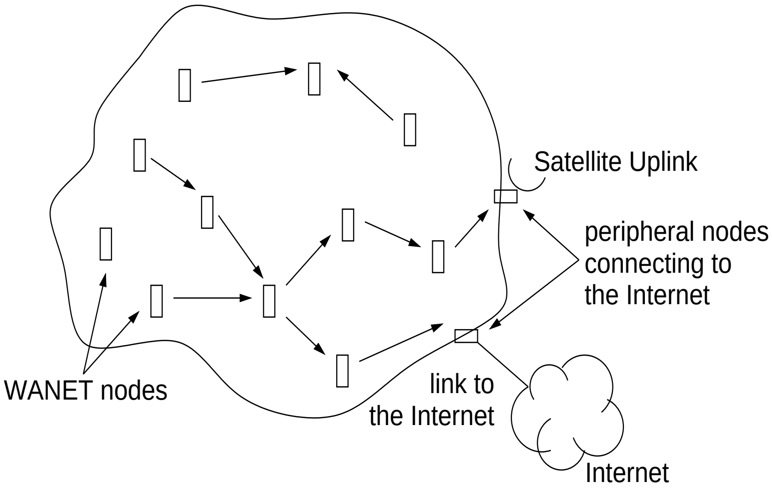

Figure 1: A schematic of a wireless ad hoc network, showing packet flow between the nodes, and attachments to the wide-area communication infrastructure via satellite or terrestrial links.

topology depends on the way the network is operated. Evidently, unlike wired and fixed topology networks, understanding and optimising the performance of WANETs is a difficult undertaking, owing to the complex interaction between the various "layers" of the network. Nevertheless, in order to talk about the issues systematically, we can begin by developing a layered view of a WANET, akin to the one utilised so effectively by the OSI (Open System Interconnection) model for communication network protocols. This will also set down our view of a WANET, for our subsequent discussions.

1. *Wireless physical communication:* At the physical level, a WANET comprises several nodes (e.g., hand-held, laptop or personal computers) each equipped with a digital radio unit (see Figure 1 for a schematic depiction). The antennas are (typically) omni-directional, and the radios use a portion of the spectrum that does not require complicated spectrum licensing, coordination and planning. Thus, for example, the popular IEEE 802.11b standard, and the emerging Bluetooth system, use the unlicensed 2.4GHz ISM band. To combat multi-path and interference, spread spectrum modulation is usually employed. Beginning with 1 Mbps or 2 Mbps, with modulation schemes such as OFDM and better coding techniques, data rates greater than 20 Mbps are now achievable. We note that the digital communication system employed, the transmit power used, and the radio propagation characteristics of the environment determine the "links" in the network, and hence the topology of a WANET

(see Section 2). Thus the most basic performance aspects of a WANET (which nodes are neighbours, which nodes have a path between them, and at what speeds neighbour nodes can communicate) are determined by the physical communication layer.

2. *Multiple access mechanism:* Since all the radios share the same band, and the antennas are omni-directional, clearly nodes cannot arbitrarily communicate with each other. Hence a WANET employs a multiple access mechanism which permits the nodes to coordinate their transmissions in a decentralised manner. A simple random access mechanism can be used, in which the nodes just attempt, and then resend if there is a collision. Such extreme lack of coordination obviously leads to poor performance. Such a simple mechanism, however, is amenable to analysis, at least to yield performance bounds (such an analysis is presented in Section 3.2). The IEEE 802.11 standard employs the CSMA/CA multiple access algorithm. A large class of multiple access schemes is based on the 'RTS-CTS' dialogue. A node wanting to transmit first requests the destination node (Request To Send - RTS) and transmits only when it receives a grant (Clear To Send - CTS) from it. This ensures that the destination node is in the receiving mode during transmission. Other nodes that hear this dialogue refrain from transmitting for some duration, thus reducing interference. The Bluetooth system, essentially designed for ad hoc wireless interconnection of devices (laptop, cordless phone, printer, computer) in an office environment, uses a polling based multiple access mechanism. In each, so called *piconet* there is a master device that polls the other devices; all data transfer takes place through the piconet master. We will present an analysis of the Bluetooth system in Section 4.

3. *Multihop packet communication:* Nodes in an ad hoc network are usually battery-powered hand-held devices. This constrains their transmission power. Even though a node may not be able to directly send a packet to its destination, it can forward that packet to a node to which it can, hoping that the latter node has a path to the final destination; hence we have a multihop packet radio network. It is not necessarily good for a node to send as far as it possibly can, as this may increase the interference it causes to other transmissions. For effective communication (low probability of error) and also to conserve battery power, nodes need to choose appropriate transmission ranges which determine their neighbouring nodes.

4. *Adaptive routing protocols:* A routing protocol, typically derived from one of the Internet routing protocols, permits nodes to forward packets in a way that if there is a path between a pair of nodes then packets can be sent between those nodes. Since such a routing protocol is adaptive, the nodes can actually move, and new routes are learnt as the topology changes. If nodes fail (owing to damage or loss of power) the network will automatically learn new paths between nodes that can still communicate. Routing protocols vary in their handling of topology updates. Some protocols are proactive, and constantly keep learning new routes and updating routing tables. Other protocols are "on-demand", and search for routes when such a request is made. We will not touch on this topic any further in this paper. A recent book ([1]) is mainly devoted to ad hoc network routing protocols. Several simulation studies comparing various ad hoc routing protocols are available (see [2] and [3]).

5. *Ad hoc network applications:* We can broadly classify WANETS into wireless ad hoc internets, and special purpose ad hoc networks. Nodes in an ad hoc internet would like to run the same applications as any other node attached to the Internet. Hence the TCP/IP protocol suite has to be extended across such a WANET. Ad hoc internet nodes encapsulate their data into IP (Internet Protocol) packets. The transmission control protocol (TCP), in the applications in the end-systems, takes care of lost packets, and prevents network congestion. By adopting the TCP/IP networking protocols, applications on ad hoc network nodes can communicate seamlessly with applications in the Internet. The performance of TCP's end-to-end adaptive window based packet transmission mechanism has been a subject of much concern over point-to-point wireless links (we will summarise an analytical approach to such problems in Section 5). The problems with TCP over multihop packet radio links are only recently beginning to be understood (see, for simulation based performance analysis [4] and [5], and for some scaling results [6]). We will also touch upon the performance of TCP over a multihop radio path in Section 5.

   An important emerging class of applications of WANETs arises from the availability of inexpensive miniature multifunction devices. Such a device would have embedded sensors (for light, chemicals, temperature, for example), would have embedded computing, a digital radio, and a long idle-life battery. These devices could be independent units to be randomly strewn into an area, or could be deliberately embedded in common systems such as watches, appliances, building walls, etc. Functions of such ad hoc sensor networks could be

- Monitoring contamination levels after radioactive or chemical leaks
- Forest fire detection, timber management, and wild life tracking
- Materials and people tracking and security management in factories, airports, and hospitals
- Distributed instrumentation in large machines or vehicles (such as ships)

Ad hoc sensor networks would not need to use TCP. In fact, the challenge would be for them to achieve their function by only explicitly communicating with their neighbours. See [7] for a survey of this emerging area.

Ad hoc networks are hard to analyse, and most analytical models are intractable because of the various interdependencies that need to be accounted for. The earliest papers aimed at determining the network capacity for a given topology and a combination of channel access and capture schemes ([8], [9], [10], see [11] for a survey). These were essentially extensions of the analyses for single hop networks; hence the effort was made to formulate Markovian models. The general models assumed exogenous traffic as independent Poisson processes, exponentially distributed packet lengths, and transmission scheduling process for packets from any node i to any node j as independent Poisson processes which included new and rescheduled packets. The network topology was specified as a graph where two nodes were connected by a link if they could hear each other's transmissions. Throughputs of ALOHA, CSMA, CDMA schemes with various collision management and capture assumptions were analysed. Results, however, could be obtained only for simple network topologies since the state description became formidable for large networks.

Since the devices will, in general, be randomly located, the graph of a WANET is actually a *random graph*. Hence, the "specification" of the topology in the above analyses turned out to be a major limitation of these approaches. Recent papers, therefore, have focussed on obtaining bounds on the network capacity for random topologies ([12], [13],[14], [15]). These analyses have led to the so called "scaling laws" i.e. scaling of the network capacity with the number of nodes in the network. These results will be discussed in Sections 2 and 3.

The following is a summary of the results surveyed in this paper. In Section 2, we discuss the basic trade-off between spatial reuse and WANET connectivity. We discuss how rapidly the transmission range can decrease when the number of network nodes increases in a given area so that the network stays connected. In Section 3 we first review how this leads to a scaling law which shows that the per node capacity of a WANET scales poorly with the number of nodes. Then, turning to stochastic capacity, we provide a packet flow model of WANET, and show how the performance of the network depends on the physical layer and the way nodes organise themselves and operate. In Section 4, we analyse a specific WANET, namely Bluetooth and seek its scaling law, i.e., how the performance of a cluster of piconets degrades as the number of piconets increases in a given area. We answer this question in terms of the stationary outage probability, and the temporal correlation in the outage process. In Section 5 we first review the performance analysis of TCP over a single hop wireless link. Then we discuss the performance of multihop TCP connections over WANETs. Using a simple example, we illustrate the complexity involved in this task. We conclude in Section 6.

2 WANET Topology, Spatial Reuse, and Connectivity

One of the most basic questions about a network is whether any two nodes in the network are able to communicate, perhaps by routing their data through other nodes. Such a question is formally answered by representing the topology of the network by a graph on the set of nodes of the network. In the context of a WANET, where a radio spectrum is shared among several contending nodes, there is the additional, and conflicting, issue of spatial reuse. To promote spatial reuse the transmission ranges of the nodes need to be kept small, while the opposite is required for ensuring that the network is connected. We will begin by examining this issue.

2.1 The WANET Graph

Let $\mathcal{N}(= \{1, 2, \ldots, n\})$ denote the WANET nodes. In the WANET topology graph, there is a directed link (i, j) (from node i to node j) if node i can send data to node j with the desired level

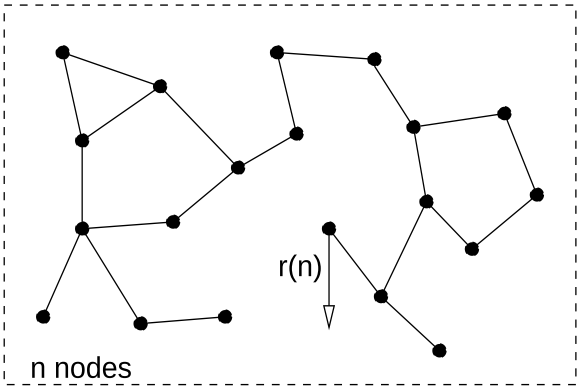

Figure 2: The graph of a wireless ad-hoc network. There are n nodes in a given area. The radio communication range is $r(n)$.

of reliability (which may, for example, be specified as a bit error rate of 10^{-4}, or perhaps a packet error rate of .01). This definition of a link is not as straightforward as it may at first seem. If several nodes are simultaneously transmitting, and if node i is attempting to transmit to node j, then this transmission may not succeed owing to excessive radio interference at j. On the other hand, if i is the only transmitting node then the signal power to receiver noise ratio at j may be sufficient to provide reliable communication. For the purpose of defining the WANET topology, we will define a link in this more "optimistic" sense. The point is that if two nodes are not connected in this graph, then they cannot be connected when arbitrary sets of nodes are allowed to transmit. Further, we will assume in our discussions that if link (i, j) exists then so does link (j, i), and hence we can take the WANET topology graph to be undirected (this assumption basically requires that the propagation channel between the two nodes is reciprocal). Let $\mathcal{L}$ denote these links, and let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ denote the WANET graph.

Thus, we say that the WANET is connected if the graph $\mathcal{G}$ is connected (there is a path between every pair of nodes of the graph).

2.2 Spatial Reuse and Connectivity

Given a particular transmitter and receiver design, and a transmitter power, there is a range (say, r) over which two nodes can communicate. Now let us consider n nodes distributed uniformly in a given area, and the effect of increasing n. If r is kept fixed as we increase n, then transmissions from any node interfere with a large number of nodes, and the number of simultaneous transmissions possible in the network (or the spatial reuse) is just $O(\frac{1}{r^2})$, which does not increase with increasing n. Evidently, in order to increase the spatial reuse the transmitter powers must be reduced so that the range (say, $r(n)$) is a decreasing function of n. Figure 2 shows a WANET in which the nodes have transmission range $r(n)$; any two nodes that are within a distance $r(n)$ of each other are shown as being neighbours in the WANET graph. Observe that, if $r(n)$ is too small then the WANET may not be connected. An important question that follows from this discussion is, "How fast can $r(n)$ decrease so that the network remains connected?"

Consider n nodes uniformly distributed over a square field of unit area, and a graph $\mathcal{G}$ on these nodes obtained by putting a link between any two nodes that are separated by a distance no more than $r(n)$. What we obtain is a probabilistic model, called a *random graph*. Let I_i denote the event that node i is isolated (i.e., has no links to any other node), and consider the probability that at least one node is isolated, i.e., $P(\cup_{i=1}^{n} I_i)$. We proceed in the spirit of the arguments in [16]. Since the area within the range of a node, and also within the square field is at least $\frac{1}{4}\pi r^2(n)$, we have, for all $i, 1 \leq i \leq n$,

$$P(I_i) \leq (1 - \frac{1}{4}\pi r^2(n))^{(n-1)}$$

Using the union bound we then have

$$\begin{aligned}
P(\cup_{i=1}^{n} I_i) &\leq \sum_{i=1}^{n} P(I_i) \\
&\leq n(1 - \frac{1}{4}\pi r^2(n))^{(n-1)} \\
&= e^{(\ln n + (n-1)\ln(1 - \frac{1}{4}\pi r^2(n))))} \\
&= e^{(\ln n - (n-1)O(\frac{1}{4}\pi r^2(n)))} \\
&= e^{\ln n(1 - \frac{(n-1)}{\ln n}O(\frac{1}{4}\pi r^2(n)))}
\end{aligned}$$

where we have used the fact that $r(n)$ decreases to zero as $n \to \infty$. It follows that if $r(n)$ shrinks slower than $\sqrt{\frac{\ln n}{n}}$ then the probability of some node being isolated goes to zero as $n \to \infty$. In fact, using some results from random graph theory, it has been shown in [16] that in order for the WANET graph to stay *connected* with probability 1, $r(n)$ must shrink slower than $\sqrt{\frac{\ln n}{n}}$. More

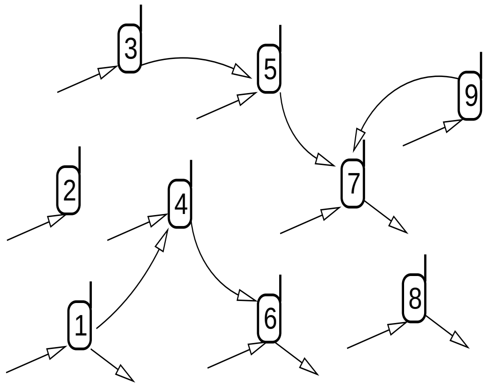

Figure 3: Packet flow in a multihop wireless ad-hoc network.

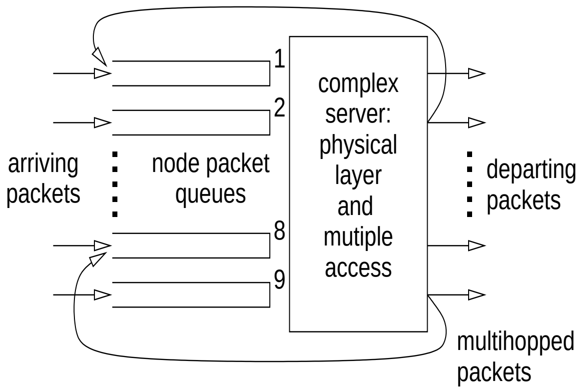

Figure 4: A queuing model representing the entire wireless ad-hoc network.

precisely, it has been shown in [17] that if the range $r(n)$ scales as $O\left(\sqrt{\frac{\ln n + c(n)}{n}}\right)$ then the graph stays connected, as $n \to \infty$, if and only if $c(n) \to \infty$. Note that $c(n)$ can go to ∞ arbitrarily slowly.

3 The Capacity of a WANET

Consider again n nodes constituting a WANET in a given area. Figure 3 is a depiction of such a network. Each node is running some applications (e.g., packet telephony, web browsing, or email) which generate packets that need to be transported to other nodes in the network. We can view these application generated packets as arrivals into the network. These arrivals are shown by the straight arrows pointing into the nodes in Figure 3. Packets are transmitted to neighbouring nodes, from where they may need to be relayed to other nodes; i.e., some packets are multihopped. Finally, each packet reaches its destination node, and this can be viewed as a departure; shown by the straight arrows pointing away from some of the nodes in Figure 3.

With the above view of the traffic flow, it is natural to think of the WANET as a service system into which customers (i.e., packets) arrive, get serviced (i.e., transported over the wireless medium) and finally depart. Figure 4 shows a queuing model for the entire WANET. The model shows one queue at each node, and this queue holds all the packets (new or transit) waiting to be transmitted out of the node. The digital radio communication, and multiple access mechanism of the network, can be viewed as a complex service mechanism that serves packets from these queues. Multihopped packets are fed back to other queues.

For this queuing model, the first question that can be asked is: "How fast can packets arrive into the nodes so that the nodal queues remain stable?" Stability could be defined in the usual sense of the joint queue length random process converging to a proper distribution.

3.1 A Scaling Law from Spatial Reuse

Let the rate of arrival of new packets at each node be $\lambda(n)$. The transmission range $r(n)$ scales with n, as discussed above. Let $h(n)$ be the average number of hops over which a packet is relayed. Hence the total rate of packets that need to be transmitted by the WANET is $n\lambda(n)h(n)$. Assuming that the nodes are uniformly distributed, and all destinations are equally likely, it follows that $h(n) = O(r(n)^{-1})$.

Let us denote the service capacity of the network by $S(n)$; i.e., $S(n)$ is the total rate at which the network can serve packets from the queues. In general, $S(n)$ will depend on the amount of spatial reuse, on the physical layer and the multiple access, on the routing protocol, and the prevailing propagation conditions. In any case, the rate of packets to be transported can be no more than $S(n)$; i.e.,

$$n\lambda(n)h(n) \leq S(n)$$

If we consider only spatial reuse, then $S(n) \leq O(r(n)^{-2})$. Hence if $r(n)$ scales down no faster than $\sqrt{\frac{\ln n}{n}}$ (in order to maintain connectivity) we can put together the above expressions to yield

$$\lambda(n) \leq O\left(\frac{1}{\sqrt{n \ln n}}\right)$$

Thus the per node capacity of a WANET scales very poorly with n, and this can be seen to be primarily because of multihopping.

It follows that dense WANETs would serve the best for applications in which there is primarily local communication between nodes. The basic wireless ad-hoc networking paradigm is extremely well suited to ad-hoc sensor networks. When strewn over an area, these devices sense their immediate environment, and can be viewed as constituting a *data space*. The sensor network can then be queried ("What is the maximum level of chemical contamination?"), and by using a distributed algorithm, that requires the devices to only communicate with their neighbours, the query can be answered and the results returned.

3.2 Stochastic Capacity

After discussing the above fairly general scaling results, we now turn to a packet flow model of a WANET. The interest in packet flow models is because various performance measures can be directly calculated from them. The question of particular interest is the following. If the network consists of n randomly distributed nodes in a given area A, then for given input traffic rates $\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ does their exist a channel access, routing and packet scheduling scheme so that the network is stable? In this paper, however, we will discuss the following simpler questions instead. For a given multiple access, routing and packet scheduling scheme, what is the throughput capacity of the network and, what is the maximum input traffic rate for which the network is stable?

We consider the following network model. Nodes are randomly distributed in the plane according to a Poisson point process of intensity λ per m^2. In the simulation results presented, we consider a Poisson field of 5,000 network nodes. Each radio node transmits on the same frequency f with power P using an omni-directional antenna. Also it can not transmit and receive simultaneously. For the propagation model, we assume only path loss with exponent $\eta = 4$. We say that a transmission can be decoded if its signal to interference ratio (SIR) exceeds a given threshold β [1]. Typical values of β are in the range of $10 - 20$dB. Channel access is random, i.e., nodes take decisions to transmit or receive independently, and time is slotted. In each slot, a node decides to transmit with probability α and decides to receive with probability $(1 - \alpha)$ independently; α is called the attempt probability. If the transmission is successful, the packet is removed from the queue (i.e., this is an assumption of instantaneous acknowledgements). A successfully received packet at a node joins the queue for next hop transmission with probability ν, otherwise the packet departs the

[1]Given a modulation and coding scheme, β actually specifies the maximum BER assuming interference to be Gaussian (which is a fair assumption with a large number of interferers). In this way, details of the actual modulation and coding scheme can be abstracted.

network. A node addresses[2] a transmission only to one of its neighbours; the neighbours are the nodes which are within a given distance R from a node. Thus, a transmission is successful when the node to which it is addressed is in receive mode, and the SIR of the transmission exceeds β at the addressed node. Note that, R need not be the radio range of a node, in fact, we show later that a smaller R improves the performance. In each slot, the 1-hop destination for the head-of-the-line (HOL) packet is chosen randomly from amongst the neighbours, i.e., given the number of neighbours K, a neighbour is chosen as the destination of the HOL packet with probability $\frac{1}{K}$ (for a typical node, the number of its neighbours is Poisson distributed with mean $\lambda \pi R^2$). Then the random selection of nodes to transmit to is basically the assumption that the routing and traffic pattern are uniform and homogeneous. Thus each packet will traverse a geometrically distributed number of hops before departing the network.

3.2.1 Interference Analysis

With the channel attempt model described above, in each slot, the nodes that are in the transmit mode form an independent Poisson process[3] (see [18] for some early work in this framework). At a typical node, the average power received in a slot, from transmitters further than a distance d_0, is proportional to $\lambda \alpha \int_{d_0}^{\infty} \left(\frac{r}{d_0}\right)^{-\eta} 2\pi r dr$, ($d_0$ denotes the near field cut off distance). The integral converges for path loss exponents $\eta > 2$, and hence even with infinite number of transmitters average power received is finite. We now assume that $\eta = 4$. It can be further shown that the characteristic function of the distribution of total received power at a node in a slot is given by

$$\psi(w) = \exp^{\lambda \pi d_0^2 jw \int_0^1 t^{-1/2} \exp^{jwt} \, dt}$$

It can be shown from this that the mean interference is $2\lambda\alpha\pi d_0^2$, and the variance is $\frac{4}{3}\lambda\alpha\pi d_0^2$. Applying the Central Limit Theorem, we approximate the interference as Gaussian $\mathcal{N}(2\lambda\alpha\pi d_0^2, \frac{4}{3}\lambda\alpha\pi d_0^2)$; where $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2. Note that, the mean is linear in both λ and α; this is shown in Figure 5, where simulation results are plotted along with the analytical values. The value of d_0 is taken to be 0.1m, and $\lambda = 1$, or 2 nodes per square metre. For the simulation, 5,000 nodes were generated, and the simulation was run for 20,000 slots. An

[2]By actually inserting a physical or IP address in the packet header.

[3]In general, the spatial point processes of transmitters and receivers depend upon the channel access scheme.

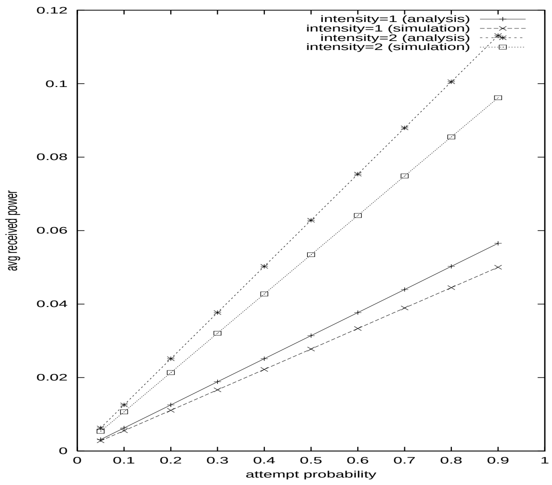

Figure 5: Variation of mean interference at a node with attempt probability, for $\lambda = 1$ and 2 per square metre.

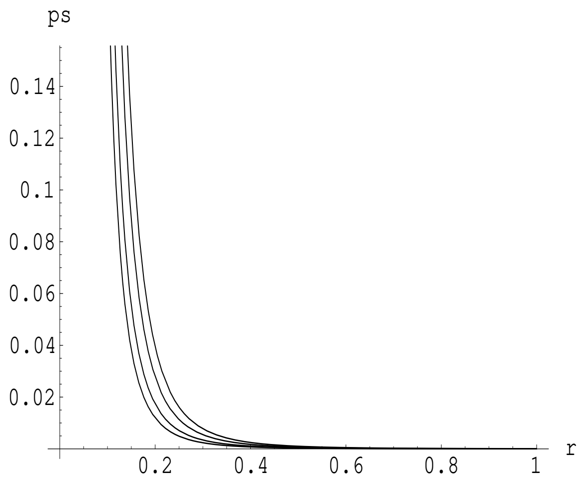

Figure 6: Variation of probability of successful transmission with distance for $\alpha = 0.1, 0.2, 0.5, 1$, and $\lambda = 1$ per m^2.

average was taken over several realisations of the node positions, and for several target nodes in the middle of the network. We observe that the analysis and simulation results are qualitatively the same, and the analysis is also a good approximation.

Consider a transmitter-receiver pair at a distance r in this Poisson field of points. Their communication is successful with probability,

$$p_s(r, \alpha) := P\left(\frac{P_s}{N + I} \geq \beta\right)$$

where $P_s = (r/d_0)^{-\eta}$ denotes the intended signal power (normalised to the power at the reference distance d_0), I the interference (distributed as $\mathcal{N}(2\lambda\alpha\pi d_0^2, \frac{4}{3}\lambda\alpha\pi d_0^2))$ and N the thermal noise at the receiver. Figure 6 shows the variation of $p_s(r, \alpha)$ with r for various values of α, for $\lambda = 1$ per m^2. For these calculations we take the transmit power to be 1mW, and the thermal noise $N = 5 \times 10^{-7}$mW. The curves are decreasing with α; here $\alpha = 1$ is just indicative of the performance if all the nodes in the network were transmitting simultaneously, and thus interfering with reception at our reference node. Note the sharp decay of $p_s(r, \alpha)$ with r. The probability of success is very poor beyond a few centimeters from a node; clearly random access is not a good way to operate a WANET.

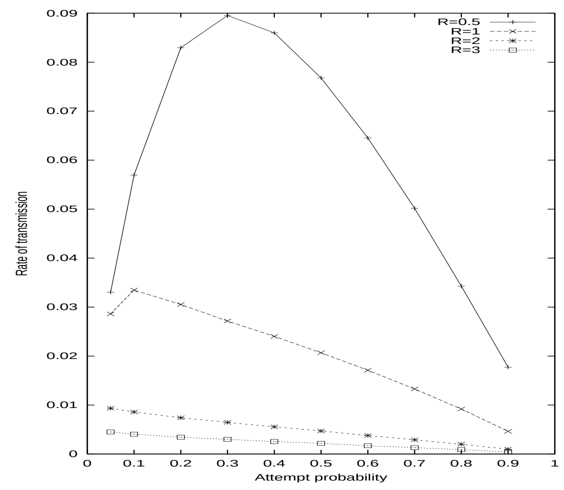

Figure 7: Saturation throughput of a node vs. attempt probability; simulation results. $R = 0.5, 1, 2, 3$m.

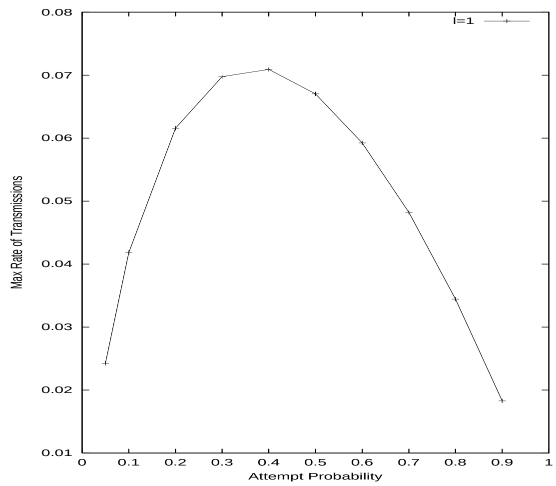

Figure 8: Capacity bound vs. attempt probability (Equation 1). $\lambda = 1$m.

3.2.2 Saturation Throughput

The saturation throughput of a network is defined as the rate of transmission in the network when a transmitted packet from the HOL of a queue at a node is immediately replaced by another packet. Saturation throughput is akin to a *service rate* provided by the complex channel *server* (see Figure 4). In some models, the saturation throughput is shown to yield a sufficient condition for network stability. Let γ_i denote the saturation throughput of the i^{th} node.[4] Then,

$$\gamma_i := \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} Z_k = p_t$$

where, $Z_k = 1$ if the transmission is successful in slot k, and p_t denotes the probability of successful transmission. Figure 7 shows the probability of successful transmission as α varies for various values of R. Recall that, we use a Poisson field of 5,000 nodes. Here $\lambda = 1$ per m^2. We have again used $\eta = 4$. R specifies the "neighbourhood" of a node; the mean number of neighbours being $\lambda\pi R^2$. Observe that, as R increases, the probability decreases. This is to be expected. A node attempts transmissions uniformly randomly to its neighbours. With a smaller R, a node's neighbours are closer, and the probability of successful transmission to these nodes is higher. We

[4]The network is assumed to be homogeneous, therefore, the subscript i can be omitted. If the network consists of N nodes, then the network throughput is $\sum_{j=1}^{n} \gamma_j$.

also see from Figure 7 that, for a given R, there is an *optimum* attempt probability. Thus, for a range of 0.5m, an attempt probability $\alpha = 0.3$ provides the best throughput. This analysis shows that the performance of the network can be optimised based on how nodes organise themselves.

The maximum rate of transmission is obtained when, instead of addressing its transmission to a particular neighbour[5], a node broadcasts it. Its transmission is, thus, successful if at least one node in its radio range decodes it. Let C denote the maximum rate of transmission of a node in saturation (normalised to average number of nodes in its radio range). Then, it can be shown that

$$C < \int_0^{\frac{1}{\beta}-N} \frac{\alpha(1-\alpha)}{\pi\sqrt{2.6\lambda\alpha d_0^2}} e^{-(y-2\lambda\alpha\pi d_0^2)^2/2.6\lambda\alpha\pi d_0^2} dy \tag{1}$$

This bound, in a sense, is a measure of network capacity of our model. Figure 8 shows its variation with α for $\lambda = 1$.

It appears from Figure 7 and Figure 8 that when $R = 0.5$, the throughput exceeds the capacity! The answer to this apparent paradox lies in the fact that when R is small, the network is not connected and consists of many isolated nodes. Since isolated nodes do not transmit at all (since, these nodes do not have neighbours to address their transmissions to), the overall interference to transmitting nodes is reduced, thereby greatly increasing the probability of success. The capacity, on the other hand, is calculated assuming every node is able to transmit. This leads us to important questions regarding the performance of the network and the organisation of nodes. We, however, do not discuss these issues in this paper.

The above analysis was all for WANETs in which nodes use random access. There appears to be little literature available on the analytical performance evaluation of RTS-CTS type protocols in dense WANETs with random node placement.

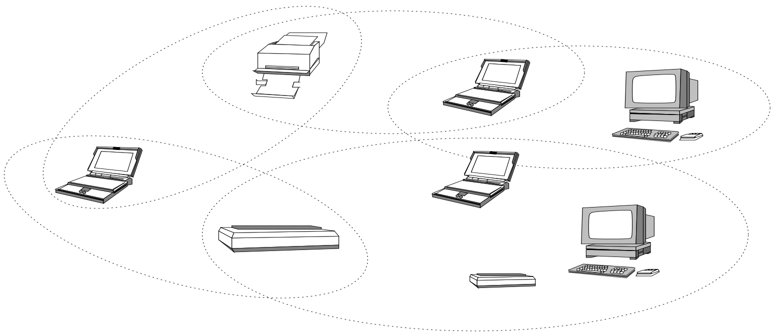

Figure 9: A system of overlapping Bluetooth piconets.

4 Performance of Bluetooth

4.1 The Model and its Analysis

We consider a single circular room (radius R) office environment (LOS propagation), with a number of Bluetooth devices distributed randomly in it to form M piconets. Because of the ad-hoc nature of the formation of piconets, the devices in a piconet are spatially randomly distributed in the room leading to overlap among the piconets. Note that, in our results M will be a parameter, i.e. we are interested in analysing the performance of the Bluetooth system as the number of piconets increases in a given area.

The propagation of radio waves inside a building is a complicated process, and depends significantly on the indoor environment (e.g., office, factory), and topography (LOS, obstructed). The statistics of the indoor channel varies with time due to motion of people and equipment. The indoor channel is characterised by high path losses (path loss exponent η is 1.5-1.8 for LOS; see [19]) and large variations in losses. Motion causes signal fading which compares well with the Rician distribution with K equal to 6.8 dB for the office environment. If the degree of motion is small (as in an office environment), the fading is extremely slow and the channel is said to be quasi-static. For practical data rates, fading is quasi-wide-sense stationary. In our analysis, since we consider LOS propagation in an open office environment, we take $\eta = 2$ and a Rician signal distribution with $K = 6$ dB.

[5] Recall that, success of a transmission depends on the addressed node being in receive mode.

Since Bluetooth operates in the ISM 2.4 GHz band, there are many sources of interference other than Bluetooth devices, e.g., microwave ovens and IEEE 802.11 Wavelans. We, however, focus on the interference generated within the Bluetooth system only. A piconet experiences interference when a frequency hit occurs, i.e., the transmission frequency of a packet in a piconet matches with that being used in one or more other piconets for some overlapping duration. We assume an ideal frequency hopping pattern (with N_f frequency hops), and model it as a discrete time Markov chain; a state denotes a frequency. Hopping patterns of piconets are independent and identically distributed. T denotes the hop time. Note that, the piconets are not synchronised in time. We assume that the time offset of the i^{th} piconet $(2 \leq i \leq M-1)$, denoted by t_i, with respect to the reference piconet are uniformly distributed in $[0, T]$.

Since the devices in the piconets are spatially distributed, *the interfering piconet* is not a fixed location or a device but a random device in that piconet transmitting at a given time. We assume a physically stationary environment (little or infrequent movement) so that the channel is quasi-static and quasi-wide-sense stationary. Hence, for the *duration of a frequency hit* on the single frequency, constant interference power can be assumed to be received from the individual interferers.

We consider a reference piconet and denote by $\{\Gamma(t), t \geq 0\}$, the SIR process.

$$\Gamma(t) = \frac{P_S(t)}{\sum_{j=1}^{N(t)} P_I^j(t) + \frac{N_0 W}{2}}$$

where, at time t, $P_S(t)$ is the desired signal power, $P_I^j(t)$ is the interference power due to the j^{th} interfering piconet and $N(t)$ is the number of interferers. Recall that, in Bluetooth a device can transmit in alternate slots. In addition, a slave can transmit only when polled. Hence, the receiver is not a fixed device and the SIR process we characterise is of the reference piconet as a whole.

Outage occurs when the bit SIR falls below the resistance ratio (denoted by β) specified by the standard (11-14 dB). We denote by Γ_b the bit SIR and seek the stationary outage probability P_{out}.

$$P_{out} := P\left(\frac{P_S}{\sum_{j=1}^{N_b} P_I^j + \frac{N_0 W}{2}} < \beta\right) \tag{2}$$

P_S is the desired signal power and P_I^j denotes the power received from the j^{th} interferer. N_b is a random variable denoting the number of interferers in a bit duration. We assume that the devices are distributed uniformly in the room *with respect to the center of the room.* [6]

In order to characterise the temporal correlation in the outage process, we consider a bit b and obtain the one step joint distribution of bit b and $b+1$ being in outage. This outage process then approximated by a Markov process. This gives us a 2-state Markov model as follows. We say that the channel is in *good* state, denoted by 'g', if $\Gamma_b \geq \beta$ or BER on the channel is below the specified value and is in *bad* state, denoted by 'b', when $\Gamma_b < \beta$, i.e., when the outage occurs. Let P_{ij} denote the transition probability of going from state i to state j. Then, the outage duration (in number of bits) is geometrically distributed with mean $1/(1 - P_{bb})$. Also the outage probability calculated from the Markov model, denoted by P_{out}^M, is given by $P_{gb}/(P_{gb} + P_{bg})$.

Details of the modelling assumptions and the analysis can be found in [20].

4.2 Results and Discussion

We take $R = 5$m. Since the calculation of exact outage probability is complicated, even numerically, we resort to finding bounds. Table 1 shows lower bounds on P_{out} with signal fading. Using

M-1	Lower bound on P_{out}	
	$\beta = 14$ dB	$\beta = 11$ dB
1	0.0114	0.0103
2	0.0227	0.0204

Table 1: Lower bounds on outage probability for $M = 2, 3$ piconets

the 2-state Markov chain approximation and and lower bounds on the outage probability, we calculate approximate values of the P_{gb} and P_{bg} for $\beta =$14dB and 400 bits packet size. We also calculate the outage probability from this Markov model (P_{out}^M). Results are shown in Table 2. Table 2 also shows in bits the mean outage duration (EB) and the mean duration in good state (EG).

[6] A Bluetooth receiver has -70dBm sensitivity level to meet BER < 0.001. With 1 Mbps bit rate, the noise power is approximately 5×10^{-7}mW.

M-1	P_{gb}	P_{bg}	P^{M}_{out}	EB	EG
1	0.00005	0.005	0.010	200	20000
2	0.000055	0.0035	0.016	285	18018

Table 2: Parameters of the 2-state Markov model

It is very complicated to calculate even the approximate values of P_{out} as the number of piconets increases. We, therefore, have results only for $M \leq 3$ piconets. However, the insight gained with these results allows us to predict the performance for larger values of M. The results indicate that when a frequency hit occurs it is difficult to maintain the resistance ratio of 14dB as specified by the standard. Hence, co-channel interference is avoided mainly through the use of large number of frequencies. For larger values of M, we expect that P_{out} will increase almost linearly, in the form of $\frac{(M-1)}{N_f}$. Though the Markov model is only an approximation (as seen from Table 2), we expect that when number of piconets is small outages are infrequent (once in 20000 bits) This is because piconets are not time-synchronised and frequency hits occur with probability $\frac{1}{N_f}$. However, outages persist for approximately 200 bits which is considerable duration considering $1/3$ and $2/3$ FEC used in Bluetooth. Also in certain packet types FEC is not mandatory. As M increases, the duration of the good state will reduce at the cost of longer outage durations.

5 Performance of TCP Controlled Transfers over a WANET

One of the applications of WANETS is to create ad-hoc internets. Each WANET node in such a network is an end-point, as well as a packet forwarding device (even a fully-fledged router, participating in a distributed routing protocol). In an internet, most store and forward applications (email, file transfers, web browsing) operate over the end-to-end TCP (Transmission Control Protocol). The TCP protocol serves three functions in the Internet:

1. TCP converts the nonsequential and unreliable packet transport service provided at the IP layer, into a sequential and reliable service.

2. TCP implements end-to-end flow control, thereby preventing a fast sender (e.g., a high per-

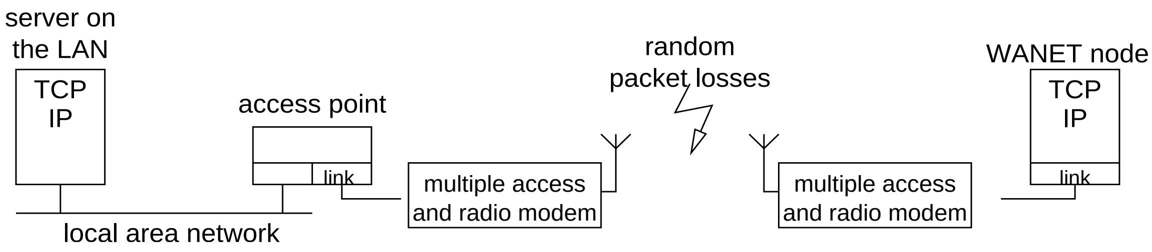

Figure 10: A WANET node transferring data over a TCP connection from a server on a LAN.

formance computer) from swamping out a slow receiver (e.g., a slow mechanical device such as a printer).

3. The applications that use TCP's services are *elastic*, in the sense that they can work satisfactorily even if the network offers them a time-varying packet transfer rate. TCP determines the way the network shares bandwidth among the competing flows.

Since TCP sessions comprise over 90% of the Internet traffic, it can be said to a large extent TCP governs the dynamics and the performance of the Internet.

Since elastic applications will continue to be the most popular ones to be deployed on ad-hoc Internets, it is important to understand the impact of TCP on the performance of these networks.

We divide our discussion into two parts. First we consider the situation in which the TCP session is over a single hop between neighbouring nodes. This will be by far the most common situation within offices and homes, as one can expect that the WANET node will be near a wired access point. On the other hand, ad-hoc internets can be expected to be deployed in situations where there is no wired infrastructure (e.g., in disaster zones). The performance of multihop TCP sessions over such a network will, therefore, also be of interest.

5.1 Single Hop Performance

The motivating scenario is shown in Figure 10. A WANET node is downloading a large file from a server on a high-speed LAN. The propagation delay over this wireless link is much smaller than the packet transmission times. We identify random epochs $(T_1, T_2, \cdots, T_k, \cdots)$ as follows. Let $T_0 = 0$,

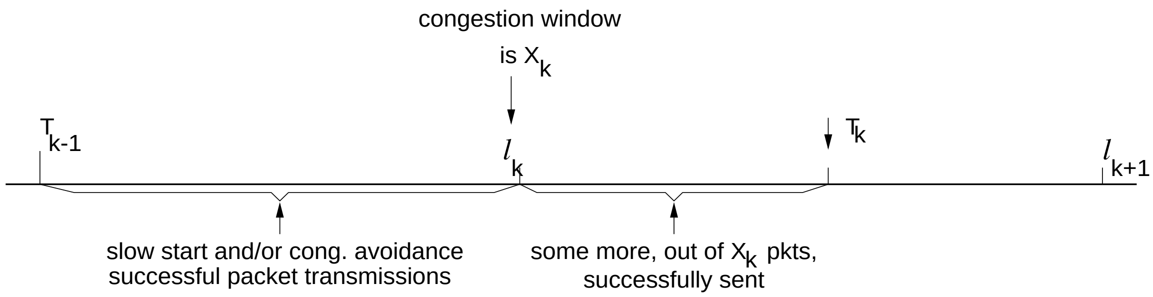

Figure 11: Analysis of TCP performance over a lossy link. The epochs $\{T_k\}$ and the Markov chain $\{X_k\}$.

and the connection starts off with the congestion window $W = 1$. As packets are transmitted and acked, the window grows (slow-start, if necessary, and then congestion avoidance). Eventually a loss occurs on the lossy link at the epoch l_1; we denote the window achieved at this epoch by X_1. Owing to the local area and high speed LAN assumptions, this window includes all increments due to any successful transmissions before the lost packet. Some random time later (depending on the version of the protocol and the recovery method used), at the epoch T_1, the normal congestion window algorithm resumes with a slow-start threshold of $\lceil \frac{X_1}{2} \rceil$. In this way we identify the sequence of random vectors $((X_0, T_0), (X_1, T_1), \cdots, (X_k, T_k), \cdots)$; see Figure 11. With Bernoulli packet loss [21], or Markov modulated packet loss [22] and appropriate assumptions (the window increase during congestion avoidance is taken as a probabilistic increase), we can show that (i) $\{X_k\}$ is a Markov chain, and (ii) $\{(X_k, T_k)\}$ is a Markov renewal process. Denoting the number of successful packets in the kth cycle by a "reward" V_k, we find that the V_k depends only on $X_{(k-1)}$, and we have a Markov renewal reward process. Letting U denote the random variable for the cycle length, and taking expectations under the stationary distribution $\pi(\cdot)$ of $\{X_k\}$, the throughput in packets per second is given by

$$\gamma = \frac{E_\pi V}{E_\pi U}$$

The various versions of TCP are distinguished by the structure of the Markov chain $\{X_k\}$, and the cycle times $\{T_k\}$; e.g., for the original Van Jacobson algorithm (TCP "Old"Tahoe) $T_k = l_k + \mathrm{timeout}$.

In spite of many approximations, the analysis is quite accurate as demonstrated in Figure 12 (taken

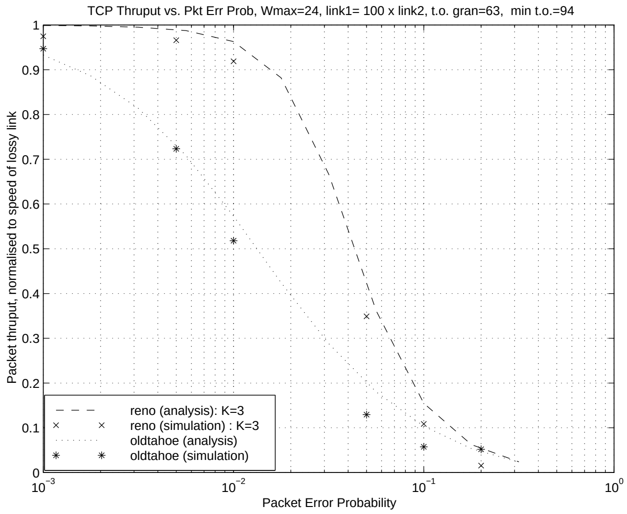

Figure 12: Throughput of TCP OldTahoe and TCP Reno vs. packet loss probability; comparison of simulation and analysis; (maximum congestion window is 24 packets; the LAN is 100 times faster than the lossy link; K is the duplicate ack threshold for fast retransmit).

from [21]), where analytically obtained results are compared with those obtained from actual TCP code running over an emulated lossy link.

5.2 Performance of Ad Hoc Internets with Multihop TCP Connections

A detailed model, such as the one above, for multihop TCP connections in a WANET is not yet available. In fact, several basic issues that have been well studied for TCP over wired networks are not yet clear in the context of TCP over multihop wireless links. For example, if there is a single TCP connection over a h hop wired connection, with each link's propagation delay being very small compared to the packet transmission time, then the TCP window should optimally be h (ignoring the acknowledgement transmission times). In a multihop wireless connection, however, the links are half-duplex, and furthermore arbitrary links cannot be simultaneously used owing to interference.

In Figures 13 and 14 we show the evolution of end-to-end window controlled packet transmission, over a 3 hop connection, with the nodes being labelled a, b, c, d. This would be the case if TCP

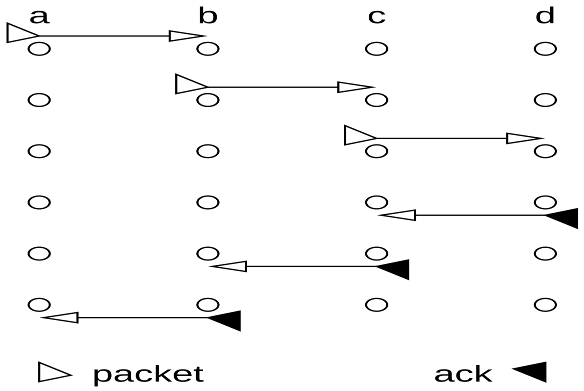

Figure 13: Window controlled transmission over a 3 hop wireless route; window=1.

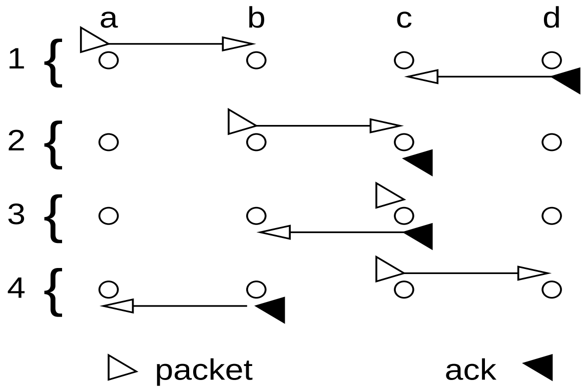

Figure 14: Window controlled transmission over a 3 hop wireless route; window=2

with a fixed window is used. It is assumed that each transmission takes one slot, whether it is a TCP packet or a TCP ack, and there is perfect centralised scheduling of transmissions (i.e., which nodes should transmit and to whom). We further assume that nodes more than two hops away (e.g., a and c, or b and d) are out of range; this implies that if a transmits to b, and d to c, the interference at b, caused by the transmission from d to c, is acceptable. The numbers to the left of Figure 14 are the slot numbers. The open triangles are packets, and the shaded ones are acknowledgements. A triangle with an arrow indicates that the packet or ack is being transmitted. A triangle without an arrow means that the packet or ack is queued. The triangles "point" in the direction in which the packet is travelling.

In Figure 13 the window is 1, and it takes 6 slots to transmit one packet and get back its acknowledgement, thus yielding a connection throughput of $1/6$ packets per slot. Can we use a window of 2, and thus improve the throughput? We note that not all hops can be active at the same time. For example, node a cannot send to node b, while node c sends to node d, as the transmission from node c will interfere at node b with the transmission from node a. Figure 14 shows a transmission schedule with a window of 2. The schedule is assumed to have "started" some time back, and will continue even after slot 4. At the end of schedule, the acknowledgement reaching the TCP transmitter, and the packet reaching the TCP receiver would generate a new packet and an acknowledgement, thus bringing the schedule back to the state that we had in slot 1. Notice that 2 "half" packets were transmitted in 4 slots, thus yielding a throughput of $1/4$.

The above simple example illustrates the complexity of the question "What is the optimal TCP window, and what is the corresponding throughput?" The answer will depend on the topology of the path, the way the links interfere, and the multiple access protocol (i.e., link scheduling mechanism).

We notice from the above discussion that even if a link is scheduled, and if TCP is being used, this link may not be able to be used; see the example in Figure 13. Consider a WANET confined to a fixed (say, unit) area. There are n nodes. It follows, by continuing the arguments in Section 3.1, that the per node throughput will scale as

$$\lambda(n) \leq O\left(\frac{1}{\sqrt{n \ln n}}\right) \Phi(n)$$

Some directions to determining $\Phi(n)$ have been provided in [6]. In [14] the authors have reported empirical results, where it has been found that the per node throughput in an ad hoc internet (of UDP controlled sessions, i.e., no error recovery) scales as $O\left(\frac{1}{n^{1.68}}\right)$. Since the transfers do not take place over TCP, this scaling probably captures the inefficiencies of the multiaccess protocol. One would expect that the performance would be even worse with TCP.

Recent research (see [4] and [5]) has shown that TCP over multihop wireless connections has serious performance limitations. A single TCP connection provides good throughput only if the window is properly tuned; several TCP connections sharing a WANET tend to lock each other out (almost completely) over long periods of time, and these lock-outs occur at random times. We may wish to conclude that dense WANETS should not be used for building traditional Internet-like packet networks.

6 Conclusion

In the performance evaluation of ad hoc networks we need to work with models that incorporate the properties of all the layers. Models which are general and address these interdependencies are hard to analyse. We have surveyed some of the recent progress in the area. Much recent work has emphasised network scaling laws, which show how the network performance changes with increasing node density. We have also provided a model for stochastic capacity, and a detailed analysis of a

Bluetooth scatternet, taking into account propagation phenomena. The stochastic model for a general WANET assumed network synchronisation along slots; although this is impractical, the model provides useful insights. This analysis shows that in order for the network to operate efficiently, optimisation of performance needs to be done simultaneously at various levels. The performance of the network depends on the physical layer and the way nodes organise themselves and operate. Similar insights were obtained from our discussion of TCP over multihop wireless routes. The most challenging task is to build upon such results and insights, and develop distributed algorithms for efficient resource management of WANETs.

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