Optimal Association of Stations and APs in an IEEE 802.11 WLAN

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Abstract— **We propose a maximum utility based formulation for the problem of optimal association of wireless stations (STAs) with access points (APs) in an IEEE 802.11 wireless local area network. Each STA can associate with one or more APs at different physical bit rates. Each such association yields certain STA throughputs, depending on the MAC scheduling used. We use simple formulas for the throughputs obtained by the STAs in each possible association, for two MAC scheduling policies (namely, random polling, which is an approximation to the DCF mechanisms in IEEE 802.11, and proportional fair sharing). We then evaluate the quality of an association by summing the utilities of these throughputs, where the utility is the logarithm of the throughput. Then we seek the sum utility maximising association. In this paper, we develop the problem formulation, and provide the optimal association results for some simple cases.**

Keywords: wireless LANs, utility maximisation, proportional fairness, random polling,

I. Introduction

In an IEEE 802.11 WLAN, a station (STA) can be in the range of several access points (APs), and can associate with each one of these at a certain maximum physical bit rate, depending on radio channel conditions. For example, let us consider 4 STAs in the vicinity of 2 APs, such that 2 of the STAs can associate with either of the APs at 11 Mbps and the other 2 can associate with either of the APs at 2 Mbps. Evidently several associations are possible. For example (i) both the STAs that can associate at 2 Mbps do so with one AP and the other 2 STAs associate with the other AP, or (ii) with each AP we associate 1 STA at 11 Mbps and another at 2 Mbps. Is there any reason to prefer one of these associations over the other, and if so what is the optimal association? This is the problem that we consider in this paper. Our work anticipates the future possibility of a sophisticated association protocol that will facilitate the implementation of such an optimal association, but we do not pursue this matter in this paper.

Our approach is the following. When several stations associate with an AP at various rates then the medium access control mechanism determines the time average throughputs that each one of the stations obtains. Thus, given several APs and STAs, an association between them can be characterised by the throughputs obtained by each of the STAs. We use a utility function to evaluate the "value" of the individual throughputs. Then the global "value" of the association is taken as the sum of the individual STA utilities. The optimal association is the one that achieves the maximum sum utility over all the possible associations.

In this paper the above problem is formulated as an optimisation problem with 0-1 variables, a nonlinear objective function (i.e., for a given association, the objective is the sum of the logarithm of the rates obtained by each STA), and linear constraints on the 0-1 variables. We develop the formulation, and motivate the form of the throughput formulas and the log utility. Then we show that in two special cases a linear relaxation of the problem can be solved and the solution provides important insights. We also provide numerical results from a general example.

The problem that we address can also be placed in the context of the research on bandwidth sharing in wired networks (see, for example, Chapter 7 of [2] for a survey of such research). The important difference is that, whereas in wired networks the topology (which is akin to the association) is a given, in wireless networks we have some flexibility in being able to optimise over the topology as well. During the preparation of this final manuscript, it has been brought to our attention that a very similar approach has also been reported in [1].

II. The Optimal Association Problem

There is a wireless local area network (WLAN) with n Access Points (APs), indexed $1 \leq j \leq n$. *It is assumed that the AP placement and channel allocation are such that the interference between cochannel APs can be ignored.* Hence each AP and its associated stations (STAs) work as a single isolated BSS, which we call a *cell*. There are m STAs, indexed by $1 \leq i \leq m$. STA i can be associated with AP j at the physical bit rate r_{ij}, where $r_{ij} \in \mathcal{C}$, a finite rate set. For example, in IEEE 802.11b, $\mathcal{C} = \{1, 2, 5.5, 11\}$ Mbps. If STA i cannot be associated with AP j because even the lowest rate in $\mathcal{C}$ is not sustainable between them then we define $r_{ij} = 0$. We define $\mathbf{R}$ as the *rate matrix* with r_{ij} as elements; thus $\mathbf{R}$ is an $m \times n$ matrix.

We denote by $m^{(j,k)}$ the number of STAs that can associate with AP j at rate $C_k \in \mathcal{C}$. Then $m^{(j)} := \sum_{C_k \in \mathcal{C}} m^{(j,k)}$ is the number of STAs that can be associated with AP j; i.e., $m^{(j)}$ is the number of positive entries in the jth column of $\mathbf{R}$.

If STA i can be associated with AP j, i.e. $r_{ij} \neq 0$, then we define a 0-1 variable a_{ij}. An association is an assignment of a 0 or 1 value to each of the a_{ij}. We express an association compactly as a 0-1 matrix $\mathbf{A}$ which is necessarily 0 everywhere except for the elements ij for which $r_{ij} > 0$. $\mathbf{A}$ has m rows and n columns, the rows corresponding to STAs and columns to APs.

Not every such $\mathbf{A}$ will be feasible. Because each STA must be associated with exactly one AP, we have for all $i, 1 \leq i \leq m$,

$$\sum_{j=1}^{n} a_{ij} = 1 \tag{1}$$

i.e., the sum of each row of a feasible $\mathbf{A}$ is 1.

We will develop a performance measure for evaluating the quality of an association. Let us denote the set of all associations in a given problem by $\mathcal{A}$. Let $\Omega(\mathbf{A})$ denote a quality

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measure for the association $\mathbf{A}$ (see Section III). Then the optimal association problem is to find an association, $\mathbf{A}^*$, such that

$$\mathbf{A}^* = \arg\max_{\mathbf{A}\in\mathcal{A}} \Omega(\mathbf{A}) \quad (2)$$

and the corresponding quality measure is $\Omega(\mathbf{A}^*)$.

Let us define some additional notation. Given an association $\mathbf{A}$, let us denote by $m_{j,k}$, $1 \le j \le m$ and k such that $C_k \in \mathcal{C}$, the number of STAs that are associated with AP j at rate C_k. Then $m_j := \sum_{C_k\in\mathcal{C}} m_{j,k}$ is the number of STAs that are associated with AP j. Note that $m_{j,k} = \sum_{i=1}^m a_{ij} \cdot I_{\{r_{ij}=C_k\}}$, $\sum_{i=1}^m a_{ij} = m_j$, and $\sum_{j=1}^n \sum_{i=1}^m a_{ij} = m$. Clearly $m_{j,k} \le m^{(j,k)}$.

III. Measures of Association Performance

Consider an IEEE 802.11 cell, comprising m_j STAs associated with AP j, and assume that there is no interference from any other cell. In this paper we measure the quality of this association in terms of the up-link UDP bulk transfer throughput obtained by the STAs when their MAC queues are saturated. We do this so as to obtain a simple objective function. More complex traffic scenarios in each cell can be modelled but they will yield complex formulas that may only be possible to evaluate numerically. All the packet lengths are assumed to be the same, denoted by L bits.

We denote by $\boldsymbol{\theta_{SC}}(\mathbf{A})$ the throughput vector for an association $\mathbf{A}$ and with the MAC scheduling SC. We consider two MAC schedules: random polling (RP) and proportional fair scheduling (PF); thus SC will be either RP or PF. The elements of $\boldsymbol{\theta_{SC}}(\mathbf{A})$ are denoted by $\theta_{SC,i}(\mathbf{A})$, $1 \le i \le m$.

Given an association $\mathbf{A}$, let $\mathcal{C}_j$ denote the set of rates of the STAs associated with AP j, each rate appearing as many times in $\mathcal{C}_j$ as the number of STAs associated at that rate. Now for a MAC scheduling policy SC the individual throughputs of the STAs will be given as follows. For STA i such that $a_{ij} = 1$

$$\theta_{SC,i}(\mathbf{A}) = \tau(\mathcal{C}_j, r_{ij})$$

where $\tau_{SC}(\mathcal{C}_j, r)$ is the throughput of a node that is associated at rate r and is in a cell with the associated STAs comprising the rate set $\mathcal{C}_j$. Thus the throughput of a node depends only on the set of rates of stations that are associated with the same AP and its own rate (for a justification see, for example, [3]).

We also denote the measure of association performance for schedule SC and association $\mathbf{A}$ as $\Omega_{SC}(\mathbf{A}) = f(\boldsymbol{\theta_{SC}}(\boldsymbol{A}))$; in the following sections we will develop the form of the function $f(\cdot)$. Note that $\Omega_{SC}(\mathbf{A})$ is a scalar function of throughput vector $\boldsymbol{\theta_{SC}}(\mathbf{A})$.

A. Random Polling Access – Cell Performance

Consider now AP j. For simplicity of notation in this section, index the STAs associated with AP j by $i, 1 \le i \le m_j$, and these are associated at rate $r_i, 1 \le i \le m_j$. The set C_j is this set of rates, with each rate appearing as many times as there are STAs associated at that rate. It can be observed from the analysis of the standard IEEE 802.11 Distributed Coordinated Function (DCF) access mechanism in [3] that the aggregate up-link throughput has the form

$$\frac{1}{\frac{1}{m_j}\sum_{i=1}^{m_j}\frac{1}{r_i} + h_{m_j}} \quad (3)$$

where h_{m_j} depends only on m_j, the IEEE 802.11 DCF parameters, and the packet length L. Notice that if ideal random polling was used to schedule transmissions from the STAs, and if one packet was transmitted from each STA at each poll, then the aggregate up-link throughput would have the following form,

$$\mathcal{T}_{RP}(\mathcal{C}_j) := \frac{m_j}{\sum_{i=1}^{m_j}\frac{1}{r_i}} \quad (4)$$

The formula in Equation (3) differs because of the overhead term h_{m_j}. The ideal polling formula is obtained by reducing the overheads to 0. For simplicity in developing the association algorithm, we will adopt $\mathcal{T}_{RP}(\mathcal{C}_j)$ as the measure of aggregate throughput of the cell when STAs with the capacity set $\mathcal{C}_j$ are associated with an AP. The throughput per node is then given by

$$\tau_{RP}(\mathcal{C}_j) := \frac{1}{\sum_{i=1}^{m_j}\frac{1}{r_i}} \quad (5)$$

the throughput being the same for all the nodes.

B. Motivating the Quality Measure for Comparing Associations

We motivate the quality measure we will use by an example. There are 4 STAs and 2 APs; 2 STAs can connect to either AP at 2 Mbps and the other 2 STAs can connect to either AP at, say, 10 Mbps. There are 5 possible associations, denoted by $\mathbf{A}_i, 1 \le i \le 5$, in Table I. The two 2 Mbps STAs can be associated either one to each AP or both to one of the APs. For each such association of the 2 Mbps STAs, the 10 Mbps STAs can be associated one to each AP or both to one of the APs. This yields the 5 distinct associations shown in Table I. Since the throughput depend only on the rates between the associated STAs and APs the identity of the STAs that associate with each AP are irrelevant; only the rates matter.

TABLE I
THE POSSIBLE ASSOCIATIONS OF 4 STAS WITH 2 APS IN THE EXAMPLE.

$\mathbf{A}$	AP1	AP2
$\mathbf{A}_1$	10, 2	10, 2
$\mathbf{A}_2$	2	10, 10, 2
$\mathbf{A}_3$	10	10, 2, 2
$\mathbf{A}_4$	10, 10	2, 2
$\mathbf{A}_5$	–	10, 10, 2, 2

Table II then shows the values of $\theta_{RP,i}(\mathbf{A})$ and $\ln\theta_{RP,i}(\mathbf{A})$ per node for each node. Also shown in the last column are $\sum_i \theta_{RP,i}(\mathbf{A})$ and $\sum_i \ln\theta_{RP,i}(\mathbf{A})$ for each association. We can use this example to compare three measures (see also Chapter 7 of the book [2]).

TABLE II

PERFORMANCE MEASURES FOR THE ASSOCIATIONS IN THE EXAMPLE WITH 4 STAS AND 2 APS.

Node →	10	10	2	2	Σ
	AP1	AP2	AP1	AP2	
$\theta_{RP,i}(\mathbf{A}_1)$	1.67	1.67	1.67	1.67	6.67
$\ln\theta_{RP,i}(\mathbf{A}_1)$	0.51	0.51	0.51	0.51	2.04
	AP2	AP2	AP2	AP1	
$\theta_{RP,i}(\mathbf{A}_2)$	1.43	1.43	1.43	2.00	6.29
$\ln\theta_{RP,i}(\mathbf{A}_2)$	0.36	0.36	0.36	0.69	1.76
	AP1	AP2	AP2	AP2	
$\theta_{RP,i}(\mathbf{A}_3)$	10.0	0.91	0.91	0.91	12.72
$\ln\theta_{RP,i}(\mathbf{A}_3)$	2.30	-0.095	-0.095	-0.095	2.02
	AP1	AP1	AP2	AP2	
$\theta_{RP,i}(\mathbf{A}_4)$	5.00	5.00	1.00	1.00	12.0
$\ln\theta_{RP,i}(\mathbf{A}_4)$	1.61	1.61	0.00	0.00	3.22
	AP2	AP2	AP2	AP2	
$\theta_{RP,i}(\mathbf{A}_5)$	0.83	0.83	0.83	0.83	3.33
$\ln\theta_{RP,i}(\mathbf{A}_5)$	-0.18	-0.18	-0.18	-0.18	-0.73

1) *Max-Min Fair (MMF) rates*: A rate vector is MMF if there is no other vector for which the rate of some node, say i, is higher without the rate of some other node, say j, that has a smaller or equal rate than i, being made lower. Consider Association $\mathbf{A}_4$ (see Table I). Table II shows that the 10 Mbps nodes obtain an average throughput of 5 Mbps and the 2 Mbps nodes obtain an average throughput of 1 Mbps. In Association $\mathbf{A}_1$ the throughputs of the 2 Mbps nodes are increased to 1.67 Mbps, while the throughputs of the 10 Mbps nodes are *reduced* to 1.67 Mbps. It can be seen that Association $\mathbf{A}_1$ is MMF.
2) $\max \sum_{i=1}^{m} \theta_{RP,i}(\mathbf{A})$: This is provided by Association $\mathbf{A}_3$
3) $\max \sum_{i=1}^{m} \ln\theta_{RP,i}(\mathbf{A})$: This is provided by Association $\mathbf{A}_4$

Notice that the MMF association yields equal rates but low throughputs for all the nodes and also a low aggregate throughput. The approach of maximising the aggregate throughput yields very unfair rates, substantially penalising one of the 10 Mbps nodes. The third approach yields a high aggregate throughput and also a certain fairness between nodes with the same physical link rates.

With the above discussion in mind, we propose to evaluate the quality of an association $\mathbf{A}$ by the measure

$$\Omega_{SC}(\mathbf{A}) = \sum_{i=1}^{m} \ln\theta_{SC,i}(\mathbf{A})$$

where $\theta_{SC,i}(\mathbf{A})$ is the rate obtained by node i in the association $\mathbf{A}$. Thus we now have the form of the function $f(\boldsymbol{\theta_{SC}(A)})$ mentioned earlier in this section.

With these ideas we see immediately that the association performance measure for RP is written as

$$\begin{aligned}\Omega_{RP}(\mathbf{A}) &= \sum_{j=1}^{n} m_j \ln \frac{1}{\sum_{C_k\in\mathcal{C}} m_{j,k}\frac{1}{C_k}} \\ &= -\sum_{j=1}^{n} m_j \ln\left(\sum_{C_k\in\mathcal{C}} m_{j,k}\frac{1}{C_k}\right) \qquad (6)\end{aligned}$$

C. Proportionally Fair Access

Notice that with random polling all nodes associated with an AP obtain the same time average throughput irrespective of their physical link rates. Thus if one STA with low speed access is associated with an AP that otherwise has STAs with high speed access, then the throughputs of all the latter STAs will be pulled down to values less than the physical bit rate of the STA with low speed access. This is clearly not a desirable situation. One solution is to not let the STA with low speed access associate with an AP that is already handling STAs with high speed access. This is not a viable option if network coverage is such that there is no other AP that this STA can associate with. The alternative is to somehow modify channel access so that STAs obtain time average throughputs that are *proportional* to their physical bit rates. We will now show that this desirable objective arises naturally owing to the log utility formulation.

Now consider the performance measure

$$\begin{aligned}\Omega_{SC}(\mathbf{A}) &= \sum_{i=1}^{m} \ln\theta_{SC,i}(\mathbf{A}) \\ &= \sum_{j=1}^{n}\sum_{i=1}^{m} I_{\{a_{ij}=1\}} \ln \tau_{SC}(\mathcal{C}_j, r_{ij})\end{aligned}$$

We are interested in maximising Ω_{SC}. It is reasonable to ask whether the MAC scheduler can itself be chosen so as to increase Ω_{SC}. So consider the term corresponding to an AP (say, AP 1) in the right hand side of the previous equation. Suppose m_1 STAs, $1 \le i \le m_1$, are associated at rates in the set $\mathcal{C}_1$ with AP 1. The MAC schedule allocates a fraction ϕ_i of the time to STA i. Now consider the problem

$$\begin{aligned} \max \quad & \sum_{i=1}^{m_1} \ln \tau_i \\ \text{subject to} \quad \tau_i &\le \phi_i r_i \\ \sum_{i=1}^{m_1} \phi_i &= 1 \\ \tau_i \ge 0, \quad \phi_i \ge 0, & \qquad 1 \le i \le m_1. \end{aligned}$$

Again, for simplicity, we have ignored multiple access overheads in this formulation. It can be shown that the optimal solution is $\phi_i = \frac{1}{m_1}, 1 \le i \le m_1$, and $\tau_i = \frac{1}{m_1}\cdot r_i$. This is called proportionally fair scheduling; notice that the throughputs are proportional to the physical link rates. We define

$$\tau_{PF}(\mathcal{C}_j, r) = \frac{1}{|\mathcal{C}_j|}\cdot r,$$

where, of course, $|\mathcal{C}_j| = m_j$, and hence we get

$$\Omega_{PF}(\mathbf{A}) = \sum_{j=1}^{n}\sum_{i=1}^{m}\left(I_{\{a_{ij}=1\}}.\ln\left(\frac{r_{ij}}{m_j}\right)\right) \qquad (7)$$

or, equivalently,

$$\Omega_{PF}(\mathbf{A}) = \sum_{j=1}^{n}\sum_{C_k=\mathcal{C}}\left(m_{j,k}\cdot\ln\left(\frac{C_k}{m_j}\right)\right) \qquad (8)$$

Proportionally fair scheduling has also been called time fairness in the literature.

IV. Optimal Association: Analysis and Special Cases

In this section we analyse some special cases of the association problem.

A. Random Polling

Using the form of the objective function in Equation 6, we can write the optimal association problem for random polling as

$$\min \quad \sum_{j=1}^{n}\left(\sum_{C_k\in\mathcal{C}} m_{j,k}\right)\ln\left(\sum_{C_k\in\mathcal{C}} m_{j,k}\frac{1}{C_k}\right) \tag{9}$$

$$\begin{aligned} \text{subject to} \quad m_{j,k} &= \sum_{i=1}^{m} a_{ij} I_{\{r_{ij}=C_k\}} \\ \sum_{j=1}^{n} a_{ij} &= 1 \quad \text{for all } i \\ a_{ij} &\le I_{\{r_{ij}>0\}} \\ a_{ij} &\in \{0,1\} \end{aligned}$$

Example IV.1:
$\mathcal{C} = \{C_1\}, m^{(j,1)} = m$, for all $j, 1 \le j \le n$: This means that all STAs can associate with any of the APs at the one rate C_1. Without loss of generality, let us take $C_1 = 1$. The optimisation problem (9) then simplifies to

$$\min \quad \sum_{j=1}^{m} m_j \ln m_j$$

$$\begin{aligned} \text{subject to} \quad \sum_{j=1}^{n} m_j &= m \\ m_j &\in \mathbb{Z}^+ \end{aligned}$$

This simplification can be seen directly, since each STA can associate with every AP.

Let us now divide the objective function inside and outside the $\ln$ by m, and the constraints by m (where we recall that m is the number of nodes). This amounts to scaling the objective by a known positive constant, and subtracting a constant. This yields the following *relaxed problem* whose solution (appropriately scaled and shifted) will provide a lower bound to the above $\min$ problem, and hence an upper bound to the original $\max$ problem defined in (2).

$$\min \quad \sum_{j=1}^{n} x_j \ln x_j \tag{10}$$

$$\begin{aligned} \text{subject to} \quad \sum_{j=1}^{n} x_j &= 1 \\ x_j &\ge 0 \end{aligned}$$

The following result is easily proved.

Lemma IV.1: Given $h_j \ge 0, 1 \le j \le n$, the function $\sum_{j=1}^{n} x_j \ln h_j x_j$ is a convex function in its argument $(x_1, ..., x_n)$ for $x_j \ge 0, 1 \le j \le n$. ■

It follows that (10) is a standard convex programming problem with linear constraints and it is necessary and sufficient that the optimum solution is a KKT point. Taking $\mu \ge 0$ to be the Lagrange multiplier for the single equality constraint, we obtain the Langrangian function

$$\Lambda(x_1, \cdots, x_n, \mu) = \sum_{j=1}^{n} x_j \ln x_j + \mu\left(\sum_{j=1}^{n} x_j - 1\right)$$

Differentiating $\Lambda(x_1, \cdots, x_n, \mu)$ with respect to each x_j and setting equal to zero we obtain

$$x_j = e^{-(\mu+1)}$$

from which we conclude that the optimal association is given by

$$x_j = \frac{1}{n}$$

for $1 \le j \le n$; i.e., *the STAs must be equally divided over the APs*. Also note that if m is divisible by n then we have an integer solution, and the relaxed problem solves the original integer problem. ■

Example IV.2:
$\mathcal{C} = (C_1, C_2, \cdots, C_n)$, $m^{(j,j)} = m$, for all $1 \le j \le n$: This means that all STAs can associate with AP j at rate C_j. There are as many rates as there are APs. This could model a situation in which the nodes are in a cluster and there are several APs. All the nodes can associate with the nearest AP at one rate, with an AP further away at a lower rate, etc. Now the optimisation problem (9) simplifies to

$$\min \quad \sum_{j=1}^{n} x_j \ln\left(x_j \frac{1}{C_j}\right)$$

$$\begin{aligned} \text{subject to} \quad \sum_{j=1}^{n} x_j &= 1 \\ x_j &\ge 0 \end{aligned}$$

Again we have a convex programming problem with linear constraints and we seek a KKT point. As in the previous example, it is easily shown that the solution has the form

$$x_j = \frac{C_j}{\sum_{j'=1}^{n} C_{j'}}$$

i.e., *the allocations to the APs are proportional to the physical rates achievable to the APs.* If there are two APs and two rates, e.g., 5.5 Mbps and 11 Mbps, the optimal association is to assign twice as many STAs to the AP with which they can sustain 11 Mbps as to the other AP (with which they can only sustain 5.5 Mbps). Again if the solution of this relaxed problem is integral, then we also have the solution to the original integer problem.

Note that *the load is not balanced across the APs in any sense, and hence "load balancing" is not always the correct solution.* The number of STAs assigned to the APs are unequal, and the aggregate throughput at each AP is the physical link rate sustained to that AP (of course, assuming our simplified, overhead-free model). ■

B. Proportionally Fair Access

With proportionally fair access, for a given association $\mathbf{A}$ we can write using Equation 8

$$\Omega_{PF}(\mathbf{A}) \quad = \quad \sum_{j=1}^{n} \sum_{C_k \in \mathcal{C}} m_{j,k} \ln \frac{C_k}{m_j}$$

Dividing by m and multiplying inside the $\ln$ by m, the right side of the equation becomes

$$\sum_{j=1}^{n} \sum_{C_k \in \mathcal{C}} x_{j,k} \ln \frac{C_k}{x_j}$$

where $x_{j,k}$ is the fraction of nodes with bit rate k assigned to AP j, and x_j is the fraction of nodes assigned to AP j. This objective function has to be maximised over a constraint set.

Remark: It can be seen that, even with this objective, the two special problems in the previous section have the same form and hence they have the same solutions. Thus even if we enforce proportional fairness the following holds:

1) If all STAs can associate with all APs at the same rate then distribute the STAs equally over the APs.
2) If all STAs can associate with each AP at the same rate, but with different APs at different rates then distribute the STAs over the APs in proportion to the rates they achieve to the APs.

V. A General Example

In order to demonstrate a more general situation, we consider a simple association problem with four STAs and four APs with the possible associated rates given in Table III (i.e., this is the matrix $\mathbf{R}$). Table IV provides the results obtained by using the Branch-and-Bound method. This example problem was solved using the versatile optimisation package, *LINGO* from *LINDO SYSTEMS* [4]. The optimal association is shown in the first group of rows in Table IV; STA i, $1 \leq i \leq 4$, is associated with AP j, $1 \leq j \leq 4$. We also show the value of the throughput $\theta_{RP,i}$ obtained by STA i, the values of $\ln \theta_{RP,i}$, $\sum_{i=1}^{4} \theta_{RP,i}$ and $\sum_{i=1}^{4} \ln \theta_{RP,i}$; observe that 7.8867 is the optimal value of Ω_{RP}.

The default association algorithm in implementations associates an STA with an AP with which it has the strongest signal and hence the highest rate. Table IV also shows the calculation for two such associations. The second group of rows correspond to the association STA 1 $\rightarrow$ AP 1, STA 2 $\rightarrow$ AP 2, STA 3 $\rightarrow$ AP 2, STA 4 $\rightarrow$ AP 4, and the third group correspond to the association STA 1 $\rightarrow$ AP 1, STA 2 $\rightarrow$ AP 1, STA 3 $\rightarrow$ AP 4, STA 4 $\rightarrow$ AP 4. Of course, these associations perform worse than the optimal ones shown in first group.

TABLE III
EXAMPLE: MATRIX OF MAXIMUM LINK RATE BETWEEN STA_i AND AP_j

Access Points $\rightarrow$ Stations $\downarrow$	AP_1	AP_2	AP_3	AP_4
STA_1	11	11	2	11
STA_2	11	11	2	2
STA_3	2	5.5	2	5.5
STA_4	2	2	2	11

TABLE IV
COMPARISON OF SOME ASSOCIATIONS FOR THE EXAMPLE IN TABLE III

STA $i \rightarrow$	STA 1	STA 2	STA 3	STA 4	$\sum$
AP j	AP 1	AP 2	AP 3	AP 4	
$\theta_{RP,i}$	11	11	2	11	35
$\ln \theta_{RP,i}$	2.3979	2.3979	0.693	2.3979	7.8867 (optimal)
AP j	AP 1	AP 2	AP 2	AP 4	
$\theta_{RP,i}$	11	3.667	3.667	11	29.334
$\ln \theta_{RP,i}$	2.3979	1.2993	1.2993	2.3979	7.3944
AP j	AP 1	AP 1	AP 4	AP 4	
$\theta_{RP,i}$	5.5	5.5	3.667	3.667	18.334
$\ln \theta_{RP,i}$	1.7047	1.7047	1.2993	1.2993	6.0080

VI. Conclusion

In this paper we have taken the first steps in studying the problem of optimal association of STAs and APs in a WLAN. We have developed a formulation of the problem and have provided solutions for some simple cases. In particular, we have shown that the sometimes assumed objective of "load balancing" may not always be the correct solution.

Much work remains to be done on this problem. Insights are needed into the solution of the association problem which is a problem with 0-1 variables, a nonlinear objective, and linear constraints. In practice, STAs simply associate with the AP from which the received signal is the strongest. In order to implement a globally optimal association for a WLAN, a central system will need to know the matrix $\mathbf{R}$ (which can be achieved if each STA provides its signal and noise measurements to the system), and then the calculated optimal association will need to be enforced. Additionally, there will be the issue of STAs leaving and new ones joining.

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