Fixed Point Analysis of the Saturation Throughput of IEEE 802.11 WLANs with Capture

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***Abstract*— In this paper we study the performance of a single cell IEEE 802.11 WLAN with the possibility of frame capture at the receiver. In [9], we studied in detail the performance of single cell IEEE 802.11e WLANs under ideal channel conditions (without capture, fading or frame error), using a fixed point framework developed in [7]. Here, we extend the work presented in [9] to model WLANs with capture. We provide a basic framework to analyse capture. We then obtain the fixed point equations for the infrastructure model. We show that these equations accurately explain some experimental observations. Then we provide conditions under which the system has a unique and balanced fixed point solution corresponding to the competing stations even with capture.**

Key-words: IEEE 802.11, performance evaluation, fixed point equation, capture

I. Introduction

In [9], we studied the saturation throughput analysis of single cell IEEE 802.11e type networks with nonhomogeneous nodes. We considered an ideal channel, without capture, fading or frame error. According to the model, a node succeeds in reserving the channel for its transmission, only if no other node attempts in the same slot. Simultaneous transmissions always result in all the transmissions being corrupted, i.e., we worked with the pure collision model. However, in practice, the power levels of different transmissions as heard at the receiver are different, due to path loss, shadowing and fading. Hence, it is possible that the receiver is able to receive one of the simultaneous transmissions successfully, i.e., one of the nodes is able to *capture* the receiver. This is particularly true when spread spectrum communication is used, as is the case in the IEEE 802.11b physical layer.

In this paper, we extend the analysis in [9] to include the possibility of capture at the receiver. We consider a single cell model with IEEE 802.11e type nodes; i.e., the nodes use Distributed Coordination Function (DCF) with possibly different back-off parameters (we do not, however, model the AIFS mechanism in this paper). Every transmission is heard by every other node, i.e., there are no hidden nodes in the system. We consider an *infrastructure* model, wherein, each node has traffic only with the access point (a similar framework can be developed for the *ad hoc* model, but the system becomes intractable, as an ad hoc model with capture would allow multiple receptions simultaneously). Also, we assume that all the nodes have at least one packet to transmit at any time (saturation throughput analysis).

An important concern in [9] is the importance of the uniqueness of the solution of the fixed point equation, as it was observed therein that when several fixed points exist then the fixed point analysis does not capture the steady state system performance. Hence, in this paper, in addition to formulating the fixed point equations with capture, and showing that the fixed point is *balanced* (in the sense that all the coordinates of the vector fixed point are the same), we also provide a uniqueness result.

A. Literature Survey

There have been a number of analytical and simulation studies modeling the performance of IEEE 802.11e networks under the pure collision assumption (refer [9] and [7]). However, there is very little and partial work studying the effect of capture for IEEE 802.11 networks. The study of the effect of capture on slotted ALOHA systems dates back to the 1980's. In [2], the capture probability in a Slotted ALOHA system with poisson traffic for a Rayleigh fading channel was obtained, for coherent and incoherent addition of signals. Lau and Leung generalised the study of capture phenomenon using different capture models and spatial distribution of nodes in [4]. A similar study was done in [1] as well. [10] obtained the throughput of Slotted ALOHA in a Nakagami fading channel. In [6], the capture probability of an access point (AP) in a channel with Rayleigh fading, shadowing, and near far effects is obtained and they derived the throughput and packet delay for the various protocols with CSMA/CA system under simplistic assumptions. [5] developed a framework to analyse IEEE 802.11 systems with hidden nodes and capture. However, the model was mathematically intractable. Based on the Markov chain model proposed by Bianchi ([3]) and the general capture analysis for wireless systems, Velkov and Spasenovski studied the performance of IEEE 802.11 WLANs with capture in [12], [13] and [14]. They partially analyze the system for a homogeneous system of nodes (similar backoff parameters and equal capture probabilities). However, due to complex modeling assumptions, they do not analyze the system completely and use values obtauned from simulation in their analysis. Using the simplification proposed in [7] and [9] to model IEEE 802.11e WLANs, in this paper we provide a general framework using which we are able to completely characterise the system with capture. Together with [9], the analysis of IEEE 802.11 systems using the fixed point framework becomes more complete and tractable. Also, using our approach we are able to model

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the asymmetry in backoff parameters as well as capture probabilities in IEEE 802.11e WLANs, which has not been attempted earlier.

II. The Generalised Back-Off Model

There are n IEEE 802.11e nodes, indexed by $i, 1 \leq i \leq n$. In [7], the authors consider a generalisation of the back-off behaviour of the nodes, and define the following back-off parameters (for node i)

$K_i :=$ At the $(K_i + 1)$th attempt either the packet being attempted by node i succeeds or is discarded

$b_{i,k} :=$The mean back-off (in slots) at the kth attempt for a packet being attempted by node i, $0 \leq k \leq K_i$

We call a system of nodes with identical (non-identical) back-off parameters as **homogeneous** (**nonhomogeneous**).

Remark: IEEE 802.11e permits different backoff parameters to differentiate channel access obtained by the nodes in an attempt to provide QoS. The above definitions capture the possibility of having different $CW_{\min}$ and $CW_{\max}$ values, different exponential back-off multiplier values and even different number of permitted attempts.

It has been shown in [7] that under the decoupling assumption, introduced by Bianchi in [3], the attempt rate of node i (conditioned on being in back-off (see [7])) for given collision probability γ_i is given by,

$$G_i(\gamma_i) := \frac{1 + \gamma_i + \gamma_i^2 \cdots + \gamma_i^{K_i}}{b_{i,0} + \gamma_i b_{i,1} + \gamma_i^2 b_{i,2} + \cdots + \gamma_i^k b_{i,k} + \cdots + \gamma_i^{K_i} b_{i,K_i}} \quad (1)$$

In [7] it was shown that, for every i, $G_i(\gamma_i)$ is monotone decreasing with γ_i if the $b_{i,k}$ are nondecreasing with k.

III. Fixed Point Equation without Capture

We now review the basic fixed point equations from [9]. Focusing on the back-off and attempt process of node i, and being given the collision probability γ_i, the attempt rate is provided by $G_i(\gamma_i)$ in Equation 1. It is important to note that in the present discussion all rates are conditioned on being in the back-off periods; i.e., we have eliminated all durations other than those in which nodes are counting down their back-off counters. As explained in [7] this suffices to obtain the collision probabilities. Now consider a nonhomogeneous system of n nodes. Let $\boldsymbol{\gamma}$ be the n-dimensional vector of collision probabilities of the nodes. With the slotted model for the back-off process and the decoupling assumption, the natural mapping of the attempt probabilities of other nodes to the collision probability of a node for an ideal channel without frame error or capture is given by

$$\gamma_i = \Gamma_i(\beta_1, \beta_2, \ldots, \beta_n) := 1 - \prod_{j=1, j\neq i}^{n} (1 - \beta_j)$$

where := is, as usual, read as "is defined to be." Now we can, in turn, write the attempt probabilities in terms of collision probabilities as $\beta_j = G_j(\gamma_j)$. Hence we can expect that the equilibrium behaviour of the system will be characterised by the solutions of the following system of equations. For $1 \leq i \leq n$,

$$\gamma_i = \Gamma_i(G_1(\gamma_1), \cdots, G_n(\gamma_n))$$

We write these n equations compactly in the form of the following multidimensional fixed point equation.

$$\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma})) \quad (2)$$

Since $\boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ is a composition of continuous functions it is continuous. We thus have a continuous mapping from $[0,1]^n$ to $[0,1]^n$. Hence by Brouwer's fixed point theorem there exists a fixed point in $[0,1]^n$ for the equation $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$.

Consider the i^{th} component of the fixed point equation, i.e.,

$$\gamma_i = 1 - \prod_{1 \leq j \leq n, j \neq i} (1 - G_j(\gamma_j))$$

or equivalently,

$$(1 - \gamma_i) = \prod_{1 \leq j \leq n, j \neq i} (1 - G_j(\gamma_j))$$

Multiplying both sides by $(1 - G_i(\gamma_i))$, we get,

$$(1 - \gamma_i)(1 - G_i(\gamma_i)) = \prod_{1 \leq j \leq n} (1 - G_j(\gamma_j))$$

Thus a *necessary and sufficient condition* for a vector of collision probabilities $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_n)$ to be a fixed point solution is that, for all $1 \leq i \leq n$,

$$(1 - \gamma_i)(1 - G_i(\gamma_i)) = \prod_{j=1}^{n} (1 - G_j(\gamma_j)) \quad (3)$$

where the right-hand side is seen to be independent of i.

Define $F_i(\gamma) := (1 - \gamma)(1 - G_i(\gamma))$. From Equation 3 we see that if $\boldsymbol{\gamma}$ is a solution of Equation 2, then for all $i, j, 1 \leq i, j \leq n$,

$$F_i(\gamma_i) = F_j(\gamma_j) \quad (4)$$

Notice that this is only a *necessary condition*. For example, in a homogeneous system of nodes, the vector $\boldsymbol{\gamma}$ such that $\gamma_i = \gamma$ for all $1 \leq i \leq n$, satisfies Equation 4 for any $0 \leq \gamma \leq 1$, but not all such points are solutions of the fixed point Equation 2.

Definition III.1: **Balanced fixed point**: We say that a fixed point $\boldsymbol{\gamma}$ (i.e., a solution of $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$) is balanced if $\gamma_i = \gamma_j$ for all $1 \leq i, j \leq n$; ∎

Studying the above (and similar) system of equations for IEEE 802.11e type networks, in [9], we showed that there exists a unique fixed point solution for the equations. We also showed that the function $F(\cdot)$ is one-to-one for typical values of IEEE 802.11e parameters. Also, we observed that the unique fixed point of the system charactersises the actual performance very well.

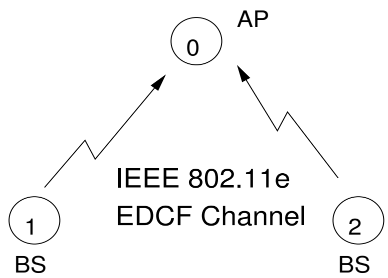

Fig. 1. Example System: A 3 node system with two STAs and an AP. UDP traffic is sent from the STAs to the AP (no downlink traffic). The sending rate is such that the STA MAC queues are saturated.

IV. Capture Phenomenon

Capture in wireless systems is a well-studied phenomenon and different models have been proposed to analyze such systems. Consider a scenario with n transmitters competing for a common receiver. The system is time-slotted and the transmitters attempt with some probability at each slot (this corresponds to a Slotted ALOHA system as well as a single cell IEEE 802.11 WLAN without hidden nodes).

Under the vulnerability region model for capture, a competing node successfully captures the receiver if no other node transmits (simultaneously) from within a distance zr from the receiver, where r is the distance of the tagged node from the receiver and z is called the capture ratio. Under the interfering-power model for capture, the receiver captures the frame of the tagged node only if the frame's detected power sufficiently exceeds the joint interfering power of the other contenders by z, capture parameter. For example, it has been shown in [8] that, for slotted ALOHA protocol in the presence of Rayleigh fading and log-normal shadowing, the conditional capture probability of a tagged node with n other contending nodes is given by,

$$P_c(r_t|\underline{\xi}, \underline{r}) = \prod_{i=1}^{n} \frac{1}{1 + z e^{\xi_i - \xi_t} (\frac{r_i}{r_t})^{-\eta}}$$

where t is the tagged node, r_i is the distance of node i from the common receiver, ξ_i models the log-normal shadowing for node i and η is the power loss exponent. The capture probability and the throughput of a Slotted ALOHA system is then calculated by integrating over ξ and the spatial distribution of the nodes. When the system is power controlled, i.e., when the mean received power is the same for every node, the above expression simplifies to,

$$P_c = \left(\frac{1}{1+z}\right)^n \tag{5}$$

V. Fixed Point Equations with Capture: An Experimental Example

In this section, we will develop the general fixed point framework by starting with a simple experiment involving capture. Figure 1 shows the experimental setup involving three nodes: 2 stations (STAs) and an access point (AP). The actual experimental setup involved three Orinoco 802.11b Wireless Client Adapter with firmware version 8.72; see [11] for complete details. UDP traffic is set up from node 1 to node 0 and from node 2 to 0 (there is only uplink traffic). It was ensured that the nodes 1 and 2 always had packets in their buffer to transmit (corresponding to the saturation throughput case). All the nodes had identical backoff parameters ($b_0 = 16, p = 2, K = 7$). The collision probabilities and the capture probabilities were obtained using the experimental set up and are provide in Table I; 95% confidence intervals were also obtained but are not reported here. The two rows in the table correspond to different placements of the STAs in relation to the AP. Let c_i be the probability that node i captures the receiver when both the nodes transmit simultaneously. In the table, γ_i and c_i are the collision probability and capture probability of node i. Under a pure collision model, the collision probabilities of both the nodes should be the same, and capture probabilities should be 0.

Experiment No.	γ_1	γ_2	c_1	c_2
1	0.061	0.0084	0.0279	0.8623
2	0.061	0.0038	0.0301	0.3827

TABLE I

Results of the capture experiment. Each row corresponds to a different relative placement of the devices.

Now we provide a fixed point analysis of the above experimental observations. Using the decoupling assumption of the attempt process of the nodes and the constant collision probability assumption (see [7] and [3]), we have the following equations for the collision probability of the nodes,

$$\gamma_1 = (1 - c_1)\beta_2$$
$$\gamma_2 = (1 - c_2)\beta_1$$

where $\beta_i = G_i(\gamma_i)$. For this example we have, $G_i(\cdot) = G(\cdot)$. Rewriting the above equations, we have,

$$(1 - \gamma_1) = (1 - (1 - c_1)\beta_2)$$
$$(1 - \gamma_2) = (1 - (1 - c_2)\beta_1)$$

Multiplying suitably, we get,

$$(1 - \gamma_1)(1 - (1 - c_2)\beta_1) = (1 - (1 - c_1)\beta_2)(1 - (1 - c_2)\beta_1)$$
$$(1 - \gamma_2)(1 - (1 - c_1)\beta_2) = (1 - (1 - c_2)\beta_1)(1 - (1 - c_1)\beta_2)$$

Hence the fixed point solution must satisfy

$$(1 - \gamma_1)(1 - (1 - c_2)\beta_1) = (1 - \gamma_2)(1 - (1 - c_1)\beta_2)$$

Since $0 \le c_i \le 1$ and since we know that $F_i(\cdot)$ is one-to-one (for IEEE 802.11 parameters, see Section III), we see that $(1 - \gamma_1)(1 - (1 - c_2)\beta_1)$ and $(1 - \gamma_2)(1 - (1 - c_1)\beta_2)$ are one-to-one. Hence we can show that there exists a unique fixed point solution for this system (we will defer such analysis until next section). Solving numerically for (γ_1, γ_2) in the above equations, using the values of c_1 and c_2 from Table I, we get the following values for the collision probability.

1) $\gamma_1 = 0.0603$ and $\gamma_2 = 0.008$ with $c_1 = 0.0279$ and $c_2 = 0.8623$
2) $\gamma_1 = 0.0584$ and $\gamma_2 = 0.0362$ with $c_1 = 0.0301$ and $c_2 = 0.3827$

Comparing them with the observed collision probability values in Table I we see that the fixed point analysis captures the performance quite well.

VI. Fixed Point Analysis of the Infrastructure Model with Capture

In the infrastructure model for the WLAN, all the competing STAs communicate only with the AP. We will first consider the case where there is only reverse traffic (from the STAs to the AP).

Consider n homogeneous nodes with $G_i(\gamma) = G(\gamma)$ for all $1 \leq i \leq n$, where $G(\cdot)$ is defined as

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \ldots + \gamma^K}{b_0(1 + p\gamma + p^2\gamma^2 + \ldots + p^K\gamma^K)}$$

We recall that the significance of this function is that when the collision probability at node i is γ_i, then its attempt rate β_i is $G(\gamma_i)$. We assume that the function $G(\gamma)$ is monotone decreasing and the function $F(\gamma) := (1-\gamma)(1-G(\gamma))$ is one-to-one and decreasing (these conditions are satisfied for IEEE 802.11 back-off parameters, see [9]).

Assumptions: (i) We assume that all the nodes have a common destination which can receive packets (this does not prevent the receiver from sending CTS or MAC acknowledgements). (ii) At present our general equations are restricted to the situation in which the channel propagation effects are spatially homogeneous with respect to all the nodes. Hence, we include a constant probability r_k that a tagged node will capture the channel when it contends with k other transmitters. While the fixed point equations can be written down for more general situations, their analysis becomes intractable.

Then, assuming that the attempt probabilities of the nodes are $\beta_j, 1 \leq j \leq n$, the collision probabilities are given by

$$\begin{aligned}\gamma_1 = \Gamma_1(\beta_1, \cdots, \beta_n) &= 1 - (\prod_{i \neq 1}(1-\beta_i) \\ &\quad + r_1 \sum_{i \neq 1} \beta_i \prod_{j \neq (1,i)} (1-\beta_j) \\ &\quad + r_2 \sum_{i \neq 1, j > i} \beta_i\beta_j \prod_{k \neq (1,i,j)} (1-\beta_k) \\ &\quad + \cdots) \qquad (6)\end{aligned}$$

Rearranging the above equation and multiplying by $(1-\beta_1)$, and writing $\beta_1 = G(\gamma_1)$ in the left hand side, we get,

$$\begin{aligned}(1-\gamma_1)(1-G(\gamma_1)) &= \prod_i (1-\beta_i)(1 + r_1 \sum_{i \neq 1} \frac{\beta_i}{(1-\beta_i)} \\ &\quad + r_2 \sum_{i \neq 1, j > i} \frac{\beta_i}{(1-\beta_i)} \frac{\beta_j}{(1-\beta_j)} + \cdots)\end{aligned}$$

Similary, for node 2, we have,

$$\begin{aligned}(1-\gamma_2)(1-G(\gamma_2)) &= \prod_i (1-\beta_i)(1 + r_1 \sum_{i \neq 2} \frac{\beta_i}{(1-\beta_i)} \\ &\quad + r_2 \sum_{i \neq 2, j > i} \frac{\beta_i}{(1-\beta_i)} \frac{\beta_j}{(1-\beta_j)} + \cdots)\end{aligned}$$

Subtracting the above two equations, we get,

$$\begin{aligned}&(1-\gamma_1)(1-G(\gamma_1)) - (1-\gamma_2)(1-G(\gamma_2)) = \\ &\quad \prod_i (1-\beta_i)(r_1(\frac{\beta_2}{(1-\beta_2)} - \frac{\beta_1}{(1-\beta_1)}) \\ &\quad + r_2(\frac{\beta_2}{(1-\beta_2)} - \frac{\beta_1}{(1-\beta_1)}) \sum_{j > 2} \frac{\beta_j}{(1-\beta_j)} + \cdots)\end{aligned}$$

Consider the L.H.S. of the above equation. If $\gamma_1 < \gamma_2$, then $F(\gamma_1) = (1-\gamma_1)(1-G(\gamma_1)) > (1-\gamma_2)(1-G(\gamma_2)) = F(\gamma_2)$ (since $F(\cdot)$ is one-to-one and also decreasing). Hence, L.H.S. is greater than 0 . On the other hand, in R.H.S. $\prod_i (1-\beta_i) \geq 0$, $r_1, r_2 \geq 0$. If $\gamma_1 < \gamma_2$, then $\beta_1 = G(\gamma_1) > G(\gamma_2) = \beta_2$ and hence $\frac{\beta_1}{(1-\beta_1)} > \frac{\beta_2}{(1-\beta_2)}$, implies, the R.H.S. is negative. Since the nodes are identical, the reverse argument holds directly. Hence, $\gamma_1 = \gamma_2$. In other words, for any two nodes i, j, $1 \leq i, j, \leq n$, $\gamma_i = \gamma_j$ or the fixed point solution for the system is *balanced*. Let us denote this common value by γ.

A. Uniqueness of the Balanced Fixed Point

Now, with the above observation that the fixed point solution is balanced, it follows as well that the attempt probabilities β_i are all equal; denote the common value by β. The collision probability function now simplifies to

$$\Gamma(\beta) = \sum_{k=1}^{n-1} \frac{(n-1)!}{((n-1)-k)!k!} \beta^k (1-\beta)^{n-1-k} (1-r_k)$$

Then the fixed point equation becomes $\gamma = \Gamma(G(\gamma))$. To establish uniqueness, and knowing that $G(\cdot)$ is nonincreasing, it suffices to show that $\Gamma(\cdot)$ is nondecreasing in its argument.

We will show that if r_k are nonincreasing with k, then $\Gamma(\beta)$ is nondecreasing with β, which will establish that the fixed point is unique.

Since the binomial distribution for $(n-1)$ "trials" with "success" probability β is the convolution of $(n-1)$ Bernoulli distributions each with success probability β, and the Bernoulli distribution is obviously stochastically increasing with β, it follows that the binomial distribution is stochastically increasing with β. Hence $\Gamma(\beta)$ is the expectation of an increasing function of k (i.e., $(1-r_k)$) with respect to the binomial distribution that is stochastically increasing with the parameter β. Hence, $\Gamma(\beta)$ increases with β, and we have a unique fixed point.

Remarks VI.1:

1) In the spatially homogeneous case, the condition that the capture probability r_k decreases with k is reasonable to

expect. As discussed in Section IV for the case where the nodes are power controlled, $r_k = \frac{1}{(1+z)^k}$ for a Rayleigh fading channel. Here r_k strictly decreases with k.
2) [7] provides the equations relating the system throughput with the attempt rates (or collision probability) of the nodes. Hence, the collision probability analysis provided here completely characterises the system throughput as well.

B. With Downlink Traffic

Now, we will include the case where there is downlink traffic from the AP to the STAs as well (this completely characterises the infrastructure model). Let 0 corresponds to the AP and $1 \leq i \leq n$ correspond to the base stations. Using spatial homogeneity, as before, we have, for all $1 \leq t \leq n$,

$$\begin{aligned}\gamma_t = \Gamma_t(\beta_1, \cdots, \beta_n) \quad &= \quad 1 - (1-\beta_0)(\prod_{i \neq t}(1-\beta_i) \\ &\quad + r_1 \sum_{i \neq t} \beta_i \prod_{j \neq (t,i)} (1-\beta_j) \\ &\quad + r_2 \sum_{i \neq t, j > i} \beta_i \beta_j \prod_{k \neq (t,i,j)} (1-\beta_k) \\ &\quad + \cdots)\end{aligned}$$

Notice that the equation is similar to Equation 6 except for the $(1-\beta_0)$ term. Since $0 \leq (1-\beta_0) \leq 1$, we see (by a similar proof) that $\gamma_i = \gamma_j$ for all $1 \leq i, j \leq n$ for this case as well. Thus in this case all the STAs will have the same collision probability γ, and the AP will have a possible different collision probability γ_0. Uniqueness of the fixed point would require that the vector $(\gamma_0, \gamma, \cdots, \gamma)$ is unique for the system. However, we would require more information to solve this problem (the capture probability at an STA when the AP is the transmitter when some other STAs also transmit).

VII. Conclusion

In this paper we have studied a multidimensional fixed point equation arising from the model of the back-off process of the IEEE 802.11 and 11e EDCF access mechanism with the possibility of capture. We developed a general framework to analyze such systems with capture. We first considered the case when the STAs send UDP traffic to the AP, and their queues are saturated. For this case, we showed that when the propagation is spatially homogeneous, and when all STAs use the same back-off parameters, then the fixed point solution for the collision probabilities is balanced. A balanced solution implies that the probabilities of success of the STAs are equal, and in this sense their access is fair (though it does not imply throughput fairness). Further, if the capture probabilities decrease with the number of colliding nodes, then the fixed point is also unique. As argued in [9], establishing such uniqueness is important for the applicability of the fixed point analysis to predict performance. We also showed that when the AP is also transmitting packets then the result that the fixed point is balanced across the STAs continues to hold.

The fixed point approach is quite general and can be extended to different capture models and different spatial distribution of nodes. As pointed out in [9], however, it would be important to establish the uniqueness of the fixed point in each case, as we have done for some special cases in this paper.

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