Transmission Policies for Outage Minimization in Energy Harvesting Multi-hop Links

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Outline

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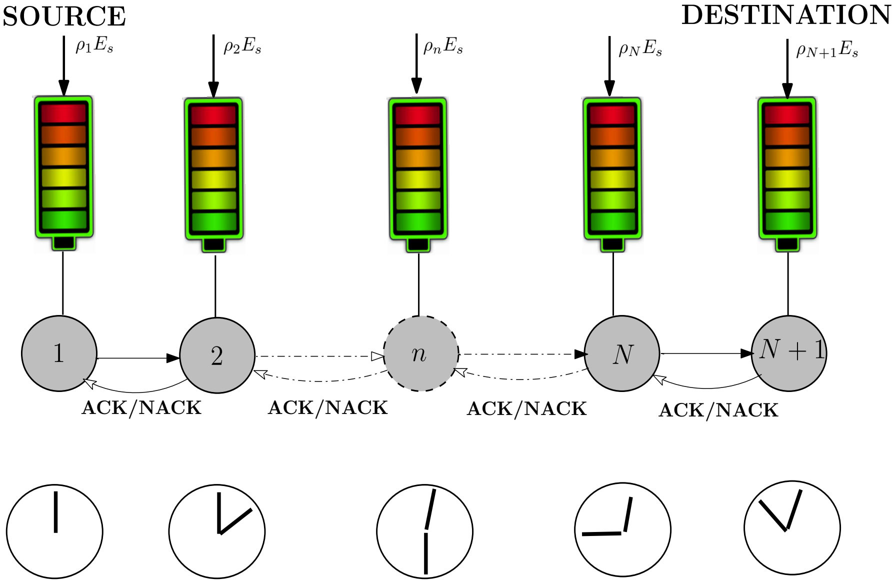
Introduction

- *Sporadically* energy is harvested from environment for eg. solar, wind etc
- *Energy neutrality constraint* (ENC): cumulative energy used cannot exceed the total harvested energy
- Energy neutrality constraint: infinite number of constraints
- Central issue: design of energy management policies to optimize a utility function
- Policy: prescription of the transmit power on the basis of available system-state information

System Model



System Model

- N-hop EH link with block fading channel
- All nodes are EH nodes (EHN)
- Periodically gets a packet, to be delivered by a deadline (multi-hop frame duration T_f)
- NK slots of duration T_p, $NK \triangleq \lfloor T_f / T_p \rfloor$
- Known $\rho_n E_s$ per slot, $\forall n$
- Retransmission protocol: ARQ
- Rx sends ACK/NACK, for decoding success/failure
- Tx does not have access to CSI
- A packet remains in outage if not decoded correctly
- Packet is dropped if doesn't reach $N+1^{\text{th}}$ node by the end of the frame

Problem Statement

- The *drop probability*

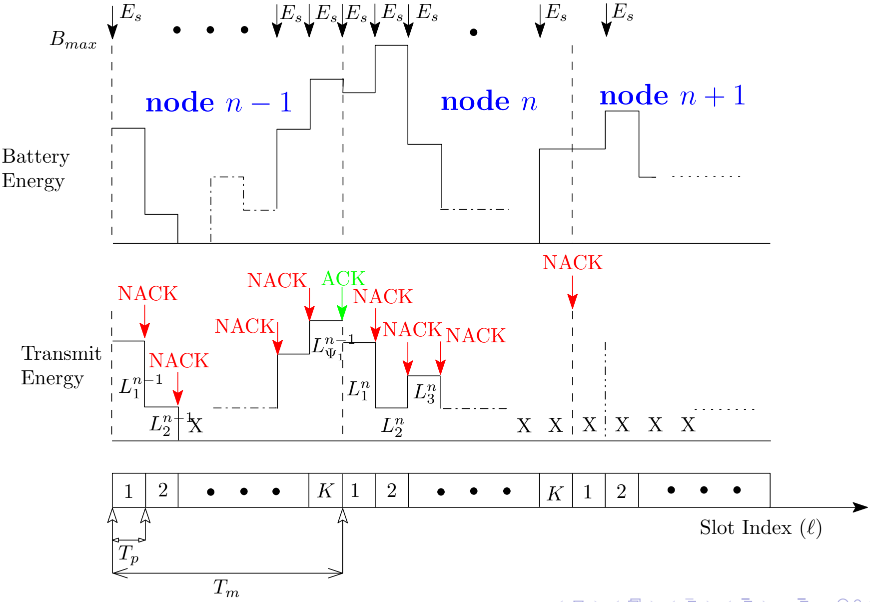
$$
\begin{aligned}
P_{\mathrm{D}} &= 1 - \Pr[N+1] \\
\text{s.t.,} \quad \sum_{t=1}^{t_0} \rho_n E_s &\geq \sum_{t=1}^{t_0} E_t^n \qquad \forall n, t_0
\end{aligned}
$$

$$
\Pr[N+1] : \Pr\left[N + 1^{\text{th}} \text{ node receive the packet correctly}\right]
$$

- Design goal:

$$
\min_{\{E_1^n, E_2^n, \ldots, E_K^n \geq 0\}_{n=1}^{N}} P_{\mathrm{D}}
$$

System Dynamics



Modified Energy Neutrality Constraint

- Average Power Constraint (APC): on average, EHN consumes energy lower than the harvesting rate
- APC with large battery capacity:
  - Battery evolution has a net positive drift
  - Battery has sufficient energy to make all K attempts
  - It is throughput optimal
- For large battery system operating under APC, ENC is *equivalent* to APC
- Infinite battery approximation
  - Maximum transmit power is limited by the RF front-end hardware
  - Finite number of attempts per packet

Problem Statement: Single-hop

- The *outage probability*

$$P_{\text{out}} = \mathbb{E}_\gamma\left\{\prod_{i=1}^{K} P_e(E_i,\gamma)\right\}$$

$$\text{where,} \quad P_e(E_i,\gamma) = \exp\left(-\frac{E_i\gamma}{N_0}\right)$$

- Energy neutrality constraint

$$\sum_{k=1}^{K} E_k \cdot \mathbb{E}_\gamma\left\{\prod_{i=1}^{k-1} P_e(E_i,\gamma)\right\} \leq K\rho E_s$$

γ : Channel State

E_i : Energy used in i^{th} attempt

- Design goal:

$$\min_{E_1,E_2,\ldots,E_K\geq 0} P_{\text{out}}$$

Transmit Power Policy

Algorithm 1 To find $E_k^*, k = 1, \ldots, K$, for a block fading channel

for $k = 1$ to K **do**

$$\text{Set } E_k^* = \left[\mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^*, \gamma) \right\} \right]^{-1} \rho E_s$$

end for

Optimality for Rayleigh Fading

- The problem statement

$$\min_{E_1,E_2,\ldots,E_K \geq 0} P_{\text{out}} = \left(1 + \sum_{k=1}^{K} \frac{E_k}{N_0}\right)^{-1}$$

$$\text{subject to } \sum_{k=1}^{K} E_k \left(1 + \sum_{i=1}^{k-1} \frac{E_i}{N_0}\right)^{-1} = K\rho E_s$$

- Equivalently the objective function is

$$\max_{E_1,E_2,\ldots,E_K \geq 0} E_{\text{sum}} = \sum_{k=1}^{K} E_k$$

- Necessary conditions for $\mathbf{E}^*$ to be optimal

$$\begin{aligned} \mathbf{E}^* = [E_1^*, \ldots, E_K^*] \succeq 0 \\ \nabla E_{\text{sum}}^* \succeq 0 \\ E_i^* (\nabla E_{\text{sum}}^*)_i = 0,\ 1 \leq i \leq K \end{aligned} \quad (1)$$

Optimality for Rayleigh Fading

Proposition

The optimal transmit energy vector (EV) allots nonzero values to all K slots

Proof Sketch

Proof is by contradiction

- Suppose the optimal EV $\mathbf{A} = [A_1, \ldots, A_K]$ has $A_{k'} = 0$ for some $1 \leq k' < K$
- Let another EV $\mathbf{B} = [A_1, \ldots, A_{k'-1}, A_{k'+1}, \ldots, B, B]$, with $B > 0$, having the same average energy consumption as $\mathbf{A}$
- Equate the average energy consumption of both policies to get $0 < A_K < 2B$
- Hence, $P_{\text{out}}(\mathbf{A}) > P_{\text{out}}(\mathbf{B})$

Optimality for Rayleigh Fading

Theorem

For Rayleigh fading channels the optimal energy vector is

$$E_k^* = \rho E_s \left(1 + \frac{\rho E_s}{N_0}\right)^{k-1} \quad (2)$$

Proof Sketch

- Convert the problem into unconstrained optimization problem by substituting for E_K
- Use induction to prove that the solution

$$E_k^* = \rho E_s \left(1 + \sum_{i=1}^{k-1} \frac{E_i^*}{N_0}\right) \quad (3)$$

satisfies $\quad \frac{\partial E_{\text{sum}}}{\partial E_k^*} = 0, \quad 1 \le k \le K-1.$

- To show uniqueness use induction again

Finite battery: Heuristic

- $\delta \downarrow 0 \Rightarrow \delta' \downarrow 0$
- For large battery:
  - Design a policy for infinite battery
  - $E_{tx} = \min(B_i, E_k^*)$

Finite battery: Exact Analysis

$$\begin{aligned} \delta' &= \Pr(B_i = B_{\max}) \text{ x } \rho E_s \\ \bar{E}_{tx} &= \bar{E}'_f = K\rho E_s - \delta' \end{aligned}$$

Algorithm 2 To find $E_k^*, k = 1, \ldots, K$, for a block fading channel

Initialize: $\Pr(B_i = B_{\max}|\bar{E}'_f) = 0$

repeat $\bar{E}'_f = K\rho E_s - \Pr(B_i = B_{\max}|\bar{E}'_f) \text{ x } K\rho E_s$

- Evaluate $E_1^*, \ldots, E_K^*$ for $\bar{E}'_f$
- Evaluate corresponding $\Pr(B_i = B_{\max}|\bar{E}'_f)$

until $\bar{E}_{tx} \neq K\rho E_s - \Pr(B_i = B_{\max}|\bar{E}'_f) \text{ x } K\rho E_s$

Simulation Results

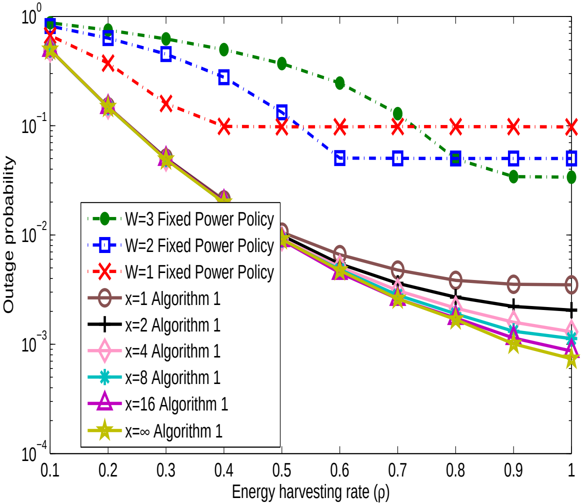


Figure : Comparison of the performance of Algorithm 1 with the fixed-energy scheme, with Es = 12 dB, K = 4, and uncoded BPSK transmission

Simulation Results

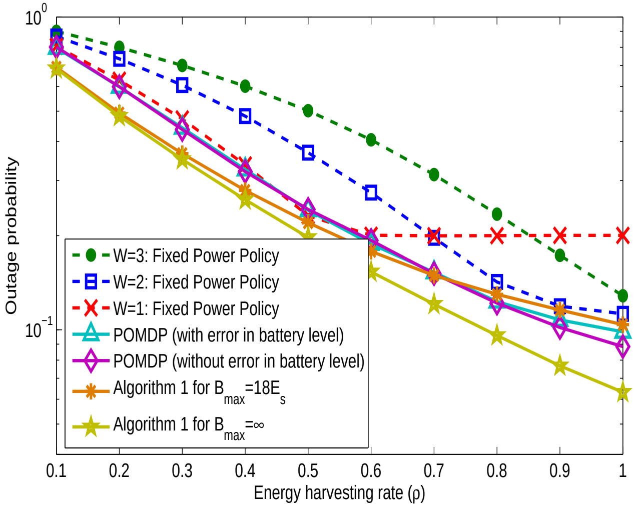


Figure : Comparison of the performance of Algorithm 1 with the POMDP solution, with Es = 0 dB, K = 4.

Back to Multihop

- **CASE 1**: Node n is assigned fixed K_n slots for transmission

$$\kappa = \sum_{n=1}^{N} K_n = NK$$

- **CASE 2**: There is no restriction on as how many, out of NK, slots each node uses
- **Assumptions**:
  - Channel remain constant throughout the NK frame
  - No energy is consumed in reception of a packet

CASE 1: Problem Statement

$$\min_{\left\{\left\{E_n^k\right\}_{k=1}^{K_n}\right\}_{n=1}^{N}} P_{\text{out}} = 1 - \max_{\left\{\left\{E_n^k\right\}_{k=1}^{K_n}\right\}_{n=1}^{N}} \Pr\left[N+1\right]$$

where

$$\Pr\left[N+1\right] = \prod_{n'=1}^{N} \mathbb{E}_\gamma \left\{ \sum_{i=1}^{K_{n'}} \prod_{\ell=1}^{i-1} \left(1 - P_e(E_i^{n^*}, \gamma)\right) P_e(E_{\ell-1}^{n^*}, \gamma) \right\}$$

subject to

$$\Pr\left[n-1\right] . \sum_{i=1}^{K_n} E_k^{n^*} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^{n^*}, \gamma) \right\} = \kappa \rho_n E_s$$

for all $n = 1, 2, \ldots, N+1$

CASE 1: Policy

- Infinite Battery

$$E_k^{n^*} = \frac{\left[\mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^{n^*}, \gamma) \right\}\right]^{-1}}{\Pr[n-1]} \text{ x } \frac{\kappa \rho_n E_s}{K_n}$$

- Finite Battery

$$\bar{E}_{tx} = \min B_n^k, E_n^{k^*}$$

CASE 2: Problem Statement

$$\min_{\left\{\left\{E_n^k\right\}_{k_{r_n}=1}^{NK}\right\}_{n=1}^{N}} P_{\text{out}} = 1 - \max_{\left\{\left\{E_n^k\right\}_{k_{r_n}=1}^{NK}\right\}_{n=1}^{N}} \Pr\left[N+1\right]$$

where

$$\begin{aligned}
\Pr\left[n\right] \quad = \quad & \sum_{i_1=1}^{KN-N+1} \mathbb{E}_{\gamma}\left\{\left(1 - P_e\left(E(1,i_1,1)^*,\gamma\right)\right) \prod_{j_1=1}^{i_1-1} P_e(E(1,i_1,1)^*,\gamma)\right\} \\
\sum_{i_2=i_1+1}^{KN-N+2} \quad & \mathbb{E}_{\gamma} \quad \left\{\left(1 - P_e\left(E(2,i_2,i_1+1)^*,\gamma\right)\right) \prod_{j_2=i_1+1}^{i_2-1} P_e(E(2,j_2,i_1+1)^*,\gamma)\right\} \\
& \vdots \\
\sum_{i_{n-1}=i_{n-2}+1}^{KN-N+n-1} \quad & \mathbb{E}_{\gamma} \quad \left\{\left(1 - P_e\left(E(n-1,j_{n-1},i_{n-2}+1)^*,\gamma\right)\right)\right. \\
& \left.\prod_{j_{n-1}=i_{n-2}+1}^{i_{n-1}-1} P_e(E(n-1,j_{n-1},i_{n-2}+1)^*,\gamma)\right\}
\end{aligned}$$

CASE 2: Problem Statement (contd.)

subject to

$$\Pr[n] \text{ x } \sum_{k=k_{r_n}}^{K-k_{r_n}-N+n+1} E(n,k,k_{r_n})^* \mathbb{E}_\gamma \left\{ \prod_{\ell=k_{r_n}}^{K-1} \mathrm{P}_e(E(n,\ell,k_{r_n})^*, \gamma) \right\}$$

for all $n = 1, 2, \ldots, N+1$, and
$k_{r_n} = n-1, \ldots, KN - k_{r_n} - N + n + 1$

CASE 2: Policy

$$
\begin{aligned}
E(n,k,k_{r_n})^* \quad = \quad & \frac{KN\rho_n E_s}{KN - k_{r_n} - N + n + 1} \text{ x } [\Pr(n)]^{-1} \\
\text{x} \quad & \left[\mathbb{E}_\gamma \left\{ \prod_{i=k_{r_n}}^{KN-k_{r_n}-N+n+1} \mathrm{P}_e(E(n,i,k_{r_n})^*, \gamma) \right\} \right]^{-1}
\end{aligned}
$$

for all $n = 1, 2, \ldots, N+1$, $k = k_{r_n}$ to NK,and
$k_{r_n} = n - 1, \ldots, KN - k_{r_n} - N + n + 1$

Conclusions

- Proposed a novel harvesting-rate optimized power management policy for EHS with ARQ-based packet (re)transmissions
- Outage-optimality of proposed algorithm is theoretically established for Rayleigh fading channels
- By design, the policy operates independent of the current battery state
- The proposed algorithm outperforms existing state-of-the-art policies, especially in the scenarios when battery state is not known accurately
- Provided it is large enough, the finiteness of battery capacity has only a minor effect on the performance.