On Outage Optimal Transmission Policies for Energy Harvesting Multi-hop Links with Retransmissions

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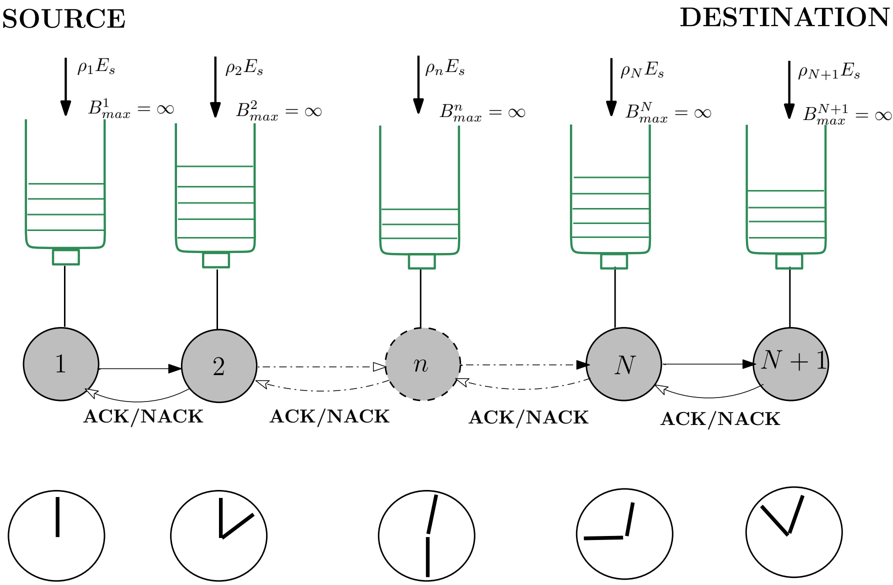
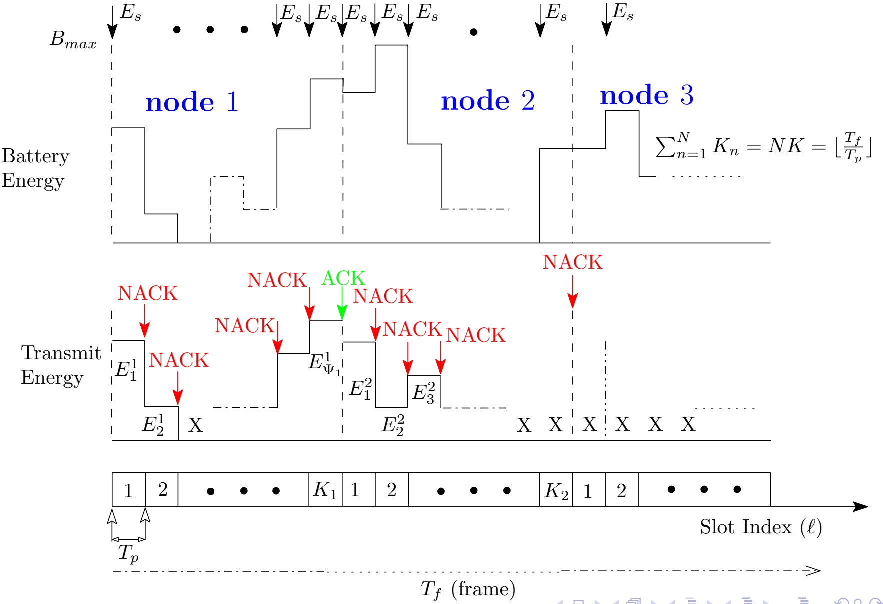
Outline

- System Model
- Problem Statement
- Geometric Programming
- Proposed Solution
- Conclusions

Multi-hop EH Links: ARQ & HARQ-CC



System Dynamics



Problem Statement (Informal)

- The *drop probability*

$$P_{\mathrm{D}} = 1 - \Pr[N+1]$$

$$\Pr[N+1] : \Pr\left[N+1^{\text{th}} \text{ node receive the packet correctly}\right]$$

- Design goal:

$$\min_{\{E_1^n, E_2^n, \ldots, E_K^n \geq 0\}_{n=1}^N} \quad P_{\mathrm{D}}$$

$$\text{s.t.,} \quad B_0^n + \sum_{t=1}^{t_0} \mathbb{1}_{\{E_t^n \neq 0\}} E_s \;\geq\; \sum_{t=1}^{t_0} E_c^n(t) \qquad \forall n, t_0$$

Modified Energy Neutrality Constraint

- PDP of a dual EH link with finite sized buffers converges to 'optimal' as $\Theta(e^{r^* B^t_{\max}}) + \Theta(e^{r^* B^r_{\max}})$, with $r^* < 0$.
- Modified energy neutrality constraint (MENC): on average, EHN consumes energy lower than the harvesting rate
- MENC with large battery capacity:
  - Battery evolution has a net positive drift
  - Battery has sufficient energy to make all K attempts
- For large battery system operating under MENC, ENC is *equivalent* to MENC

Modified Problem Statement: ARQ Slow Fading

$$\min_{\left\{\left\{E_k^n\right\}_{k=1}^{K_n}\right\}_{n=1}^{N}} \quad P_{\text{out}} = 1 - \Pr\left[N+1\right]$$

subject to

$$\underbrace{\Pr\left[n\right]. \sum_{k=1}^{K_n} E_k^{n^*} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^{n^*}, \gamma]) \right\}}_{\text{Average enrgy consumed for transmission}} \; + \; \Pr\left[n-1\right].$$

$$\underbrace{RE_s. \sum_{k=1}^{K_{n-1}} \mathbb{1}_{\left\{E_k^{(n-1)^*} \neq 0\right\}} . \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^{(n-1)^*}, \gamma) \right\}}_{\text{Average energy consumed for reception}} \; \leq \; NK\rho_n E_s$$

$\forall \quad n = 1, \ldots, N+1,$ and where

$$\Pr\left[n\right] = \overbrace{\prod_{n'=1}^{n-1} \mathbb{E}_\gamma \left\{ \sum_{i=1}^{K_{n'}} \left(1 - P_e(E_i^{n^*}, \gamma)\right) \prod_{\ell=1}^{i-1} P_e(E_\ell^{n^*}, \gamma) \right\}}^{\text{Prob. that packet reaches } n^{th} \text{ node}}$$

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$\forall \quad n = 1, \ldots, N+1,$ and where

$$\Pr[n] = \overbrace{\prod_{n'=1}^{n-1} \mathbb{E}_\gamma\left\{\sum_{i=1}^{K_{n'}}\Big(1 - P_e(E_i^{n^*},\gamma)\Big)\prod_{\ell=1}^{i-1} P_e(E_\ell^{n^*},\gamma)\right\}}^{\text{Prob. that packet reaches } n^{th} \text{ node}}$$

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$\forall \quad n = 1, \ldots, N+1$,and where

$$\Pr\left[n\right] = \overbrace{\prod_{n'=1}^{n-1}\mathbb{E}_\gamma\left\{\sum_{i=1}^{K_{n'}}\left(1 - P_e(E_i^{n^*},\gamma)\right)\prod_{\ell=1}^{i-1} P_e(E_\ell^{n^*},\gamma)\right\}}^{\text{Prob. that packet reaches } n^{th} \text{ node}}$$

Case I. Special Case
(Negligible Reception Cost)

Problem Statement: Case I

$$\min_{\left\{\left\{E_k^n\right\}_{k=1}^{K_n}\right\}_{n=1}^{N}} \quad P_{\text{out}} = 1 - \Pr\left[N+1\right]$$

subject to

$$\Pr\left[n\right] . \sum_{k=1}^{K_n} E_k^{n^*} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^{n^*}, \gamma) \right\} \leq N K \rho_n E_s \quad \forall \quad n = 1, \ldots, N+1$$

where,

$$\Pr\left[n\right] = \prod_{n'=1}^{n-1} \mathbb{E}_\gamma \left\{ \sum_{i=1}^{K_{n'}} \left(1 - P_e(E_i^{n^*}, \gamma)\right) \prod_{\ell=1}^{i-1} P_e(E_\ell^{n^*}, \gamma) \right\}$$

Problem Statement: Mono EH link with slow fading and ARQ

$$\min_{E_1,\ldots,E_K} \frac{1}{1+\sum_{i=1}^{K} E_i}$$

$$\text{s.t.} \sum_{i=1}^{K} E_i \frac{1}{1+\sum_{\ell=1}^{i-1} E_\ell} \leq K\rho E_s$$

Single hop: Transmit Power Policy [A Devraj, MS, C Murthy] GlobalSIP 14

Algorithm 1 *

Proposed Algorithm: To find $E_k^*, k = 1, \ldots, K$, for a block fading channel

Initialize: $E_1^* = \rho E_s$

for $k = 2$ **to** K **do**

$$\textbf{Set } E_k^* = \left[\mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^*, \gamma) \right\} \right]^{-1} \rho E_s$$

end for

Simulation Results

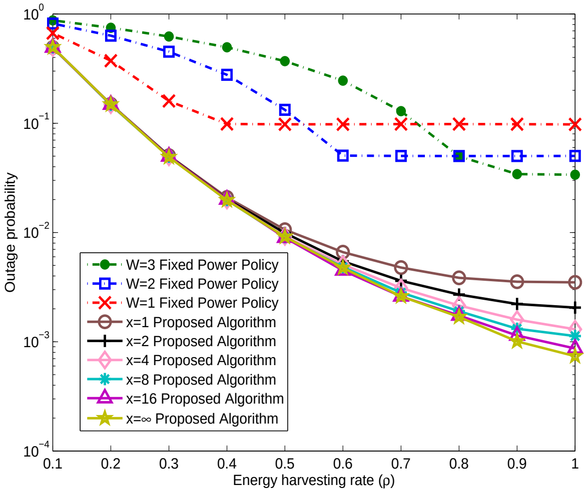

Figure: Comparison of the performance of Proposed Algorithm with the fixed-energy scheme, with $E_s = 12$ dB, $K = 4$, $B_{\max} = \mathrm{x}E_{\mathrm{av}}^{c}$, and uncoded BPSK transmission

Simulation Results

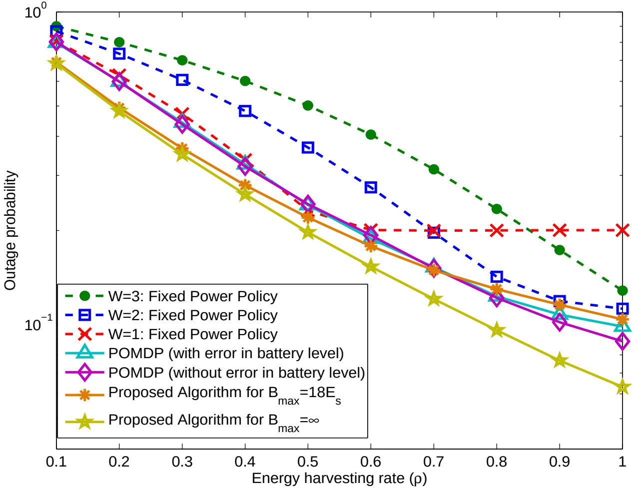

Figure: Comparison of the performance of Proposed Algorithm with the POMDP solution, with $E_s = 0$ dB, $K = 4$.

Multi-hop Case I: Optimal Transmit Power Policy

Algorithm 2 *

Proposed Algorithm: To find $E_k^{n^*} \quad \forall n, \text{and} k = 1, \ldots, K_n$, for a block fading channel

for $n = 1$ to N **do**
Initialize: $E_1^{n^*} = \rho E_s$

for $k = 2$ **to** K_n **do**

$$\textbf{Set } E_k^{n^*} = \frac{\left[\mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^*, \gamma) \right\}\right]^{-1}}{\mathbf{Pr}[n-1]} x \frac{K \rho E_s}{K_n}$$

end for
end for

Mono EH links with HARQ-CC and slow fading

$$\min_{E_1,\ldots,E_K} \frac{1}{1+\sum_{i=1}^{K}(K-i+1)E_i}$$

$$\text{s.t.} \sum_{i=1}^{K} E_i \frac{1}{1+\sum_{\ell=1}^{i-1}(i-\ell)E_\ell} \leq K\rho E_s$$

Optimal Solution satisfies

$$\frac{E_2}{1+E_1} = \cdots = \frac{E_{K-1}}{1+\sum_{i=1}^{K-2}(K-1-i)E_i} = \frac{K\rho E_s - 2E_1 + 1}{K-2}$$

$$\text{and } \frac{E_K}{1+\sum_{i=1}^{K-1}(K-i)E_i} = E_1 - 1$$

$$\text{where } E_1 = \frac{(K\rho E_s - 1)}{2K}\left(1+\sqrt{1+\frac{4K(K\rho E_s+1)}{(K\rho E_s-1)^2}}\right)$$

Mono EH links with ARQ and fast fading

$$\min_{E_1,\ldots,E_K} \frac{1}{\prod_{i=1}^{K}(E_i+1)}$$

$$\text{s.t. } \sum_{i=1}^{K} E_i \frac{1}{\prod_{\ell=1}^{i-1}(E_\ell+1)} \leq K\rho E_s$$

Optimal Solution satisfies

$$E_{i+1} = \frac{E_i(E_i+2)}{2}$$

$$E_1 \approx L\left(1+\sqrt{1+\frac{f(K)}{L^2(2^K-1)}}\right)$$

$$\text{where } L = \frac{(1+K\rho E_s)2^{K-1}}{2(2^K-1)} \text{ and } f(K) = ?$$

Case II. General Case (Non-negligible Reception Cost)

Alternative Formulation: Case II

- Subproblem 1:

$$\min_{\{(Tx_1, Rx_2), (Tx_2, Rx_3), \ldots, (Tx_N, Rx_{N+1})\}} P_{\text{out}}$$

s.t.

$$\begin{aligned} Tx_1 &\leq NK\rho_1 E_s \\ Rx_2 + Tx_2 &\leq NK\rho_2 E_s \\ &\vdots \\ Rx_N + Tx_N &\leq NK\rho_N E_s \\ Rx_{N+1} &\leq NK\rho_{N+1} E_s \end{aligned}$$

where, Rx_n, and Tx_n are average energy spent for transmission and reception by nth node

Subproblem 2 (SP2)

For all $n = 1, \ldots, N$

$$\min_{\{E_1^n, \ldots, E_{K_n}^n\}} P_{\text{out}}^n = \mathbb{E}_\gamma \left\{ \prod_{\ell=1}^{K_n} P_e(E_\ell^n, \gamma) \right\}$$

s.t.

$$\sum_{k=1}^{K_n} E_k^n \; \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^n, \gamma) \right\} \leq \frac{Tx_n}{\Pr[n]} = T_n$$

$$RE_s. \sum_{k=1}^{K_n} \mathbb{1}_{\{E_k^n \neq 0\}} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^n, \gamma) \right\} \leq \frac{Rx_{n+1}}{\Pr[n]} = R_n$$

$$0 \leq E_k^n \leq E_{\max} \qquad \forall \quad k = 1, \ldots, K_n$$

SP2 for Rayleigh fading

$$\min_{\{E_1,\dots,E_K\}} P_{\text{out}} = \left(1 + \sum_{\ell=1}^{K} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1}$$

s.t.

$$\sum_{k=1}^{K} E_k \left(1 + \sum_{\ell=1}^{k-1} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1} \leq T_n$$

$$RE_s \cdot \left[1 + \sum_{k=2}^{K} \mathbb{1}_{\{E_k \neq 0\}} \left(1 + \sum_{\ell=1}^{k-1} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1}\right] \leq R_n$$

$$0 \leq E_k \leq E_{\max} \qquad \forall \quad k = 1, \dots, K$$

Geometric Programming: Terminology

- Monomial: $f : \mathbf{R}^{\mathbf{n}}_{++} \to \mathbf{R}$:

$$f(\mathbf{x}) = c x_1^{a^{(1)}} x_2^{a^{(2)}} \dots x_n^{a^{(n)}}$$

  where, $c \geq 0$, and $a^{(j)} \in \mathbf{R}, j = 1, \dots, n$

- Posynomial: Sum of monomials

$$f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}}$$

  where, $c_k \geq 0$, $a_k^{(j)} \in \mathbf{R}$, $k = 1, 2, \dots, K$, $j = 1, 2, \dots, n$

- Examples:

$$\text{Posynomial :} \quad 2x_1^{-\pi} x_2^{0.5} + 3x_1 x_3^{100}, \frac{x_1}{x_2}$$

$$\text{Not a Posynomial :} \quad x_1 - x_2, \frac{x_1 + x_2}{x_3 + x_1}$$

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Not a Posynomial : $x_1 - x_2, \frac{x_1 + x_2}{x_3 + x_1}$

Geometric Programming: Standard form

Standard form: (non convex problem)

$$
\begin{array}{rlll}
\min & f_0(\mathbf{x}) & & \\
\text{subject to} & f_i(\mathbf{x}) & \leq 1 & \forall \quad i = 1, 2, \ldots, m \\
& h_\ell(\mathbf{x}) & = 1 & \forall \quad i = 1, 2, \ldots, M
\end{array}
$$

where,

$$
f(\mathbf{x}) = \sum_{k=1}^{K_i} c_{ik} x_1^{a_{ik}^{(1)}} x_2^{a_{ik}^{(2)}} \ldots x_n^{a_{ik}^{(n)}}
$$

$$
\text{and,} \quad h_\ell(\mathbf{x}) = c_\ell x_1^{a_\ell^{(1)}} x_2^{a_\ell^{(2)}} \ldots x_n^{a_\ell^{(n)}}
$$

Geometric Programming: Convex form

- Let $y_i = \log x_i$, $b_{ik} = \log c_{ik}$, and $b_\ell = \log d_\ell$

$$
\begin{aligned}
\min \quad P_0(\mathbf{y}) &= \log \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \mathbf{y} + b_{0k}) \\
\text{s. t.} \quad P_i(\mathbf{y}) &= \log \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \mathbf{y} + b_{ik}) \leq 0 \quad \forall \quad i = 1, 2, \ldots, m \\
q_\ell(\mathbf{y}) &= \mathbf{a}_\ell^T \mathbf{y} + b_\ell = 0 \quad \forall \quad \ell = 1, 2, \ldots, M
\end{aligned}
$$

SP2 as a Geometric Program

Let $z_i = 1 + \frac{1}{\mathcal{N}_0} \sum_{\ell=1}^{i} E_\ell \implies \quad E_i = (z_i - z_{i-1})\mathcal{N}_0$, and $Z_0 = 1$

$$\max_{\{z_1,\ldots,z_K\}} (z_K - 1)$$

s.t.

$$\sum_{\ell=1}^{K} z_\ell z_{\ell-1}^{-1} \leq \frac{T_n}{\mathcal{N}_0} + K$$

$$\sum_{k=1}^{K-1} \mathbb{1}_{\{z_{i+1} \neq z_i\}} z_i^{-1} \leq \frac{R_n}{RE_s} - 1$$

$$0 \leq (z_\ell - z_{\ell-1})\mathcal{N}_0 \leq E_{\max} \qquad \forall \quad \ell = 1, \ldots, K$$

Approximation for posynomial ratio

- Let $g(\mathbf{x}) = \sum_i u_i(\mathbf{x})$. Approximate a ratio of polynomials $\frac{f(\mathbf{x})}{g(\mathbf{x})}$ with $\frac{f(\mathbf{x})}{\tilde{g}(\mathbf{x})}$ where

$$\tilde{g}(\mathbf{x}) = \prod_i \left(\frac{u_i(\mathbf{x})}{\alpha_i} \right)^{\alpha_i} \leq g(\mathbf{x})$$

- Directly follows from AM-GM inequality $\sum_i \alpha_i v_i \geq \prod_i v_i^{\alpha_i}$
- If, $\alpha_i = \frac{u_i(\mathbf{x_0})}{g(\mathbf{x_0})} \quad \forall \quad i$, for any fixed $\mathbf{x}_0 > 0$, then

$$\tilde{g}(\mathbf{x}_0) = g(\mathbf{x}_0)$$

- $\tilde{g}(\mathbf{x}_0)$, it is the best local monomial approximation $g(\mathbf{x}_0)$ near x_0, in the sense of first order Taylor approximation.

Algorithm: finds locally optimal power allocation

Initialize with feasible $\mathbf{z} = \{z_1, \ldots, z_K\}$

1. Evaluate the denominator posynomial with the given z
2. Compute for each term i in this posynomial

$$\alpha_i = \frac{\text{value of } i\text{th term in posynomial}}{\text{value of posynomial}}$$

3. Approximate the denominator of the posynomial ratio by $\tilde{g}(\mathbf{z})$ using weights α_i
4. Solve the resulting GP
5. Go to step 1, using $\mathbf{z}$ of step 4
6. Terminate the k^{th} loop if

$$||\mathbf{z}^{(k)} - \mathbf{z}^{(k-1)}|| \leq \epsilon$$

Outputs: A locally optimal $\mathbf{z}$

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Subproblem 2 for HARQ-CC with slow fading

$$P_e\left(\sum_{\ell=1}^{i} E_\ell, \gamma\right) = \exp\left(\frac{-\sum_{\ell=1}^{i} E_\ell . \gamma}{\mathcal{N}_0}\right)$$

$$P_{\text{out}} = \mathbb{E}_\gamma\left\{\prod_{\ell=1}^{K} P_e\left(\sum_{\ell=1}^{i} E_\ell, \gamma\right)\right\}$$

for Rayleigh fading

$$P_{\text{out}} = \frac{1}{\sum_{\ell=1}^{i}(K-\ell+1)E_\ell}$$

Subproblem 2 for HARQ-CC with slow fading

$$
\begin{aligned}
\max_{\{E_1,\ldots,E_K\}} \quad & \sum_{\ell=1}^{K} (K-\ell+1)E_\ell \\
\text{s.t.} \quad & \sum_{\ell=1}^{K} E_\ell \left(\sum_{i=1}^{\ell-1} (\ell-i+1)E_i + 1 \right)^{-1} \leq E_{av}^t \\
& \sum_{\ell=1}^{K} \mathbb{1}_{\{E_{\ell \neq 0}\}} E_\ell \left(\sum_{i=1}^{\ell-1} (\ell-i+1)E_i + 1 \right)^{-1} \leq \frac{E_{av}^r}{RE_s} \\
& 0 \leq E_i \leq E_{\max}
\end{aligned}
$$

Converting to GP

Let $z_1 = 1 + E_1$, $z_2 = 1 + 2E_1 + E_2$, $z_3 = 1 + 3E_1 + 2E_2 + E_3$

$$E_i = z_i - 2z_{i-1} + z_{i-2}$$

$$\begin{aligned} \max_{\{z_1,\ldots,z_K\}} \quad & z_K \\ \text{s.t.} \quad & \sum_{\ell=1}^{K} z_{\ell-1}^{-1}(z_\ell + z_{\ell-2}) \le E_{av}^t - 2K + 1 \\ & \sum_{\ell=1}^{K} z_{\ell-1}^{-1} \le \frac{E_{av}^r}{RE_s} \\ & 0 \le z_i - 2z_{i-1} + z_{i-2} \le E_{\max} \end{aligned}$$

- Problem for fast fading with HARQ-CC can also be converted to GP

Subproblem 1

- Can be solved using merit-based sequential quadratic programming (MSQP)
- MSQP guarantees global convergence to local optimum under some weak conditions.

Future Work

- Numerical evaluation of proposed policies
- Extension to dynamic slot allocation case