Through-optimal Power Control Policies for Uncoordinated Energy Harvesting Links

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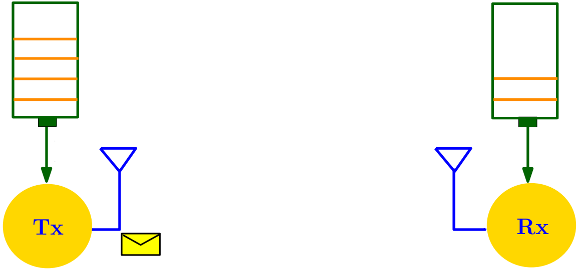
Introduction

- Energy Neutrality Constraint:

  Cumulative energy consumed $\leq$ Cumulative energy harvested

- Must be satisfied always.
- Dual EH links: P2P links with EH transmitter and receiver.

System Model: uncoordinated case



- Tx and Rx does not have the BSI of the other node.

CASE-I: Constant Decoding Energy

- Receiver uses R amount of energy to sample and decode a packet.
- Goal: Maximize the long-term time averaged utility

$$\max_{e_n^t, e_n^r} \quad \mathcal{U} = \max_{e_n^t, e_n^r} \left(\liminf_{T \to \infty} \frac{1}{T} \sum_{n=1}^{T} \mathbb{1}_{\{e_n^r \neq 0\}} U(e_n^t) \right)$$

where $U(.)$ is a concave non-decreasing function.

Bound 1

- **Lemma**
  For an uncoordinated dual EH link the time-averaged utility is upper bounded as

$$\mathcal{U} \leq \min\left(\frac{U(\mu_t)}{U(B^t_{\max})}, \frac{\mu_r}{R}\right) U(B^t_{\max}).$$

  - Proof:

$$\begin{aligned}
\mathcal{U} &= \liminf_{T\to\infty} \sum_{n=1}^{T} \mathbb{1}_{\{e^r_n \neq 0\}} U(e^t_n) \overset{(a)}{=} \mathbb{E}\left[\mathbb{1}_{\{e^r_n \neq 0\}} U(e^t_n)\right], \\
&\overset{(b)}{=} \min\left(\frac{\mathbb{E}\left[U(e^t_n)\right]}{U(B^t_{\max})}, \mathbb{E}\left[\mathbb{1}_{\{e^r_n \neq 0\}}\right]\right) U(B^t_{\max}), \\
&\overset{(c)}{\leq} \min\left(\frac{U\left[\mathbb{E}(e^t_n)\right]}{U(B^t_{\max})}, \mathbb{E}\left[\mathbb{1}_{\{e^r_n \neq 0\}}\right]\right) U(B^t_{\max}), \\
&\overset{(d)}{\leq} \min\left(\frac{U(\mu_t)}{U(B^t_{\max})}, \frac{\mu_r}{R}\right) U(B^t_{\max}).
\end{aligned}$$

Optimal Policy

- SCENARIO I: $\frac{\mu_r}{R} \geq 1$

$$e_n^t = \begin{cases} \mu_t + \delta, & \text{if } B_n^t \geq \frac{B_{\max}^t}{2}, \\ \min\{\mu_t - \delta, B_n^t\}, & \text{if } B_n^t < \frac{B_{\max}^t}{2}. \end{cases} \quad (1)$$

- SCENARIO II($\frac{\mu_r}{R} < 1$): The receiver employs a policy where it turns on after every N_r slot which is given as follows

$$N_r = \begin{cases} N = \lceil \frac{R}{\mu_r} \rceil, & \text{if } B_n^r < \frac{B_{\max}^r}{2}, \\ N = \lfloor \frac{R}{\mu_r} \rfloor, & \text{if } B_n^r \geq \frac{B_{\max}^r}{2}. \end{cases}$$

In each slot transmitter allocates the energy according to (1), and transmits the accumulated energy in the next slot when the receiver is on.

Performance: SCENARIO I

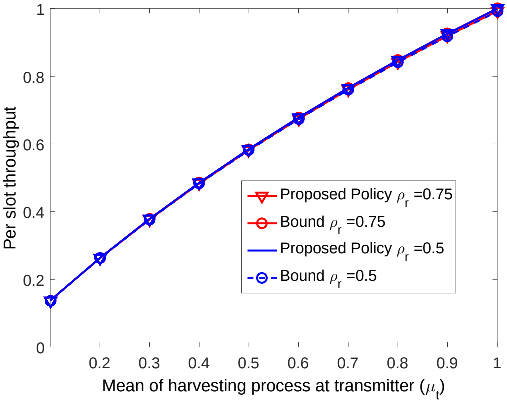


Figure: Comparison of upper bound with policy in (1). The parameters chosen are $B^t_{max} = B^r_{max} = 50, R = 0.5$

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Performance: SCENARIO II

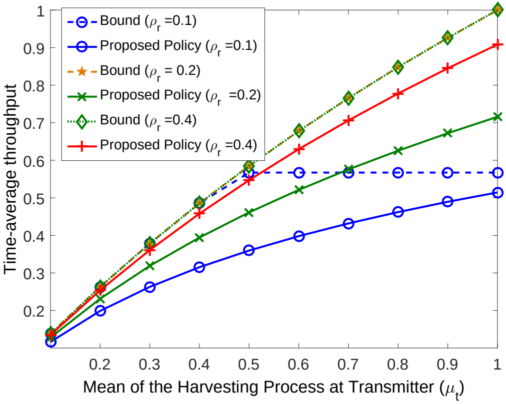

Figure: Case II: Comparison of upper bound and policy in (6). The parameters chosen are $B^t_{\max} = B^r_{\max} = 50$

Bound 2

- SCENARIO I $\left(\frac{\mu_r}{R} \geq 1\right)$: Receiver is always on

$$\mathcal{U} \leq U(\mu_t)$$

- SCENARIO II $\left(\frac{\mu_r}{R} < 1\right)$: Receiver turns on intermittently
  1. Consider a system with infinite size battery
  2. Assume all the energy at both nodes arrive at the start.

$$\begin{aligned} \text{Total energy at transmitter} &= T\mu_t \\ \text{Total energy at receiver} &= T\mu_r \\ \text{Number of slots coummunication is possible} &= \left\lfloor \frac{T\mu_r}{R} \right\rfloor \end{aligned}$$

Bound 2

- We equally divide the power over the slots in which communication happen, i.e., $e_n^t = \frac{T\mu_t}{N'}$ where $N' \triangleq \left\lfloor \frac{T\mu_r}{R} \right\rfloor$

$$\begin{aligned} \mathcal{U} &\leq \frac{1}{T}\sum_{n=1}^{N'} \mathbb{1}_{\{e_n^r \neq 0\}} U(e_n^t) \\ &= \frac{1}{T}\sum_{n=1}^{N'} \mathbb{1}_{\{e_n^r \neq 0\}} U\left(\frac{T\mu_t}{N'}\right) \\ &= \frac{1}{T}\left\lfloor \frac{T\mu_r}{R} \right\rfloor U\left(\frac{T\mu_t}{N'}\right) \end{aligned}$$

Performance: SCENARIO II

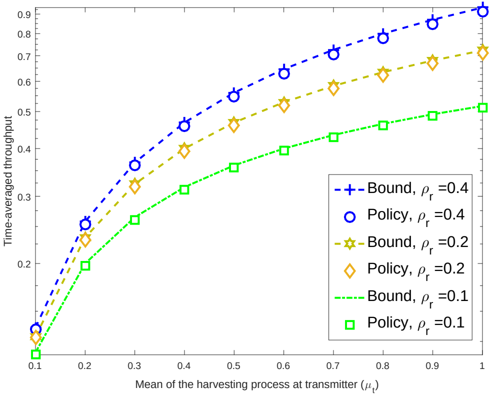

Figure: Case II: Comparison of modified upper bound and policy in (6). The parameters chosen are $B^t_{\max} = B^r_{\max} = 50$

Analysis of the battery size

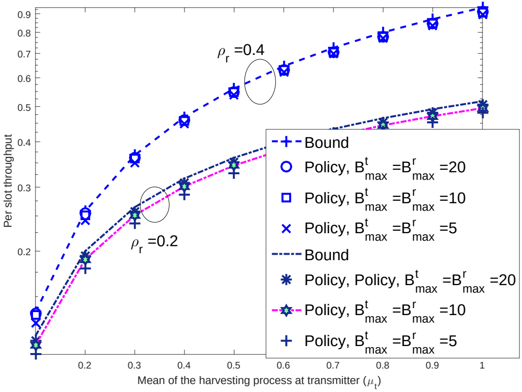

Figure: Case II: Comparison of modified upper bound and policy in (6). The parameters chosen are $R = 0.5$