Harvesting Rate Optimized Transmit Power Control Policies in Wireless Energy Harvesting Sensors with Retransmissions

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Transmission Timeline

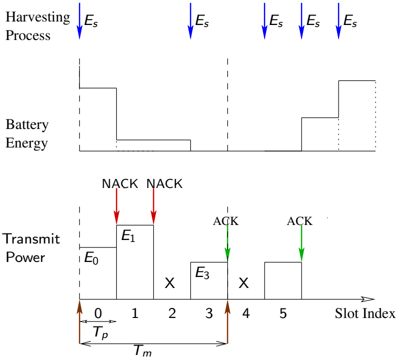

Figure : Transmission timeline of the EH node for $K = 4$, showing the random energy harvesting process $(\downarrow)$ and periodic data arrival $(\uparrow)$.The marker “X” denotes slots where the EHS does not transmit data

- Energy Injection Process:
  - Every slot, E_s energy is harvested with probability ρ
  - No energy is harvested with probability 1-ρ
- Channel:
  - Slow fading: $T_c = T_m$
  - Fast fading: $T_c = T_p$
- Packet error probability:

$$\text{For BPSK: } P_e(E_i, \gamma) = 1 - \left(1 - Q\left(\sqrt{\frac{2\gamma E_i}{N_0}}\right)\right)^L \quad (1)$$

$$\text{Convex approximation: } P_e(E_i, \gamma) \approx \exp\left(-\frac{E_i \gamma}{N_0}\right) \quad (2)$$

- Feedback:
  - If packet is in error: (NAK) is sent
  - If packet is successfully decoded: (ACK) is sent
- Performance metric:

$$\text{Outage probability} = \frac{\text{number of packets discarded}}{\text{number of frames}} \quad (3)$$

Previous Works

- Heuristic policies
- Decision-theoretic policies (using dynamic programming):
  - Completely observable MDP
  - Partially observable MDP (POMDP)
  - Channel independent MDP
- Issues with decision-theoretic policies:
  - Hard to compute; except under certain conditions
  - Accurate knowledge of the battery energy level
  - Lack of insight into the structure of optimal solution

This Work

- *Harvesting rate optimized fixed energy transmission scheme*
  - Optimal fixed energy transmission scheme for a given ρ
  - Transmit energy fixed through all K slots for all the frames
- *Harvesting rate optimized variable transmit energy scheme*
  - Transmit energy varies from one retransmission index to the other
  - The transmit energy vector for a given ρ remains same throughout

Infinite Battery Capacity Assumption

- Replace instantaneous power constraint with an average energy constraint
  - Valid only when we have an infinite energy buffer
  - Completely absorbs the randomness in energy harvesting
- As long as $\sum_{k=1}^{K} E_k \ll B_{\max}$, infinite battery capacity is a reasonable approximation

Harvesting Rate Optimized Fixed Energy Transmission Scheme

- Transmit energy is fixed through out the transmission process
- Every slot, node harvests E_s with probability ρ
- Average energy harvested per frame: $\bar{E}_f = K\rho E_s$
- Fixed transmit energy: $E_{\text{tx}} = \epsilon K\rho E_s$
    - Find the optimum ϵ value

Slow Fading

- Outage probability:

$$P_{\text{out}} = \mathbb{E}_\gamma \left\{ P_e^K(\epsilon K \rho E_s, \gamma) \right\} \quad (4)$$

- Average energy used per frame:

$$\begin{aligned} \bar{E_{\text{tx}}} =& \mathbb{E}_\gamma \Big\{ \epsilon K \rho E_s (1 - P_e(\epsilon K \rho E_s, \gamma)) \\ & + 2\epsilon K \rho E_s (P_e(\epsilon K \rho E_s, \gamma))(1 - P_e(\epsilon K \rho E_s, \gamma)) + \ldots \\ & + k\epsilon K \rho E_s (P_e(\epsilon K \rho E_s, \gamma))^{k-1}(1 - P_e(\epsilon K \rho E_s, \gamma)) + \ldots \\ & + K^2 \epsilon \rho E_s (P_e(\epsilon K \rho E_s, \gamma))^{K-1} \Big\} \end{aligned} \quad (5)$$

- Energy unconstrained regime ($\bar{E_{\text{tx}}} \leq \bar{E_f}$):

$$\mathbb{E}_\gamma \left\{ \sum_{k=0}^{K-1} \epsilon K \rho E_s \Big(P_e(\epsilon K \rho E_s, \gamma) \Big)^k \right\} \leq K \rho E_s \quad (6)$$

Remark 1:

Given that we make all K attempts of transmission in a frame with non-zero transmit energy, for an optimum value of $\epsilon \triangleq \epsilon^*$ resulting in the minimum outage probability, (6) holds by equality:

$$\mathbb{E}_\gamma\left\{\sum_{k=0}^{K-1}\epsilon^* K\rho E_s\Big(P_e(\epsilon^* K\rho E_s,\gamma)\Big)^k\right\} = K\rho E_s \quad (7)$$

- If $\bar{E}_{\text{tx}} < \bar{E}_f$:
  - Not using enough energy
- If $\bar{E}_{\text{tx}} > \bar{E}_f$:
  - Unable to make K transmissions

Fast Fading

- Outage probability:

$$P_{\text{out}} = \left(\mathbb{E}_{\gamma}\left\{P_e(\epsilon K \rho E_s, \gamma)\right\}\right)^K \tag{8}$$

- Average energy used per frame:

$$\bar{E_{\text{tx}}} = \sum_{k=0}^{K-1} \epsilon K \rho E_s \left(\mathbb{E}_{\gamma}\{P_e(\epsilon K \rho E_s, \gamma)\}\right)^k \tag{9}$$

- Energy unconstrained regime ($\bar{E_{\text{tx}}} \leq \bar{E_f}$):

$$\sum_{k=0}^{K-1} \epsilon K \rho E_s \left(\mathbb{E}_{\gamma}\{P_e(\epsilon K \rho E_s, \gamma)\}\right)^k \leq K \rho E_s \tag{10}$$

- *Remark 1* follows

Harvesting Rate Optimized Variable Transmit Energy Scheme

- Choose different ϵ for different k
  - Transmit with $E_k = \epsilon_k K \rho E_s$ in the k^{th} slot
- ϵ_k for a given k remains constant for all the frames
- Find the optimum transmit energy vector: $[\epsilon_1 K \rho E_s, \ldots, \epsilon_K K \rho E_s]$ that minimizes the outage

Slow Fading

- Outage probability:

$$P_{\text{out}} = \mathbb{E}_\gamma \Big\{ \prod_{i=1}^{K} P_e(\epsilon_i K \rho E_s, \gamma) \Big\} \tag{11}$$

- Average energy used per frame:

$$\bar{E_{\text{tx}}} = \sum_{k=1}^{K} \epsilon_k K \rho E_s \left\{ \mathbb{E}_\gamma \Big\{ \prod_{i=1}^{k-1} P_e(\epsilon_i K \rho E_s, \gamma) \Big\} \right\} \tag{12}$$

- Constraint:

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \left\{ \mathbb{E}_\gamma \Big\{ \prod_{i=1}^{k-1} P_e(\epsilon_i K \rho E_s, \gamma) \Big\} \right\} \leq K \rho E_s \tag{13}$$

Remark 2:

The optimal transmit energy vector to minimize the outage probability given by (11) satisfies (13) by equality:

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \left\{ \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(\epsilon_i K \rho E_s, \gamma) \right\} \right\} = K \rho E_s \tag{14}$$

- Optimization problem:
  - minimize:

$$P_{\text{out}} = \mathbb{E}_\gamma \left\{ \prod_{i=1}^{K} P_e(\epsilon_i K \rho E_s, \gamma) \right\}$$

  - subject to:

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \left\{ \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(\epsilon_i K \rho E_s, \gamma) \right\} \right\} = K \rho E_s$$

- Need a closed form expression for $P_e(E_i, \gamma)$ to evaluate the optimization problem
- Even if $P_e(E_i, \gamma)$ is approximated to a strict convex function (2), Hessian of the constraint $\bar{E}_{\text{tx}}$ is not necessarily positive semidefinite
  - Can not use standard convex optimization tools (e.g., Lagrange multiplier method)
- Following algorithm can be used to obtain the transmit energy values:

Algorithm 1 To find ϵ_k^* values for a slow fading channel

Define: $\epsilon_0^* = 0$; Therefore, $P_e(\epsilon_0 K \rho E_s, \gamma) = 1$;
for $k = 1$ to K **do**
$\triangleright$ Solve for ϵ_k^* using the values of $\epsilon_0^*, \ldots, \epsilon_{k-1}^*$ in:

$$\epsilon_k K \rho E_s \left\{ \mathbb{E}_\gamma \left\{ \prod_{i=0}^{k-1} P_e(\epsilon_i^* K \rho E_s, \gamma) \right\} \right\} = \rho E_s \tag{15}$$

end for

- Based on the fact that on an average, ρE_s arrives every slot
- The solution in closed form:

$$E_k^* = \epsilon_k^* K \rho E_s = \frac{\rho E_s}{\mathbb{E}_\gamma \left\{ \prod_{n=0}^{k-1} f_{\text{Pe}}(n, \gamma) \right\}} \tag{16}$$

  - Where:

$$f_{\text{Pe}}(n, \gamma) = P_e \left(\frac{\rho E_s}{\mathbb{E}_\gamma \left\{ \prod_{i=1}^{n-1} f_{\text{Pe}}(i, \gamma) \right\}}, \gamma \right) \quad \forall n \geq 1 \tag{17}$$

with $f_{\text{Pe}}(0, \gamma) = 1$;

Proposition

For a slow fading channel and the approximate probability of error function $P_e(E_i, \gamma) \approx \exp\left(-\frac{E_i \gamma}{N_0}\right)$, the optimal transmit energy level for each (re)transmission attempt is obtained using (16).

- For the approximation, the optimization problem to be solved is:

$$\text{constraint: } \sum_{k=1}^{K} \frac{E_k}{\left(1 + \sum_{i=1}^{k-1} E_i / N_0\right)} = \bar{E}_f \tag{18}$$

$$\text{minimize: } P_{\text{out}} = \frac{1}{1 + \sum_{k=1}^{K} E_k / N_0} \tag{19}$$

$$\text{or maximize: } E_{\text{sum}} = \sum_{k=1}^{K} E_k \tag{20}$$

- To prove that the optimal solution to the problem is given by:

$$E_k^* = \frac{\bar{E}_f}{K}\left(1 + \sum_{i=1}^{k-1} E_i^* / N_0\right) \tag{21}$$

- The problem is a K dim. constrained optimization problem
- We can write E_K in terms of $\bar{E}_f, E_1, \ldots, E_{K-1}$
  - Reduces to $K-1$ dim. unconstrained optimization problem
- For optimality, it is sufficient to prove that (21) holds with:

$$\frac{\partial E_{\text{sum}}}{\partial E_k^*} = 0 \quad \forall \;\; 1 \leq k \leq K-1 \tag{22}$$

- Proof is by mathematical induction; consider $K = 2$:

$$\bar{E}_f = E_1 + \frac{E_2}{1 + E_1/N_0}$$
$$E_2 = (\bar{E}_f - E_1)(1 + E_1/N_0) \quad (23)$$

- Substituting in (20) for $K = 2$:

$$E_{\text{sum}} = E_1 + (\bar{E}_f - E_1)(1 + E_1/N0) \quad (24)$$

- Differentiating E_{sum} w.r.t E_1:

$$\frac{\partial E_{\text{sum}}}{\partial E_1} = 1 - (1 + E_1/N_0) + (\bar{E}_f - E_1)/N_0 = 0 \quad (25)$$
$$\Rightarrow E_1 = \frac{\bar{E}_f}{2} \quad (26)$$
$$\Rightarrow E_2 = \frac{\bar{E}_f}{2}(1 + E_1/N_0) \quad (27)$$

- (21) holds for $K = 2$

- Assume that (21) holds for $K = K'$; the problem for $K = K' + 1$ is:

$$\max_{E_1,\ldots,E_{K'+1}} \sum_{k=1}^{K'+1} E_k \tag{28}$$

$$\text{subject to: } \sum_{k=1}^{K'+1} \frac{E_k}{\left(1 + \sum_{i=1}^{k-1} E_i / N_0\right)} = \bar{E}_f \tag{29}$$

- Fix E_1: $0 \leq E_1 \leq \bar{E}_f$; above problem reduces to:

$$\max_{E_2,\ldots,E_{K'+1}} \sum_{k=2}^{K'+1} E_k \tag{30}$$

$$\text{subject to: } \sum_{k=2}^{K'+1} \frac{E_k}{\left(1 + \sum_{i=2}^{k-1} E_i / N_0'\right)} = (\bar{E}_f - E_1)(1 + E_1/N_0) \tag{31}$$

$$\text{where: } N_0' = N_0(1 + E_1/N_0)$$

- The problem is a $K = K'$ case, with $k = 2, \ldots, K'+1$
  - Solution (21) holds for $K = K'$:

$$E_k^* = \frac{(\bar{E}_f - E_1)(1 + E_1/N_0)}{K'} \left(1 + \sum_{i=2}^{k-1} E_i^*/N_0'\right) \forall \;\; 2 \leq k \leq K'+1 \quad (32)$$

  - Reduces to:

$$E_k^* = \frac{(\bar{E}_f - E_1)(1 + E_1/N_0)}{K'} \left(1 + \frac{\bar{E}_f - E_1}{K'N_0}\right)^{k-2} \quad (33)$$

- Only variable left to be maximized in (28) is E_1:

$$\max_{E_1,\ldots,E_{K'+1}} \sum_{k=1}^{K'+1} E_k = \max_{0 \leq E_1 \leq \bar{E}_f} E_1 + \sum_{k=2}^{K'+1} E_k^* \quad (34)$$

$$= \max_{0\leq E_1 \leq \bar{E}_f} E_1 + \frac{(\bar{E}_f - E_1)(1 + E_1/N_0)}{K'} \times \sum_{k=2}^{K'+1} \left(1 + \frac{\bar{E}_f - E_1}{K' N_0}\right)^{k-2} \quad (35)$$

$$= \max_{0\leq E_1 \leq \bar{E}_f} E_1 + \frac{(\bar{E}_f - E_1)(1 + E_1/N_0)}{K'} \times \sum_{k=0}^{K'-1} \left(1 + \frac{\bar{E}_f - E_1}{K' N_0}\right)^{k} \quad (36)$$

$$= \max_{0\leq E_1 \leq \bar{E}_f} E_1 - N_0(1 + E_1/N_0) \times \left(1 - \left(1 + \frac{\bar{E}_f - E_1}{K' N_0}\right)^{K'}\right) \quad (37)$$

Differentiating w.r.t E_1:

$$\frac{\partial E_{\text{sum}}}{E_1} = \left(1 + \frac{\bar{E}_f - E_1}{K' N_0}\right)^{K'} - (1 + E_1/N_0)\left(1 + \frac{\bar{E}_f - E_1}{K' N_0}\right)^{K'-1} = 0 \quad (38)$$

$$E_1^* = \frac{\bar{E}_f}{K' + 1} \quad (39)$$

Remarks

Remark 3:

For an infinite battery capacity, the optimal transmit energy vector depends only on the average energy arrival per slot ρE_s and not on the individual values of ρ and E_s.

Remark 4:

For an optimal policy, the energy with which a (re)transmission attempt is made, is simply the expected energy that will be available in the battery at the start of that attempt.

Remark 5:

In any given frame, the optimal transmit energy level at a higher retransmission attempt is higher; i.e., the optimal transmit energy policy is a *strictly* increasing policy $(E_1^* < E_2^* \ldots < E_K^*)$, with non-zero transmit energy levels.

Fast Fading

- Outage probability:

$$P_{\text{out}} = \prod_{i=1}^{K} \mathbb{E}_{\gamma}\left\{P_e(\epsilon_i K \rho E_s, \gamma)\right\} \tag{40}$$

- Average energy used per frame:

$$\bar{E_{\text{tx}}} = \sum_{k=1}^{K} \epsilon_k K \rho E_s \prod_{i=1}^{k-1} \mathbb{E}_{\gamma}\left\{P_e(\epsilon_i K \rho E_s, \gamma)\right\} \tag{41}$$

- Introducing the constraint ($\bar{E_{\text{tx}}} \leq \bar{E_f}$):

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \prod_{i=1}^{k-1} \mathbb{E}_{\gamma}\left\{P_e(\epsilon_i K \rho E_s, \gamma)\right\} \leq K \rho E_s \tag{42}$$

- Remark 2 follows; (42) satisfied by equality:

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \prod_{i=1}^{k-1} \mathbb{E}_{\gamma} \Big\{ P_e(\epsilon_i K \rho E_s, \gamma) \Big\} = K \rho E_s \tag{43}$$

- Optimization problem:
  - minimize:

$$P_{\text{out}} = \prod_{i=1}^{K} \mathbb{E}_{\gamma} \Big\{ P_e(\epsilon_i K \rho E_s, \gamma) \Big\}$$

  - subject to:

$$\sum_{k=1}^{K} \epsilon_k K \rho E_s \prod_{i=1}^{k-1} \mathbb{E}_{\gamma} \Big\{ P_e(\epsilon_i K \rho E_s, \gamma) \Big\} = K \rho E_s$$

Dual Problem [H. Seo and B. G. Lee, 2007]

- Define $P_{e_{\text{avg}}}(E_i) = \mathbb{E}_\gamma\left\{P_e(E_i, \gamma)\right\}$
- Optimization problem addressed:
  - minimize: $\bar{E_{\text{tx}}} = \sum_{k=1}^{K} E_k \prod_{i=1}^{k-1} P_{e_{\text{avg}}}(E_i)$
  - subject to: $\text{FDR} = \prod_{i=1}^{K} P_{e_{\text{avg}}}(E_i) = c$
- Solution:

$$\frac{P_{e_{\text{avg}}}(E_{i+1}^*)}{P'_{e_{\text{avg}}}(E_{i+1}^*)} - E_{i+1}^* - \frac{1}{P'_{e_{\text{avg}}}(E_i^*)} = 0 \tag{44}$$

Remark 5:

The solution to the problem of minimizing the average energy used per frame subject to a given outage probability constraint is equivalent to that of the dual problem of minimizing the outage probability subject to a given average energy constraint.

- Policy requires $P_{e_{\text{avg}}}(E_i)$ to be analytic and strictly convex
- Alternative suboptimal policy:

Algorithm 2 To find ϵ_k^* values for a fast fading channel

Define: $\epsilon_0^* = 0$; Therefore, $P_e(\epsilon_0 K \rho E_s, \gamma) = 1$;
for $k = 1$ to K **do**
▷ Solve for ϵ_k^* using the values of $\epsilon_0^*, \ldots, \epsilon_{k-1}^*$ in:

$$\epsilon_k K \rho E_s \prod_{i=1}^{k-1} \mathbb{E}_\gamma \Big\{ P_e(\epsilon_i^* K \rho E_s, \gamma) \Big\} = \rho E_s \tag{45}$$

end for

- Does not require $P_{e_{\text{avg}}}(E_i)$ to be analytic or strictly convex
- Provides insight into the optimal policy

- The solution in closed form is given as:

$$E_k^* = \epsilon_k^* K \rho E_s = \frac{\rho E_s}{\prod_{n=0}^{k-1} \mathbb{E}_\gamma \left\{ f_{\mathrm{Pe}}(n,\gamma) \right\}} \tag{46}$$

  - Where:

$$f_{\mathrm{Pe}}(n,\gamma) = P_e \left(\frac{\rho E_s}{\prod_{i=1}^{n-1} \mathbb{E}_\gamma \left\{ f_{\mathrm{Pe}}(i,\gamma) \right\}}, \gamma \right) \quad \forall n \geq 1 \tag{47}$$

with $f_{\mathrm{Pe}}(0,\gamma) = 1$;

Finite Battery Capacity

- In the case of infinite energy buffer:
  - No energy lost because of the battery being full
  - Randomness in harvesting is completely absorbed
- Infinite battery capacity assumption for a battery with finite energy buffer is valid when:

$$\left(\sum_{k=1}^{K} \epsilon_k K \rho E_s\right) \ll B_{\max} \quad (48)$$

- What if the battery is not that large?
  - But large enough to allow all K transmission attempts:

$$\left(\sum_{k=1}^{K} \epsilon_k K \rho E_s\right) < B_{\max} \quad (49)$$

  - Number of instances when the battery hits its upper limit
    - Energy arriving at such instances can not be harvested
    - Average energy harvested:

$$\bar{E}'_f = \bar{E}_f - \delta = K \rho E_s - \delta \quad (50)$$

Exact Modification

- Optimal transmit energy vector should satisfy:

$$\bar{E_{\text{tx}}} = \bar{E_f'} = K\rho E_s - \delta \tag{51}$$

- Where $\delta = \Pr(B_i = B_{\max}) \times \rho \times E_s$
  - $\Pr(B_i = B_{\max})$: steady state probability of the battery being full.[1]
  - $\Pr(B_i = B_{\max})$ and $\bar{E_{\text{tx}}}$ are interrelated
- Transmit energy vector for a finite battery buffer can be obtained as:

Algorithm 3 To find ϵ_k^* values for a finite battery capacity

Initialize: $\Pr(B_i = B_{\max}|\bar{E_f'}) = 0$

do

▷ $\bar{E_f'} = K\rho E_s - \Pr(B_i = B_{\max}|\bar{E_f'}) \times \rho E_s$

▷ Evaluate $\epsilon_1^*, \ldots, \epsilon_K^*$ for the given $\bar{E_f'}$

▷ Evaluate corresponding $\Pr(B_i = B_{\max}|\bar{E_f'})$

while $(\bar{E_{\text{tx}}} \neq K\rho E_s - \Pr(B_i = B_{\max}|\bar{E_f'}) \times \rho E_s)$

[1] 2012, A. Aprem and C. R. Murthy

Heuristic Modification

- Exact modification: does not take into account the sporadicity of harvesting
- Following heuristic modification can be used:
  - Transmit energy at the k^{th} slot and i^{th} instant:

$$E_{\text{tx}} = \min(B_i, \epsilon_k K \rho E_s) \tag{52}$$

  - Heuristically takes into account the loss of energy
  - Does not allow the battery to reach its full state too often

Simulations and Results

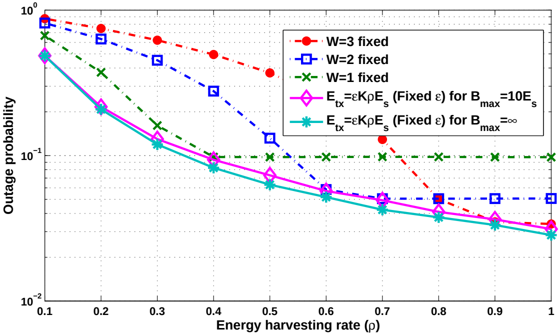

Figure : Comparison of the performance of the harvesting rate optimized fixed energy transmission scheme with the standard fixed energy transmission schemes for $B_{\max} = 10E_s$ for a slow fading channel. An additional plot of outage probability obtained using the same policy but for an infinite energy buffer is also plotted.

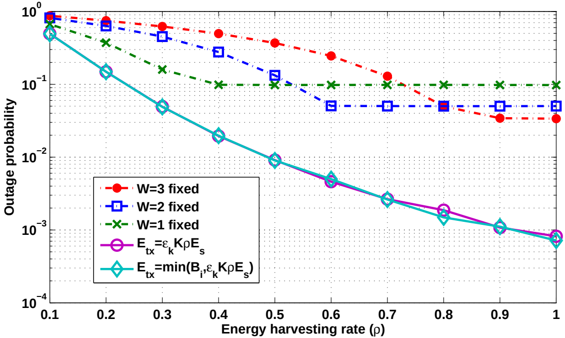

Figure : Performance evaluation of the harvesting rate optimized variable transmit energy policy for a slow fading channel and a transmitter with infinite energy buffer. Here, $E_s = 12$dB, $D = 50$ and $K = 4$.

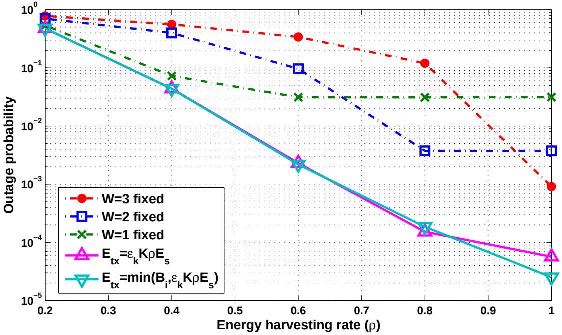

Figure : Performance evaluation of the harvesting rate optimized variable transmit energy policy for a fast fading channel and a transmitter with infinite energy buffer. Here, $E_s = 5\text{dB}$, $D = 20$ and $K = 4$.

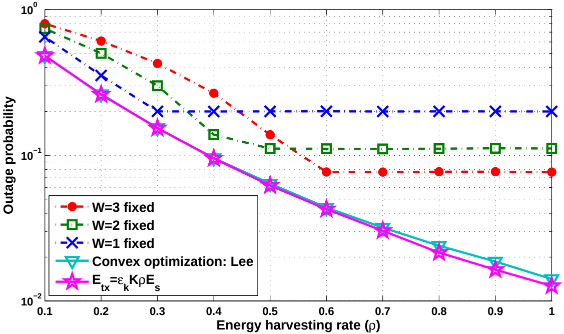

Figure : Comparison of $E_{\text{tx}} = \epsilon_k K \rho E_s$ policy with policy obtained in [H. Seo and B. G. Lee, 2007] for a slow fading channel. Here, $E_s = 1.2\text{dB}$ and $K = 4$.

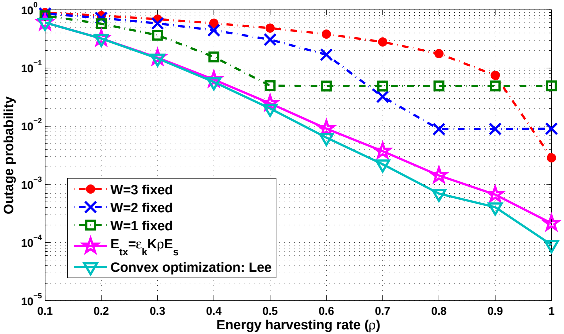

Figure : Comparison of $E_{\text{tx}} = \epsilon_k K \rho E_s$ policy with policy obtained in [H. Seo and B. G. Lee, 2007] for a fast fading channel. Here, $E_s = 0.5$dB and $K = 4$.

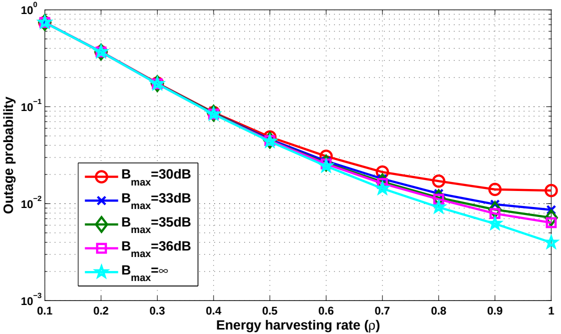

Figure : Comparison of the performance of the harvesting rate optimized variable transmission energy scheme for different finite battery capacities when the channel is slow fading. Here, $E_s = 10$dB, $D = 50$, $K = 4$ and $\sum_{k=1}^{K} \epsilon_k K \rho E_s \approx 30$dB.

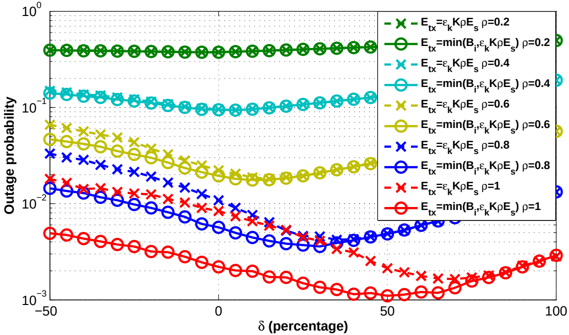

Figure : $\bar{E}_f' = K\rho E_s - \delta \times \rho E_s$ for $B_{\max} = 80E_s$ and $E_s = 0$ dB; δ varying from -50% to +100%. Converged δ values obtained using Algorithm 3: $\rho = 0.2$: 0.2%, $\rho = 0.4$: 0.71%, $\rho = 0.6$: 2.02%, $\rho = 0.8$: 5.75%, $\rho = 1$: 13.95%.

Thank you!