Spectrum Cartography

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Overview

Spectrum Cartography

- Estimating power distribution in space
- Applications in Wireless Cognitive Radio (CR) network
- Goals
    - Spatial reuse of frequency
    - Transmit power estimation
    - Tracking activities of primary users

Problem

- A set of sources transmitting at same frequncy
- Randomly deployed sensors measures the power radiated in the system
- Sensors co-operate to estimate the PSD map
- Unknowns
    - Transmitter location
    - Transmit power

Sparsity

- Sparsely located transmitters in space
- Compressed Sensing Techniques?
- How to form a suitable basis?

Problem Setup

- N_S transmitting sources
  - Stationary
  - Mutually uncorrelated
- Transmitter locations $\mathcal{X} = \{\boldsymbol{x}_s\}_{s=1}^{N_S}$
- Transmit power $\mathcal{P} = \{P_s\}_{s=1}^{N_s}$
- N_r sensors located at $\mathcal{Y} = \{\boldsymbol{y}_r\}_{r=1}^{N_r}$
- Measurements ϕ_r, $r = 1, 2, ..., N_r$

Literature Survey

- Assumption : Channel gains in parametric form*
  - Simple choice is path loss model
  - $\gamma(\boldsymbol{x}_s, \boldsymbol{y}_r) = \min\{1, (||\boldsymbol{x}_s - \boldsymbol{y}_r||_2/D_0)^{-\eta}\}$
- Choice of basis : Virtual grid model
  - Basis functions corresponding to all possible transmitter locations

*J. A. Bazerque and G. B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity"

Virtual Grid Model

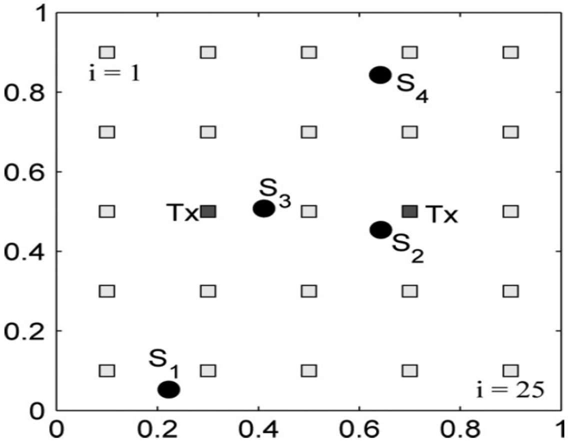

Figure: Virtual network grid with 25 candidate locations, 2 transmitters and 4 sensors

Basis Expansion Model

- Form an over-complete basis with discretized values of transmitter locations $\mathcal{Z} = \{z_i\}_{i=1}^{N}$

$$\underbrace{\begin{bmatrix} \phi_1 \\ \phi_2 \\ . \\ . \\ . \\ \phi_{N_r} \end{bmatrix}}_{\boldsymbol{\phi}} = \underbrace{\begin{bmatrix} \gamma(\boldsymbol{z}_1, \boldsymbol{y}_1) & \gamma(\boldsymbol{z}_2, \boldsymbol{y}_1) & . \; . \; . & \gamma(\boldsymbol{z}_N, \boldsymbol{y}_1) \\ \gamma(\boldsymbol{z}_1, \boldsymbol{y}_2) & \gamma(\boldsymbol{z}_2, \boldsymbol{y}_2) & . \; . \; . & \gamma(\boldsymbol{z}_N, \boldsymbol{y}_2) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ \gamma(\boldsymbol{z}_1, \boldsymbol{y}_{N_r}) & \gamma(\boldsymbol{z}_2, \boldsymbol{y}_{N_r}) & . \; . \; . & \gamma(\boldsymbol{z}_N, \boldsymbol{y}_{N_r}) \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ . \\ . \\ . \\ \theta_N \end{bmatrix}}_{\boldsymbol{\theta}} \quad (1)$$

- Sparse solution reveal the location of sources and their transmit power
- Solution via LASSO

Issues with the approach

- Only path-loss model is studied
- Shadowing is not included in the model
- Another approach : Solve for unknown spatial loss function : $l_s(x)$
- Solved using spline based technique[†]
  - Includes a roughness regularization term in the optimization problem

[†] Juan-Andres Bazerque, Gonzalo Mateos and Georgios B. Giannakis, "Group-Lasso on Splines for Spectrum Cartography"

System Model with Shadowing

- Received power at sensor r

$$\phi(r) = \sum_{s=1}^{N_S} P_s \left(\frac{D_0}{||\boldsymbol{y}_r - \boldsymbol{x}_s||} \right)^{\eta} \xi(r) \tag{2}$$

  - η is the path loss exponent
  - ξ is the shadowing component

- In dB scale

$$\phi(r) = 10 \log_{10} \left(\sum_{s=1}^{N_S} P_s \left(\frac{D_0}{||\boldsymbol{y}_r - \boldsymbol{x}_s||} \right)^{\eta} \right) + \xi_{\mathrm{dB}}(r) \tag{3}$$

Model for Shadowing

- ξ_{dB} spatially correlated Gaussian random process
- Widely accepted Gudmundson model for correlation

$$R(\Delta x) = \sigma^2 e^{-\Delta x / d_{\text{cor}}} \tag{4}$$

  - d_{cor} is the decorrelating distance
  - σ^2 is the shadowing variance

Solution via Greedy Approach

- Similar to BEM approach : Candidate locations $\mathcal{Z} = \{z_i\}_{i=1}^{N}$
- Exploits the spatial correlation
- Measurement vector in dB scale $\phi \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
  - $\mu_i = 10\log_{10}\left(\sum_{s=1}^{N_s} P_s \left(\frac{D_0}{||\mathbf{y_i} - \mathbf{x_s}||}\right)^{\eta}\right), \ i = 1, 2, .., N_r$
  - $\boldsymbol{\Sigma}_{ij} = \sigma^2 e^{-\Delta||\mathbf{y_i} - \mathbf{y_j}||/d_{cor}}$

Approach

- Locate a source in each iteration
  - Candidate location that maximizes the likelihood function
- After each iteration update the set candidate locations
  - Exclude the candidate locations where the the measured value and the transmit power due to all sources revealed so far are close enough
- Repeat the procedure N_s times

Likelihood Function

- For i th candidate location,

$$L_i = \max_{\gamma_i} (\phi - \psi_i - \gamma_i \mathbf{1})^{\mathsf{T}} \mathbf{C}_i (\phi - \psi_i - \gamma_i \mathbf{1}) \qquad (5)$$

- Pathloss vector :$\psi_i \in \mathbb{R}^{N_r}$, with r th entry $\psi_i(r) = 10\eta \log_{10}(\frac{D_0}{||\mathbf{y}_r - \mathbf{z}_i||})$
- Weighted inverse covariance matrix $\mathbf{C}_i = \mathbf{F}_i^{1/2} \mathbf{\Sigma}^{-1} \mathbf{F}_i^{1/2}$
- Forgetting factor matrix gives more weightage to measurements near the candidate location $\mathbf{F}_i = \text{diag}\{e^{-\lambda ||\mathbf{y}_r - \mathbf{z}_i||}, r = 1, 2, ..., N_r\}$
- Transmit power estimate that maximizes L_i is $\gamma_i = \frac{\mathbf{1}^{\mathsf{T}} \mathbf{C}_i (\phi - \psi_i)}{\mathbf{1}^{\mathsf{T}} \mathbf{C}_i \mathbf{1}}$

Algorithm : Inputs

- Sensor network
  - Measurements in dB scale: $\boldsymbol{\phi} \in \mathbb{R}^{N_r \times 1}$
  - Sensor locations $\{\boldsymbol{y}_r\}_{r=1}^{N_r}$
- Environment
  - Shadowing variance σ^2
  - Shadowing decorrelation distance d_{cor}
  - Pathloss exponent η
  - Reference Distance D_0
- Primary Network
  - Number of sources N_s

Algorithm : Parameters and Output

- Algorithm parameters
  - Candidate locations $\mathcal{Z} = \{\boldsymbol{z}_i\}_{i=1}^{N}$
  - Forgetting factor parameter λ
  - Threshold α
- Output
  - Source locations $\boldsymbol{T} \in \mathbb{R}^{N_s}$
  - Transmit powers $\boldsymbol{\theta} \in \mathbb{R}^{N_s}$

Algorithm

- Initialization: $\mathcal{S} = \{1, 2, ..., N\}$
- For $l = 1, 2, ...N_s$
  - $w \rightarrow \underset{j \in \mathcal{S}}{\text{argmax}}\ L_j$
  - $\boldsymbol{T}(l) \rightarrow \boldsymbol{z}_w$
  - $\boldsymbol{\theta}(l) \rightarrow 10^{\gamma_w/10}$
  - For $r = 1, 2, ..., N_r$
    - $\rho_r \rightarrow 10 \log_{10} \left(\sum_{s=1}^{l} \boldsymbol{\theta}(s) \left(\frac{D_0}{||\boldsymbol{y}_r - \boldsymbol{T}(s)||} \right)^{\eta} \right)$
  - $\mathcal{Q} \rightarrow \{\boldsymbol{y}_r : |\phi_r - \rho_r| < \alpha\phi_r + \sigma^2\} \cup \boldsymbol{z}_w$
  - $\mathcal{S} \rightarrow \mathcal{S} \setminus \{i : \boldsymbol{z}_i \in \text{Conv}(\mathcal{Q})\}^{\ddagger}$

‡Conv(.) represents the convex hull of the finite set

Simulation Result

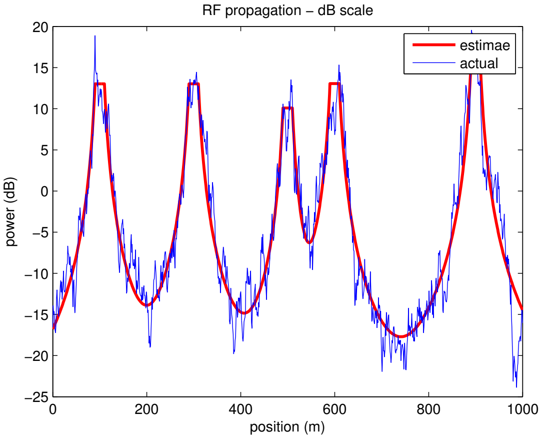

Figure: Recontructed Power Map:5 sources, 100 sensors

Conclusion

- Discussed popular approaches for Spectrum Cartography in literature
- Proposed a greedy algorithm for transmitter localization and power estimation when spatially correlated shadowing is present