Resource Allocation in OFDMA Cellular Networks

An Iterative Re-weighted Minimization Framework

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Outline

Goal !

Resource allocation in OFDMA cellular network :

As found in plethora of OFDMA literature, resource allocation can be broadly classified into 2 types :

- Rate Adaptive (RA) problem :
  - Maximize system utility function - maintaining QoS/power constraints etc.,.
- Margin Adaptive (MA) problem :
  - Minimize total transmit power - maintaining QoS/power constraints etc.,.

Our Goal :

- General framework for optimal resource allocation in OFDMA based cellular networks.

Single-Cell OFDMA Network

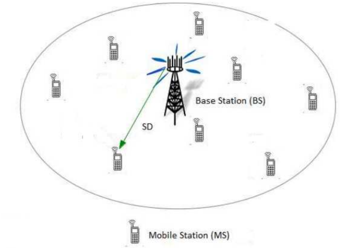


FIGURE – Single-cell multi-user OFDMA Network

Notations

$\mathcal{M} \in \{1, 2, \ldots, M\} \Rightarrow$ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\} \Rightarrow$ Sub-carrier indices
$\gamma_m^n \Rightarrow$ Sub-carrier gain
$p_m^n \Rightarrow$ Allotted power
$y_m^n \Rightarrow$ Binary indicator variable
$R_m^n \Rightarrow$ Rate achieved

Rate achieved by m^{th} user

$$R_m^n = log_2(1 + SNR_m^n); \quad SNR_m^n = \gamma_m^n p_m^n \tag{1}$$

$$\text{Total rate achieved} \Rightarrow R_m = \sum_{n \in \mathcal{N}} log_2(1 + \gamma_m^n p_m^n); \tag{2}$$

OFDMA constraint

$$\sum_{m \in \mathcal{M}} y_m^n \leq 1 \quad , \forall n \in \mathcal{N} \qquad (3)$$

$$y_m^n \in \{0, 1\} \quad , \forall m, n$$

Maximum power on each sub-carrier constraint

$$0 \leq p_m^n \leq y_m^n P^{max} \quad \forall m, n \qquad (4)$$

Total transmit power constraint

$$\sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{M}} p_m^n \leq P_T \qquad (5)$$

Problem formulation

Rate-Adaptive (RA) optimization problem :

$$
\begin{aligned}
\max_{\{p_m^n, y_m^n\}} \quad & U(R_1, R_2, \ldots, R_M) \\
\text{s.t.} \quad \mathrm{c}_1 : \quad & \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p_m^n \leq P_T, \\
\mathrm{c}_2 : \quad & 0 \leq p_m^n \leq y_m^n P^{\max} \quad, \forall m, n, \\
\mathrm{c}_3 : \quad & \sum_{m \in \mathcal{M}} y_m^n \leq 1 \quad, \forall n \in \mathcal{N}, \\
\mathrm{c}_4 : \quad & y_m^n \in \{0, 1\} \quad, \forall m, n.
\end{aligned} \tag{6}
$$

Problem formulation

Rate-Adaptive(RA) optimization problem :

$$
\begin{aligned}
\max_{\{p_m^n, y_m^n\}} \quad & U(R_1, R_2, \ldots, R_M) \\
s.t. \quad c_1 : \quad & \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p_m^n \leq P_T, \\
c_2 : \quad & 0 \leq p_m^n \leq y_m^n P^{\max} \quad , \forall m, n, \\
c_3 : \quad & \sum_{m \in \mathcal{M}} y_m^n \leq 1 \quad , \forall n \in \mathcal{N}, \\
c_4 : \quad & y_m^n \in \{0, 1\} \quad , \forall m, n.
\end{aligned} \tag{7}
$$

Bad News :

- The above problem in its raw form is NP-Hard.

Proposed algorithm based on IRM framework

Smooth concave utility function

Sum-rate utility function $\Rightarrow \sum_{m \in \mathcal{M}} R_m$

Let $x_m^n = \frac{p_m^n}{P^{max}}$

$$
\begin{aligned}
& \max_{\{x_m^n, y_m^n\}} \quad \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} log_2(1 + P^{max} x_m^n \gamma_m^n) \\
s.t. \quad & c_1 : \quad \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} P^{max} x_m^n \leq P_T, \\
& c_2 : \quad 0 \leq x_m^n \leq y_m^n \quad , \forall m, n, \\
& c_3 : \quad \sum_{m \in \mathcal{M}} y_m^n = 1 \quad , \forall n \in \mathcal{N}, \\
& c_4 : \quad y_m^n \in \{0, 1\} \quad , \forall m, n.
\end{aligned} \tag{8}
$$

The above optimization problem is equivalent to the previous optimization problem.

Optimal solution of y_m^n

$$\begin{aligned} \min_{\{y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_m^n + \epsilon)^q \\ s.t. \quad & \sum_{m\in\mathcal{M}} y_m^n = 1 \quad, \forall n \in \mathcal{N}, \\ & y_m^n \geq 0 \quad, m \in \mathcal{M}, n \in \mathcal{N}. \end{aligned} \tag{9}$$

Relaxed optimization problem

$$\begin{aligned} \min_{\{x_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} -log_2(1 + P^{max} x_m^n \gamma_m^n) + \lambda \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_m^n + \epsilon)^q \\ s.t. \quad c_1: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \leq P_T, \\ c_2: \quad & 0 \leq x_m^n \leq y_m^n \quad, \forall m, n, \\ c_3: \quad & \sum_{m\in\mathcal{M}} y_m^n = 1 \quad, \forall n \in \mathcal{N}, \end{aligned} \tag{10}$$

Relaxed optimization problem

$$\begin{aligned}
\min_{\{x_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{-log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{(y_m^n + \epsilon)^q}_{\text{concave}} \\
s.t. \quad c_1 : \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \leq P_T, \\
c_2 : \quad & 0 \leq x_m^n \leq y_m^n \quad, \forall m, n, \\
c_3 : \quad & \sum_{m\in\mathcal{M}} y_m^n = 1 \quad, \forall n \in \mathcal{N},
\end{aligned} \tag{11}$$

Difference of convex (DC) programming/ CCCP

Convex + Concave function

⇓

Keep convex part, linearise concave part

Relaxed optimization problem

$$\min_{\{x_m^n, y_m^n\}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{-log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{(y_m^n + \epsilon)^q}_{\text{concave}} \tag{12}$$

$$s.t. \quad \text{c}_1, \text{c}_2 \text{ and } \text{c}_3$$

Can be solved using DC Programming

Keep convex part as it is and linearise concave part

$$\min_{\{x_m^n, y_m^n\}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{-log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda q \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{(y_m^n(t) + \epsilon)^{q-1} y_m^n}_{\text{convex}} \tag{13}$$

$$s.t. \quad \text{c}_1, \text{c}_2 \text{ and } \text{c}_3$$

Algorithm 1

Initialization : $\lambda = NP^{\max}$, $q \in (0,1)$, $\sigma_1 \in (0,1)$, $\sigma_2 \in (0,1)$, $\delta \in (0,1)$, $\tau > 1$, $w_m^n(1) = 1 \ \forall\ m \in \mathcal{M}$ and $n \in \mathcal{N}$; $\epsilon(1) = 1$.

while (1) **do**

  for t = 1, 2, . . . , MaxItr **do**

    Solve the following convex sub-problem

$$\min_{\{x_m^n, y_m^n\}} -\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} log_2(1 + P^{max} x_m^n \gamma_m^n) + \lambda q \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} w_m^n(t) y_m^n$$

$$s.t. \quad \tilde{c}_1, \tilde{c}_2 \text{ and } \tilde{c}_3. \tag{14}$$

    Update :

$$w_m^n(t+1) = (x_m^n(t) + \epsilon(t))^{q-1}$$
$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(x_m^n(t+1))\}$$

    if $\sum_m \sum_n |x_m^n(t) - x_m^n(t-1)| < \sigma_1$ **then**

      break ;

    end if

  end for

  if $f(y_m^n(t)) < \sigma_2$ **then**

    Stop.

  else

    $\lambda = \tau\lambda$

  end if

end while

Single-Cell OFDMA REC Network

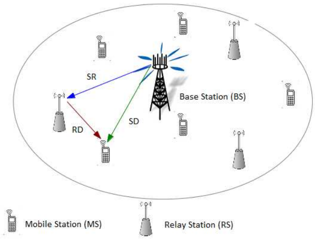
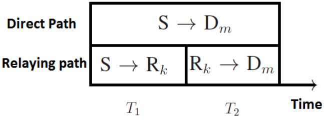


FIGURE – Single-cell multi-user OFDMA REC Network

Notations

$\mathcal{K} \in \{0, 1, \ldots, K\} \Rightarrow$ DL path indices
$\mathcal{M} \in \{1, 2, \ldots, M\} \Rightarrow$ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\} \Rightarrow$ Sub-carrier indices
$\gamma_*^n \Rightarrow$ Sub-carrier gain
$p_*^n \Rightarrow$ Allotted power
$R_m^n \Rightarrow$ Rate achieved
$* \in \{SD_m, SR_k, R_kD_m\}$



Path selection and Rate achieved

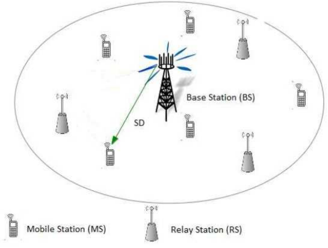


FIGURE – Direct path

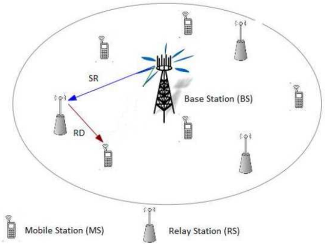


FIGURE – Relay path

Rate achieved

$$R^n_{0,m} = R^n_{SD_m} = log_2(1 + \gamma^n_{SD_m} P^n_{SD_m})$$

Rate achieved

$$R^n_{k,m} = \frac{1}{2} min\{R^n_{SR_k}, R^n_{R_k D_m}\}$$

Total achievable rate

$$R_m = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} R^n_{k,m}$$

Problem Formulation

Rate-Adaptive(RA) optimization problem :

$$
\begin{aligned}
&\max_{\{p^n_{SD_m}, p^n_{SR_k}, p^n_{R_k D_m}\}} \quad U(R_1, R_2, \ldots, R_M) \\
s.t.\quad & c_1 : \quad \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \{p^n_{SD_m} + \sum_{k\in\mathcal{K} k\neq 0} (p^n_{SR_k} + p^n_{R_k D_m})\} \leq P_T, \\
& c_2 : \quad p^n_{SD_m}, p^n_{SR_k}, p^n_{R_k D_m} \geq 0 \quad , \forall m, n, \\
& c_3 : \quad \text{OFDMA Constraint.}
\end{aligned} \tag{15}
$$

- Reformulate the above problem.

Problem re-formulation

- Let us introduce $(K+1)MN$ variables $\{p^n_{k,m}\ \forall k, m, n.\}$

$$p^n_{k,m} = \begin{cases} p^n_{SD_m}, & \text{if } k = 0 \\ (p^n_{SR_k} + p^n_{R_k D_m}), & \text{otherwise.} \end{cases} \tag{16}$$

- Relaying path rate : $R^n_{k,m} = \frac{1}{2} min\{R^n_{SR_k}, R^n_{R_k D_m}\} \Rightarrow$ is maximized if and only if

$$R^n_{SR_k} = R^n_{R_k D_m}$$
$$\Downarrow$$
$$\gamma^n_{SR_k} p^n_{SR_k} = \gamma^n_{R_k D_m} p^n_{R_k D_m} \tag{17}$$

- Using (16) and (17), we get the achievable rate of user m on n^{th} sub-carrier through k^{th} downlink path as

$$R^n_{k,m} = \alpha_k log_2(1 + \beta^n_{k,m} p^n_{k,m}) \tag{18}$$

$$\alpha_k = \begin{cases} 1, & \text{if } k = 0 \\ \frac{1}{2}, & \text{otherwise.} \end{cases} \qquad \beta^n_{k,m} = \begin{cases} \gamma^n_{SD_m}, & \text{if } k = 0 \\ \frac{\gamma^n_{SR_k} \gamma^n_{R_k D_m}}{\gamma^n_{SR_k} + \gamma^n_{R_k D_m}}, & \text{otherwise.} \end{cases}$$

Reformulated RA optimization problem

RA problem after reformulation

$$
\begin{aligned}
\max_{\{p_{k,m}^n\}} \quad & U(R_1, R_2, \dots, R_M) \\
s.t. \quad c_1 : \quad & \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{k,m}^n \leq P_T, \\
c_2 : \quad & 0 \leq p_{k,m}^n \leq P^{\max} \quad , \forall k, m, n, \\
c_3 : \quad & p_{k,m}^n p_{k',m'}^n = 0 \quad , \forall k \neq k', m \neq m' \\
& k, k' \in andn, n' \in \mathcal{N}.
\end{aligned} \tag{19}
$$

Introduce binary indicator variable $y_{k,m}^n$

$$
\begin{aligned}
&\max_{\{p_{k,m}^n, y_{k,m}^n\}} \quad U(R_1, R_2, \ldots, R_M) \\
\text{s.t.} \quad & c_1: \quad \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{k,m}^n \leq P_T, \\
& c_2: \quad 0 \leq p_{k,m}^n \leq y_{k,m}^n P^{\max} \quad, \forall k, m, n, \\
& c_3: \quad \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} y_{k,m}^n \leq 1 \quad, \forall n \in \mathcal{N} \\
& c_4: \quad y_{k,m}^n \in \{0, 1\}
\end{aligned}
\tag{20}
$$

Non-smooth concave utility function

Min-rate utility function $\Rightarrow \min_{1\leq m\leq M} R_m$

Introduce binary indicator variable $y_{k,m}^{n}$

$$\begin{aligned} \max_{\{p_{k,m}^{n}, y_{k,m}^{n}\}} \quad & \min_{1\le m\le M} R_m \\ \text{s.t.} \quad c_1 : \quad & \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{k,m}^{n} \le P_T, \\ c_2 : \quad & 0 \le p_{k,m}^{n} \le y_{k,m}^{n} P^{\max} \quad , \forall k, m, n, \\ c_3 : \quad & \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} y_{k,m}^{n} \le 1 \quad , \forall n \in \mathcal{N} \\ c_4 : \quad & y_{k,m}^{n} \in \{0, 1\} \end{aligned} \tag{21}$$

Problems to take care of :

- Combinatorial nature of the problem.
- Non-smoothness in objective function.

Transformation of non-smooth to smooth function

Introduce a new variable ϕ

$$
\begin{aligned}
&\max_{\{p_{k,m}^n, y_{k,m}^n, \phi\}} \quad \phi \\
\text{s.t.} \quad & \mathrm{c}_1: \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{k,m}^n \leq P_T, \\
& \mathrm{c}_2: 0 \leq p_{k,m}^n \leq y_{k,m}^n P^{\max} \quad, \forall k, m, n, \\
& \mathrm{c}_3: \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} y_{k,m}^n \leq 1 \quad, \forall n \in \mathcal{N} \\
& \mathrm{c}_4: y_{k,m}^n \in \{0,1\} \quad, \forall k, m, n, \\
& \mathrm{c}_5: R_m \geq \phi \quad, \forall m.
\end{aligned}
\tag{22}
$$

Problem solved

- Non-smooth $\Rightarrow$ smooth.
- Combinatorial problem $\Rightarrow$ Same method as used in previous section.

Steps

- Relaxation.
- Convex + Concave part ⇒ Solve it using DC programming

Relaxed optimization problem

$$\min_{\{x_{k,m}^n, y_{k,m}^n, \phi\}} \quad -\phi + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_{k,m}^n + \epsilon)^q \quad \text{s.t.} \quad c_1, c_2, c_3 \text{ and } c_5. \tag{23}$$

Keep convex part as it is and linearise concave part

$$\min_{\{x_{k,m}^n, y_{k,m}^n, \phi\}} \quad -\phi + \lambda q \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_{k,m}^n(t) + \epsilon)^{q-1} y_{k,m}^n \quad \text{s.t.} \quad c_1, c_2, c_3 \text{ and } c_5. \tag{24}$$

Algorithm 2

Initialization : $\lambda = NP^{\max}$, $q \in (0,1)$, $\sigma_1 \in (0,1)$, $\sigma_2 \in (0,1)$, $\delta \in (0,1)$, $\tau > 1$, $w^n_{k,m}(1) = 1 \; \forall \; k \in \mathcal{K} \; m \in \mathcal{M}$ and $n \in \mathcal{N}$; $\epsilon(1) = 1$.

while (1) **do**

    for t = 1, 2, . . . , MaxItr **do**

        Solve the following convex sub-problem

$$\min_{\{x^n_{k,m}, y^n_{k,m}, \phi\}} -\phi + \lambda q \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} w^n_{k,m}(t) y^n_{k,m}$$
$$s.t. \quad \tilde{c}_1, \tilde{c}_2, \tilde{c}_3 \text{ and } \tilde{c}_5. \tag{25}$$

        Update :

$$w^n_{k,m}(t+1) = (x^n_{k,m}(t) + \epsilon(t))^{q-1}$$
$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(x^n_{k,m}(t+1))\}$$

        if $\sum_k \sum_m \sum_n |x^n_{k,m}(t) - x^n_{k,m}(t-1)| < \sigma_1$ **then**

            break ;

        end if

    end for

    if $f(y^n_{k,m}(t)) < \sigma_2$ **then**

        Stop.

    else

        $\lambda = \tau \lambda$

    end if

end while

Simulation result

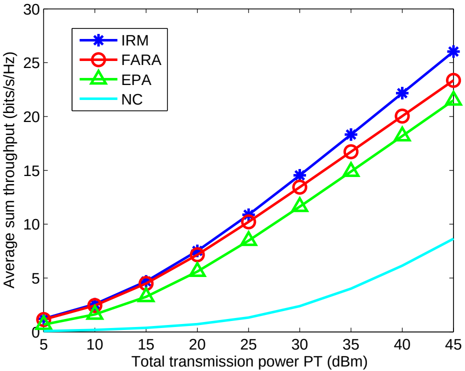


FIGURE – Comparison of the average sum throughput of different algorithms with the max-min resource allocation as the utility function versus the total transmission power. The system parameters are $K = 3$, $M = 8$ and $N = 16$.

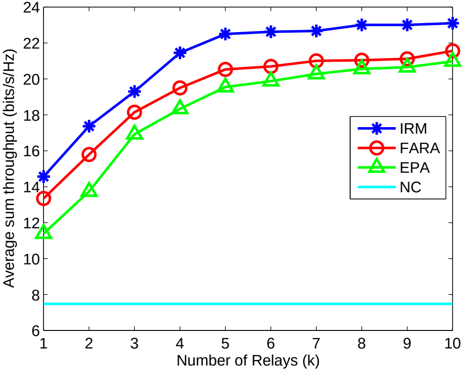


FIGURE – Illustration of the throughput improvement obtainable by using cooperative relaying. The system parameters : $M = 4$, $N = 8$ and $\mathrm{P}_T = 10$ dB.

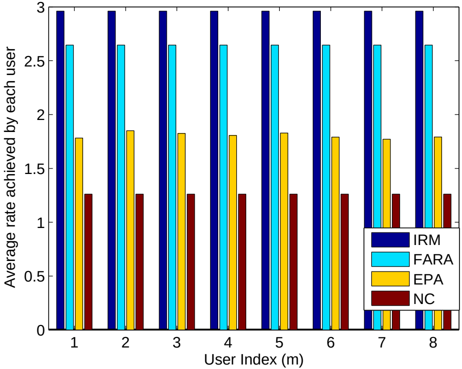


FIGURE – Average rate of each user, with $K = 3$, $M = 8$, $N = 16$ and $P_T = 10$ dB. All the schemes compared are fair resource allocation schemes.

Multi-cell OFDMA network

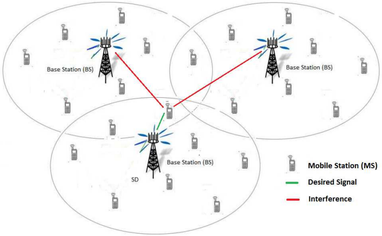


FIGURE – Multi-cell multi-user OFDMA Network

Notations

$\mathcal{K} \in \{1, 2, \ldots, M\} \Rightarrow$ BS indices
$\mathcal{M} \in \{1, 2, \ldots, M\} \Rightarrow$ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\} \Rightarrow$ Sub-carrier indices
$\gamma^n_{k,m} \Rightarrow$ Sub-carrier gain
$p^n_{k,m} \Rightarrow$ Allotted power
$y^n_{k,m} \Rightarrow$ Binary indicator variable
$R^n_{k,m} \Rightarrow$ Rate achieved

Rate-Adaptive (RA) optimization problem :

$$
\begin{aligned}
\max_{\{p_{k,m}^{n}, y_{k,m}^{n}\}} \quad & \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} log_2(1+SINR_{k,m}^{n}) \\
s.t. \quad \mathrm{c}_1 : \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \le P_T \quad, \forall k \in \mathcal{K} \\
\mathrm{c}_2 : \quad & 0 \le p_{k,m}^{n} \le y_{k,m}^{n} P^{\max} \quad, \forall k, m, n, \\
\mathrm{c}_3 : \quad & \sum_{m\in\mathcal{M}} y_{k,m}^{n} \le 1 \quad, \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \\
\mathrm{c}_4 : \quad & y_{k,m}^{n} \in \{0,1\} \quad, \forall k, m, n.
\end{aligned}
\tag{26}
$$

Signal-to-interference-noise ratio :

$$
SINR_{k,m}^{n} = \frac{\beta_{k,m}^{n} p_{k,m}^{n}}{\sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \alpha_{i,m}^{n} p_{i,j}^{n} + N_0}
$$

Solution

Steps

- Relaxation.
- Convex + Concave part ⇒ Solve it using DC programming

Relaxed optimization problem

$$\begin{aligned} \min_{\{x^n_{k,m}, y^n_{k,m}\}} \quad & -\sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} log_2(P^{max}\beta^n_{k,m}x^n_{k,m} + \sum_{i\in\mathcal{K},i\neq k}\sum_{j\in M_i} P^{max}\alpha^n_{i,m}x^n_{i,j} + N_0) \\ & + \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} log_2(\sum_{i\in\mathcal{K},i\neq k}\sum_{j\in M_i} P^{max}\alpha^n_{i,m}x^n_{i,j} + N_0) \\ & + \lambda\sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} (y^n_{k,m} + \epsilon)^q \\ s.t. \quad & c_1, c_2 \text{ and } c_3. \end{aligned} \tag{27}$$

Solution Contd.

1^{st} term → f(x)

- f(x) ⇒ convex, keep as it is.

2^{nd} term → g(x)

- g(x) ⇒ concave, linearise it.

$$g(x^n_{k,m}(t)) + \mathcal{D}g(x^n_{k,m}(t))(x^n_{k,m} - x^n_{k,m}(t)) \tag{28}$$

where $\mathcal{D}g(x^n_{k,m})$ is the derivative matrix of $g(x)$ at $x^n_{k,m}$ and is given by

$$\mathcal{D}g(x^n_{k,m}) = \mathcal{W}1^n_{k,m} = \frac{1}{\sum_{i\in\mathcal{K},i\neq k}\sum_{j\in M_i}\alpha^n_{i,m}P^{max}x^n_{i,j} + N_0}e^n_{i,m}$$

$$\text{Where, } e^n_{i,m} = \begin{cases} 0 & \text{, if } i = k, \\ \frac{P^{max}\alpha^n_{i,m}}{ln2} & \text{, if } i \neq k. \end{cases}$$

3^{rd} term → h(y)

- h(y) ⇒ concave, linearise it.

$$h(y^n_{k,m}(t)) + \mathcal{D}h(y^n_{k,m}(t))(y^n_{k,m} - y^n_{k,m}(t)) \tag{29}$$

where $\mathcal{D}h(y^n_{k,m})$ is the derivative matrix of $h(y)$ at $y^n_{k,m}$ and is given by

$$\mathcal{D}h(y^n_{k,m}) = \mathcal{W}2^n_{k,m} = q(y^n_{k,m})^{q-1}$$

Convex sub-problem

$$\min_{\{x^n_{k,m}, y^n_{k,m}\}} \quad f(x^n_{k,m}) + \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} \mathcal{W}1^n_{k,m} x^n_{k,m} + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} \mathcal{W}2^n_{k,m} y^n_{k,m}$$
$$s.t. \quad \tilde{c}_1, \tilde{c}_2, \text{ and } \tilde{c}_3. \tag{30}$$

Algorithm 3

Initialization
while (1) **do**
  for t = 1, 2, . . . , MaxItr **do**
    Solve the following convex sub-problem

$$\min_{\{x^n_{k,m}, y^n_{k,m}\}} \quad f(x^n_{k,m}) + \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} \mathcal{W}1^n_{k,m} x^n_{k,m} + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} \mathcal{W}2^n_{k,m} y^n_{k,m}$$

$$s.t. \quad \tilde{c}_1, \tilde{c}_2 \text{ and } \tilde{c}_3. \tag{31}$$

    Update :

$$\mathcal{W}1^n_{k,m}(t) = \mathcal{D}g(x^n_{k,m}(t))$$
$$\mathcal{W}2^n_{k,m}(t) = \mathcal{D}h(y^n_{k,m}(t))$$
$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(y^n_{k,m}(t+1))\}$$

    if $\sum_k \sum_m \sum_n |x^n_{k,m}(t) - x^n_{k,m}(t-1)| < \sigma_1$ **then**
      break ;
    end if
  end for
  if $f(y^n_{k,m}(t)) < \sigma_2$ **then**
    Stop.
  else
    $\lambda = \tau\lambda$
  end if
end while

Multi-cell OFDMA REC network

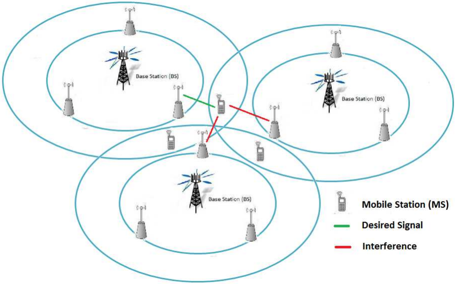
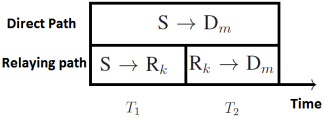


FIGURE – Multi-cell multi-user OFDMA REC Network

Notations

$\mathcal{K} \in \{1, 2, \ldots, K\}$ ⇒ BS indices
$\mathcal{L} \in \{1, 2, \ldots, L\}$ ⇒ RS indices
$\mathcal{M} \in \{1, 2, \ldots, M\}$ ⇒ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\}$ ⇒ Sub-carrier indices
$\gamma^n_{k,l,m}$ ⇒ Sub-carrier gain
$p^n_{k,l,m}$ ⇒ Allotted power
y^n ⇒ Binary indicator variable

Problem formulation :



Assumptions :

- We assume, all BS-RS works in sync.
- We focus on cell-edge users.
- Rate achieved by each cell-edge user

$$R^n_{k,l,m} = \frac{1}{2} min\{R^n_{S_k R_l}, R^n_{R_l D_m}\} \tag{32}$$

- We assume the RS-MS link have dominated influence on the transmission rate.

$$R^n_{k,l,m} = \frac{1}{2} R^n_{R_l D_m} \tag{33}$$

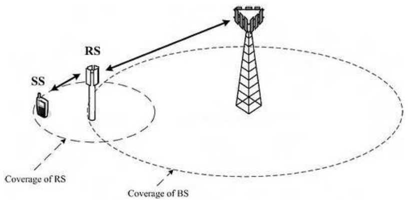
Reason behind strong assumption :

BS-RS Link

- Strong LoS propogation channel.
- Interference is negligible.

RS-MS Link

- NLoS communication link.
- Interference is dominant.



Total transmission rate

$$\text{Thus,} \quad R^n_{k,l,m} = \frac{1}{2} R^n_{R_l D_m} \tag{34}$$

$$\Downarrow$$

Dependent on $p^n_{l,m}$ and $y^n_{l,m}$, Greatly reduces problem complexity !

Rate-Adaptive (RA) optimization problem :

$$\begin{aligned} \max_{\{p^n_{l,m}, y^n_{l,m}\}} \quad & \sum_{k\in\mathcal{K}} \sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} log_2(1 + SINR^n_{l,m}) \\ s.t. \quad \mathrm{c}_1: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p^n_{l,m} \leq P_{T-RS} \quad , \forall l \in \mathcal{L} \text{ and } k \in \mathcal{K} \\ \mathrm{c}_2: \quad & 0 \leq p^n_{l,m} \leq y^n_{l,m} P^{\max} \quad , \forall k, l, m, n, \\ \mathrm{c}_3: \quad & \sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}} y^n_{l,m} \leq 1 \quad , \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \\ \mathrm{c}_4: \quad & y^n_{l,m} \in \{0, 1\} \quad , \forall k, l, m, n. \end{aligned} \tag{35}$$

Signal-to-interference-noise ratio :

$$SINR^n_{l,m} = \frac{\beta^n_{l,m} p^n_{l,m}}{\sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \alpha^n_{i,m} p^n_{i,j} + N_0}$$

Solution

Steps

- Relaxation.
- DC programming.

Summary of our work

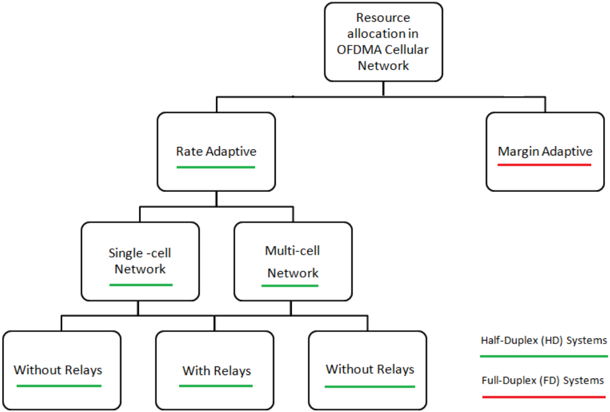


FIGURE – Summary of our work

Thank You :)