Resource Allocation Under Channel Uncertainties for Relay-Aided Device-to-Device Communication Underlaying LTE-A Cellular Networks

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Problem and Contributions

Problem:

- Joint resource allocation for a relay-assisted D2D Communication underlaying LTE-A cellular network
  - Under Perfect CSI
  - Under Imperfect CSI

Contributions:

- Semi-distributed joint resource allocation algorithms
  - Under Perfect CSI: Nominal Optimization Problem
  - Under Imperfect CSI: Robust Optimization Problem
    - Using worst-case approach
  - Closed Form expressions for RB and power allocation for both the cases

System Model

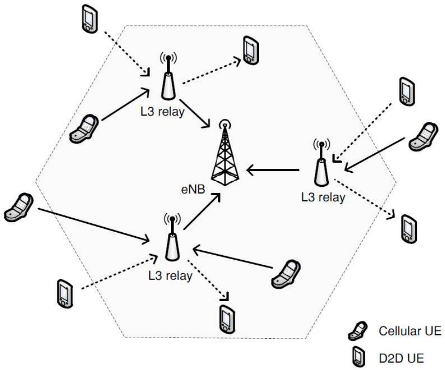

FIGURE – Relay aided D2D communication model

Notation:

- $\mathcal{L} \in \{1, \ldots, \mathbf{L}\} \rightarrow$ Set of RSs
- $\mathcal{N} \in \{1, \ldots, \mathbf{N}\} \rightarrow$ Set of subcarriers
- $\mathcal{C} \in \{1, \ldots, \mathbf{C}\} \rightarrow$ Set of CUEs
- $\mathcal{D} \in \{1, \ldots, \mathbf{D}\} \rightarrow$ Set of D2D Pairs
- $\mathcal{U}_l \subseteq \{\mathcal{C} \cup \mathcal{D}\} \rightarrow$ Set of UEs $\Rightarrow$ Relay l

Operation

- First hop (T_1): Transmission between UE (CUE or D2D) and RS
- Second hop (T_2): Transmission between RS and eNB or to a D2D UE

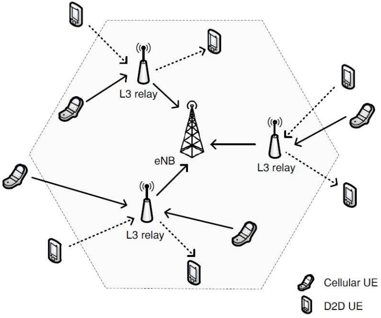

FIGURE – Relay aided D2D communication model

First hop link SINR:

$$\gamma^n_{u_l,l,1} = \frac{h^n_{u_l,l}}{\sum\limits_{\forall u_j \in \mathbb{U}_j, j \neq l, j \in \mathcal{L}} P^n_{u_j,j} g^n_{u_j,l} + \sigma^2}$$

Second hop link SINR:

$$\gamma^n_{l,u_l,2} = \frac{h^n_{l,eNB}}{\sum\limits_{\forall u_j \in \{\mathcal{D} \cap \mathbb{U}_j\}, j \neq l, j \in \mathcal{L}} P^n_{j,u_j} g^n_{l,eNB} + \sigma^2}$$

$$\gamma^n_{l,u_l,2} = \frac{h^n_{l,u_l}}{\sum\limits_{\forall u_j \in \mathbb{U}_j, j \neq l, j \in \mathcal{L}} P^n_{j,u_j} g^n_{j,u_l} + \sigma^2}$$

Achievable Data Rate:

$$r^n_{u_l,1} = B_{RB} \log_2(1 + P^n_{U_l,l} \gamma^n_{u_l,l,1})$$
$$r^n_{u_l,2} = B_{RB} \log_2(1 + P^n_{l,U_l} \gamma^n_{l,u_l,2})$$

$$R^n_{u_l} = \frac{1}{2}\{r^n_{u_l,1}, r^n_{u_l,2}\}$$

Total Achievable Rate:

$$R_{u_l} = \sum_{n=1}^{N} x^n_{u_l} R^n_{u_l}$$

Formulating Optimization Problem

Constraints:

- Maximum allowable transmit power for UE is $P_{u_l}^{\max}$ and for RS is $P_l^{\max}$

$$\sum_{n\in\mathcal{N}} x_{u_l}^n P_{u_l,l}^n \leq P_{u_l}^{\max}, \quad \forall u_l \in \mathcal{U}_l$$

$$\sum_{u_l\in\mathcal{U}_l}\sum_{n\in\mathcal{N}} x_{u_l}^n P_{l,u_l}^n \leq P_l^{\max}$$

- Minimum QoS requirement

$$R_{u_l} \geq Q_{u_l}, \quad \forall u_l \in \mathcal{U}_l$$

- Maximum allowable interference during First and Second hop is $\mathcal{I}_{th,1}^n$ and $\mathcal{I}_{th,2}^n$, respectively

$$\sum_{u_l\in\mathcal{U}_l} x_{u_l}^n P_{u_l,l}^n g_{u_l^\star,l,1}^n \leq \mathcal{I}_{th,1}^n, \quad \forall n \in \mathcal{N}$$

$$\sum_{u_l\in\mathcal{U}_l} x_{u_l}^n P_{l,u_l}^n g_{l,u_l^\star,2}^n \leq \mathcal{I}_{th,2}^n, \quad \forall n \in \mathcal{N}$$

where, First Hop: $u_l^\star = \arg\max_j g_{u_l,j}^n, \quad u_l \in \mathcal{U}_l, j \neq l, j \in \mathcal{L}$

Second Hop: $u_l^\star = \arg\max_{u_j} g_{l,u_j}^n, \quad j \neq l, j \in \mathcal{L}, u_j \in \{\mathcal{D} \cap \mathcal{U}_j\}$

Optimization Problem

$$\max_{\{x^n_{u_l}, P^n_{U_l,l}, P^n_{l,U_l}\}} \sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} x^n_{u_l} R^n_{u_l}$$

$$\begin{aligned}
s.t. \quad C_1 &: \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} \leq 1, \quad \forall n \in \mathcal{N} \\
C_2 &: \sum_{n \in \mathcal{N}} x^n_{u_l} P^n_{u_l,l} \leq P^{\max}_{u_l}, \quad \forall u_l \in \mathcal{U}_l \\
C_3 &: \sum_{u_l \in \mathcal{U}_l} \sum_{n \in \mathcal{N}} x^n_{u_l} P^n_{l,u_l} \leq P^{\max}_{l} \\
C_4 &: \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} P^n_{u_l,l} g^n_{u^\star_l,l,1} \leq \mathcal{I}^n_{th,1}, \quad \forall n \in \mathcal{N} \\
C_5 &: \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} P^n_{l,u_l} g^n_{l,u^\star_l,2} \leq \mathcal{I}^n_{th,2}, \quad \forall n \in \mathcal{N} \\
C_6 &: R_{u_l} \geq Q_{u_l}, \quad \forall u_l \in \mathcal{U}_l \\
C_7 &: P^n_{u_l,l} \geq 0, P^n_{l,u_l} \geq 0, \quad \forall n \in \mathcal{N}, u_l \in \mathcal{U}_l \\
C_8 &: x^n_{u_l} \in \{0, 1\}
\end{aligned}$$

Reformulation:

$$R^n_{u_l} = \frac{1}{2} \min\{r^n_{u_l,1}, r^n_{u_l,2}\}$$

$R^n_{u_l}$ is maximized iff

$$r^n_{u_l,1} = r^n_{u_l,2}$$

$$P^n_{u_l,l} \gamma^n_{u_l,l} = P^n_{l,u_l} \gamma^n_{l,u_l,2}$$

$$P^n_{u_l,l} = \frac{\gamma^n_{l,u_l,2}}{\gamma^n_{u_l,l}} P^n_{l,u_l}$$

- Express $P^n_{u_l,l}$ in terms of P^n_{l,u_l}

$$R^n_{u_l} = \frac{1}{2} B_{RB} \log_2(1 + P^n_{u_l,l} \gamma^n_{u_l,l,1})$$

- Introduce new variable

$$S^n_{u_l,l} = x^n_{u_l} P^n_{u_l,l}$$

Reformulated Optimization Problem

$$\max_{\{x^n_{u_l}, S^n_{U_l,l}\}} \sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{\gamma^n_{u_l,l,1}}{x^n_{u_l}})$$

$$\text{s.t.} \quad C_1: \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} \leq 1, \quad \forall n \in \mathcal{N}$$

$$C_2: \sum_{n \in \mathcal{N}} S^n_{u_l,l} \leq P^{\max}_{u_l}, \quad \forall u_l \in \mathcal{U}_l$$

$$C_3: \sum_{u_l \in \mathcal{U}_l} \sum_{n \in \mathcal{N}} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{l,u_l} \leq P^{\max}_l$$

$$C_4: \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l} g^n_{u^\star_l,l,1} \leq \mathcal{I}^n_{th,1}, \quad \forall n \in \mathcal{N}$$

$$C_5: \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} g^n_{l,u^\star_l,2} \leq \mathcal{I}^n_{th,2}, \quad \forall n \in \mathcal{N}$$

$$C_6: \sum_{n \in \mathcal{N}} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{\gamma^n_{u_l,l,1}}{x^n_{u_l}}) \geq Q_{u_l},$$

$$C_7: S^n_{u_l,l} \geq 0 \quad \forall n \in \mathcal{N}, u_l \in \mathcal{U}_l$$

$$C_8: x^n_{u_l} \in \{0, 1\}$$

where,

$$\gamma^n_{u_l,l,1} = \frac{h^n_{u_l,l}}{I_{u_l,l,1} + \sigma^2}$$

$$I_{u_l,l,1} = \sum_{\forall u_j \in \mathbb{U}_j, j \neq l, j \in \mathcal{L}} P^n_{u_j,j} g^n_{u_j,l}$$

Issue:

- MINLP Problem
  - Computationally intractable

Solution:

- Relax the integer constraint to Box constraint

Nominal Optimization Problem

$$\max_{\{x^n_{u_l}, S^n_{U_l,l}, w^n_{u_l}\}} \quad \sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{h^n_{u_l,l,1}}{x^n_{u_l} w^n_{u_l}})$$

$$s.t. \quad C_1 : \quad \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} \leq 1, \quad \forall n \in \mathcal{N}$$

$$C_2 : \quad \sum_{n \in \mathcal{N}} S^n_{u_l,l} \leq P^{\max}_{u_l}, \quad \forall u_l \in \mathcal{U}_l$$

$$C_3 : \quad \sum_{u_l \in \mathcal{U}_l} \sum_{n \in \mathcal{N}} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{l,u_l} \leq P^{\max}_l$$

$$C_4 : \quad \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l} g^n_{u_l^\star,l,1} \leq \mathcal{I}^n_{th,1}, \quad \forall n \in \mathcal{N}$$

$$C_5 : \quad \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} g^n_{l,u_l^\star,2} \leq \mathcal{I}^n_{th,2}, \quad \forall n \in \mathcal{N}$$

$$C_6 : \quad \sum_{n \in \mathcal{N}} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{\gamma^n_{u_l,l,1}}{x^n_{u_l}}) \geq Q_{u_l},$$

$$C_7 : \quad S^n_{u_l,l} \geq 0 \quad \forall n \in \mathcal{N}, u_l \in \mathcal{U}_l$$

$$C_8 : \quad x^n_{u_l} \in (0, 1]$$

$$C_9 : \quad I^n_{u_l,l} + \sigma^2 \leq w^n_{u_l}$$

Solving Issues:

- MINLP Problem:

  Relax integer constraint c_8
  i.e., $x^n_{u_l} \in (0, 1]$

- Introduce auxiliary variable $w^n_{u_l}$ s.t.

$$I^n_{u_l,l} + \sigma^2 \leq w^n_{u_l}$$

Convex Problem:

- There exists a unique optimal solutior

Closed form solution

Lagrangian:

$$L_l(x, S, w, \mu, \rho, v_l, \psi, \phi, \lambda, \varrho) = -\sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} \frac{1}{2} x_{u_l}^n B_{RB} \log_2(1 + S_{u_l,l}^n \frac{h_{u_l,l,1}^n}{x_{u_l}^n w_{u_l}^n})$$

$$+ \sum_{n=1}^{N} \mu_n (\sum_{u_l \in \mathcal{U}_l} x_{u_l}^n - 1) + \sum_{u_l \in \mathcal{U}_l} \rho_{u_l} (\sum_{n \in \mathcal{N}} S_{u_l,l}^n - P_{u_l}^{\max}) + v_l (\sum_{u_l \in \mathcal{U}_l} \sum_{n \in \mathcal{N}} \frac{h_{u_l,l,1}^n}{h_{l,u_l,2}^n} S_{l,u_l}^n - P_l^{\max})$$

$$+ \sum_{n=1}^{N} \psi_n (\sum_{u_l \in \mathcal{U}_l} S_{u_l,l}^n g_{u_l^\star,l,1}^n - \mathcal{I}_{th,1}^n) + \sum_{n=1}^{N} \phi_n (\sum_{u_l \in \mathcal{U}_l} \frac{h_{u_l,l,1}^n}{h_{l,u_l,2}^n} S_{u_l,l}^n g_{l,u_l^\star,2}^n - \mathcal{I}_{th,2}^n)$$

$$+ \sum_{u_l \in \mathcal{U}_l} (Q_{u_l} - \sum_{n \in \mathcal{N}} \frac{1}{2} x_{u_l}^n B_{RB} \log_2(1 + S_{u_l,l}^n \frac{\gamma_{u_l,l,1}^n}{x_{u_l}^n w_{u_l}^n}))$$

- Upon applying KKT conditions

$$P_{u_l,l}^{n^\star} = \frac{S_{u_l,l}^{n^\star}}{x_{u_l}^{n^\star}} = \left[\delta_{u_l,l}^n - \frac{w_{u_l}^n}{h_{u_l,l,1}^n} \right]^+ \quad (1)$$

where,

$$\delta_{u_l,l}^n = \frac{\frac{1}{2} B_{RB} \frac{1+\lambda_{u_l}}{\log 2}}{\rho_{u_l} + \frac{h_{u_l,l,1}^n}{h_{l,u_l,2}^n} v_l + g_{u_l^\star,l,1}^n \psi_n + \frac{h_{u_l,l,1}^n}{h_{l,u_l,2}^n} g_{l,u_l^\star,2}^n \phi_n}$$

Closed Form Solution Contd.,

- RB allocation is given by

$$x_{u_l}^{n\star} = \begin{cases} 1, & \mu_n \leq \mathcal{X}_{u_l,l}^n \\ 0, & otherwise \end{cases} \tag{2}$$

where,

$$\mathcal{X}_{u_l,l}^n = \frac{1}{2}(1+\lambda_{u_l})B_{RB}\left[\log_2(1+\frac{S_{u_l,l}^n h_{u_l,l,1}^n}{x_{u_l}^n w_{u_l}^n}) - \frac{S_{u_l,l}^n \gamma_{u_l,l,1}^n}{(x_{u_l}^n w_{u_l}^n + S_{u_l,l}^n \gamma_{u_l,l,1}^n)\log(2)}\right]$$

- Update Lagrange multipliers using gradient descent method
  - Example:

$$\mu_n(t+1) = \left[\mu_n(t) + \Delta_{\mu_n(t)}\left(\sum_{u_l \in \mathcal{U}_l} x_{u_l}^n - 1\right)\right]^+$$

  Similarly, Update $\rho, v_l, \psi, \phi, \lambda, \varrho$ using Gradient descent algorithm.

- Solution of the relaxed problem is **Asymptotically optimal**

Imperfect CSI !

- Estimated link gains are erroneous !

$$\bar{g} \neq g$$
$$\Downarrow$$
$$g = \bar{g} + \hat{g}$$

where,

$$\bar{g} \rightarrow \text{Estimated value}$$
$$\hat{g} \rightarrow \text{Error in estimation}$$

- As found in the literature, there are two ways to tackle it
  - Bayesian Approach
  - Worst-Case Approach

Worst Case Approach

Let

$$g_{l,1}^n = [g_{1^\star,l,1}^n, g_{2^\star,l,1}^n, \dots, g_{|\mathcal{U}_l|^\star,l,1}^n]$$
$$g_{l,2}^n = [g_{1^\star,l,2}^n, g_{2^\star,l,2}^n, \dots, g_{|\mathcal{U}_l|^\star,l,2}^n]$$

Assumption: Link gains and aggregated interfernece are unknown but are bounded in a region

$$g_{l,1}^n = \bar{g}_{l,1}^n + \hat{g}_{l,1}^n$$
$$g_{l,1}^n \in \mathcal{R}_{g_{l,1}^n}$$

Similarly,

$$g_{l,2}^n \in \mathcal{R}_{g_{l,2}^n}$$
$$I_{u_l,l}^n \in \mathcal{I}_{u_l,l}^n$$

Uncertainty Set:

$$\mathcal{R}_{g_l,1}^n = \{g_{l,1}^n | \quad ||M_{g_l,1}^n.(g_{l,1}^n - \hat{g}_{l,1}^n)^T|| \leq \psi_{l,1}^n\}$$
$$\mathcal{R}_{g_l,2}^n = \{g_{l,2}^n | \quad ||M_{g_l,2}^n.(g_{l,2}^n - \hat{g}_{l,2}^n)^T|| \leq \psi_{l,2}^n\}$$
$$\mathcal{R}_{I_{u_l},1}^n = \{I_{u_l,1}^n | \quad ||M_{I_{u_l},1}^n.(I_{l,1}^n - \hat{I}_{l,1}^n)^T|| \leq \Gamma_{u_l}^n\}$$

$$\max_{\{x^n_{u_l}, S^n_{U_l,l}, w^n_{u_l}\}} \sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{h^n_{u_l,l,1}}{x^n_{u_l} w^n_{u_l}})$$

$$\text{s.t.} \quad C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$$

$$\text{and} \quad g^n_{l,1} \in \mathcal{R}_{g^n_{l,1}}$$

$$g^n_{l,2} \in \mathcal{R}_{g^n_{l,2}}$$

$$I^n_{u_l,l} \in \mathcal{R}^n_{I_{u_l,l}}$$

Uncertainty Constraints:

$$C_4 : \max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l} g_{u^\star_l,l,1} \leq I^n_{th,1}, \quad \forall n$$

$$C_5 : \max_{g^n_{l,2} \in \mathcal{R}^n_{g_l,2}} \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} g_{u^\star_l,l,2} \leq I^n_{th,1}, \quad \forall n$$

$$C_9 : \max_{I^n_{u_l,l} \in \mathcal{R}^n_{I_{u_l,l}}} I^n_{u_l,l} + \sigma^2 \leq w^n_{u_l}, \quad \forall n, u_l$$

- Uncertainty constraint can be equivalently rewritten as

$$C_4 : \max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l}(g_{u^\star_l,l,1} - \bar{g}_{u^\star_l,l,1} + \bar{g}_{u^\star_l,l,1}) \leq I^n_{th,1}, \quad \forall n$$

$$\Downarrow$$

$$\sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l}(\bar{g}_{u^\star_l,l,1}) + \underbrace{\max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l}(g_{u^\star_l,l,1} - \bar{g}_{u^\star_l,l,1})}_{\Delta^n_{gl,1}} \leq I^n_{th}$$

$$C4 : \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l}(\bar{g}_{u^\star_l,l,1}) + \Delta^n_{gl,1} \leq I^n_{th,1}$$

Reformulation of Robust Resource Allocation Problem

Similarly,

$$C_5 : \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} \bar{g}_{u_l^\star,l,2} + \Delta^n_{gl,2} \leq I^n_{th,1}, \quad \forall n$$

$$C_9 : I^n_{u_l,l} + \Delta_{I^n_{u_l,l}} \leq w^n_{u_l}, \quad \forall n, u_l$$

where,

$$\Delta^n_{gl,2} = \max_{g^n_{l,2} \in \mathcal{R}^n_{g_l,2}} \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} (g_{u_l^\star,l,2} - \bar{g}_{u_l^\star,l,2}) \leq I^n_{th,1}, \quad \forall n$$

$$\Delta_{I^n_{u_l,l}} = \max_{I^n_{u_l,l} \in \mathcal{R}^n_{I_{u_l},l}} I^n_{u_l,l} + \sigma^2$$

- Protection functions in terms of general norm

$$\begin{aligned} \Delta^n_{gl,1} &= \max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l} (g_{u_l^\star,l,1} - \bar{g}_{u_l^\star,l,1}) \\ &= \max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} S^n_{l,1} (g^n_{l,1} - \bar{g}^n_{l,1})^T \\ &= \max_{g^n_{l,1} \in \mathcal{R}^n_{g_l,1}} S^n_{l,1} (M^{-1}_{g_l,1} w^n_{l,1}) = \psi^n_{l,1} ||M^{-1}_{gl,1}(S^T_{l,1})||^\star \end{aligned}$$

Dual Norm:
Given a norm $||y||$ for a vector y, its dual norm induced over the dual space of linear functionals z is

$$||z||^{\star} = \max_{||y|| \leq 1} z^T y$$

Note: If y is a linear norm of order $\alpha \geq 2$ then dual norm is a linear norm of order $\beta = 1 + \frac{1}{\alpha - 1}$

$$\Delta^n_{gl,1} = \psi^n_{l,1} ||M^{-1}_{gl,1}(S^T_{l,1})||^{\star} = \psi^n_{l,1} ||M^{-1}_{gl,1}(S^T_{l,1})||_{\beta} \tag{3}$$

Similarly,

$$\begin{aligned} \Delta^n_{gl,2} &= \psi^n_{l,2} ||M^{-1}_{gl,2}(S^T_{l,2})||^{\star} = \psi^n_{l,2} ||M^{-1}_{gl,2}(S^T_{l,2})||_{\beta} \\ \Delta_{I^n_{u_l,l}} &= \Gamma^n_{u_l} ||M^{-1}_{gl,2} I^n_{u_l,l}||^{\star} = \Gamma^n_{u_l} ||M^{-1}_{gl,2} I^n_{u_l,l}||_{\beta} \end{aligned} \tag{4}$$

Final Optimization Problem

$$\max_{\{x^n_{u_l}, S^n_{U_l,l}, w^n_{u_l}\}} \quad \sum_{u_l \in \mathcal{U}_l} \sum_{n=1}^{N} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{h^n_{u_l,l,1}}{x^n_{u_l} w^n_{u_l}})$$

$$s.t. \quad C_1 : \quad \sum_{u_l \in \mathcal{U}_l} x^n_{u_l} \leq 1, \quad \forall n \in \mathcal{N}$$

$$C_2 : \quad \sum_{n \in \mathcal{N}} S^n_{u_l,l} \leq P^{\max}_{u_l}, \quad \forall u_l \in \mathcal{U}_l$$

$$C_3 : \quad \sum_{u_l \in \mathcal{U}_l} \sum_{n \in \mathcal{N}} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{l,u_l} \leq P^{\max}_l$$

$$C_4 : \quad \sum_{u_l \in \mathcal{U}_l} S^n_{u_l,l} \bar{g}^n_{u^\star_l,l,1} + \psi^n_{l,1} \left(\sum_{k=1}^{|\mathcal{U}_l|} (M^n_{gl,l})^{-1}(k,:) S^n_{l,1})^\beta \right)^{1/\beta} \leq \mathcal{I}^n_{th,1}, \quad \forall n \in \mathcal{N}$$

$$C_5 : \quad \sum_{u_l \in \mathcal{U}_l} \frac{h^n_{u_l,l,1}}{h^n_{l,u_l,2}} S^n_{u_l,l} \bar{g}^n_{l,u^\star_l,2} + \psi^n_{l,2} \left(\sum_{k=1}^{|\mathcal{U}_l|} (M^n_{gl,2})^{-1}(k,:) S^n_{l,2})^\beta \right)^{1/\beta} \leq \mathcal{I}^n_{th,2}, \quad \forall n \in \mathcal{N}$$

$$C_6 : \quad \sum_{n \in \mathcal{N}} \frac{1}{2} x^n_{u_l} B_{RB} \log_2(1 + S^n_{u_l,l} \frac{\gamma^n_{u_l,l,1}}{x^n_{u_l}}) \geq Q_{u_l},$$

$$C_8 : \quad x^n_{u_l} \in (0, 1]$$

$$C_9 : \quad I^n_{u_l,l} + \Delta_{I^n_{u_l,l}} + \sigma^2 \leq w^n_{u_l}$$

Closed Form Expression

- Power allocation is given by

$$P_{u_l,l}^{n\star} = \frac{S_{u_l,l}^{n\star}}{x_{u_l}^{n\star}} = \left[\delta_{u_l,l}^{n} - \frac{w_{u_l}^{n}}{h_{u_l,l,1}^{n}}\right]^{+} \quad (5)$$

where,

$$\delta_{u_l,l}^{n} = \frac{\frac{1}{2} B_{RB} \frac{1+\lambda_{u_l}}{\log 2}}{\rho_{u_l} + \frac{h_{u_l,l,1}^{n}}{h_{l,u_l,2}^{n}} v_l + (\bar{g}_{u_l^\star,l,1}^{n} + \psi_{l,1}^{n} m_{u_l,u_l g_l,1}^{n})\psi_n + \frac{h_{u_l,l,1}^{n}}{h_{l,u_l,2}^{n}} (\bar{g}_{l,u_l^\star,2}^{n} + \psi_{l,2}^{n} m_{u_l,u_l g_l,2}^{n})\phi_n}$$

- RB allocation is given by

$$x_{u_l}^{n\star} = \begin{cases} 1, & \mu_n \leq \mathcal{X}_{u_l,l}^{n} \\ 0, & otherwise \end{cases} \quad (6)$$

where,

$$\mathcal{X}_{u_l,l}^{n} = \frac{1}{2}(1 + \lambda_{u_l}) B_{RB} \left[\log_2(1 + \frac{S_{u_l,l}^{n} h_{u_l,l,1}^{n}}{x_{u_l}^{n} w_{u_l}^{n}}) - \frac{S_{u_l,l}^{n} \gamma_{u_l,l,1}^{n}}{(x_{u_l}^{n} w_{u_l}^{n}) + S_{u_l,l}^{n} \gamma_{u_l,l,1}^{n}} \log(2)\right]$$

Joint RB and Power Allocation Algorithm

Algorithm

1. Each relay l $\in \mathcal{L}$ estimated the reference gain $\bar{g}^n_{u_l^\star,l,1}$ and $\bar{g}^n_{u_l^\star,l,1}$ from previous time slots $\forall u_l \in \mathcal{U}_l$ and $n \in \mathcal{N}$.
2. Initialize Lagrange Multipliers to some positive value and set $t = 0, S^n_{u_l,l} = \frac{P^{\max}_{u_l}}{N} \quad \forall u_l, n.$
3. **Repeat**
4. Set $t = t + 1$
5. Calculate $x^n_{u_l}$ and $S^n_{u_l,l}$ from closed form solution.
6. Update Lagrange multipliers using Gradient descent algorithm.
7. **Until**: Convergence or Maximum Iteration