SBL for Cluster-sparse Channel Estimation in OFDM Systems

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Outline

1. OFDM Channel
   - Block-sparsity
2. Algorithm: Derivation
   - Parallel Cluster-SBL Algorithm
3. Complexity
4. Time Varying SISO-OFDM Channels

Sparse Bayesian Learning

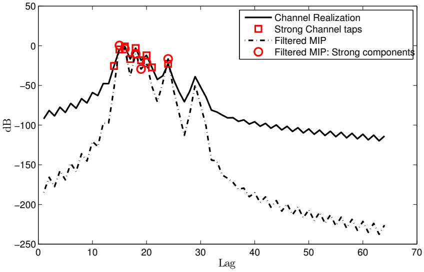


Figure: OFDM channel

- $\mathbf{h} \in \mathbb{C}^{L\times 1}$: Sampled from $\mathbf{h}[t] = \mathbf{g}_t[t] * \tilde{\mathbf{h}}[t] * \mathbf{g}_r[t]$, where $\tilde{\mathbf{h}}[t]$: Sparse, $\mathbf{g}_t[t]$ and $\mathbf{g}_r[t]$: baseband transmit and receive filters

Block Sparsity

$\mathbf{h}(1)$	$\mathbf{h}(2)$	$\dots$	$\mathbf{h}(M)$	$\dots$	$\mathbf{h}((B-1)M+1)$	$\dots$	$\mathbf{h}(BM)$

BSBL: $\mathbf{h}_1 \in \mathcal{R}^M$ $\quad$ $\mathbf{h}_B \in \mathcal{R}^M$

$\mathbf{h}_1 \sim \mathcal{N}(0, \gamma_1 \mathbf{I}_M)$ $\quad$ $\mathbf{h}_B \sim \mathcal{N}(0, \gamma_B \mathbf{I}_M)$

SBL: M = 1

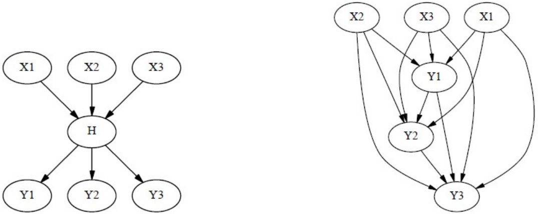
- SBL: $\mathbf{h}(1) \sim \mathcal{CN}(0, \gamma(1))$
- Block SBL: $[\mathbf{h}(1), \dots, \mathbf{h}(B)] \sim \mathcal{CN}(0, \gamma(1))$

Why use the notion of Block-sparsity

- Model Accuracy: Underwater acoustic channels (UWA)[1], Ultra wideband channels (UWB) (Saleh-Valenzuela (S-V) model)
- Complexity
  - $\gamma \in \mathbb{R}_+^{B \times 1}$ is the vector to be estimated (in SBL $\gamma \in \mathbb{R}_+^{L \times 1}$)
  - Can we devise techniques that exploit the decrease in problem dimension?

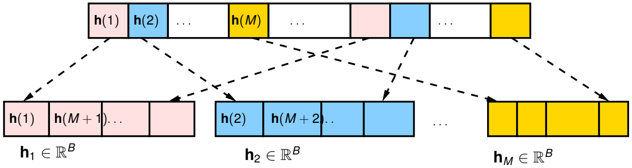
[1] Clustered Adaptation for Estimation of Time-Varying Underwater Acoustic Channels, Z. Wang, S. Zhou, J. C. Preisig, K. R. Pattipati, and P. Willett, *IEEE Trans. on Sig. Proc.*, Vol. 60, No. 6, June 2012

Hidden variables



(a) with hidden variable (b) no hidden variable

Can we use the same trick?



Received vector: $\mathbf{y} = \Phi\mathbf{h} + \mathbf{n}$

$$\mathbf{y} = \sum_{m=1}^{M} \mathbf{t}_m, \quad \text{where} \quad \mathbf{t}_m \triangleq \boldsymbol{\Phi}_m \mathbf{h}_m + \mathbf{n}_m, \quad 1 \leq m \leq M.$$

EM Algorithm

EM algorithm for estimating γ, as follows:

$$\text{E-step} : Q\left(\gamma|\gamma^{(r)}\right) = \mathbb{E}_{\mathbf{t},\mathbf{h}|\mathbf{y};\gamma^{(r)}}[\log p(\mathbf{y},\mathbf{t},\mathbf{h};\gamma)]$$

$$\text{M-step} : \gamma^{(r+1)} = \underset{\gamma\in\mathbb{R}_{+}^{B\times 1}}{\arg\max}\, Q\left(\gamma|\gamma^{(r)}\right).$$

$$\begin{aligned} p(\mathbf{t},\mathbf{h}|\mathbf{y};\gamma^{(r)}) &= p(\mathbf{h}|\mathbf{t},\mathbf{y};\gamma^{(r)})p(\mathbf{t}|\mathbf{y};\gamma^{(r)}) \\ &= \underbrace{p(\mathbf{h}|\mathbf{t};\gamma^{(r)})}_{\mathbb{E}_{\mathbf{h}|\mathbf{t};\gamma^{(r)}}}\underbrace{p(\mathbf{t}|\mathbf{y};\gamma^{(r)})}_{\mathbb{E}_{\mathbf{t}|\mathbf{y};\gamma^{(r)}}}. \end{aligned}$$

Finding the posterior

To compute $Q(\gamma|\gamma^{(r)})$: compute posterior distribution $p(\mathbf{h}|\mathbf{t};\gamma^{(r)})$

- Likelihood: $p(\mathbf{t}_m|\mathbf{h}_m) = \mathcal{N}(\Phi_m\mathbf{h}_m, \beta_m\sigma^2\mathbf{I}_N)$ for $1 \leq m \leq M$
- Prior $p(\mathbf{h}_m;\gamma) = \mathcal{N}(0,\Gamma)$, $\Gamma = \text{diag}(\gamma)$

To compute $p(\mathbf{t}|\mathbf{y};\gamma^{(r)})$

- Define $\mathbf{H} = \mathbf{1}_M \otimes \mathbf{I}_N$
- Hence, $\mathbf{y} = \mathbf{H}\mathbf{t}$

We obtain $p(\mathbf{t}|\mathbf{y};\gamma^{(r)}) = \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$, where

$$\boldsymbol{\mu}_t = (\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T)\mathbf{H}^T(\mathbf{H}(\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T)\mathbf{H}^T)^{-1}\mathbf{y}$$

$$\boldsymbol{\Sigma}_t = (\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T) - (\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T)\mathbf{H}^T (\mathbf{H}(\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T)\mathbf{H}^T)^{-1}\mathbf{H}(\mathbf{R} + \boldsymbol{\Phi}_B \Gamma_B \boldsymbol{\Phi}_B^T).$$

- $\boldsymbol{\Phi}_B \in \mathbb{R}^{NM \times BM}$ is a block diagonal matrix with $\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_M$ along the diagonal
- $\Gamma_B = \mathbf{B} \otimes \Gamma$, where $\Gamma = \text{diag}(\gamma)$
- $\mathbf{R}$ is a diagonal matrix: m^{th} diagonal entry $\mathbf{R}_m = \beta_m \sigma^2 \mathbf{I}_N$

M-step

the update for γ is obtained as follows:

$$\gamma^{(r+1)} = \arg\max_{\gamma \in \mathbb{R}_+^{B\times 1}} \mathbb{E}_{\mathbf{t},\mathbf{h}|\mathbf{y};\gamma^{(r)}}[\log p(\mathbf{t},\mathbf{h};\gamma)]$$

$$= \arg\max_{\gamma \in \mathbb{R}_+^{B\times 1}} (c' - \mathbb{E}_{\mathbf{t}|\mathbf{y};\gamma^{(r)}} \mathbb{E}_{\mathbf{h}|\mathbf{t};\gamma^{(r)}} [\frac{\mathbf{h}^T \Gamma_B^{-1} \mathbf{h}}{2} + \frac{1}{2} \log |\Gamma_B|])$$

$$\gamma^{(r+1)} = \arg\min_{\gamma \in \mathbb{R}_+^{B\times 1}} (c' + \frac{M}{2} \log |\Gamma| + \frac{1}{2} \sum_{m=1}^{M} \mathrm{Tr}(\Gamma^{-1} \boldsymbol{\Sigma}_{\mathbf{h}_m}) + \mathrm{Tr}\left(\Gamma^{-1} \frac{\boldsymbol{\Sigma}_{\mathbf{h}_m} \boldsymbol{\Phi}_m^T \mathbf{R}_m \boldsymbol{\Phi}_m \boldsymbol{\Sigma}_{\mathbf{h}_m}}{\beta_m^2 \sigma^4}\right))$$

$$\gamma^{(r+1)} = \frac{1}{M} \sum_{m=1}^{M} \mathrm{diag}\left(\boldsymbol{\Sigma}_{\mathbf{h}_m} + \frac{\boldsymbol{\Sigma}_{\mathbf{h}_m} \boldsymbol{\Phi}_m^T \mathbf{R}_m \boldsymbol{\Phi}_m \boldsymbol{\Sigma}_{\mathbf{x}_m}}{\beta_m^2 \sigma^4}\right).$$

B-SBL

$$\text{E-step}: Q\left(\gamma_l|\gamma_l^{(r)}\right) = \mathbb{E}_{\mathbf{h}|\mathbf{y};\gamma_l^{(r)}}[\log p(\mathbf{y},\mathbf{h};\gamma_l)]$$

$$\text{M-step}: \gamma_l^{(r+1)} = \underset{\gamma_l \in \mathbb{R}_+^{L\times 1}}{\arg\max}\, Q\left(\gamma_l|\gamma_l^{(r)}\right).$$

Complexity

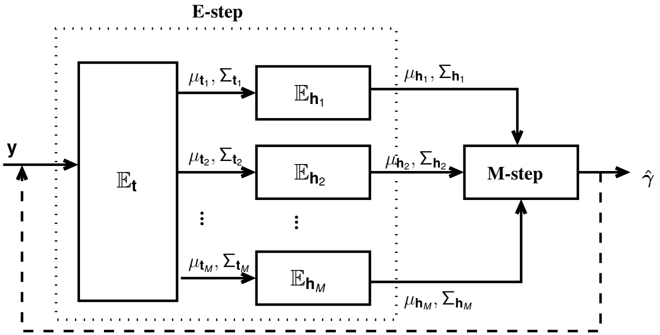
Complexity of BSBL:

- Inversion complexity: $\mathcal{O}(N^3)$
- Multiplication complexity: $\mathcal{O}(N^2L)$

Complexity of P-CSBL:

- Inversion complexity: $\mathcal{O}(N^3)$
- Multiplication complexity: $\mathcal{O}(N^2M)$ or $\mathcal{O}(NBL)$?

Parallel Implementation



- Time varying channel: First order AR model
- Use Kalman based tracking and smoothing
- Can be implemented as M parallel Kalman filters