Co-operative Spectrum Sensing from Sparse Observations

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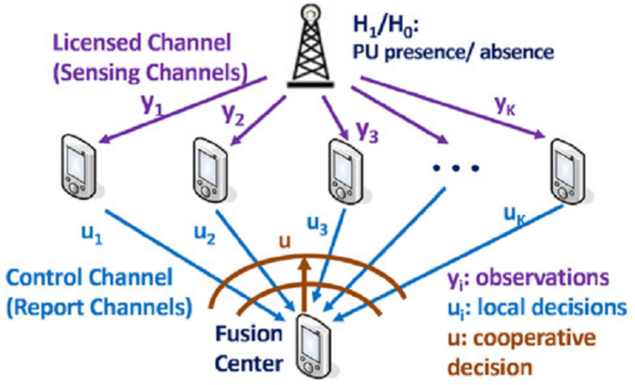
Outline

Introduction



Co-operative spectrum sensing in CR Network

System Model - I

- Consider a Cognitive radio network with m CR nodes monitoring a subset of n channels
- A channel is either occupied or unoccupied by a primary $\Rightarrow$ state 1 or 0 respectively
- Assumption: Number of channels in state $1 \Rightarrow s$ and $s < n$
- Goal: To identify the occupied channels in a co-operative manner
- Constraint: limited number of CR nodes to observe n channels simultaneously

Problem Formulation

- Conventional co-operative spectrum sensing technique: Spectrum sensing in all the n channels and total number of observations from m CR nodes $\Rightarrow mn$ samples
- Each CR node has to support wide-band spectrum sensing $\Rightarrow$ high RF signal acquisition costs
- If the monitored spectrum has very high bandwidth then each CR node has to sample Nyquist rate for signal recovery, which is infeasible

Problem Formulation

- Instead of sending all channels and sending each measurements only a small subset of measurements are sent
- Using frequency-selective filters, a CR node forms a subset of measurements using linear combinations of multiple channels
- The filter co-efficients are designed to be random numbers and can be implemented easily
- Suppose there are p frequency selective filters in each CR node transmitting p reports for n channels and if $pm < n$, then the number of reports from all CR nodes are less than the total number of channels.

Problem Formulation

- The sensing process at each node is represented by the filter co-efficient matrix **F**
- Let $n \times n$ matrix **R** represent the channel state information with diagonal entries representing the channel state information
- Let **G** represent the channel gains matrix represented as

$$G_{i,j} = P_i(d_{i,j})^{-\frac{\alpha}{2}}|h_{i,j}| \tag{1}$$

  - $P_i \Rightarrow$ transmit power of primary user i
  - $d_{i,j} \Rightarrow$ distance between primary using j^{th} channel
  - $h_{i,j} \Rightarrow$ is the channel fading gain
  - $\alpha \Rightarrow$ is the propagation loss factor

Problem Formulation

- The measurement report sent to the fusion center can be expressed as a $p \times m$ matrix

$$M_{p\times m} = F_{p\times n} R_{n\times n} G^{T}_{m\times n} \tag{2}$$

- Due to loss or errors some of the entries of **M** are possibly missing.
- The binary channels states (diagonal entries of **R** are to be estimated from the available entries of M

Matrix Completion

- Fusion center observes only subset of entries of **M**, i.e., **E** $\subseteq [p] \times [m]$ of **M**
- It is possible to recover the missing entries of **M** using matrix completion if the following two properties hold, i.e.,
- **Low Rank**: rank(M) is equation to s, which is the number of occupied channels in the network
- **Incoherent property**: If **F** is random, then from the expression for **M**, **R** has only s non-zeros on the diagonal, the SVD factors of **M**, i.e., **U**, Σ, and **V** satisfy the incoherence condition

Incoherent Property

- If the SVD of $\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^H$ then there exists a constant μ_0 such that for all $i \in [p]$, $j \in [m]$,

$$\sum_{k=1}^{s} U_{i,k}^2 \leq \mu_0 s$$
$$\sum_{k=1}^{s} V_{i,k}^2 \leq \mu_0 s$$

- There exists a μ_1 such that

$$\sum_{k=1}^{s} U_{i,k}\Sigma_k V_{j,k}| \leq \mu_1 s^{\frac{1}{2}} \quad (3)$$

Matrix Completion

- **M** is in general incomplete due transmission failure or because each CR collects a random (upto p) number of measurements due to hardware limitations
- The received entries are uniformly distributed with high probability, i.e., each entry shows up in **E** identically and independently manner
- Given $\mathbf{E}_{p\times m}$, the partial observation of **M** is defined as a $p \times m$ matrix $M^E_{i,j}$
- The unobserved elements of **M** from $\mathbf{M}^E$ are recovered first using matrix completion techniques
- Then, given **F** and **M**, the $\mathbf{RG}^T$ matrix is reconstructed using the fact that all but s rows are zero

Matrix Completion

- Under suitable conditions, a low rank matrix can be recovered from a random, yet small subset of its entries by nuclear norm minimization
- Fixed Point Continuation Iterative algorithm is used in the paper (More details in the paper)
- Since p and m are much smaller than n, the proposed method requires much less sensing and transmission compared to conventional methods

System Model - II

- Co-operative CR nodes equipped with narrow band filters and can tune to a subset of overall spectrum are feasible
- The exchange of spectrum occupancy decisions to fusion center results in determining the spectrum usage
- User diversity is utilized to reduce the hardware cost per CR in terms of lowering the Nyquist rates
- However the number of CRs for detecting the same frequency decreases, i.e., reduces the channel diversity against fading
- Given the same user diversity, can a desired trade-off between channel diversity and sampling costs be achieved?

System Model - II

- Consider a wide-band spectrum with a total bandwidth of B Hz consisting of n non-overlapping channels of equal bandwidth B/n Hz
- As in the previous model at any instant of time the band is occupied with only s channels
- Let s^f denote the unknown transmitted spectrum of the primary user with a sparsity order s
- m CR nodes are spatially distributed and co-operate during the sensing stage. Each CR node can sense only a segment of the spectrum of bandwidth mB/n

Problem Formulation

- The Nyquist sampling rate is reduced by m/n, where $m < n$
- The spectra that can be perceived at CR receivers is represented as r_j in the noise free case, where $j = 1, \dots, m$

$$r_j = H_j s_f \tag{4}$$

- H_j is an $n \times n$ matrix whose diagonal elements are the fading co-efficients
- r_j and s_f share the same non-zero support provided that deep fade cases are not considered

Problem Formulation

- Each CR only monitors m out of n channels, i.e., the received spectrum after frequency selective filter B_j
- $r_{s,j} = B_j r_j$ and $B_j \in 0, 1^{m \times n}$
- If Nyquist sampling is adapted at the CR node then $x_j = F^{-1} r_{s,j}$, F is the DFT matrix
- When compressive sampling is used, a $k \times m$ matrix Φ_j can be used as a measurement matrix
- k/m is the compression ratio. In the presence of noise the signal model is

$$x_j = \Phi_j F^{-1} r_{s,j} + w_j, \forall j \quad (5)$$

Matrix Rank Minimization

- The discrete time sample vector, x_j has an averaging sampling rate of kb/n
- When multiple $\{x_j\}$s are used collectively to decide the common non-zero support of r_j, it becomes a co-operative support detection problem
- The multiple measurements $\{x_j\}$ allow sparse representations due to low spectrum utilization and possess a low rank property
- The low rank property can be used to estimate r_j from x_j using matrix rank minimization(MRM) formulation
- Rank of $R^f = [r_1, r_2, \ldots, r_m]$ has a rank of order $r = \min(s, m)$

Matrix Rank Minimization

- The MRM is NP-hard and can be replaced by the convex problem, i.e., the nuclear norm
- After obtaining the estimate $\hat{R}^f$, the fusion center can detect the spectrum occupancy based on the total energy collected from all CR nodes on each channel, i.e.,

$$d^f[.] = \sum_{j=1}^{M} |\hat{r}_j[.]|^2 > \eta \tag{6}$$

- $\hat{r}_j$ is the column of $\hat{R}^f$ and η is the detection threshold

References

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