DMT in a training based TDD-SIMO system

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Feb 12th, 2011

OUTLINE

- Introduction to DMT
- System Model
- Training
- Power Control
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- Conclusion

- The multiplexing gain and the diversity is defined as follows:

$$g_m \triangleq \lim_{\bar{P}\to\infty} \frac{R(\bar{P})}{\log \bar{P}}, \quad (1)$$

where $R(\bar{P})$ is the rate adapted as a function of $\bar{P}$

$$d \triangleq -\lim_{\bar{P}\to\infty} \frac{\log \Pi}{\log \bar{P}} \quad (2)$$

where Π is the outage probability.

System Model

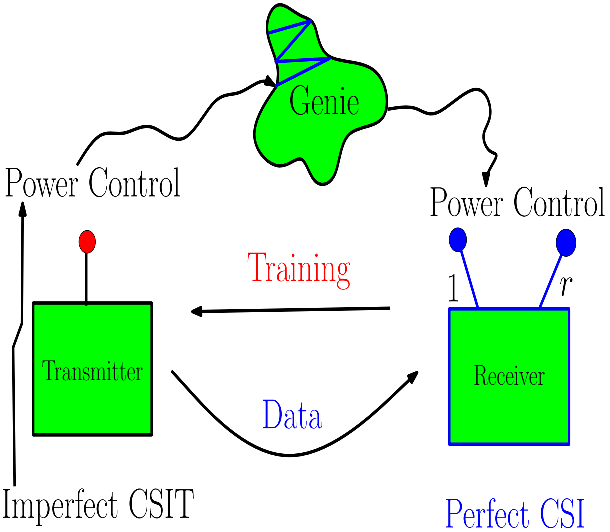

Figure: SYSTEM MODEL

System Model

- The input Output relation is:

$$Y = H_T x + W, \tag{3}$$

  where $H_T \in \mathbb{C}^{r \times 1}$, $W \sim \mathbb{CN}(0, I_r)$.
- $H_T = \lambda V$ denote the singular value decomposition (SVD), where $\lambda = \sqrt{\sum_{i=1}^{r} |h_i|^2}$ is the singular value of the channel and $V \in \mathbb{C}^{r \times 1}$ is a unit vector, i.e., $V^H V = 1$.
- TDD $\rightarrow \quad H_T = H_R^T$.

Training

- $$Y = \lambda V^T S_\tau + W, \quad (4)$$
  $$S_\tau = \sqrt{\bar{P}L}V^*, \quad (5)$$
- $$y_T = \sqrt{\bar{P}L}\lambda + w, \quad (6)$$
- $$\hat{\lambda} = \lambda + \bar{n}, \quad (7)$$

where $\bar{n} = \frac{w+w^*}{\sqrt{2\bar{P}L}}$ is the symmetrized training noise.

Power Control

- $$\mathbb{P}(\hat{\lambda}) = \begin{cases} k \times P(\hat{\lambda}) & \hat{\lambda} \geq \theta(\bar{P}) \\ \bar{P}^{l} & \hat{\lambda} < \theta(\bar{P}) \end{cases} \tag{8}$$

  $$P(\hat{\lambda}) \triangleq \frac{\exp(\frac{TR}{T-L}) - 1}{\hat{\lambda}^2}, \tag{9}$$

  where $\theta(\bar{P}) = \frac{1}{\bar{P}^n}$ for some n and $l > 0$.

- $$\int_0^\infty \mathbb{P}(\hat{\lambda}) f_{\hat{\lambda}}(\hat{\lambda}; \bar{P}) d\hat{\lambda} = \bar{P}, \tag{10}$$

  where $f_{\hat{\lambda}}(\hat{\lambda}; \bar{P})$ is the probability density function of the estimated channel gain $\hat{\lambda}$.

Power Constraint

- $$\mathbb{E}\left(\mathbb{P}(\hat{\lambda})\right) = k\left(\exp(\frac{TR}{T-L}) - 1\right) F(\bar{P}) + \underbrace{\bar{P}^l \int_{-\infty}^{\theta(\bar{P})} f_{\hat{\lambda}}(x;\bar{P})dx}_{A}, \quad (11)$$

  where,

  $$F(\bar{P}) \triangleq \int_{\theta(\bar{P})}^{\infty} \frac{1}{x^2} f_{\hat{\lambda}}(x;\bar{P})dx. \quad (12)$$

- $$F(\bar{P}) = \int_{\theta(\bar{P})}^{1} \frac{1}{x^2} f_{\hat{\lambda}}(x;\bar{P})dx + \int_{1}^{\infty} \frac{1}{x^2} f_{\hat{\lambda}}(x;\bar{P})dx, \quad (13)$$

Power Constraint

Theorem

$\mathbb{E}\left(\mathbb{P}(\hat{\lambda})\right) \leq$ *constant* $< \infty$ *if*

Case 1: $0 \leq n \leq r - \frac{1}{2}$ *and* $\forall l \geq 0$ *and*

- *Proof*: Idea is to find the distribution of $f_{\hat{\lambda}}(x; \bar{P})$ and find the condition under which the integral converges.

Outage analysis

- Let $l = 2$

$$\Pi \triangleq \Pr\left(\frac{T-L}{T}\log(1+\gamma\mathbb{P}(\hat{\lambda})) < R\right) \tag{14}$$

- where,

$$\Pi = \Pi_1 \Pr(\hat{\lambda} > \theta(\bar{P})) + \Pi_2 \Pr(\hat{\lambda} \leq \theta(\bar{P})), \tag{15}$$

$$\Pi_1 = \Pr\left(\frac{T-L}{T}\log(1+P(\hat{\lambda})\gamma) < R\right), \tag{16}$$

$$\text{and} \;\; \Pi_2 \triangleq \Pr\left(\frac{T-L}{T}\log(1+\bar{P}^2\gamma) < R\right). \tag{17}$$

- Now,

$$\Pi_2 = \Pr\left(\gamma < \frac{\left(\exp\left(\frac{RT}{T-L}\right)-1\right)}{\bar{P}^2}\right) \approx \frac{C_1}{\bar{P}^{2r}}. \tag{18}$$

where $C_1 = \frac{\left(\exp\left(\frac{RT}{T-L}\right)-1\right)^r}{r!} > 0.$

Outage analysis

Theorem

With $\bar{P}$ sufficiently large and if R is such that outage probability equal to zero is achievable in the presence of perfect CSIT and CSIR, a system with imperfect CSIT and perfect CSIR will be in outage if and only if

$$P(\hat{\gamma}) < P_{opt}(\gamma).$$

- The equivalent condition here is,

Lemma

At high SNR, the outage probability, Π, of a system with power control function (8) is given by,

$$\Pi = \Pr(k\gamma < |\bar{n}|^2) \Pr(\hat{\lambda} > \theta(\bar{P})) + \Pi_2 \Pr(\hat{\lambda} \leq \theta(\bar{P})). \quad (19)$$

Continued...

-

$$\Pr(k\gamma < |\bar{n}|^2) = \int_0^\infty f_{|\bar{n}|}(y) \underbrace{\int_0^{y^2/k} \frac{1}{(r-1)!} x^{r-1} e^{-x} dx}_{B}\, dy. \quad (20)$$

$$\Upsilon(r, \frac{y^2}{k}) = \frac{e^{(-y^2/k)}}{r!} \sum_{i=0}^{\infty} \frac{y^{2(r+i)}}{k^{r+i} b_i} \quad (21)$$

$$b_i = 1 \qquad b_i = \prod_{j=0}^{i} (r+j), \qquad i > 0, \quad (22)$$

$$\frac{\beta}{\sigma} \frac{2}{\sqrt{2\pi}\beta} \int_0^\infty y^{2(r+i)} e^{-\frac{y^2}{2\beta^2}} dy = \frac{\beta}{\sigma} \frac{1}{\sqrt{2\pi}\beta} \int_{-\infty}^\infty y^{2(r+i)} e^{-\frac{y^2}{2\beta^2}} dy, \quad (23)$$

where $\beta^2 = \frac{k\sigma^2}{2\sigma^2+k} \propto \frac{1}{P}$.

Continued...

- Integral in (23) is the $2(r+i)^{th}$ moment of a Gaussian random variable, resulting in $\frac{\beta^{2(r+i)+1}}{\sigma}$. Now, substituting (21) in (20) leads to:

$$\Pr(k\gamma < |\bar{n}|^2) = \frac{\beta}{2\sigma r!} \sum_{i=0}^{\infty} \frac{\beta^{2(r+i)}}{k^{r+i} b_i} \propto \frac{1}{\bar{P}^{2r}}. \tag{24}$$

Combining the terms in (19), an upper bound on the outage probability Π is thus given by,

$$\Pi \leq \frac{1}{\bar{P}^{2r}} \left(C_2 \Pr(\hat{\gamma} \geq \bar{P}^{-1}) + C_1 \Pr(\hat{\gamma} < \bar{P}^{-1}) \right) + \mathcal{O}(\frac{1}{\bar{P}^{4r}}) \tag{25}$$

$$\Rightarrow \log \Pi \leq c_1 - 2r \log \bar{P}, \tag{26}$$

The diversity gain d is,

$$d \triangleq - \lim_{\bar{P} \to \infty} \frac{\log \Pi}{\log \bar{P}} = 2r. \tag{27}$$

DMT

Theorem

Given r receive antennas and L training symbols being used per coherence interval T to estimate CSIT in a SIMO system with a perfect CSIR, we have the following equation for diversity order as a function of multiplexing gain g_m,

$$d(r) = r\left(2 - \left(\frac{g_m T}{T - L}\right)\right). \tag{28}$$

- *Proof*: See paper.

Simulation Results

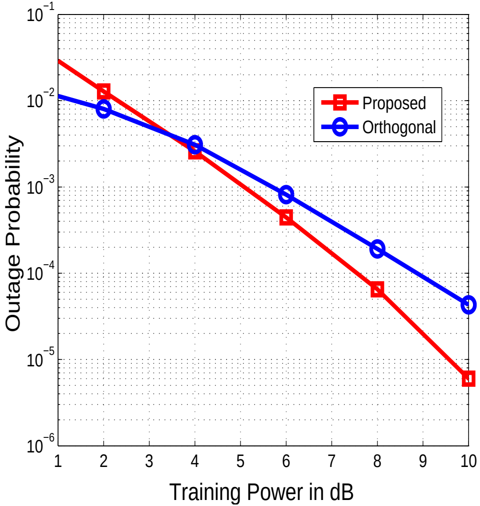

Figure: Probability of Outage vs SNR

Improved Power control

- Using the following power control,

$$P(\hat{\lambda}) \triangleq \frac{\exp(\frac{TR}{T-L}) - 1}{\hat{\lambda}^{2r}}, \tag{29}$$

- results in a diversity order of $r = r(r+1)$.

Infinite diversity order using power controlled training

- $$S_\tau = (r) V^* \frac{1}{\lambda}\sqrt{P(\lambda)} \tag{30}$$

- where $\sqrt{P(\lambda)} = \frac{\sqrt{\bar{P}}}{\lambda}$. Also, $\mathbb{E}(Tr(S_\tau S_\tau^H)) = \bar{P}$. The corresponding estimate of the power control $\sqrt{P(\lambda)}$ at the transmitter is obtained as,

$$\mathbf{P_c} \triangleq \left| \frac{\Re\{y_T\}}{\sqrt{r}} \right| = \left| \sqrt{P(\lambda)} + \frac{\Re(w_T)}{\sqrt{r}} \right|. \tag{31}$$

$$\mathbb{E}\left\{ |\mathbf{P_c}|^2 \right\} = \frac{\bar{P}}{r(r-1)} + \frac{1}{2r}. \tag{32}$$

- With proper power scaling $C(\bar{P})$, we have

$$\tilde{Y}_R = \sigma \mathbf{P}_c C(\bar{P}) x + w_R, \tag{33}$$

Outage

- $$P_{out} \triangleq \Pr\left\{\frac{T-L}{T}\log_2\left(1+\sigma^2\mathbf{P}_c^2 C(\bar{P})^2\right) < R\right\}. \quad (34)$$

$$\begin{aligned} P_{out} &= \Pr\left\{\sigma^2\mathbf{P}_c^2 < \frac{2^{\frac{TL}{T-L}}-1}{C(\bar{P})^2}\right\}, && (35)\\ &= \Pr\left\{\sigma^2\left(\sqrt{P(\sigma)}+\frac{\Re\{w_T\}}{\sqrt{r}}\right)^2 < \frac{2^{\frac{TL}{T-L}}-1}{C(\bar{P})^2}\right\}, && (36)\\ &\leq \Pr\left\{\left|\sqrt{\bar{P}}+\frac{\Re\{w_T\}\sigma}{\sqrt{r}}\right| < \sqrt{\frac{2^{\frac{TL}{T-L}}-1}{C(\bar{P})^2}}\right\}, && (37)\\ &\leq \Pr\left\{\Re\{w_T\}\sigma < \bar{R}_0\right\}, && (38) \end{aligned}$$

Continuation of the messy calculation:

- where $\bar{R}_0 \triangleq \sqrt{r}\left(-\sqrt{\bar{P}} + \sqrt{\frac{2^{\frac{TL}{T-L}}-1}{C(\bar{P})^2}}\right)$

$$
\begin{aligned}
P_{out} \leq\ & \Pr\left\{\Re\{w_T\}\sigma < \bar{R}_0 \bigcap |\Re\{w_T\}| > \sigma \bigcap \Re\{w_T\} < 0\right\} \\
+\ & \Pr\left\{\Re\{w_T\}\sigma < \bar{R}_0 \bigcap \Re\{w_T\} > \sigma \bigcap \Re\{w_T\} \geq 0\right\} \\
+\ & \Pr\left\{\Re\{w_T\}\sigma < \bar{R}_0 \bigcap \Re\{w_T\} \leq \sigma \bigcap \Re\{w_T\} < 0\right\} \\
+\ & \Pr\left\{\Re\{w_T\}\sigma < \bar{R}_0 \bigcap \Re\{w_T\} \leq \sigma \bigcap \Re\{w_T\} \geq 0\right\},
\end{aligned} \tag{39}
$$

At Last!

- $$\begin{aligned} P_{out} &\leq \Pr\left\{\Re\{w_T\}^2 > -\bar{R}_0\right\} + \Pr\left\{\sigma^2 < \bar{R}_0\right\} && (40)\\ &+ \Pr\left\{\sigma^2 > -\bar{R}_0\right\} + \Pr\left\{\Re\{w_T\}^2 < \bar{R}_0\right\} 1_{\bar{R}_0 > 0}, && (41)\\ &\leq 2Q(\sqrt{-\bar{R}_0}) + \Pr\left\{\sigma^2 < \bar{R}_0\right\} + \Pr\left\{\sigma^2 > -\bar{R}_0\right\} && (42)\\ &+ \Pr\left\{\Re\{w_T\} < \sqrt{\bar{R}_0}\right\} 1_{\bar{R}_0 > 0}, && (43) \end{aligned}$$

$$\begin{aligned} \Pr\{\sigma^2 > -\bar{R}_0\} &= \int_{-\bar{R}_0}^{\infty} x^{r-1} e^{-x} dx, && (44)\\ &= e^{\bar{R}_0} \sum_{k=0}^{r-1} \frac{(-1)^k \bar{R}_0^k}{k!}, && (45) \end{aligned}$$

Infinite Diversity

- Thus, $P_{out} \doteq e^{-\bar{P}}$. Now it is clear that the diversity order,

$$d(r) = -\lim_{\bar{P}\to\infty} \frac{\log P_{out}}{\log \bar{P}} = \infty.$$

Simulation Results

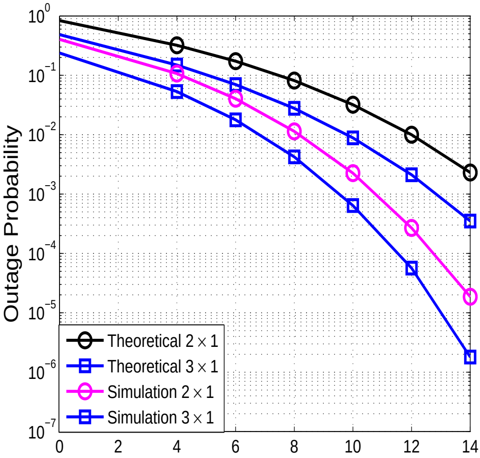

Figure: Probability of Outage vs SNR

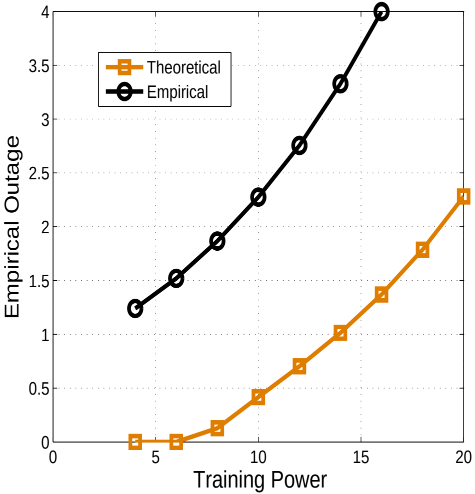

Figure: Empirical Outage vs SNR

Two-Way Training

- Phase 1: $\hat{H} = H + \tilde{H}$
- Phase 2: Transmitted training signal

$$S_\tau = \sqrt{P_R L_R}\hat{V}, \tag{46}$$

where P_R and L_R are the receiver training power and the training duration respectively, and,

$$\hat{V} = \frac{\hat{H}}{||\hat{H}||_F}. \tag{47}$$

- Received signal $Y_T = \sqrt{P_R L_R}\lambda V^H \hat{V} + w_T$

$$\hat{\lambda} \triangleq \frac{\Re\{Y_T\}}{\sqrt{P_R L_R}} = \lambda V^H \hat{V} + \frac{\bar{w}_T}{\sqrt{P_R L_R}} \tag{48}$$

where $\bar{w}_T \sim \mathbb{N}(0, \frac{1}{2})$ is the real part of w_T.

Power Control

- The singular value estimate at the transmitter is used for power control, as follows:

$$\mathbb{P}(\hat{\lambda}) = \begin{cases} P(\hat{\lambda}) & \hat{\lambda} \geq \theta(\bar{P}) \\ \bar{P}^{l} & \hat{\lambda} < \theta(\bar{P}), \end{cases} \tag{49}$$

where, unlike one way training,

$$P(\hat{\lambda}) = \frac{1}{\hat{\lambda}^2}. \tag{50}$$

The threshold $\theta(\bar{P}) \triangleq 1/\bar{P}^n$ is chosen such that $\mathbb{E}\mathbb{P}(\hat{\lambda}) < \infty$.

Composite Channel Estimate

- With the power control in (49), the receiver, after some processing, gets the following signal,

$$Y_R^{(\tau)} = \bar{G} + \frac{W_R^{(\tau)}}{\sqrt{P_s L_s}}, \tag{51}$$

where $\bar{G} \triangleq H\sqrt{P(\hat{\lambda})}1_{\{\hat{\lambda} \geq \theta(\bar{P})\}} + H\sqrt{\bar{P}^l}1_{\{\hat{\lambda} < \theta(\bar{P})\}}$, $W_R^{(\tau)} \sim \mathcal{CN}(0, I_{r\times r})$ and, P_s and L_s are the training power and training duration, respectively.

- Let $G = \sqrt{P(\hat{\lambda})}H$ and define $Q \triangleq \mathbb{E}\{\bar{G}\bar{G}^H\}$.
- MMSE estimate: $\hat{G}_{mmse}$, where $\bar{G} = \hat{G}_{mmse} + \tilde{G}_{mmse}$, where $\hat{G}_{mmse}$ is uncorrelated with $\tilde{G}_{mmse}$.

Composite Channel Estimate

- Linear Minimum Mean Square Error Estimate (LMMSE) of the effective channel which is given by,

$$\hat{G} = \mathbb{E}\bar{G} + M\left(Y_R^{(\tau)} - \mathbb{E}\bar{G}\right), \quad (52)$$

  where $M \triangleq Q\left(Q + \frac{1}{P_s L_s} I\right)^{-1}$.

- $$Y_d = \hat{G}_{mmse} k x_s + \tilde{G}_{mmse} k x_s + W, \quad (53)$$

  where $W \sim \mathcal{CN}(0, I_{r \times r})$ is the noise at the receiver during data transmission, and x_s is the unit variance Gaussian data signal which is scaled by k in order to satisfy the average power constraint, $\bar{P}$.

Power Constraint

- Power Constraint:

$$\bar{P} = \frac{L_T P_T}{T} + \frac{L_R P_R}{T} + \left(\frac{L_S P_S}{T} + \frac{(T - L_T - L_R - L_s)k^2}{T} \right) \mathbb{E}(\mathbb{P}(\hat{\lambda})). \quad (54)$$

We will choose P_T, P_R and P_s to be proportional to $\bar{P}$ and study the outage behavior as $\bar{P}$ goes to ∞. Also, note that $k^2 \doteq 1/\bar{P}$.

Main Result on Two-Way Training

Theorem

- *For a SIMO system with r receive antennas with three phases of two-way training, the DMT is achievable,*

$$d(r) = r\left(2 - \frac{g_m}{\alpha}\right), \tag{55}$$

where $\alpha \triangleq \frac{T - L_s - L_T - L_R}{T}$.

Outage

- Capacity Lower Bound:

$$C_L \triangleq \underbrace{\frac{T - L_s - L_T - L_R}{T}}_{\alpha} \log_2 \left(1 + \frac{\hat{\beta}_{mmse} k^2}{k^2 \mathbb{E}\tilde{\beta}_{mmse} + r}\right) \quad (56)$$

- Outage Probability:

$$\Pi \triangleq \Pr(C_L < R) \quad (57)$$

- $\bar{R} \triangleq \frac{k^2 \mathbb{E}\tilde{\beta}_{mmse} + r}{k^2}(e^{\frac{R}{\alpha}} - 1)$.
- $\tilde{G} = (I - M)(\bar{G} - \mathbb{E}\bar{G}) - \frac{M W_T^{\mathcal{T}}}{\sqrt{P_s L_s}}$

Main Result on Two-Way Training

- It is clear that the mean square error using the MMSE estimate is lesser than the mean square error using suboptimal LMMSE, it follows that:

$$\mathbb{E}(\tilde{\beta}_{mmse}) \leq \mathbb{E}(\|\tilde{G}\|_F^2) \leq \mathbb{E}\|\bar{G}-\mathbb{E}\bar{G}\|_F^2\|I-M\|_F^2+\frac{\|M\|_F^2}{P_s L_s}. \quad (58)$$

Note that

$$\|I-M\|_F^2 = \|I-Q\bar{P}(I+\bar{P}Q)^{-1}\|_F^2, \quad (59)$$
$$= \|(I+Q\bar{P})^{-1}\|_F^2, \quad (60)$$
$$= \sum_{i=0}^{r} \frac{1}{(\lambda_i \bar{P}+1)^2}, \quad (61)$$
$$\doteq \frac{1}{\bar{P}^2}, \quad (62)$$

- $\mathbb{E}\|\tilde{G}\|_F^2 \preceq 1/\bar{P} \Longrightarrow \bar{R} \doteq \frac{1}{\bar{P}}$ as $k^2 \doteq \bar{P}$.

Continuing Messy Calculation

- Outage Probability

$$\Pr\left(\frac{\hat{\beta}_{mmse}k^2}{k^2\mathbb{E}\tilde{\beta}_{mmse}+r} < (e^{\frac{R}{\alpha}}-1)\right) = \Pr\left\{\|\hat{G}\| < \sqrt{\bar{R}}\right\}, \quad (63)$$

$$= \Pr\left\{\|G-\tilde{G}_{mmse}\|_F < \sqrt{\bar{R}}\right\}, \quad (64)$$

$$\leq \Pr\left\{|\|G\|_F - \|\tilde{G}_{mmse}\|_F| < \sqrt{\bar{R}}\right\}, \quad (65)$$

$$\leq \Pr\left\{\|G\|_F < \|\tilde{G}_{mmse}\|_F + \sqrt{\bar{R}}\right\}, \quad (66)$$

$$\leq \Pr\left\{\|G\|_F < \|\tilde{G}_{mmse}\|_F + \sqrt{\bar{R}} \bigcap \|\tilde{G}\|_F \leq \sqrt{\bar{R}}\right\}$$

$$+ \Pr\left\{\|G\|_F < \|\tilde{G}_{mmse}\|_F + \sqrt{\bar{R}} \bigcap \|\tilde{G}\|_F > \sqrt{\bar{R}}\right\}, \quad (67)$$

$$\leq \Pr\left\{\|G\|_F^2 < 4\bar{R}\right\} + \Pr\left\{\|G\|_F^2 < 4\|\tilde{G}_{mmse}\|_F^2\right\}. \quad (68)$$

Continuing Messy Calculation

- Second term:

$$\begin{aligned}
\Pr\left\{\|G\|_F^2 < 4\|\tilde{G}_{mmse}\|_F^2\right\} &= \Pr\left\{\frac{\|H\|_F^2}{\hat{\gamma}_U} < 4\|\tilde{G}\|_{mmse}^2\right\}, \\
&\doteq \Pr\left\{\frac{\|H\|_F^2}{\hat{\gamma}_U} < \frac{4\|\bar{\bar{G}}\|^2}{\bar{P}}\right\}, \\
&\leq \Pr\left\{\gamma < \frac{4\|\bar{\bar{G}}\|_F^2\,|\bar{w}_T|^2}{\bar{P}^2\left|1 - \frac{2\|\bar{\bar{G}}\|_F}{\sqrt{\bar{P}}}\right|^2}\right\}, \\
&\approx \frac{1}{\bar{P}^{2r}}\mathbb{E}\left(\frac{4\|\bar{\bar{G}}\|_F^2\,|\bar{w}_T|^2}{\bar{P}^2\left|1 - \frac{2\|\bar{\bar{G}}\|_F}{\sqrt{\bar{P}}}\right|^2}\right),
\end{aligned}$$

Continuing Messy Calculation

- $$\Pr\left\{\|G\|_F^2 < 4\|\tilde{G}_{mmse}\|_F^2\right\} \preceq \frac{1}{\bar{P}^{2r}}$$
- Upper Bound on the estimate of the singular value

$$\left|\hat{\lambda}\right| = \left|\lambda V^H \hat{V} + \frac{w_T}{\sqrt{P_R L_R}}\right| \quad (69)$$

$$\leq \lambda\left|V^H \hat{V}\right| + \left|\frac{w_T}{\sqrt{P_R L_R}}\right| \quad (70)$$

$$\leq \lambda + \left|\frac{w_T}{\sqrt{P_R L_R}}\right|, \quad (71)$$

Continuing Messy Calculation

- Define $\hat{\gamma}_U \triangleq \left(\lambda + |\frac{\bar{w}_T}{\sqrt{P_R L_R}}|\right)^2$ and $\bar{W}_T \triangleq |\frac{\bar{w}_T}{\sqrt{P_R L_R}}|$. $\Pr_g(.)$ and $\Pr_b(.)$ to means $\Pr(. \bigcap \hat{\lambda} > \theta(\bar{P}))$ and $\Pr(. \bigcap \hat{\lambda} \leq \theta(\bar{P}))$. First term:

$$\Pr_g \left\{ \|G\|_F^2 < 4\bar{R} \right\} = \Pr_g \left\{ \frac{\|H\|_F^2}{\hat{\gamma}} < 4\bar{R} \right\} \quad (72)$$

$$\leq \Pr_g \left\{ \frac{\|H\|_F^2}{\hat{\gamma}_U} < 4\bar{R} \right\}, \quad (73)$$

$$= \Pr_g \left\{ \gamma^{1/2} < 2\sqrt{\bar{R}}(\gamma^{1/2} + \bar{W}_T) \right\}, \quad (74)$$

$$\leq \Pr \left\{ \gamma^{1/2} < 2\frac{\sqrt{\bar{R}}}{1 - \sqrt{2\bar{R}}} \bar{W}_T \right\}, \quad (75)$$

$$\approx \Pr \left\{ \gamma < 4\bar{R}\bar{W}_T^2 \right\}. \quad (76)$$

Continuing Messy Calculation

- Bad channel case $\hat{\lambda} < \theta(\bar{P})$:

$$\Pr_b \left\{ \|G\|_F^2 < 4\bar{R} \right\} = \Pr_b \left\{ \|H\|_F^2 < \frac{4\bar{R}}{\bar{P}^{2l}} \right\}, \quad (77)$$

$$\leq \Pr \left\{ \gamma^2 < \frac{4\bar{R}}{\bar{P}^{2l}} \right\} \quad (78)$$

$$\doteq \frac{1}{\bar{P}^{2rl+r}} \quad \blacksquare \quad (79)$$

Conclusions

- Exploiting CSIR in designing the reverse channel training greatly improves the DMT performance.