Spectrum Sensing with a Frequency Hopping Primary

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- System model
- The FFT Averaging Ratio (FAR) algorithm
- Analysis : The optimal threshold
- Analysis : The optimum sensing duration
- Validation, implementation and benchmarking

System Model

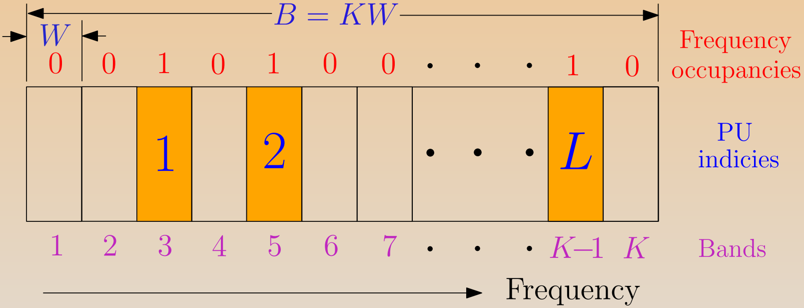

Figure: Typical band occupancies in a FHSS system.

System Model

- B (bandwidth of interest) is divided into K bands of bandwidth W each
- $L \ll K$ is the number of active PUs, and N_h be the hopping duration. Let SNR(p) denote the SNRs of each PU, assumed known
- The CR collects $N_s = NM$ samples i.e., the samples are grouped into M frames of N samples each
- N is the FFT length, chosen to be positive integer multiple of K

What We Seek

- A simple, robust detector
- Analysis (P_{FA} and P_D) $\Rightarrow$ optimal threshold
- The sensing duration (sensing + transmission dutycycle) which maximizes the secondary throughput, s.t. PU interference constraint is satisfied

FAR Algorithm

- In the time domain
$$\underbrace{\bar{y}_m}_{\text{received samples}} = \underbrace{\bar{x}_m}_{\text{PU signal samples}} + \underbrace{\bar{z}_m}_{\text{noise samples}}$$
- In the frequency domain, under $\mathcal{H}_0$
$$\bar{Y}_m(\ell) = \bar{Z}_m(\ell),\ \ell = \frac{kN}{K}, \frac{kN}{K}+1, \ldots, \frac{(k+1)N}{K}-1$$
- Under $\mathcal{H}_1$
$$\bar{Y}_m(\ell) = \bar{X}_m(\ell) + \bar{Z}_m(\ell),\ \ell = \frac{kN}{K}, \frac{kN}{K}+1, \ldots, \frac{(\ell+1)N}{K}-1,$$
- The average energy in the k^{th} band, and the total average energy $\Rightarrow$
$$P(k) \triangleq \frac{1}{M}\sum_{m=0}^{M-1}\sum_{q=0}^{\frac{N}{K}-1}\left|Y_m\left(\frac{N}{K}\times k + q\right)\right|^2,$$
$$P_{tot} = \sum_{k=0}^{K-1} P(k).$$

FAR Statistic

- In each band, the FAR statistic is given by $T_M(k) \triangleq \frac{P(k)}{P_{tot}}, \quad k = 0, \ldots, K-1.$
- Therefore, the decision strategy

$$T_M(k) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau$$

- Design:
  1. Opt. τ
  2. Opt. M, s.t. the CR throughput is maximized, with an interference constraint

P_{FA} and P_D

Lemma

For the FAR detection scheme, the signal detection and false-alarm probabilities are given by

$$P_D(k,\gamma,M) = 1 - \frac{1}{\frac{D_{1k}}{2}\mathcal{B}\left(\frac{D_{1k}}{2},\frac{F_{1k}}{2}\right)}\times \left(\frac{E_{1k}}{G_{1k}}\gamma\right)^{D_{1k}/2} {}_2\mathcal{F}_1\left(\frac{D_{1k}}{2},\frac{D_{1k}+F_{1k}}{2};1+\frac{D_{1k}}{2},-\frac{E_{1k}}{G_{1k}}\gamma\right),$$

$$P_{FA}(k,\gamma,M) = 1 - \frac{1}{\frac{MN}{K}\mathcal{B}\left(\frac{MN}{K},\frac{D_0}{2}\right)}\times (G_0\gamma)^{\frac{MN}{K}} {}_2\mathcal{F}_1\left(\frac{MN}{K},\frac{MN}{K}+\frac{D_0}{2};1+\frac{MN}{K},-G_0\gamma\right),$$

where $\gamma \triangleq \frac{\tau}{1-\tau}$.

Definitions

$$G_0 \triangleq 2 - \frac{K-1}{K-1+\frac{1}{N}\sum_{p=0}^{K-1}\mathrm{SNR}(p)}, \quad G_{1k} \triangleq 2 - \frac{1}{1+\frac{1}{N}\mathrm{SNR}(k)},$$

$$E_{1k} \triangleq 2 - \frac{K-1}{K-1+\frac{1}{N}\sum_{p=0,\neq k}^{K-1}\mathrm{SNR}(p)},$$

$$D_0 \triangleq \frac{M}{K}\sum_{p=0}^{K-1}\mathrm{SNR}(p) + MN\left\{\frac{3}{2} - \frac{3}{2K} + \frac{K-2+1/K}{2K-2+\frac{4}{N}\sum_{p=0}^{K-1}\mathrm{SNR}(p)}\right\},$$

$$F_{1k} \triangleq \frac{M}{K}\sum_{p=0,\neq k}^{K-1}\mathrm{SNR}(p) + MN\left\{\frac{3}{2} - \frac{3}{2K} + \frac{K-2+1/K}{2K-2+\frac{4}{N}\sum_{p=0,\neq k}^{K-1}\mathrm{SNR}(p)}\right\}$$

$$D_{1k} \triangleq \frac{MN}{K}\left\{\frac{3}{2} + \frac{1}{N}\mathrm{SNR}(k) + \frac{1}{2+\frac{1}{N}\mathrm{SNR}(k)}\right\}$$

Outline of the Proof

Throughput

- The utility function : Throughput

$$\begin{aligned}\Pi &\triangleq \mathbb{E}\{K^{(R)} + \alpha K^{(W)}\} W \times (N_h - N_s) \\ &= \Big[(K-L)(1 - P_{FA}(\gamma, M)) \\ &\quad + \sum_{k:U(k)=1} \alpha(k)\,(1 - P_D(k,\gamma,M))\Big] W(N_h - N_s)\end{aligned}$$

- $N_s = NM$.
- Optimization problem

$$\max_{M}\{\Pi\} \;\; s.t. \;\; \min_{k:U(k)=1} P_D(k,\gamma,M) \geq P_{min}$$

Lemma

For $\alpha(k) = 0$, the value of M which maximizes Π is the solution to

$$\frac{G_0\gamma_{min}}{{}_2\mathcal{F}_1\left(1, 1-BM; 1+AM; -G_0\gamma_{min}\right)}\left\{\frac{B}{1+AM}\,{}_2\Theta^{(1)}\left(\begin{matrix}1, & 1|1-BM, & 2-BM, & 2\\ & 2-BM|2, & 2+AM & \end{matrix}\middle|; G_0\gamma_{min}, \; G_0\gamma_{min}\right)\right.$$
$$\left.+\frac{A(1-BM)}{(1+AM)^2}\,{}_2\Theta^{(1)}\left(\begin{matrix}1, & 1|1+AM, & 2, & 2-BM\\ & 2+AM|2, & 2+AM & \end{matrix}\middle|; G_0\gamma_{min}, \; G_0\gamma_{min}\right)\right\} + (A+B)\log\left(1+G_0\gamma_{min}\right)$$
$$-A\log\left(G_0\gamma_{min}\right) + (A+B)\psi^{(0)}(AM+BM) - A\psi^{(0)}(AM) - B\psi^{(0)}(BM) - \frac{N_h}{M(N_h-NM)} = 0,$$

where

$$A \triangleq \frac{N}{K}, \quad B \triangleq \frac{\left[\left(N-\frac{N}{K}\right)+\frac{1}{K}\sum_{p=0}^{K-1} SNR(p)\right]^2}{\left(N-\frac{N}{K}\right)+\frac{2}{K}\sum_{p=0}^{K-1} SNR(p)}$$

Definitions

$$ {}_2\Theta^{(1)}\left(\begin{matrix} a_1, & a_2 \end{matrix}\middle| \begin{matrix} b_1, & b_2, & b_3 \\ c_1 \,|\, d_1, & d_2 \end{matrix}\middle|; x_1, \; x_2\right) \triangleq $$

$$ \sum_{m=0}^{\infty} \frac{(a_1)_m (b_1)_m (b_2)_m (b_3)_m}{(c_1)_m (d_1)_m (d_2)_m} \frac{x_1^m}{m!} $$

$$ \times \; {}_3\mathcal{F}_2(a_2, b_2 + m, b_3 + m; d_1 + m, d_2 + m; x_2) $$

is a Kempe de Feriet-like function

$(a)_m \triangleq \frac{\Gamma(a+m)}{\Gamma(a)}$ is the Pochhammer symbol

$\psi^{(0)}$ is the digamma function

Outline of the Proof

Simulations