Generalized Degrees of Freedom of K user Symmetric Gaussian MIMO Interference Channel

Parthajit Mohapatra

Dept. of ECE, Indian Institute of Science

Outline

- Preliminaries
- Outer bound
- Inner bound
- Results
- Conclusion

Interference in wireless networks

- Shannon provides the basis for modern day communication system
- In point-to-point communication system noise is the primary concern
- Wireless networks are interference limited rather than noise limited

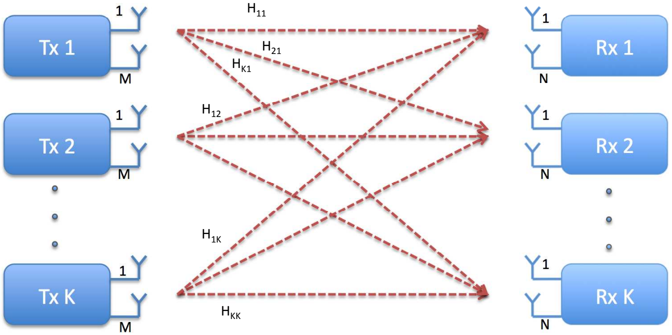
Where interference occurs?

- Cellular networks: inter cell interference, interference between macro, pico and femto cell
- Ad-hoc networks: interference from simultaneous transmissions
- Wireless LANs: interference from adjacent networks
- Cognitive networks: between primary and secondary and among secondary users

Generalized degrees of freedom (GDOF)

- GDOF is a measure of the high SNR capacity obtainable from a given channel
- For the symmetric case, it is defined as
  $$d_{\text{sym}}(\alpha) = \lim_{\rho \to \infty} \frac{1}{K} \frac{C_{\sum}(\rho, \alpha)}{\log \rho}, \text{ where } \alpha = \frac{\log \gamma}{\log \rho}$$
- Roughly measures interference free dimension accessible in a network
- When $\text{SNR}(\rho) = \text{INR}(\gamma)$, degrees of freedom (DOF) is obtained as a special case of GDOF

Interference Channel (IC)



- $\mathbf{H}_{ij}$: channel from jth transmitter to *ith* receiver
- M and N: antenna at transmitter and receiver respectively

Problem statement

- Multiple antennas help to mitigate the effect of interference e.g: When $N \geq KM$, ZF - receiving is sufficient to achieve the interference free GDOF
- When $N < KM$, trivial techniques are found to be sub-optimal
- Focus of this work: To characterize GDOF of K user symmetric MIMO Gaussian IC

Contributions

- A new outer bound is derived for the MIMO-IC
  - Cooperation
  - Providing noisy side information
- Inner bound is derived for the symmetric MIMO-IC as a combination of
  - Han-Kobayashi (HK) scheme
  - Interference Alignment (IA)
  - Treating interference as noise
  - Zero - Forcing receiving
- HK scheme is extended to multiuser MIMO scenario
- Interplay between the HK and IA schemes is explored

Outer bound

Fano's Inequality:

For any estimator $\hat{X}$ such that $X \to Y \to \hat{X}$, with $P_e = \Pr\{X \neq \hat{X}\}$, we have:

$$H(X|Y) \leq H(X|\hat{X}) \leq H(P_e) + P_e \log |X|$$

Weaker Form:

$$H(X|Y) \leq 1 + P_e \log |X|$$

- How to use:

$$\begin{aligned} nR_1 &= H(W_1) \\ &= I(W_1; Y_1^n) + H(W_1|Y_1^n) \\ &\leq I(W_1; Y_1^n) + n\epsilon_n \text{ (Fano's inequality)} \\ &\leq I(X_1^n; Y_1^n) + n\epsilon_n \text{ (Data processing inequality)} \end{aligned}$$

Outer bound based on cooperation

- Cooperation does not hurt capacity
- Outer bound is derived for a modified system
- Different possible ways of cooperation is taken in to account

Outer bound: cooperation

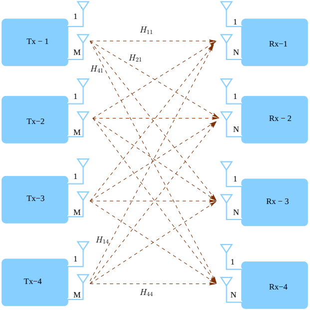
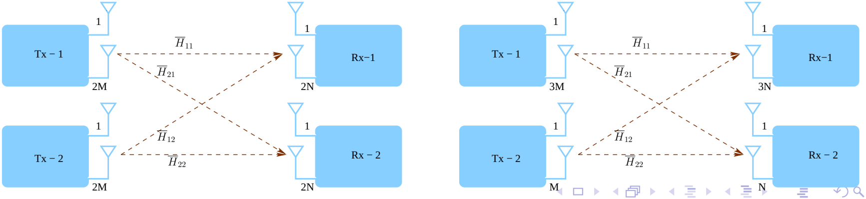
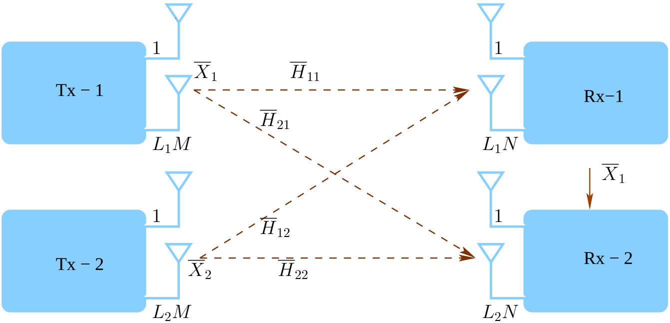


Figure: Four user Gaussian IC

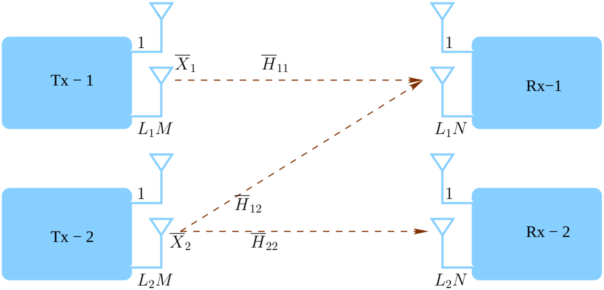


Outer bound: cooperation



- L_1 and L_2: number of users in group - 1 and group -2
- $\overline{\mathbf{H}}_{ij} \in \mathbb{C}^{L_iN \times L_jM}$, $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$: two set of messages

Outer bound: cooperation



- Equivalent to a two user MIMO Z - interference channel

Outer bound: cooperation

- System model:

$$\overline{\mathbf{Y}}_1 = \overline{\mathbf{H}}_{11}\overline{\mathbf{X}}_1 + \overline{\mathbf{H}}_{12}\overline{\mathbf{X}}_2 + \overline{\mathbf{Z}}_1,$$
$$\overline{\mathbf{Y}}_2 = \overline{\mathbf{H}}_{22}\overline{\mathbf{X}}_2 + \overline{\mathbf{Z}}_2,$$

where

$$\overline{\mathbf{Y}}_1 \triangleq \left[\ \mathbf{Y}_1, \cdots, \mathbf{Y}_{L_1}\ \right]^T, \ \overline{\mathbf{Y}}_2 \triangleq \left[\ \mathbf{Y}_{L_1+1}, \cdots, \mathbf{Y}_L\ \right]^T,$$
$$\overline{\mathbf{X}}_1 \triangleq \left[\ \mathbf{X}_1, \cdots, \mathbf{X}_{L_1}\ \right]^T, \ \overline{\mathbf{X}}_2 \triangleq \left[\ \mathbf{X}_{L_1+1}, \cdots, \mathbf{X}_L\ \right]^T,$$
$$\overline{\mathbf{Z}}_1 \triangleq \left[\ \mathbf{Z}_1, \cdots, \mathbf{Z}_{L_1}\ \right]^T \text{ and } \overline{\mathbf{Z}}_2 \triangleq \left[\ \mathbf{Z}_{L_1+1}, \cdots, \mathbf{Z}_L\ \right]^T.$$
$$\overline{\mathbf{H}}_{11} = \text{blkdiag}\left(\mathbf{H}_{11}\ \mathbf{H}_{22}\ \ldots\ \mathbf{H}_{L_1,L_1}\right)$$
$$\overline{\mathbf{H}}_{22} = \text{blkdiag}\left(\mathbf{H}_{L_1+1,L_1+1}\ \mathbf{H}_{L_1+2,L_1+2}\ \ldots\ \mathbf{H}_{L,L}\right)$$
$$\overline{\mathbf{H}}_{12} = \begin{bmatrix} \mathbf{H}_{1,L_1+1} & \mathbf{H}_{1,L_1+2} & \cdots & \mathbf{H}_{1,L} \\ \mathbf{H}_{2,L_1+1} & \mathbf{H}_{2,L_1+2} & \cdots & \mathbf{H}_{2,L} \\ \vdots & \vdots & & \\ \mathbf{H}_{L_1,L_1+1} & \mathbf{H}_{L_1,L_1+2} & \cdots & \mathbf{H}_{L_1,L} \end{bmatrix}, \ L_1 + L_2 \leq K \quad (1)$$

Outer bound: cooperation

Theorem - 1:

The sum rate of the K-user Gaussian MIMO interference channel is upper bounded as follows:

$$\sum_{i=1}^{L} R_i \leq \log\left|\mathbf{I}_{L_1N} + \overline{\mathbf{H}}_{11}\overline{\mathbf{P}}_1\overline{\mathbf{H}}_{11}^H + \overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2\overline{\mathbf{H}}_{12}^H\right| +$$

$$\log\left|\mathbf{I}_{L_2N} + \overline{\mathbf{H}}_{22}\overline{\mathbf{P}}_2^{1/2}\left\{\mathbf{I}_{L_2M} + \overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{12}^H\overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2^{1/2}\right\}^{-1}\overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{22}^H\right| + \epsilon_n$$

where $L_1 + L_2 = L \leq K$, $0 \leq L_1 \leq K$, $0 \leq L_2 \leq K$,

$\mathbf{I}_L$: $L \times L$ identity matrix

Proof outline: cooperation

By using Fano's inequality, the sum rate of the modified system is upper bounded as given below:

$$
\begin{aligned}
& n\sum_{i=1}^{L} R_i - n\epsilon_n \\
& \leq I\left(\overline{\mathbf{X}}_1^n, \overline{\mathbf{Y}}_1^n\right) + I\left(\overline{\mathbf{X}}_2^n; \overline{\mathbf{Y}}_2^n\right), \\
& \leq I\left(\overline{\mathbf{X}}_1^n, \overline{\mathbf{Y}}_1^n\right) + I\left(\overline{\mathbf{X}}_2^n; \overline{\mathbf{Y}}_2^n, \overline{\mathbf{S}}^n\right), \text{ where } \overline{\mathbf{S}} = \overline{\mathbf{H}}_{12}\overline{\mathbf{X}}_2 + \overline{\mathbf{Z}}_1, \\
& = h\left(\overline{\mathbf{Y}}_1^n\right) - h\left(\overline{\mathbf{S}}^n\right) + h\left(\overline{\mathbf{S}}^n\right) - h\left(\overline{\mathbf{Z}}_1^n\right) + h\left(\overline{\mathbf{Y}}_2^n|\overline{\mathbf{S}}^n\right) - h\left(\overline{\mathbf{Z}}_2^n\right),
\end{aligned}
$$

Proof outline: cooperation

Lemma

Let $\mathbf{x}^n = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ *and* $\mathbf{y}^n = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n\}$ *be two sequences of random vectors and let* $\mathbf{x}^*, \mathbf{y}^*, \hat{\mathbf{x}}$ *and* $\hat{\mathbf{y}}$ *be Gaussian vectors with covariance matrices satisfying*

$$Cov \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \end{bmatrix} = \frac{1}{n} \sum_{i=1}^{n} Cov \begin{bmatrix} \mathbf{x}_i \\ \mathbf{y}_i \end{bmatrix} \preceq Cov \begin{bmatrix} \mathbf{x}^* \\ \mathbf{y}^* \end{bmatrix},$$

then we get the following bound

$$h(\mathbf{x}^n) \leq nh(\hat{\mathbf{x}}) \leq nh(\mathbf{x}^*),$$
$$h(\mathbf{y}^n|\mathbf{x}^n) \leq nh(\hat{\mathbf{y}}|\hat{\mathbf{x}}) \leq nh(\mathbf{y}^*|\mathbf{x}^*).$$

Proof outline: cooperation

- Sum rate is bounded as:

$$\sum_{i=1}^{L} R_i - \epsilon_n \leq h\left(\overline{\mathbf{Y}}_1^*\right) - h\left(\overline{\mathbf{Z}}_1\right) + h\left(\overline{\mathbf{Y}}_2^* | \overline{\mathbf{S}}^*\right) - h\left(\overline{\mathbf{Z}}_2\right)$$

- It can be shown that:

$$h\left(\overline{\mathbf{Y}}_1^*\right) = \log \left| \pi e \left[\mathbf{I}_{L_1 N} + \overline{\mathbf{H}}_{11}\overline{\mathbf{P}}_1\overline{\mathbf{H}}_{11}^H + \overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2\overline{\mathbf{H}}_{12}^H \right] \right|$$

$$h\left(\overline{\mathbf{Y}}_2^* | \overline{\mathbf{S}}^*\right) = \log \left| \pi e \left[\mathbf{I}_{L_2 N} + \overline{\mathbf{H}}_{22}\overline{\mathbf{P}}_2^{1/2} \left\{ \mathbf{I}_{L_2 M} + \overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{12}^H\overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2^{1/2} \right\}^{-1} \overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{22}^H \right] \right| .$$

- Need to minimize sum rate over all possible values of L_1 and L_2.
- Difficult task !

Outer bound: cooperation

Lemma

In the symmetric case, the GDOF of the K user Gaussian MIMO-IC $M \leq N$ is upper bounded as follows:

1. *When* $0 \leq \alpha \leq 1$*:*

$$d_i(\alpha) \leq \min_{L_1, L_2} \frac{1}{L} \left[L_1 M + \min\left\{r, L_1(N-M)\right\}\alpha + (L_2 M - r)^+ + \min\left\{r, L_2 N - (L_2 M - r)^+\right\}(1-\alpha) \right],$$

2. *When* $\alpha > 1$*:*

$$d_i(\alpha) \leq \min_{L_1, L_2} \frac{1}{L} \left[r\alpha + \min\left\{L_1 M, L_1 N - r\right\} + (L_2 M - r)^+ \right],$$

where $r = \min\left\{L_2 M, L_1 N\right\}$*.*

Outer bound: Noisy side information

- Noisy side information is provided
- Define the following quantity:

$$\mathbf{S}_{j,\mathbf{B}} = \sum_{i \in \mathbf{B}} \mathbf{H}_{ji} \mathbf{X}_i + \mathbf{Z}_j$$

where $\mathbf{B} \subseteq \{1, 2, \ldots, K\}$ is the set of users.

- Consider first and third user:

$$\begin{aligned} & nR_1 + nR_3 - n\epsilon_n \\ & \leq I\left(\mathbf{X}_1^n; \mathbf{Y}_1^n\right) + I\left(\mathbf{X}_3^n; \mathbf{Y}_3^n\right) \\ & \leq I\left(\mathbf{X}_1^n; \mathbf{Y}_1^n, \mathbf{S}_{2,1}^n\right) + I\left(\mathbf{X}_3^n; \mathbf{Y}_3^n, \mathbf{S}_{4,3}^n\right) \\ & = h(\mathbf{Y}_1^n|\mathbf{S}_{2,1}^n) + h(\mathbf{Y}_3^n|\mathbf{S}_{4,3}^n) + \underbrace{h(\mathbf{S}_{2,1}^n) + h(\mathbf{S}_{4,3}^n)}_{\text{unwanted terms}} \\ & \quad - \underbrace{h(\mathbf{S}_{1,\{2,3,4\}}^n) - h(\mathbf{S}_{3,\{1,2,4\}}^n)}_{\text{unwanted terms}} - h(\mathbf{Z}_1^n) - h(\mathbf{Z}_3^n) \end{aligned}$$

Outer bound: Noisy side information

- Consider 2nd and 4th user:

$$
\begin{aligned}
& nR_2 + nR_4 - n\epsilon_n \\
& \leq I\left(\mathbf{X}_2^n; \mathbf{Y}_2^n, \mathbf{S}_{1,2}^n\right) + I\left(\mathbf{X}_4^n; \mathbf{Y}_4^n, \mathbf{S}_{3,4}^n\right) \\
& = h(\mathbf{Y}_2^n|\mathbf{S}_{1,2}^n) + h(\mathbf{Y}_4^n|\mathbf{S}_{3,4}^n) + \underbrace{h(\mathbf{S}_{1,2}^n) + h(\mathbf{S}_{3,4}^n)}_{\text{unwanted terms}} \\
& \quad \underbrace{- h(\mathbf{S}_{2,\{1,3,4\}}^n) - h(\mathbf{S}_{4,\{1,2,3\}}^n)}_{\text{unwanted terms}} - h(\mathbf{Z}_2^n) - h(\mathbf{Z}_4^n)
\end{aligned}
$$

- Summing and by conditioning further:

$$
\begin{aligned}
\sum_{i=1}^{4} R_i \quad & \leq h(\mathbf{Y}_1^n|\mathbf{S}_{2,1}^n) + h(\mathbf{Y}_3^n|\mathbf{S}_{4,3}^n) + h(\mathbf{Y}_2^n|\mathbf{S}_{1,2}^n) + h(\mathbf{Y}_4^n|\mathbf{S}_{3,4}^n) \\
& + \sum_{i=1}^{4} h(\mathbf{Z}_i^n) + n\epsilon_n
\end{aligned}
$$

Outer bound: Noisy side information

Theorem

The sum rate of the K user Gaussian MIMO-IC is upper bounded as follows:

1. *When K is even:*

$$\sum_{i=1}^{K} R_i \leq \sum_{i\ odd} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\{\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i+1,i}^H\mathbf{H}_{i+1,i}\mathbf{P}_i^{1/2}\}^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| +$$

$$\sum_{i\ even} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\{\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i-1,i}^H\mathbf{H}_{i-1,i}\mathbf{P}_i^{1/2}\}^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| + \epsilon n$$

2. *When K is odd:*

$$R_1 + 2\sum_{i=2}^{K-1} R_i + R_K$$

$$\leq \sum_{i=1}^{K-1} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\left(\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i+1,i}^H\mathbf{H}_{i+1,i}\mathbf{P}_i^{1/2}\right)^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| +$$

$$\sum_{i=2}^{K} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\left(\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i-1,i}^H\mathbf{H}_{i-1,i}\mathbf{P}_i^{1/2}\right)^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| + \epsilon n$$

Outer bound: Noisy side information

Lemma

The GDOF of the K user MIMO-IC in the symmetric case is upper bounded as follows:

$$d_j(\alpha) \leq \begin{cases} M(1-\alpha) + \min\left\{\min\left(N, (K-1)M\right), N-M\right\}\alpha & \text{if } 0 \leq \alpha \leq \frac{1}{2} \\ \min\left(N, (K-1)M\right)\alpha + \min\left\{M, N - \min\left(N, (K-1)M\right)\right\}(1-\alpha) & \text{if } \frac{1}{2} \leq \alpha \leq 1 \\ \min\left\{N, (K-1)M\right\} & \text{if } \alpha \geq 1 \end{cases}$$

when $M \leq N$.

Outer bound: Noisy side information

Theorem

The sum rate of the K user Gaussian MIMO-IC is upper bounded as follows:

$$
\begin{aligned}
R_1 + \sum_{i=2}^{K-1} R_i + R_K \quad & \leq \log \left| \mathbf{I}_{N_1} + \sum_{j=2}^{K} \mathbf{H}_{1j}\mathbf{P}_j\mathbf{H}_{1j}^H + \mathbf{H}_{11}\mathbf{P}_1^{1/2} \left\{ \mathbf{I}_{M_1} + \mathbf{P}_1^{1/2}\mathbf{H}_{K1}^H\mathbf{H}_{K1}\mathbf{P}_1^{1/2} \right\}^{-1} \mathbf{P}_1^{1/2}\mathbf{H}_{11}^H \right| + \\
& \sum_{i=2}^{K-1} \log \left| \mathbf{I}_{N_i} + \overline{\mathbf{H}}_{i1}\overline{\mathbf{P}}_{i1}^{1/2} \left\{ \mathbf{I}_{M_{r_i}} + \overline{\mathbf{P}}_{i1}^{1/2}\overline{\mathbf{H}}_{Ki}^H\overline{\mathbf{H}}_{Ki}\overline{\mathbf{P}}_{i1}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i1}^{1/2}\overline{\mathbf{H}}_{i1}^H + \right. \\
& \left. \overline{\mathbf{H}}_{i,i+1}\overline{\mathbf{P}}_{i2}^{1/2} \left\{ \mathbf{I}_{M_{s_i}} + \overline{\mathbf{P}}_{i2}^{1/2}\overline{\mathbf{H}}_{1,i+1}^H\overline{\mathbf{H}}_{1,i+1}\overline{\mathbf{P}}_{i2}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i2}^{1/2}\overline{\mathbf{H}}_{i,i+1}^H \right| + \\
& \sum_{i=2}^{K-1} \log \left| \mathbf{I}_{N_i} + \overline{\mathbf{H}}_{iK}\overline{\mathbf{P}}_{i3}^{1/2} \left\{ \mathbf{I}_{M_{r_i}} + \overline{\mathbf{P}}_{i3}^{1/2}\overline{\mathbf{H}}_{1i}^H\overline{\mathbf{H}}_{1i}\overline{\mathbf{P}}_{i3}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i3}^{1/2}\overline{\mathbf{H}}_{iK}^H + \right. \\
& \left. \overline{\mathbf{H}}_{i,K-1}\overline{\mathbf{P}}_{i4}^{1/2} \left\{ \mathbf{I}_{M_{s_i}} + \overline{\mathbf{P}}_{i4}^{1/2}\overline{\mathbf{H}}_{K,i+1}^H\overline{\mathbf{H}}_{K,i+1}\overline{\mathbf{P}}_{i4}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i4}^{1/2}\overline{\mathbf{H}}_{i,K-1}^H \right| + \\
& \log \left| \mathbf{I}_{N_K} + \sum_{j=1}^{K-1} \mathbf{H}_{Kj}\mathbf{P}_j\mathbf{H}_{Kj}^H + \mathbf{H}_{KK}\mathbf{P}_K^{1/2} \left\{ \mathbf{I}_{M_K} + \mathbf{P}_K^{1/2}\mathbf{H}_{1K}^H\mathbf{H}_{1K}\mathbf{P}_K^{1/2} \right\}^{-1} \mathbf{P}_K^{1/2}\mathbf{H}_{KK}^H \right| + \epsilon_n
\end{aligned}
$$

where $M_{r_i} = \sum_{j=1}^{i} M_j$ *and* $M_{s_i} = \sum_{j=i+1}^{K} M_j$

Outer bound: Noisy side information

- Variables are defined as follows:

$$\overline{\mathbf{H}}_{i1} = \left[\begin{array}{cccc} \mathbf{H}_{i1} & \mathbf{H}_{i2} & \ldots & \mathbf{H}_{ii} \end{array} \right], \quad \overline{\mathbf{H}}_{i,i+1} = \left[\begin{array}{cccc} \mathbf{H}_{i,i+1} & \mathbf{H}_{i,i+2} & \ldots & \mathbf{H}_{iK} \end{array} \right],$$
$$\overline{\mathbf{H}}_{Ki} = \left[\begin{array}{cccc} \mathbf{H}_{K1} & \mathbf{H}_{K2} & \ldots & \mathbf{H}_{Ki} \end{array} \right], \overline{\mathbf{H}}_{1,i+1} = \left[\begin{array}{cccc} \mathbf{H}_{1,i+1} & \mathbf{H}_{1,i+2} & \ldots & \mathbf{H}_{1K} \end{array} \right]$$
$$\overline{\mathbf{H}}_{1i} = \left[\begin{array}{cccc} \mathbf{H}_{1K} & \mathbf{H}_{12} & \ldots & \mathbf{H}_{1i} \end{array} \right], \overline{\mathbf{H}}_{K,i+1} = \left[\begin{array}{ccc} \mathbf{H}_{K1} & \mathbf{H}_{K,i+1} & \ldots \mathbf{H}_{K,K-1} \end{array} \right],$$
$$\overline{\mathbf{H}}_{iK} = \left[\begin{array}{cccc} \mathbf{H}_{iK} & \mathbf{H}_{i2} & \ldots & \mathbf{H}_{ii} \end{array} \right], \overline{\mathbf{H}}_{i,K-1} = \left[\begin{array}{cccc} \mathbf{H}_{i1} & \mathbf{H}_{i,i+1} & \ldots & \mathbf{H}_{i,K-1} \end{array} \right]$$

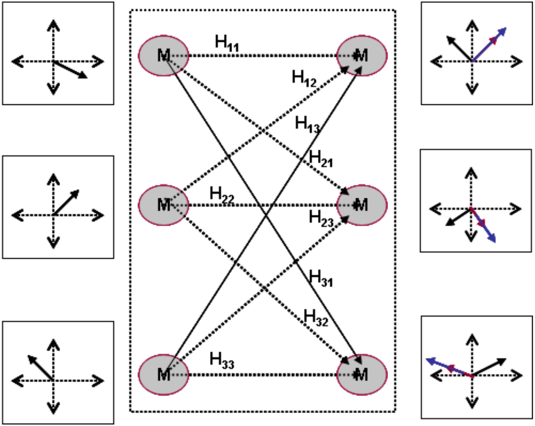
$$\overline{\mathbf{P}}_{i1} = \text{blockdiag}\left(\mathbf{P}_1\ \mathbf{P}_2\ \ldots\ \mathbf{P}_i\right), \ \overline{\mathbf{P}}_{i2} = \text{blockdiag}\left(\mathbf{P}_{i+2}\ \mathbf{P}_{i+3}\ \ldots\ \mathbf{P}_K\right)$$
$$\overline{\mathbf{P}}_{i3} = \text{blockdiag}\left(\mathbf{P}_K\ \mathbf{P}_2\ \ldots\ \mathbf{P}_i\right) \text{ and } \overline{\mathbf{P}}_{i4} = \text{blockdiag}\left(\mathbf{P}_1\ \mathbf{P}_{i+1}\ \ldots\ \mathbf{P}_{K-1}\right).$$

- Extension of SIMO-IC outer bound to MIMO-IC

Inner bound

- Inner bound is derived for the symmetric MIMO interference channel as a combination of
  - Han-Kobayashi (HK) scheme
  - Interference Alignment (IA)
  - Treating interference as noise
  - Zero - Forcing receiving
- Interplay between the HK and IA schemes is explored

Interference alignment(IA)



- Idea of IA for IC originated in the seminal work by Cadembe
- For MIMO IC, DOF achieved by IA:

$$d_j = \frac{MN}{M+N}, \; KM > N \tag{2}$$

- Requires global channel knowledge
- Relative strength between signal and interference does not matter

Han-Kobayashi (HK) scheme

- Based on the idea of splitting message in to two parts:
  1. Private part
  2. Public part
- A simple HK scheme proposed by ETW: achieves capacity with in 1 bit (two user IC)
- Different variants of HK - scheme has been proposed

Extension of HK - scheme

- HK scheme is extended to K user MIMO IC for symmetric case
- Following interference regime are considered:
  1. Strong interference case ($\alpha > 1$)
  2. Moderate interference case ($\frac{1}{2} \leq \alpha \leq 1$)
  3. Weak interference case ($0 \leq \alpha \leq \frac{1}{2}$)

HK Scheme: Strong interference case

- Every receiver tries to decode the unintended messages along with the intended one
- There is no private part
- K - user MAC channel is formed at every receiver
- Achievable rate region: intersection of K - MAC regions

HK Scheme: Strong interference case

Theorem

The following GDOD is achievable in case of K user Gaussian MIMO-IC:

1. *When* $\frac{N}{M} < K \leq \frac{N}{M} + 1$*:*

$$d_j(\alpha) = \min\left\{M, \frac{1}{K}\left[(K-1)M\alpha + N - (K-1)M\right]\right\}$$

2. *When* $K > \frac{N}{M} + 1$*:*

$$d_j(\alpha) = \min\left\{M, \frac{\alpha N}{K}\right\}$$

Proof outline: Strong interference case

- It is sufficient to consider any particular user due to symmetry of the problem
- For $S \subseteq \{1, 2, \ldots, K\}$ a MAC channel is formed
- The rate that can be achieved at user - i:

$$\sum_{j \in S} R_j = \log \left| \mathbf{I} + \rho \mathbf{H}_{11} \mathbf{H}_{11}^H + \rho^{\alpha} \sum_{j \neq 1} \mathbf{H}_{1j} \mathbf{H}_{1j}^H \right|$$
$$= \alpha \min \{(K-1)M, N\} \log \rho + \min \{M, N - \min \{(K-1)M, N\}\} \log \rho + \mathcal{O}(1)$$

- Simplified based on the value of K, M and N

HK scheme: moderate interference case

- Message is split in to private and public part
- Private power is set such that it is received at the noise floor of the unintended receiver
- Both messages are encoded using Gaussian code book
- Decoding order:
  - While decoding the common message all private messages are treated as noise
  - Private message is decoded last: treat other user's private message as noise
- Rate achieved: $R_j = R_{p,j} + R_{c,j}$

HK scheme: moderate interference case

Theorem

In case of K user Gaussian MIMO-IC, following GDOF are achievable under the following conditions.

1. *When* $\frac{N}{M} < K < \frac{N}{M} + 1$*:*

$$\begin{aligned} d_j(\alpha) &= M(1-\alpha) + \min\left\{ \frac{N\alpha}{K}, \frac{1}{K-1}\left[M\left\{\alpha(2K-1)-K\right\} + N(1-\alpha)\right], \right. \\ &\left. (2\alpha-1)M + \frac{(N-M)(1-\alpha)}{K-2} \right\} \end{aligned}$$

2. *When* $K \geq \frac{N}{M} + 1$*:*

$$\begin{aligned} d_j(\alpha) &= M(1-\alpha) + \min\left\{ \frac{N\alpha}{K}, (2\alpha-1)M + \frac{(N-M)(1-\alpha)}{K-2} \right. \\ &\left. \frac{1}{K-1}\left[N\alpha - M(1-\alpha)\right] \right\} \end{aligned}$$

HK scheme: weak interference case

Theorem

In case of Gaussian MIMO-IC, following GDOF is achievable:

$$d_j(\alpha) = M(1-\alpha) + \frac{1}{K-1}(N-M)$$

Treating interference as noise and ZF - receiving

- Trivial techniques to mitigate the effect of interference

$$d_j^{ZF} = \min\left\{M, \frac{N}{K}\right\}$$

Theorem

The following GDOF is achievable in case of Gaussian MIMO-IC

1. *When* $\frac{N}{M} < K \leq \frac{N}{M} + 1$*:*

$$d_j(\alpha) = M + \alpha(N - KM)$$

2. *When* $K > \frac{N}{M} + 1$*:*

$$d_j(\alpha) = M(1 - \alpha)$$

Comparison of different schemes

Kill it (suppression, orthogonalization)

Be friends (decode interference)

Ignore it (live with interference)

Let them fight with each other
(interference alignment)

Results

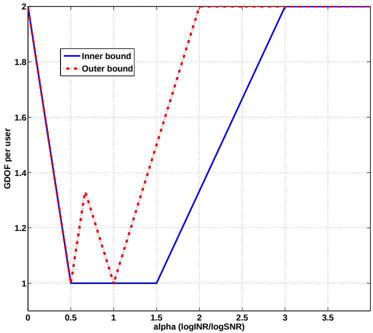


Figure: $K = 3$ user IC with $M = N = 2$

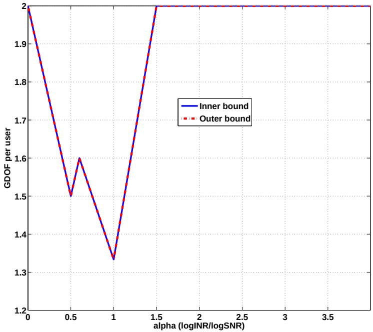


Figure: $K = 3$ user IC with $M = 2$ and $N = 4$

Some insights

- Treating interference as noise is GDOF optimal when $M = N$ in weak interference case
- When $M < N$, splitting message into private and public part helps in weak interference regime
- When $K > \frac{N}{M} + 1$, a combination of IA and HK scheme performs better
- When $\frac{N}{M} < K \leq \frac{N}{M} + 1$, HK scheme is GDOF optimal
- Unlike two user IC, ZF - receiving is found to be GDOF optimal at $\alpha = 1$ when $\frac{N}{M} < K \leq \frac{N}{M} + 1$

Conclusion

- Derived outer bound based on the notion of cooperation and providing noisy side information
- Derived achievable GDOF using a combination of HK - scheme, IA, treating interference as noise and ZF - receiving
- Explored the interplay between HK and IA

Future Work

- Proposing a scheme which combines IA and HK : deterministic model
- Relaxing the assumption of the same *SNR* and *INR* on the direct link and cross link, respectively