Iterative Matrix Decomposition Aided Block Diagonalization for mm-Wave Multiuser MIMO Systems

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Introduction

- Motivation
  - Significant interest in beamforming aided mm-wave systems due to the dearth of spectrum in the congested microwave band.
- Contributions
  - Iterative matrix decomposition based hybrid beamforming (IMD-HBF) scheme for a single-user scenario, which accurately approximates the unconstrained beamforming solution.
  - Novel subspace projection based AoD aided block diagonalization (SP-AoD-BD), which requires the knowledge of only the AoDs of the various channel paths.
  - SP-AoD-BD in the HBF architecture named SP-BD-HBF.

System Model

- Channel model

$$\mathbf{H}_j = \sqrt{N_t N_r} \sum_{i=1}^{L_j} \beta_i^{(j)} \mathbf{e}_r \left(\theta_i^{(j)}\right) \mathbf{e}_t^H \left(\phi_i^{(j)}\right), 1 \leq j \leq K. \quad (1)$$

where

- $\mathbf{L}_j$ - number of channel paths between the BS and the j^{th} user.
- $\beta_i^{(j)}$ - $CN(0,1)$ is the gain of the i^{th} path of the j^{th} user's channel.
- $\theta_i^{(j)}, \phi_i^{(j)}$ - AoD and AoA of the i^{th} path of the j^{th} user.
- $\mathbf{e}_r$, $\mathbf{e}_t$ - spatial receive and transmit signatures of a ULA, respectively.

$$\mathbf{e}_r\left(\theta\right) = \frac{1}{\sqrt{N_r}} \left[1, e^{j\frac{2\pi}{\lambda} d cos\theta}, ..., e^{j\frac{2\pi}{\lambda} d(N_r - 1) cos\theta}\right]^T \quad (2)$$

$$\mathbf{e}_t\left(\phi\right) = \frac{1}{\sqrt{N_t}} \left[1, e^{j\frac{2\pi}{\lambda} d cos\phi}, ..., e^{j\frac{2\pi}{\lambda} d(N_t - 1) cos\phi}\right]^T \quad (3)$$

where

- d - separation between the antenna elements.
- λ - carrier's wavelength.

System Model contd.

- Channel can be expressed as

$$\mathbf{H}_j = \mathbf{E}_r^{(j)} \mathbf{D}^{(j)} \mathbf{E}_t^{(j)}, \tag{4}$$

where

$$\mathbf{E}_r^{(j)} = \left[\mathbf{e}_r \left(\theta_1^{(j)} \right), \mathbf{e}_r \left(\theta_2^{(j)} \right), ..., \mathbf{e}_r \left(\theta_L^{(j)} \right) \right] \tag{5}$$

$$\mathbf{E}_t^{(j)} = \left[\mathbf{e}_t \left(\phi_1^{(j)} \right), \mathbf{e}_t \left(\phi_2^{(j)} \right), ..., \mathbf{e}_t \left(\phi_L^{(j)} \right) \right] \tag{6}$$

- $\mathbf{D}^{(j)}$ is a diagonal matrix of the channel gains.
- Unconstrained system:

$$\mathbf{y}_j = \mathbf{W}_j^H \mathbf{H}_j \mathbf{F} \mathbf{x} + \mathbf{W}_j^H \mathbf{n}_j \in \mathbb{C}^{N_s}. \tag{7}$$

- Constrained FAS system (Hybrid beamforming structure):

$$\mathbf{y}_j = \mathbf{G}_j^H \phi_j^H \mathbf{H}_j \Theta \mathbf{C} \mathbf{x} + \mathbf{G}_j^H \phi_j^H \mathbf{n}_j \in \mathbb{C}^{N_s}. \tag{8}$$

Iterative Matrix Decomposition for HBF

- For a single user scenario ($j = 1$), the optimal precoding and combining matrices for the unconstrained system are the right and left singular vectors associated with the N_s dominant singular values of the channel.
- IMD algorithm to obtain the analog and digital precoding/combining matrices.

Algorithm 1 Proposed IMD Algorithm for HBF

Require: $k = 0$, $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$, $max\_iterations$,
$\mathbf{W} = \mathbf{U}(:, [1 : M_r])$, $\mathbf{F} = \mathbf{V}(:, [1 : M_t])$,
$\mathbf{W}_{tmp} = \mathbf{W}$, $\mathbf{F}_{tmp} = \mathbf{F}$,
while $k < max\_iterations$ **do**
1. $\Phi = \measuredangle \mathbf{W}_{tmp}$, $\Theta = \measuredangle \mathbf{F}_{tmp}$,
$\Phi \leftarrow \frac{\Phi}{\sqrt{N_r}}$, $\Theta \leftarrow \frac{\Theta}{\sqrt{N_t}}$,
2. $\mathbf{G} = (\Phi^H \Phi)^{-1} \Phi^H \mathbf{W}$, $\mathbf{C} = (\Theta^H \Theta)^{-1} \Theta^H \mathbf{F}$,
3. $\mathbf{W}_{tmp} = \mathbf{W}\mathbf{G}^{-1}$, $\mathbf{F}_{tmp} = \mathbf{F}\mathbf{C}^{-1}$,
4. $\mathbf{W}' = \Phi\mathbf{G}$, $\mathbf{F}' = \Theta\mathbf{C}$,
$\mathbf{W}' \leftarrow \frac{\mathbf{W}'}{\|\mathbf{W}'\|}\sqrt{M_r}$, $\mathbf{F}' \leftarrow \frac{\mathbf{F}'}{\|\mathbf{F}'\|}\sqrt{M_t}$

end while

Iterative Matrix Decomposition for HBF contd.

- Convergence of IMD-HBF:

Definition

Let $\mathbf{A}$ and $\mathbf{B}$ selected from $\mathbb{C}^{m\times n}$, with $m \gg n$. The subspaces $\mathtt{span}(\mathbf{A})$ and $\mathtt{span}(\mathbf{B})$ are said to be *non-intersecting or parallel*, if $\mathbf{C} = [\mathbf{A}\ \mathbf{B}]$ has rank of $2n$. In other words, $\mathtt{span}(\mathbf{A}) \cap \mathtt{span}(\mathbf{B}) = \phi$.

- The residual error during the k-th iteration is

$$\mathbf{F} - \mathbf{F}'_k = \mathbf{F} - \Theta_k \mathbf{C}_k = \Delta_k, \tag{9}$$

$$\mathbf{W} - \mathbf{W}'_k = \mathbf{W} - \mathbf{\Phi}_k \mathbf{G}_k = \Gamma_k. \tag{10}$$

- The matrices $\mathbf{F}_{tmp}$ and $\mathbf{W}_{tmp}$ used in the next iteration are given by

$$\mathbf{F}\mathbf{C}_k^{-1} = \Theta_k + \Delta_k \mathbf{C}_k^{-1}, \tag{11}$$

$$\mathbf{W}\mathbf{G}_k^{-1} = \Phi_k + \Gamma_k \mathbf{G}_k^{-1}. \tag{12}$$

- It can be easily verified that $\Theta_k^H \Delta_k$ and $\Phi_k^H \Gamma_k$ are $\mathbf{0}$.

Iterative Matrix Decomposition for HBF contd.

- In the next iteration, $\Theta_{k+1} = \angle(\mathbf{F}\mathbf{C}_k^{-1})/\sqrt{N_t}$ and $\Phi_{k+1} = \angle(\mathbf{W}\mathbf{G}_k^{-1})/\sqrt{N_r}$, which satisfy

$$\Theta_{k+1} = \underset{|\Theta(i,j)|=1/\sqrt{N_t}}{\operatorname{argmin}} \|\mathbf{F}\mathbf{C}_k^{-1} - \Theta\|^2 \tag{13}$$

$$\Phi_{k+1} = \underset{|\Phi(i,j)|=1/\sqrt{N_r}}{\operatorname{argmin}} \|\mathbf{W}\mathbf{G}_k^{-1} - \Phi\|^2 \tag{14}$$

- Proof for (13): See Appendix A in the paper.
- From (13) and (14), we have

$$\|\mathbf{F}\mathbf{C}_k^{-1} - \Theta_k\|^2 > \|\mathbf{F}\mathbf{C}_k^{-1} - \Theta_{k+1}\|^2, \tag{15}$$

$$\|\mathbf{W}\mathbf{G}_k^{-1} - \Phi_k\|^2 > \|\mathbf{W}\mathbf{G}_k^{-1} - \Phi_{k+1}\|^2. \tag{16}$$

- $\mathbf{F}$, Θ_k and Θ_{k+1} form a set of mutually non-intersecting subspaces (more details about this in the paper but omitted here for brevity).

Iterative Matrix Decomposition for HBF contd.

- From (15), it is reasonable to expect $\|\mathbf{F} - \Theta_k \mathbf{C}_k\|^2 > \|\mathbf{F} - \Theta_{k+1} \mathbf{C}_k\|^2$.

Proposition

Let $\mathbf{F}, \mathbf{C}_k, \Theta_k$, *and* Θ_{k+1} *be defined as above. Let* $\mathbf{A} = \mathbf{C}_k \mathbf{C}_k^H$ *and*

$$\begin{aligned} \mathbf{B} = & \left(\mathbf{F}\mathbf{C}_k^{-1} - \Theta_k\right)^H \left(\mathbf{F}\mathbf{C}_k^{-1} - \Theta_k\right) \\ & - \left(\mathbf{F}\mathbf{C}_k^{-1} - \Theta_{k+1}\right)^H \left(\mathbf{F}\mathbf{C}_k^{-1} - \Theta_{k+1}\right). \end{aligned} \tag{17}$$

Then, we have

$$\begin{aligned} & \|\mathbf{F} - \Theta_k \mathbf{C}_k\|^2 - \|\mathbf{F} - \Theta_{k+1} \mathbf{C}_k\|^2 \\ & \geq \underbrace{\lambda_{min}(\mathbf{A})}_{\geq 0} \underbrace{\text{tr}(\mathbf{B})}_{>0} + \lambda_{min}(\mathbf{B}) \left[\text{tr}(\mathbf{A}) - M_t \lambda_{min}(\mathbf{A})\right]. \end{aligned} \tag{18}$$

Proof.

See Appendix B in paper. □

Iterative Matrix Decomposition for HBF contd.

- Furthermore,

$$\|\mathbf{F} - \Theta_{k+1}\mathbf{C}_k\|^2 > \|\mathbf{F} - \Theta_{k+1}\mathbf{C}_{k+1}\|^2 \tag{19}$$

where

$$\mathbf{C}_{k+1} = \underset{\mathbf{C}}{\operatorname{argmin}} \|\mathbf{F} - \Theta_{k+1}\mathbf{C}\|^2 \tag{20}$$

Thus $\|\Delta_k\|^2 > \|\Delta_{k+1}\|^2$.

Block Diagonalization with Unconstrained Beamforming

- AoD of various signal paths sufficient for block-diagonalizing the mm-wave MU-MIMO channel.
- Sufficient CSI for BD:
  - Composite user channel & precoding matrix:

$$\mathbf{H}_{comp} \triangleq \left[\mathbf{H}_1^T, \mathbf{H}_2^T, \ldots, \mathbf{H}_K^T\right]^T \in \mathbb{C}^{KN_r \times N_t} \quad (21)$$

$$\mathbf{F} \triangleq [\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_K] \in \mathbb{C}^{N_t \times KN_s} \quad (22)$$

Definition

A precoding matrix $\mathbf{F}$ is said to block-diagonalize the composite user channel $\mathbf{H}_{comp}$, if $\mathbf{H}_i\mathbf{F}_j = \mathbf{0}_{N_r \times N_s}$, for $1 \leq i \neq j \leq K$.

Block Diagonalization with Unconstrained Beamforming contd.

Proposition

Given a composite user channel $\mathbf{H}_{comp}$*, the knowledge of the AoDs of various users given by* $\{\mathbf{E}_t^{(1)}, \mathbf{E}_t^{(2)}, \ldots, \mathbf{E}_t^{(K)}\}$ *is sufficient for obtaining a block-diagonalizing precoder* $\mathbf{F}$*.*

Proof.

The composite user channel can be written as

$$\mathbf{H}_{comp} = \begin{bmatrix} \mathbf{E}_r^{(1)}\mathbf{D}^{(1)}{\mathbf{E}_t^{(1)}}^H \\ \mathbf{E}_r^{(2)}\mathbf{D}^{(2)}{\mathbf{E}_t^{(2)}}^H \\ \vdots \\ \mathbf{E}_r^{(K)}\mathbf{D}^{(K)}{\mathbf{E}_t^{(K)}}^H \end{bmatrix} = \overline{\mathbf{H}}_r\mathbf{E}_{t\,comp}, \tag{23}$$

where □

Block Diagonalization with Unconstrained Beamforming contd.

Proof.

$$\overline{\mathbf{H}}_r = \begin{bmatrix} \mathbf{E}_r^{(1)}\mathbf{D}^{(1)} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_r^{(2)}\mathbf{D}^{(2)} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{E}_r^{(K)}\mathbf{D}^{(K)} \end{bmatrix} \tag{24}$$

and $\mathbf{E}_{t\,comp} = [\mathbf{E}_t^{(1)}, \dots, \mathbf{E}_t^{(K)}]^H$. Let $\mathbf{Q}_j = [\mathbf{E}_t^{(1)}, \dots, \mathbf{E}_t^{(j-1)}, \mathbf{E}_t^{(j+1)}, \dots, \mathbf{E}_t^{(K)}]^H = \mathbf{U}_j \mathbf{\Sigma}_j \mathbf{V}_j^H$ for $1 \leq j \leq K$. Assuming $N_t \geq KL$ and $L = N_s$, we opt

$$\mathbf{F}_j = \mathbf{V}_j\left(:, [N_t - L + 1 : N_t]\right) \in \mathbb{C}^{N_t \times L}, \tag{25}$$

which is a subset of the nullspace basis of $\mathbf{Q}_j$. Thus

$$\mathbf{Q}_j \mathbf{F}_j = \mathbf{0}_{(K-1)L \times L}, \forall j, \tag{26}$$

Hence

$$\mathbf{H}_i \mathbf{F}_j = \mathbf{E}_r^{(i)} \mathbf{D}^{(i)} {\mathbf{E}_t^{(i)}}^H \mathbf{F}_j, \tag{27}$$

$$= \mathbf{0}_{N_r \times L}. \tag{28}$$

Block Diagonalization with Unconstrained contd.

- Subspace Projection based AoD aided BD (SP-AoD-BD)
  - Conventional BD discussed in the previous section block-diagonalizes the composite user channel, but is not aligned with the user's signal.
  - Let $\mathbf{F}_j^{(int\_null)} \triangleq \mathbf{V}_j(:, [(K-1)L+1 : N_t])$, ${\mathbf{E}_t^{(j)}}^H = \tilde{\mathbf{U}}_j \tilde{\Sigma}_j \tilde{\mathbf{V}}_j^H$, and $\mathbf{F}_j^{(sig)} = \tilde{\mathbf{V}}_j(:, [1 : L])$.
  - Let the projection matrices associated with $\mathbf{F}_j^{(int\_null)}$, $\mathbf{F}_j^{(sig)}$ be $\mathbf{P}_j^{(int\_null)}$, $\mathbf{P}_j^{(sig)}$, respectively.
  - $\mathbf{P}_j^{(eff)} = \mathbf{P}_j^{(sig)} \mathbf{P}_j^{(int\_null)} = \overline{\mathbf{U}}_j \overline{\Sigma}_j \overline{\mathbf{V}}_j^H$,
  - Precoder is given by

$$\mathbf{F}_j = \overline{\mathbf{V}}_j(:, [1 : L]) \in \mathbb{C}^{N_t \times L} \quad (29)$$

Proposition

Given a composite user channel $\mathbf{H}_{comp}$, *the precoder proposed in* (29) *satisfies* $\mathbf{H}_i \mathbf{F}_j = \mathbf{0}_{N_r \times L}$ *for* $1 \leq i \neq j \leq K$.

Block Diagonalization with Unconstrained Beamforming contd.

Proof.

- Sufficient to show that the columns of $\mathbf{F}_j$ are in the span$\left(\mathbf{F}_j^{(int_null)}\right)$ for $1 \leq j \leq K$.

$$\mathbf{P}_j^{(eff)}{}^H \mathbf{P}_j^{(eff)} = \mathbf{F}_j^{(int\_null)} \mathbf{Z}_j \mathbf{F}_j^{(int\_null)}{}^H \in \mathbb{C}^{N_t \times N_t}, \tag{30}$$

where

$$\mathbf{Z}_j = \mathbf{F}_j^{(int\_null)}{}^H \mathbf{F}_j^{(sig)} \mathbf{F}_j^{(sig)}{}^H \mathbf{F}_j^{(int\_null)} \in \mathbb{C}^{L \times L}. \tag{31}$$

- Eigenvalue decomposition of $\mathbf{Z}_j = \mathbf{A}_j \Lambda_j \mathbf{A}_j^H$. Substituting this into (30),

$$\mathbf{P}_j^{(eff)}{}^H \mathbf{P}_j^{(eff)} = \mathbf{F}_j^{(int\_null)} \mathbf{A}_j \Lambda_j \mathbf{A}_j^H \mathbf{F}_j^{(int\_null)}{}^H \tag{32}$$

$$\Rightarrow \mathbf{P}_j^{(eff)}{}^H \mathbf{P}_j^{(eff)} \mathbf{F}_j^{(int\_null)} \mathbf{A}_j = \mathbf{F}_j^{(int\_null)} \mathbf{A}_j \Lambda_j. \tag{33}$$

- From (33), we can see that the eigenvectors of $\mathbf{P}_j^{(eff)}{}^H \mathbf{P}_j^{(eff)}$ are in the span$\left(\mathbf{F}_j^{(int_null)}\right)$.

□

Block Diagonalization with Constrained Beamforming

- Subspace Projection Based AoD aided BD combined with HBF (SP-BD-HBF):
  - Given SP-AoD-BD precoder in (29) for the j^{th} user, Θ_j and $\mathbf{C}_j$ are obtained by using IMD based HBF, i.e., $\overline{\mathbf{V}}_j(:,[1:L]) \approx \Theta_j \mathbf{C}_j$.
  - Due to the residual errors in the approximation, $\mathbf{H}_i \Theta_j \mathbf{C}_j \neq \mathbf{0}_{N_r \times L}$.
  - Let the baseband composite user channel be defined as

$$\mathbf{K}_{comp} = \begin{bmatrix} \mathbf{K}_1 \\ \mathbf{K}_2 \\ \vdots \\ \mathbf{K}_K \end{bmatrix} \in \mathbb{C}^{KL \times KL} \tag{34}$$

where $\mathbf{K}_i \triangleq \left[{\mathbf{E}_t^{(i)}}^H \Theta_1 \mathbf{C}_1, {\mathbf{E}_t^{(i)}}^H \Theta_2 \mathbf{C}_2, \ldots, {\mathbf{E}_t^{(i)}}^H \Theta_K \mathbf{C}_K \right], \forall i.$

  - $\mathbf{R}_j = [\mathbf{K}_1^T, \ldots, \mathbf{K}_{j-1}^T, \mathbf{K}_{j+1}^T, \ldots, \mathbf{K}_K^T]^T = \breve{\mathbf{U}}_j \breve{\Sigma}_j \breve{\mathbf{V}}_j^H$ and $\mathbf{J}_j = \breve{\mathbf{V}}_j(:,[(K-1)L+1 : KL]) \in \mathbb{C}^{KL \times L} \forall j.$
  - Effective preprocessing for achieving BD at the BS is given by

$$\left[\Theta_1, \ldots, \Theta_K \right] \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{C}_K \end{bmatrix} \left[\mathbf{J}_1, \ldots, \mathbf{J}_K \right] \tag{35}$$

Block Diagonalization with Constrained Beamforming contd.

- The effective channel as seen by the j^{th} user becomes

$$\mathbf{H}_j^{(eff)} = \mathbf{E}_r^{(j)}\mathbf{D}^{(j)}\mathbf{K}_j\mathbf{J}_j \in \mathbb{C}^{N_r \times L}. \quad (36)$$

- The optimal combining and precoding matrix conditioned for the j^{th} user corresponds to the left and right singular vectors associated with dominant singular values of $\mathbf{H}_j^{(eff)}$, respectively.
- $\mathbf{H}_j^{(eff)} = \breve{\mathbf{U}}_j\breve{\Sigma}_j\breve{\mathbf{V}}_j^H$, then the unconstrained combining matrix is given by $\breve{\mathbf{U}}_j(:,[1:L])$.
- IMD used to get the analog and digital combining matrices.
- UE estimates $\breve{\mathbf{V}}_j$ and feeds it back to the BS which does the preprocessing for BD and user channel diagonalization as

$$[\Theta_1\mathbf{C}_1,\ldots,\Theta_K\mathbf{C}_K]\left[\mathbf{J}_1\breve{\mathbf{V}}_1,\ldots,\mathbf{J}_K\breve{\mathbf{V}}_K\right] \quad (37)$$

Block Diagonalization with Constrained Beamforming contd.

Summary of the steps to establish a reliable downlink

- BS acquires the AoD knowledge of the channel paths of each user by uplink channel sounding.
- BS obtains the effective preprocessing matrix given by (35) that allows to establish an interference free channel to each of the users.
- With the aid of DL channel training over interference free channels, each user acquires the knowledge of $\mathbf{H}_j^{(eff)}$ and obtains the precoding and combining matrices. UE feeds back the precoding matrix to the BS.
- BS uses the preprocessing matrix of (37) for DL data transmission.

Simulation Results

- Sum rate comparison of unconstrained and constrained beamforming algorithms. Nearly zero loss in the achievable rates with IMD.
- Comparison of the achievable sum rate as a function of the number of users.
- Comparison of the achievable sum rate in the conventional BD and the proposed SP-AoD-BD with unconstrained beamforming.
- Future work: Design of mm-wave communication systems with the aid of partial CSI.