Interference Management in Wireless Networks

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Overview

- On the generalized degrees of freedom of the K-user symmetric MIMO Gaussian interference channel

- Interference alignment algorithms for the K-user constant MIMO interference channel: (Joint work with Nissar K.E., M.E. student, 2008-2010)

- Three User Cognitive Radio Networks: An Information Theoretic Perspective: (Joint work with K.G. Naganada, Lehigh University, USA)

On the generalized degrees of freedom of the K-user symmetric MIMO Gaussian interference channel

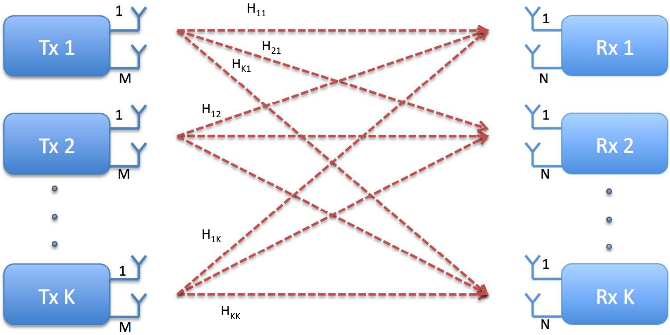
Outline

- Preliminaries
  1. Interference channel
  2. Notion of Generalized Degrees of Freedom (GDOF)
  3. Brief review on past works
- Outer bound
  1. Outer bound 1: cooperation among the users
  2. Outer bound 2 and 3: providing side information to receivers
- Inner bound
  1. Brief review on: Interference Alignment (IA) and Han-Kobayashi (HK) scheme
  2. Extension of HK-scheme to multiuser case
- Results and Discussion
- Summary

Interference in wireless networks

- Wireless networks are interference limited rather than noise limited
- Cellular networks: inter cell interference, interference between macro, pico and femto cell
- Ad-hoc networks: interference from simultaneous transmissions
- Wireless LANs: interference from adjacent networks
- Cognitive network: between primary and secondary and among secondary users
- One of the challenging task in wireless networks is interference management

Interference Channel (IC)



- $\mathbf{H}_{ij}$: channel from j^{th} transmitter to i^{th} receiver
- M and N: antennas at transmitter and receiver respectively

Generalized Degrees of Freedom (GDOF)

- GDOF is a measure of the high SNR capacity obtainable from a given channel
- For the symmetric scenario, it is defined as $d_{\text{sym}}(\alpha) = \lim_{\rho \to \infty} \frac{C_{\text{sym}}(\rho, \alpha)}{\log \rho}$, where $\alpha = \frac{\log \gamma}{\log \rho}$
- Roughly measures interference free dimension accessible in a network
- When $\text{SNR}(\rho) = \text{INR}(\gamma)$, Degrees of Freedom (DOF) is obtained as a special case of GDOF

Past works

- Capacity of two user IC is characterized within 1-bit: a simple HK-scheme [Etkin, Tse and Wang, 2008]
- DOF of K-user IC is characterized using the novel idea of IA [Cadambe and Jafar, 2008]
- GDOF of K-user SISO-IC and SIMO-IC $(K = N + 1)$ are also characterized [Jafar and Vishwanath, 2008; Gou and Jafar, 2011]

Problem statement

- Multiple antennas can help to mitigate the effect of interference

  e.g: When $N \geq KM$, Zero-Forcing (ZF) receiving is sufficient to achieve interference free GDOF

- When $N < KM$, trivial techniques are found to be sub-optimal

- Focus: To characterize GDOF of K-user symmetric MIMO Gaussian IC

Contributions

- A new outer bound is derived for the K-user MIMO Gaussian IC using
  - Cooperation
  - Providing noisy side information
- Inner bound is derived for the symmetric MIMO IC ($M \leq N$) as a combination of
  - Han-Kobayashi (HK) scheme
  - Interference Alignment (IA)
  - Treating interference as noise
  - ZF-receiving
- HK-scheme is extended to multiuser MIMO scenario
- Interplay between the HK and IA schemes is explored

Outer bound: cooperation

- Cooperation does not hurt capacity
- System is reduced to a two user MIMO Z-IC and then outer bound is derived for the modified system
- Different possible ways of cooperation is taken in to account

Outer bound: cooperation

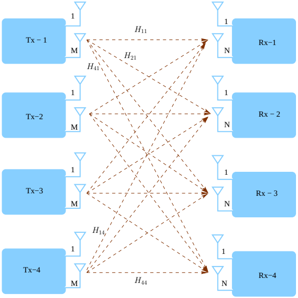
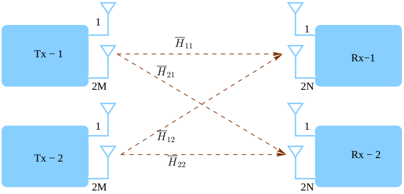
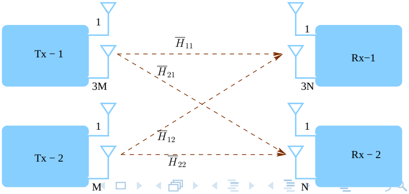
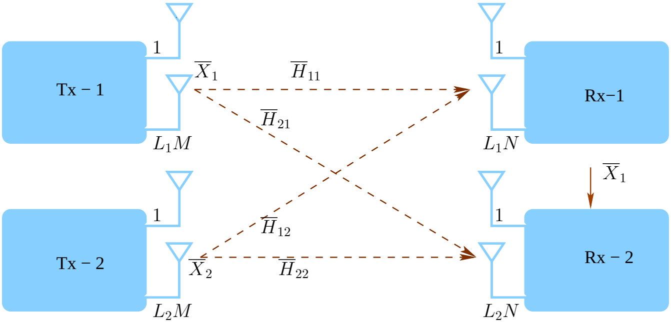


Figure: Four user Gaussian IC



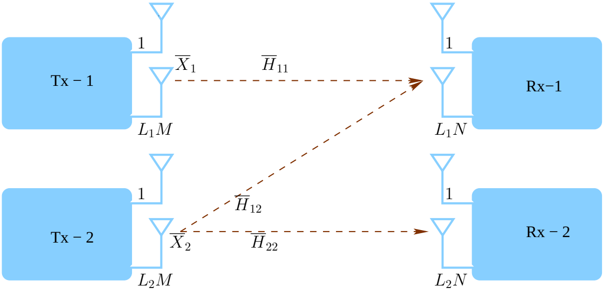


Outer bound: cooperation



- L_1 and L_2: number of users in group-1 and group-2
- $\overline{\mathbf{H}}_{ij} \in \mathbb{C}^{L_iN \times L_jM}$, $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$: two set of messages

Outer bound: cooperation



- Equivalent to a two user MIMO Z-IC

Outer bound: cooperation

- Modified system model:

$$\overline{\mathbf{Y}}_1 = \overline{\mathbf{H}}_{11}\overline{\mathbf{X}}_1 + \overline{\mathbf{H}}_{12}\overline{\mathbf{X}}_2 + \overline{\mathbf{Z}}_1,\ \overline{\mathbf{Y}}_2 = \overline{\mathbf{H}}_{22}\overline{\mathbf{X}}_2 + \overline{\mathbf{Z}}_2$$

where

$$\overline{\mathbf{Y}}_1 \triangleq \left[\ \mathbf{Y}_1, \cdots, \mathbf{Y}_{L_1}\ \right]^T,\ \overline{\mathbf{Y}}_2 \triangleq \left[\ \mathbf{Y}_{L_1+1}, \cdots, \mathbf{Y}_L\ \right]^T,$$

$$\overline{\mathbf{X}}_1 \triangleq \left[\ \mathbf{X}_1, \cdots, \mathbf{X}_{L_1}\ \right]^T,\ \overline{\mathbf{X}}_2 \triangleq \left[\ \mathbf{X}_{L_1+1}, \cdots, \mathbf{X}_L\ \right]^T,$$

$$\overline{\mathbf{Z}}_1 \triangleq \left[\ \mathbf{Z}_1, \cdots, \mathbf{Z}_{L_1}\ \right]^T \text{ and } \overline{\mathbf{Z}}_2 \triangleq \left[\ \mathbf{Z}_{L_1+1}, \cdots, \mathbf{Z}_L\ \right]^T.$$

$$\overline{\mathbf{H}}_{11} = \text{blkdiag}\left(\mathbf{H}_{11}\ \mathbf{H}_{22}\ \ldots\ \mathbf{H}_{L_1,L_1}\right)$$

$$\overline{\mathbf{H}}_{22} = \text{blkdiag}\left(\mathbf{H}_{L_1+1,L_1+1}\ \mathbf{H}_{L_1+2,L_1+2}\ \ldots\ \mathbf{H}_{L,L}\right)$$

$$\overline{\mathbf{H}}_{12} = \begin{bmatrix} \mathbf{H}_{1,L_1+1} & \mathbf{H}_{1,L_1+2} & \cdots & \mathbf{H}_{1,L} \\ \mathbf{H}_{2,L_1+1} & \mathbf{H}_{2,L_1+2} & \cdots & \mathbf{H}_{2,L} \\ & \vdots & \vdots & \\ \mathbf{H}_{L_1,L_1+1} & \mathbf{H}_{L_1,L_1+2} & \cdots & \mathbf{H}_{L_1,L} \end{bmatrix},\ L_1 + L_2 \leq K$$

Outer bound: cooperation

Theorem 1:

The sum rate of the K-user MIMO Gaussian IC is upper bounded as follows

$$\sum_{i=1}^{L} R_i \leq \log \left| \mathbf{I}_{L_1 N} + \overline{\mathbf{H}}_{11}\overline{\mathbf{P}}_1\overline{\mathbf{H}}_{11}^H + \overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2\overline{\mathbf{H}}_{12}^H \right| +$$

$$\log \left| \mathbf{I}_{L_2 N} + \overline{\mathbf{H}}_{22}\overline{\mathbf{P}}_2^{1/2} \left\{ \mathbf{I}_{L_2 M} + \overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{12}^H\overline{\mathbf{H}}_{12}\overline{\mathbf{P}}_2^{1/2} \right\}^{-1} \overline{\mathbf{P}}_2^{1/2}\overline{\mathbf{H}}_{22}^H \right| + \epsilon_n$$

where $L_1 + L_2 = L \leq K,\ 0 \leq L_1 \leq K,\ 0 \leq L_2 \leq K$,

$\mathbf{I}_L : L \times L$ identity matrix .

- Need to be minimized over all possible values of L

Outer bound: cooperation

Lemma 1:

In the symmetric case, the outer bound in Theorem 1 reduces to following form

- When $M \leq N$ and $0 \leq \alpha \leq 1$

$$d(\alpha) \leq \min_{L_1,L_2} \frac{1}{L}\left[L_1 M + \min\{r, L_1(N-M)\}\alpha + (L_2 M - r)^+ + \min\left\{r, L_2 N - (L_2 M - r)^+\right\}(1-\alpha)\right],$$

- When $M \leq N$ and $\alpha > 1$

$$d(\alpha) \leq \min_{L_1,L_2} \frac{1}{L}\left[r\alpha + \min\{L_1 M, L_1 N - r\} + (L_2 M - r)^+\right],$$

where $r = \min\{L_2 M, L_1 N\}$.

Outer bound: noisy side information

Theorem 2:

The sum rate of the K-user MIMO Gaussian IC is upper bounded as follows

$$R_1 + 2\sum_{i=2}^{K-1} R_i + R_K$$

$$\leq \sum_{i=1}^{K-1} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\left(\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i+1,i}^H\mathbf{H}_{i+1,i}\mathbf{P}_i^{1/2}\right)^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| +$$

$$\sum_{i=2}^{K} \log |\mathbf{I}_{N_i} + \sum_{j=1,\, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{P}_j\mathbf{H}_{ij}^H + \mathbf{H}_{ii}\mathbf{P}_i^{1/2}\left(\mathbf{I}_{M_i} + \mathbf{P}_i^{1/2}\mathbf{H}_{i-1,i}^H\mathbf{H}_{i-1,i}\mathbf{P}_i^{1/2}\right)^{-1}\mathbf{P}_i^{1/2}\mathbf{H}_{ii}^H| + \epsilon_n$$

- Noisy version of the message is provided to receivers in a careful manner
- Outer bound is simplified for the symmetric case

Outer bound: noisy side information

Lemma 2:

The GDOF outer bound in Theorem 2 in the symmetric case is upper bounded as follows

1. When $M \leq N$

$$d(\alpha) \leq \begin{cases} M(1-\alpha) + \min\left\{\min\left(N, (K-1)M\right), N-M\right\}\alpha & \text{if } 0 \leq \alpha \leq \frac{1}{2} \\ \min\left(N, (K-1)M\right)\alpha + \min\left\{M, N - \min\left(N, (K-1)M\right)\right\}(1-\alpha) & \text{if } \frac{1}{2} \leq \alpha \leq 1 \\ \min\left\{N, (K-1)M\right\}\alpha & \text{if } \alpha \geq 1 \end{cases}$$

2. When $M > N$

$$d(\alpha) \leq \begin{cases} M - N + (2N-M)^{+}(1-\alpha) & \text{if } 0 \leq \alpha \leq \frac{1}{2} \\ (M-N) + \min\left\{N, \{2N-M)\}^{+}\right\}\alpha & \text{if } \frac{1}{2} \leq \alpha \leq 1 \\ N\alpha & \text{if } \alpha \geq 1 \end{cases}.$$

Outer bound: noisy side information

Theorem 3:

The sum rate of the K-user MIMO IC is upper bounded as follows

$$
\begin{aligned}
R_1 + \sum_{i=2}^{K-1} R_i + R_K \quad & \leq \log \left| \mathbf{I}_{N_1} + \sum_{j=2}^{K} \mathbf{H}_{1j}\mathbf{P}_j\mathbf{H}_{1j}^H + \mathbf{H}_{11}\mathbf{P}_1^{1/2} \left\{ \mathbf{I}_{M_1} + \mathbf{P}_1^{1/2}\mathbf{H}_{K1}^H\mathbf{H}_{K1}\mathbf{P}_1^{1/2} \right\}^{-1} \mathbf{P}_1^{1/2}\mathbf{H}_{11}^H \right| + \\
& \sum_{i=2}^{K-1} \log \left| \mathbf{I}_{N_i} + \overline{\mathbf{H}}_{i1}\overline{\mathbf{P}}_{i1}^{1/2} \left\{ \mathbf{I}_{M_{r_i}} + \overline{\mathbf{P}}_{i1}^{1/2}\overline{\mathbf{H}}_{Ki}^H\overline{\mathbf{H}}_{Ki}\overline{\mathbf{P}}_{i1}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i1}^{1/2}\overline{\mathbf{H}}_{i1}^H + \right. \\
& \left. \overline{\mathbf{H}}_{i,i+1}\overline{\mathbf{P}}_{i2}^{1/2} \left\{ \mathbf{I}_{M_{s_i}} + \overline{\mathbf{P}}_{i2}^{1/2}\overline{\mathbf{H}}_{1,i+1}^H\overline{\mathbf{H}}_{1,i+1}\overline{\mathbf{P}}_{i2}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i2}^{1/2}\overline{\mathbf{H}}_{i,i+1}^H \right| + \\
& \sum_{i=2}^{K-1} \log \left| \mathbf{I}_{N_i} + \overline{\mathbf{H}}_{iK}\overline{\mathbf{P}}_{i3}^{1/2} \left\{ \mathbf{I}_{M_{r_i}} + \overline{\mathbf{P}}_{i3}^{1/2}\overline{\mathbf{H}}_{1i}^H\overline{\mathbf{H}}_{1i}\overline{\mathbf{P}}_{i3}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i3}^{1/2}\overline{\mathbf{H}}_{iK}^H + \right. \\
& \left. \overline{\mathbf{H}}_{i,K-1}\overline{\mathbf{P}}_{i4}^{1/2} \left\{ \mathbf{I}_{M_{s_i}} + \overline{\mathbf{P}}_{i4}^{1/2}\overline{\mathbf{H}}_{K,i+1}^H\overline{\mathbf{H}}_{K,i+1}\overline{\mathbf{P}}_{i4}^{1/2} \right\}^{-1} \overline{\mathbf{P}}_{i4}^{1/2}\overline{\mathbf{H}}_{i,K-1}^H \right| + \\
& \log \left| \mathbf{I}_{N_K} + \sum_{j=1}^{K-1} \mathbf{H}_{Kj}\mathbf{P}_j\mathbf{H}_{Kj}^H + \mathbf{H}_{KK}\mathbf{P}_K^{1/2} \left\{ \mathbf{I}_{M_K} + \mathbf{P}_K^{1/2}\mathbf{H}_{1K}^H\mathbf{H}_{1K}\mathbf{P}_K^{1/2} \right\}^{-1} \mathbf{P}_K^{1/2}\mathbf{H}_{KK}^H \right| + \epsilon_n
\end{aligned}
$$

where $M_{r_i} = \sum_{j=1}^{i} M_j$ and $M_{s_i} = \sum_{j=i+1}^{K} M_j$.

Inner bound

- Inner bound is derived for the symmetric MIMO ($M \leq N$) Gaussian IC as a combination of
  - Interference Alignment (IA)
  - Han-Kobayashi (HK) scheme
  - Treating interference as noise
  - ZF-receiving

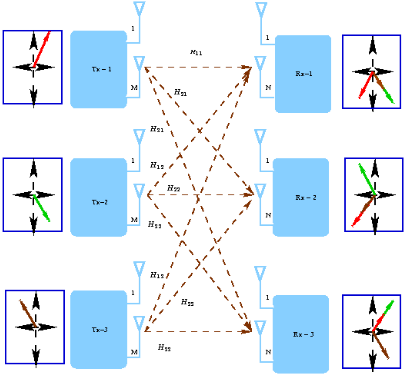
Interference Alignment (IA)

- Idea of IA for IC originated in the seminal work by Cadambe et al. in 2008
- For MIMO IC, DOF achieved by IA:

$$d_j = \frac{MN}{M+N}, \text{ if } KM > N \quad \text{[Gou and Jafar, 2010]}$$

- Requires global channel knowledge and channel to be time-varying
- Relative strength between signal and interference does not matter

IA in case of $K = 3$ user IC



Han-Kobayashi (HK) scheme

- Based on the idea of splitting message in to two parts:
  1. Private part
  2. Public part
- A simple HK scheme proposed by Etkin et al.: achieves capacity with in 1-bit
- HK scheme is extended to K-user MIMO IC for symmetric case
- Following interference regime are considered:
  1. Strong interference case ($\alpha > 1$)
  2. Moderate interference case ($\frac{1}{2} \leq \alpha \leq 1$)
  3. Weak interference case ($0 \leq \alpha \leq \frac{1}{2}$)

HK-scheme: strong interference case

- Every receiver tries to decode the unintended messages along with the intended one
- There is no private part
- K-user MAC channel is formed at every receiver
- Achievable rate region: intersection of K-MAC regions

HK-scheme: strong interference case

Theorem 4:

The following GDOF is achievable in case of K-user MIMO Gaussian IC

1. When $\frac{N}{M} < K \leq \frac{N}{M} + 1$, then HK scheme can achieve

$$d(\alpha) \geq \min\left\{M, \frac{1}{K}\left[(K-1)M\alpha + N - (K-1)M\right]\right\}$$

2. When $K > \frac{N}{M} + 1$, then HK scheme can achieve

$$d(\alpha) \geq \min\left\{M, \frac{\alpha N}{K}\right\}.$$

HK-scheme: moderate interference case

- Message is split in to private and public part
- Private power is set such that it is received at the noise floor of the unintended receiver
- Both messages are encoded using Gaussian code book
- Decoding order
  - Private message is decoded last: treat other user's private message as noise
  - While decoding the common message, all private messages are treated as noise

HK-scheme: moderate interference case

Theorem 5:

In case of K-user MIMO Gaussian IC, following GDOF are achievable under the following conditions

1. When $\frac{N}{M} < K \leq \frac{N}{M} + 1$

$$d(\alpha) \geq M(1-\alpha) + \min\left\{\frac{N\alpha}{K}, \frac{1}{K-1}\left[M\{\alpha(2K-1)-K\} + N(1-\alpha)\right], (2\alpha-1)M + \frac{(N-M)(1-\alpha)}{K-2}\right\}$$

2. When $K > \frac{N}{M} + 1$

$$d(\alpha) \geq M(1-\alpha) + \min\left\{\frac{N\alpha}{K}, (2\alpha-1)M + \frac{(N-M)(1-\alpha)}{K-2}, \frac{1}{K-1}\left[N\alpha - M(1-\alpha)\right]\right\}.$$

HK-scheme: weak interference case

Theorem 6:

In case of MIMO Gaussian IC, following per user GDOF is achievable

$$d(\alpha) \geq M(1-\alpha) + \frac{1}{K-1}(N-M).$$

Treating interference as noise and ZF-receiving

- Trivial techniques to mitigate the effect of interference

$$d^{ZF} = \min\left\{M, \frac{N}{K}\right\}$$

Theorem 7:

The following per user GDOF is achievable in case of K-user MIMO Gaussian IC:

1. When $\frac{N}{M} < K \leq \frac{N}{M} + 1$

$$d(\alpha) \geq M + \alpha(N - KM)$$

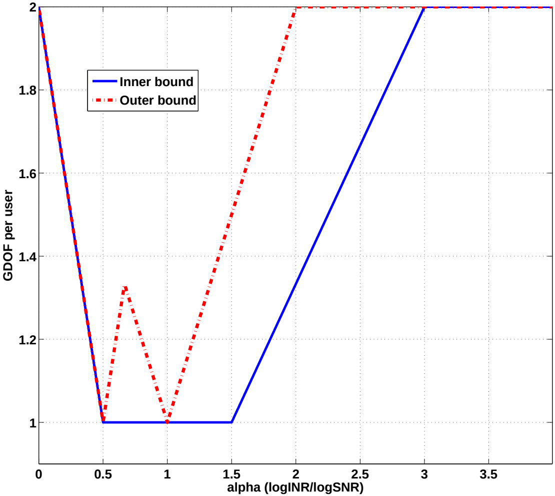
2. When $K > \frac{N}{M} + 1$

$$d(\alpha) \geq M(1 - \alpha).$$

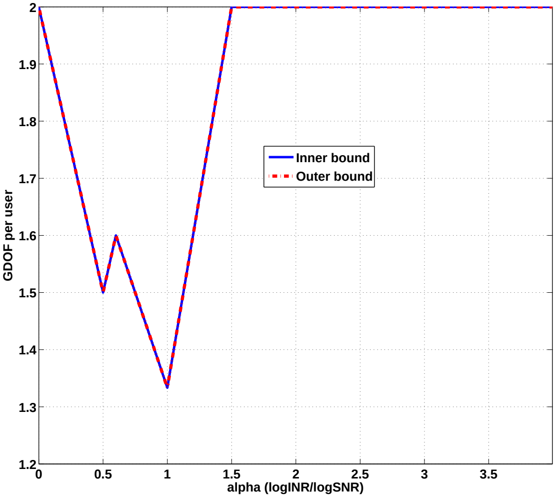
Achievable GDOF

- HK and IA schemes: different approaches to mitigate interference
- Still there exists similarity as well as difference between these two schemes
- Inner bound is obtained by taking maximum of various achievable schemes considered in this work
- Depending on α, K, M and N, performance of various schemes is characterized

GDOF plot: $K = 3$ user IC with $M = 2$ and $N = 2$



GDOF plot: $K = 3$ user IC with $M = 2$ and $N = 4$



Some insights

- Treating interference as noise is GDOF optimal when $M = N$ in weak interference case
- When $M < N$, splitting message into private and public part helps in weak interference regime
- When $K > \frac{N}{M} + 1$, a combination of IA and HK-scheme performs better
- When $\frac{N}{M} < K \leq \frac{N}{M} + 1$, HK-scheme is GDOF optimal
- Unlike two user IC, ZF-receiving is found to be GDOF optimal at $\alpha = 1$ when $\frac{N}{M} < K \leq \frac{N}{M} + 1$
- When channel is constant and $\frac{N}{M} < K \leq \frac{N}{M} + 1$, HK-scheme is also GDOF optimal

Summary

- Derived outer bound based on the notion of cooperation and providing noisy side information
- Derived achievable GDOF using a combination of HK-scheme, IA, treating interference as noise and ZF-receiving
- Explored the interplay between HK and IA

Interference alignment algorithms for the K-user constant MIMO interference channel

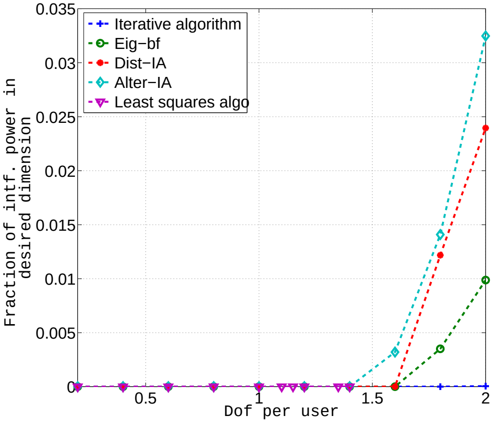
Problem considered

- In IA, precoders need to be designed such that
  1. Interference occupies less dimension at unintended receivers
  2. Desired message must remain linearly independent of the interference
- To preserve the desired signal dimension when channel is constant
- Need to design the precoders such that above conditions are satisfied

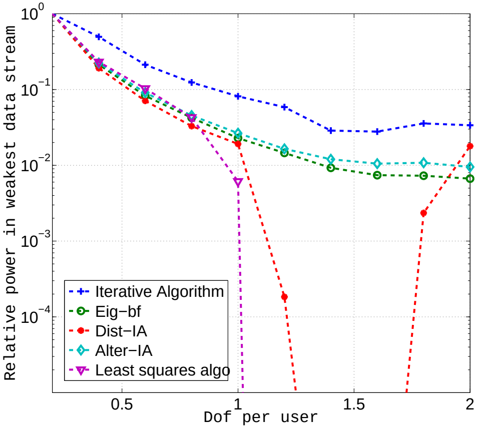
Contributions

- New performance metric: To account for possible loss in desired signal dimension
- Inspired by this metric two algorithms are proposed
  1. Iterative algorithm
  2. Eigen-beamforming algorithm
- Iterative algorithm: Tries to minimize the interference caused at each receivers along with an additional constraint on desired signal dimension
- Eigen-beamforming algorithm: Based on aligning the streams of interfering users (Non-iterative)
- Convergence to local minima is established in case of iterative algorithm
- Feasibility conditions are derived in case of Eigen-beamforming algorithm

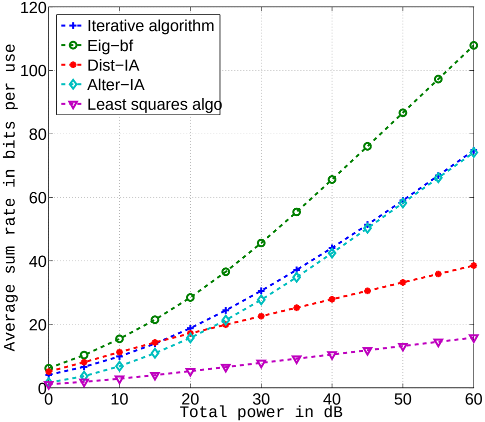
Fraction of Interference power in desired signal dimension: $K = 4$, $M = 3$, $N = 6$ and $S = 5$



Relative power in weakest desired data stream: $K = 4$, $M = 3$, $N = 6$ and $S = 5$



Average Sum rate: $K = 4$, $M = 3$, $N = 6$ and $S = 5$



Summary

- Performance metric that measures the interference suppression and DOF achieved by the algorithm were proposed
- Two numerical algorithms were proposed for designing the interference alignment precoding and receive filtering matrices
- Simulation results confirmed the superior performance of the proposed algorithms compared to existing methods

Three User Cognitive Radio Networks: An Information Theoretic Perspective

Overview of the work

- Goal: Establish the information theoretic performance limits of multi-user cognitive radio (CR) networks
- System model: 3-user CR network with 1 primary and 2 CRs (CR_1,CR_2) following cumulative message sharing (CMS), primary-only message sharing (PMS) and cognitive-only message sharing (CoMS)
- Coding scheme: CR_1 and CR_2 perform a combination of superposition & Gel'fand-Pinsker coding
- Rate region: Rate triple (R_1, R_2, R_3) is shown to be achievable for the 3-user CR channel with CMS, PMS and CoMS.
- Outer bound: Based on the notion of transmitter cooperation
- Results: Gaussian channel results in case of CMS

Channel schematic

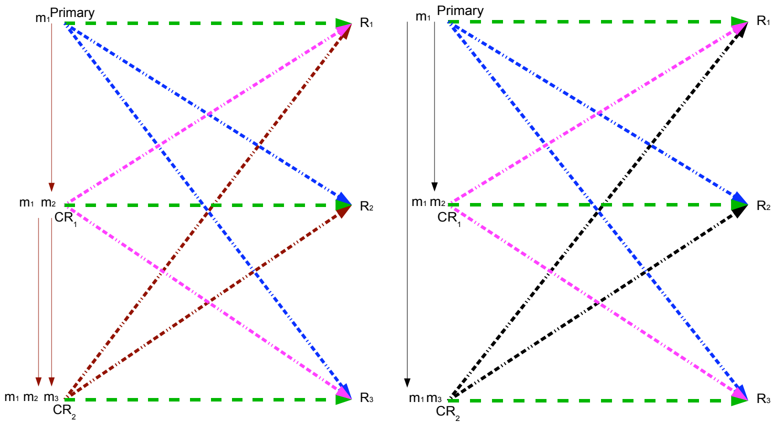


Figure: 3-user CR network with CMS (left) and PMS

Simulation results

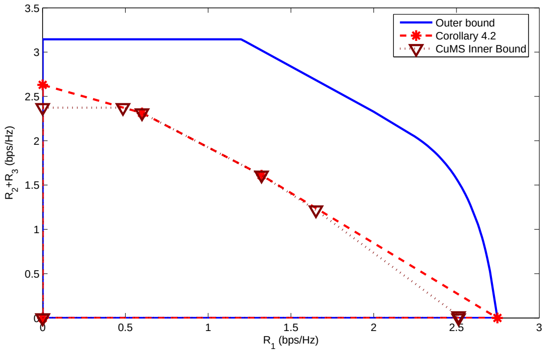


Figure: Rate of S_1 (R_1) versus the sum rate of S_2 and S_3 ($R_2 + R_3$) for the channel C^2_{CMS}

Simulation results Contd.

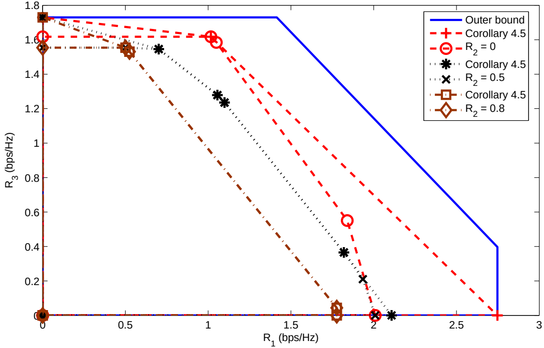


Figure: Rate of S_1 (R_1) versus the rate of S_3 (R_3) when S_2 is guaranteed to achieve a minimum rate $R_2 = 0, 0.5$ and 0.8 bps/Hz, for the channel C^2_{CMS}

Summary

- Defined a three-user cognitive interference channel and introduced cumulative message sharing, primary-only message sharing and cognitive-only message sharing
- Derived an achievable rate region by performing a combination of superposition and Gel'fand-Pinsker coding techniques, assuming different decoding capabilities at the receiver
- Derived an outer bound based on the notion of transmitter cooperation
- Illustrated results for the Gaussian channel

Future work

- GDOF of K-user MIMO Gaussian IC:
  - Proposing a scheme which combines IA and HK : deterministic model
  - To provide constant bit result: gap between the inner bound and outer bound
- Precoding algorithms for IA:
  - Interference alignment design with individual user power constraints
  - The performance of interference alignment with imperfect CSIT, or with imperfect IA
  - A study of the finite-SNR performance of the IA algorithms

Future work

- Three user CR network:
  - Extend the achievable scheme developed in case of 3 - user CR network to the case of 2 - user CR network with a cognitive relay
  - Develop outer bounds which take account of the decoding capability of the receivers
  - Characterize gap between the outer bound and inner bound

Publications

- Parthajit Mohapatra, K.E. Nissar and Chandra R. Murthy, "Interference Alignment Algorithms for the K-User Constant MIMO Interference Channel", accepted in IEEE transactions on signal processing
- Parthajit Mohapatra and Chandra R. Murthy, "On the Generalized Degrees of Freedom of the K-User Symmetric MIMO Gaussian Interference Channel", to be presented at ISIT 2011, St. Petersburg, Russia

Thank you!

Proof outline: strong interference case

- Due to the symmetry of the problem considered, it is sufficient to consider the GDOF achieved by any particular user
- Consider the user subset $S \subseteq \{2, \ldots, K\}$, and let $S' \triangleq S \cup \{1\}$, i.e., S is a subset of users excluding user 1
- Using the MAC channel formed at the receiver of user 1 with the signals from S, the achievable sum rate is bounded as

$$\begin{aligned}\sum_{j \in S} R_j &\leq \log |\mathbf{I} + \rho^{\alpha} \sum_{j \in S} \mathbf{H}_{1j} \mathbf{P}_j \mathbf{H}_{1j}^H| \\ &= \begin{cases} \alpha M \log \rho + \mathcal{O}(1), & |S| M \leq N, \\ N\alpha / |S| \log \rho + \mathcal{O}(1), & |S| M > N, \end{cases} \end{aligned} \quad (1)$$

Proof outline: strong interference case

- Using the MAC channel formed at the receiver of user 1 with the signals from the user set S':

$$\sum_{j \in S'} R_j \leq \log |\mathbf{I} + \rho \mathbf{H}_{11} \mathbf{P}_1 \mathbf{H}_{11}^H + \rho^\alpha \sum_{j \in S} \mathbf{H}_{1j} \mathbf{P}_j \mathbf{H}_{1j}^H|. \quad (2)$$

- Above equation is simplified further:
  1. When $N/M \leq |S| \leq K - 1$:

$$R_j \leq (\alpha N)/(|S| + 1) \log \rho + \mathcal{O}(1). \quad (3)$$

  2. When $N/M - 1 < |S| < N/M$:

$$R_j \leq \left[(\alpha - 1)M + \frac{N + M(1 - \alpha)}{|S| + 1} \right] \log \rho + \mathcal{O}(1). \quad (4)$$

  3. When $0 \leq |S| \leq N/M - 1$:

$$R_j \leq M(|S|\alpha + 1)/(|S| + 1) \log \rho + \mathcal{O}(1). \quad (5)$$

- Taking the minimum of (1) - (5) yields the desired result.