Journal Watch:

Transanctions on Signal Processing, May15, June-1, 2016

Saurabh Khanna,
Signal Processing for Communication, ECE, IISc

Generalized Interference Alignment - Part 1: Theoretical Framework

Authors: Lianzhong Ruan, Vincent Lau, Moe Win

- **Generalized interference alignment (GIA) problem** considers a very general setup comprising multiple transmitters, recievers, legitimate jammers and eavesdroppers.
  - Determine feasibility conditions for interference alignment
  - Design of transreceivers (precoder and decoder matrices)
- **System model:** Received signal at receiver-k:

$$\mathbf{y}_k^{(\iota)} = (\mathbf{U}_k^{(\iota)})^\dagger \left(\mathbf{H}_{kk}^{(\iota)} \mathbf{V}_k \mathbf{x}_k + \sum_{j=1,\neq k}^{K} \mathbf{H}_{kj}^{(\iota)} \mathbf{V}_j \mathbf{x}_j + \mathbf{z}_k^{(\iota)} \right)$$

- **GIA transceiver design problem:** Design $\{\mathbf{U}_k^{(\ell)}, \mathbf{V}_j\}$'s, such that

$$\mathrm{Rn}\left((\mathbf{U}_k^{(\ell)})^\dagger \mathbf{H}_{kk}^{(\ell)} \mathbf{V}_k \right) = d_k, \ \forall k \in \{1, 2, \ldots, K\},$$

$$\mathrm{Rn}(\mathbf{V}_j) = d_j, \ \forall j \in \{K+1, K+2, \ldots, K\}, \qquad \text{and } (\mathbf{U}_k^{(\ell)})^\dagger \mathbf{H}_{kj}^{(\ell)} \mathbf{V}_j = \mathbf{0}, \ \forall (k,j) \in \mathcal{A}$$

  Feasibily constraints in GIA are nonlinear and non-convex.
- *Feasibility of transceiver design problem $\equiv$ algebraic independence of GIA constraints $\equiv$ full rankness of associated Jacobian.*
- Using **algebraic geometry** concepts, shows that local and global optimums have no performance gap.

Optimal Joint Detection and Estimation Based on Decision-Dependent Bayesian Cost

Authors: Shang-Li and Xiaodong Wang, Columbia Univ.

- **Joint detection and estimation problem**:

$$\begin{aligned} \mathcal{H}_0 : \quad & \mathbf{y} \sim f_0(\mathbf{y}|\theta_0), \quad && \text{with } \theta_0 \sim \pi_0(\theta_0) \\ \mathcal{H}_1 : \quad & \mathbf{y} \sim f_1(\mathbf{y}|\theta_1), \quad && \text{with } \theta_1 \sim \pi_1(\theta_1) \end{aligned}$$

  $\mathbf{y}$ are observations and $\theta_{0/1}$ are unknown parameters under hypothesis $\mathcal{H}_{0/1}$.

- **Goal**: Decide between $\mathcal{H}_0$ and $\mathcal{H}_1$, and at the same time, also estimate the unknown parameter θ.

- **Coupling** between the underlying estimation and detection problems:
  - The quality of parameter estimate depends upon the correctness of selected hypothesis
  - Estimate of parameter helps in deciding between the competing hypothesis

- **Naive approaches:**
  - Composite hypothesis testing followed by MAP estimation.
  - Neyman-Pearson formulation: Minimize the estimation cost subject to constraint on detection performance.

Optimal Joint Detection and Estimation Based on Decision-Dependent Bayesian Cost

- **Optimal joint detection and estimation**:
  A Bayes estimation cost function is proposed:

$$\mathcal{C}(\hat{\theta}_0, \hat{\theta}_1, \delta) = \sum_{i,j \in \{0,1\}} c_{i,j} P_{i,j}(\delta = j) \times \mathbb{E}_i \left(||\hat{\theta}_{c,j} - \theta_c||^2 + ||\hat{\theta}_{s,j} - \theta_{s,i}||^2 \cdot \mathbb{I}_{\{j=i\}} \mid \delta = j \right)$$

  where δ is the detection output.

- If prior on hypothesis is <u>not available</u>:

$$\underset{\hat{\theta}_0, \hat{\theta}_1, \delta}{\text{minimize}} \; \mathcal{C}(\hat{\theta}_0, \hat{\theta}_1, \delta)$$
$$\text{subject to } \mathbb{P}_0(\delta = 1) \leq \alpha, \quad \mathbb{P}_1(\delta = 0) \leq \beta$$

- If prior on hypothesis is <u>available</u>:

$$\underset{\hat{\theta}_0, \hat{\theta}_1, \delta}{\text{minimize}} \; \mathcal{C}(\hat{\theta}_0, \hat{\theta}_1, \delta)$$
$$\text{subject to } \mathbb{P}(H_0)\mathbb{P}_0(\delta = 1) + \mathbb{P}(H_1)\mathbb{P}_1(\delta = 0) \leq \gamma$$

- Closed form solution for above problems is provided.

- Extension to multiple hypothesis is also discussed.

Design and Analysis of a Greedy Pursuit for Distributed Compressed Sensing

Authors: Dennis Sundman, Saikat Chatterjee, Mikael Skoglund, KTH Sweden

- **Distributed Compressed Sensing:**
  Estimate $\mathbf{x}_j$ from $\mathbf{y}_j$ in distributed manner.

$$\mathbf{y}_j = \mathbf{A}_j \mathbf{x}_j + \mathbf{e}_j \qquad j = 1, 2, \ldots, L.$$

- **Mixed support set model**: Each $\mathbf{x}_j$ can be decomposed as: $\mathbf{x}_j = \mathbf{x}_j^C + \mathbf{z}_j$.
  - $\mathbf{x}_1^C, \mathbf{x}_2^C, \ldots, \mathbf{x}_L^C$ share a common sparse support.
  - $\mathbf{z}_j$ is sparse local innovation component.
- **Distributed Parallel Pursuit (DIPP)**: Distributed Co-SAMP type algorithm with Co-SAMP inspired fusion of common support estimates across nodes.
- **Main steps in single iteration of DIPP**:
  - Generate local support estimate by CoSamp like update
  - Exchange current support estimates with neighboring nodes
  - Apply majority rule to obtain external support estimate
  - Refine local support by using external support estimate, once again using CoSamp like update
- RIP constant based stable signal recovery guarantees under measurement noise is provided.

Analytical Derivation of the Inverse Moments of One-Sided Correlated Gram Matrices With Aplications

Authors: K. Elkhalil, A. Kammoun, T. Y. Al Naffouri, and M. S Alouini

- **Moments of one-sided correlated gram matrices**
  - Let $\mathbf{H}$ be $n \times m$ random matrix with i.i.d zero-mean unit variance complex Gaussian random entries.
  - Let Λ be deterministic PSD matrix with distinct eigenvalues.
  - Then, consider the Gram matrix $\mathbf{S}$ as

$$\mathbf{S} = \mathbf{H}^* \Lambda \mathbf{H}.$$

  - The r^{th} moments $\mu_\Lambda(r)$ is defined as:

$$\mu_\Lambda(r) \triangleq \frac{1}{m} trace \left(\mathbb{E}_{\mathbf{H}} (\mathbf{S}^r) \right).$$

- **Application-1**: BLUE estimation error in ($\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e}$) problem can be rewritten in terms of inverse moments.

$$\mathbb{E}_{\mathbf{H}} ||\hat{\mathbf{x}}_{\text{blue}} - \mathbf{x}||^2 = \mathbb{E}_{\mathbf{H}} tr(\mathbf{H}^* \Sigma_{\text{noise}}^{-1} \mathbf{H})^{-1} = m \mu_{\Sigma_{\text{noise}}^{-1}}(-1).$$

Analytical Derivation of the Inverse Moments of One-Sided Correlated Gram Matrices With Aplications

Authors: K. Elkhalil, A. Kammoun, T. Y. Al Naffouri, and M. S Alouini

- **Application-2**: Average estimation error in LMMSE for ($\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e}$) problem:

$$\mathbb{E}_{\mathbf{H}}||\hat{\mathbf{x}}_{\text{lmmse}} - \mathbf{x}||^2 = \mathbb{E}_{\mathbf{H}} tr(\Sigma_{\mathbf{x}}^{-1} + \mathbf{H}^* \Sigma_{\text{noise}}^{-1} \mathbf{H})^{-1}.$$

  - High SNR regime

$$\mathbb{E}_{\mathbf{H}}\{||\hat{\mathbf{x}}_{\mathrm{lmmse}} - \mathbf{x}||^2\} = m \sum_{k=0}^{l} \frac{(-1)^k}{\sigma_x^{2k}} \mu_{\Sigma_{\text{noise}}^{-1}}(-k-1) + o\left(\sigma_x^{-2r}\right)$$

  where $l \leq p - 1$ with $p = \min(m, n - m)$.

  - Low SNR regime

$$\mathbb{E}_{\mathbf{H}}\{||\mathbf{x}_{\mathrm{lmmse}} - \mathbf{x}||^2\} = m \sum_{k=0}^{\infty} (-1)^k \sigma_x^{2k+2} \mu_{\Sigma_{\text{noise}}(k)}.$$

Analytical Derivation of the Inverse Moments of One-Sided Correlated Gram Matrices With Aplications

Authors: K. Elkhalil, A. Kammoun, T. Y. Al Naffouri, and M. S Alouini

- **Application-3**: Accuracy of sample covariance matrix $\hat{\mathbf{S}}$:

$$\text{Loss} = \mathbb{E}||\mathbf{R}^{1/2}\hat{\mathbf{S}}^{-1}\mathbf{R}^{1/2} - \mathbf{I}||_F^2$$

- Once again, the Loss can be written in terms of inverse moments of sample covariance matrix $\hat{\mathbf{S}}$.

- **Stieltjes Transform**: For a Hermitian matrix $\mathbf{A}$, its Stieltjes transform is given by:

$$m_{\mathbf{A}}(z) \triangleq \int \frac{1}{\lambda - z} dF^{\mathbf{A}}(\lambda) = \frac{1}{m}\mathrm{tr}\left(\mathbf{A} - z\mathbf{I}_m\right)^{-1}.$$

  where $F^{\mathbf{A}}(.)$ is empirical spectral distribution of $\mathbf{A}$.

- **Main result**: The r^{th} inverse moment can be written in terms of derivative of the Stieltjes transform.

Other Interesting Papers:

- Sequence Set Design With Good Correlation Properties Via Majorization-Minimization
- A General Design Framework for MIMO Wireless Energy Transfer With Limited Feedback
- Massive MIMO for Decentralized Estimation of a Correlated Source
- Bayes-Optimal Joint Channel-and-Data Estimation for Massive MIMO With Low-Precision ADCs
- Uplink Downlink Rate Balancing and Throughput Scaling in FDD Massive MIMO Systems
- Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems
- Decentralized Sum Rate Maximization With QoS Constraints for Interfering Broadcast Channel Via Successive Convex Approximation
- SINR Constrained Beamforming for a MIMO Multi-User Downlink System: Algorithms and Convergence Analysis
- Robust Pilot Decontamination Based on Joint Angle and Power Domain Discrimination