Packet Scheduling for Priority Based Transmission in Energy Harvesting Sensors

Chandra R Murthy[1], Joseph Joseph Cherukara[2]

[1]Department of Electrical Communication Engineering
Indian Institute of Science Bangalore, India

[2]Department of Electrical Engineering
Indian Institute of Technology Madras, Chennai, India

Outline

- Introduction
- The Problem
- The Model
- Analysis
- Form of the optimal solution
- Subdivisions of the problem
- Final notes
- Simulations
- Conclusions

Introduction

- Energy Harvesting Sensors (EHS) absorb energy (solar, vibrational, etc.) from the environment to power themselves
- Advantages:
  - Do not require cables for power/data - can be placed in inaccessible places
  - Infrequent maintenance and longer lifetimes
  - More environmental friendly
- Disadvantages:
  - Dependence on often unpredictable external sources of energy
  - Low quanta of energy available
- Proposed applications:
  - Monitoring the structural integrity of bridges and buildings
  - Body area networks
  - Environmental studies
  - Intrusion detection

The Problem

Definition

If the incoming packets are of different priorities, how do we schedule transmission for maximum benefit?

The Model

- The Packets:
  - Two priorities: h and l
  - Every time slot T_s, the source sends/creates a packet of high (low) priority to the node with probability varying as a Bernoulli distribution with parameter q_h (q_l)
  - Stored in infinite FIFO buffers b_h and b_l
- Rayleigh fading with CSI obtained by pilot signals each T_s
- Energy harvested per interval ρT_s
- Single MCS - Constant on rate of transmission
- Quality of Service (QOS): Weighted sum of expected waiting time $w_h t_h + w_l t_l$

Analysis

- With MCS such that one packet is transmitted per interval,
  - Let $p(h|\gamma, Q)$,$p(l|\gamma, Q)$, $p(0|\gamma, Q)$ be probabilities that a packet from b_h, b_l, no packet is transmitted, Q is the set of queues with packets in them, γ is channel gain
  - $Pr\{\text{tx from } b_j | b_j \neq \phi\} = P_j = \sum_{Q:j \in Q} \frac{Pr\{Q\}}{Pr\{j\}} \int_0^\infty p(j|\gamma, Q) e^{-\gamma} d\gamma \; ;(j = h, l \;)$
  - Expected power consumed is $\rho_0 \sum_Q Pr\{Q\} \sum_{j=h,l} \int_0^\infty p(j|\gamma, Q) \frac{e^{-\gamma}}{\gamma} d\gamma;$ ρ_0- Power required when $\gamma = 1$
- Each queue b_j can now be modelled as a Markov chain and by Little's law, expected waiting time becomes $t_j = \frac{1-P_j}{P_j - q_j} T_s$

Analysis

The problem we need to solve therefore becomes:

$$\min_{p(j|\gamma,Q)} w_h \frac{1-P_h}{P_h-q_h} T_s + w_l \frac{1-P_l}{P_l-q_l} T_s$$

$$\text{subject to } \sum_Q Pr\{Q\} \sum_{j=h,l} \int_0^{\infty} p(j|\gamma, Q) \frac{e^{-\gamma}}{\gamma} d\gamma \leq \frac{\rho T_s}{\rho_0}$$

Analysis

Decoupling Approximation:

- We make the approximation that

$$
\begin{aligned}
Pr\{Q\} &\approx \prod_{j:j\in Q} Pr\{b_j \neq \phi\} \prod_{j:j\notin Q} Pr\{b_j = \phi\} \\
&= \prod_{j:j\in Q} \frac{q_j}{P_j} \prod_{j:j\notin Q} \frac{P_j - q_j}{P_j}
\end{aligned}
$$

- High priority messages are typically associated with danger or variables out of their usual range. So we expect $q_h \ll q_l$, and hence probability of b_l being empty does not depend much on b_h(and vice versa)

Form of the Optimal solution

Consider any non-zero $p(i|\gamma, Q)$, $(i \neq 0)$ of any feasible solution. Consider $0 < \gamma_k < \gamma_j$ such that $p(i|\gamma_k, Q) > 0$ and $p(0|\gamma_j, Q) > 0$. We construct a new solution $p'(i|\gamma, Q)$ [and $p'(0|\gamma, Q) = p(0|\gamma, Q) + p(i|\gamma, Q) - p'(i|\gamma, Q)$]

$$p'(i|\gamma, Q) = \begin{cases} p(i|\gamma, Q) & \gamma \neq \gamma_j, \gamma_k \\ p(i|\gamma_j, Q) + \epsilon & \gamma = \gamma_j \\ p(i|\gamma_k, Q) - \epsilon e^{-\gamma_j + \gamma_k} & \gamma = \gamma_k \end{cases}$$

where $\epsilon = min(p(0|\gamma_j, Q), p(i|\gamma_j)e^{-\gamma_k + \gamma_j})$. The difference in energy consumed by old and new solutions is proportional to

$$\rho_0 Pr\{Q\}[-\epsilon \frac{e^{-\gamma_j}}{\gamma_j} + \epsilon \frac{e^{-\gamma_j}}{\gamma_k}] \geq 0$$

Truncated Channel Inversion (TCI) and the standard form

- This solution does as well as the old but uses less energy.
- By proof by contradiction, for a given performance, the solution which uses least energy has $p(0|\gamma, Q) = 0 \forall \gamma > \gamma_{min}$, 1 otherwise. This is a form of *Truncated Channel Inversion* .
- Noting that energy consumed depends on only γ_{min} and performance only on $\int_{\gamma_{min}}^{\infty} p(i|\gamma, Q) e^{-\gamma} d\gamma$, we can write $p(i|\gamma, Q) = \chi_i \forall \gamma > \gamma_{min}$.
- We call this the **standard form of the solution.**
- Let the variables we optimize over be $(\gamma_{hl}, \chi_h, \chi_l = 1 - \chi_h)$ for $Q = \{h, l\}$, γ_h for $Q = \{h\}$ and γ_l for $Q = \{l\}$.

Equating thresholds

- Consider a solution $(\gamma_h, \gamma_l, \gamma_{hl}, \chi_h, \chi_l)$ with $\gamma_h < \gamma_{hl}$. We can always define γ'_h such that:

$$(1 - Pr\{l\})e^{-\gamma_h} + Pr\{l\}e^{-\gamma_{hl}} = e^{-\gamma'_h}$$

- If we use γ'_h as the threshold when $Q = \{h\}$, transmit only h packets when $Q = \{h, l\}$ and $\gamma'_h < \gamma < \gamma_{hl}$ and follow the original solution for other values of Q, γ , we can show that this new solution uses less energy while delivering the same performance.
- It is not in the standard form, but can be standardized to give a new solution $(\gamma'_h, \gamma_l, \gamma'_h, \chi'_h, \chi'_l)$

Equating thresholds

- We can use similar arguments for other values of γ_h, γ_l, γ_{hl}.
- Intuitively, we can understand these arguments by noting that it takes less energy to transmit at higher γs
- If one threshold is less than the other, we try to "shift" the probability of transmission from the first queue state to the second, weighted by the probability of the states occurring.
- If we apply such arguments iteratively, the solution will converge to one of these 3 forms:
  - $\gamma_{hl} = \gamma_h = \gamma_l$, $0 \leq \chi_h, \chi_l \leq 1$
  - $\gamma_{hl} = \gamma_h \leq \gamma_l$, $\chi_l = 0$
  - $\gamma_{hl} = \gamma_l \leq \gamma_h$, $\chi_h = 0$
- The last two (symmetrical) cases occur when we can't "shift" probability from one queue state to the other since the probability of transmission is 0 in the first state.

$\gamma_{hl} = \gamma_h = \gamma_l$

The optimization problem becomes:

$$\min_{P_h, P_l} w_h \frac{1 - P_h}{P_h - q_h} T_s + w_l \frac{1 - P_l}{P_l - q_l} T_s$$

subject to

$$\begin{aligned} P_h &= e^{-\gamma_h}(1 - \chi_l \frac{q_l}{P_l}) \\ P_l &= e^{-\gamma_h}(1 - \chi_h \frac{q_h}{P_h}) \\ 1 &= \chi_h + \chi_l \end{aligned}$$

$$P_h > q_h,\ P_l > q_l,\ 0 \leq \chi_h, \chi_l \leq 1$$

where $\gamma_h = f^{-1}(\frac{\rho T_S}{\rho_0(q_h+q_l)})$, $f(\gamma) = e^{\gamma} E_i(\gamma)$, $E_i(\gamma) = \int_{\gamma}^{\infty} \frac{e^{-\gamma}}{\gamma} d\gamma$

$\gamma_{hl} = \gamma_h = \gamma_l$

We can rewrite the constraints by writing P_l in the form of a quadratic: $P_l^2 + P_l(\chi_h q_h - \chi_l q_l - e^{-\gamma_h}) + e^{-\gamma_h}\chi_l q_l = 0$. Imposing $P_l > q_l$ and $P_h > q_h$, we can show that P_l will be the greater root of the above equation and even if it is written as a less than or equal to constraint, at optimum it is as an equality. So:

$$
\begin{aligned}
P_l^2 + P_l(\chi_h q_h - \chi_l q_l - e^{-\gamma_h}) + e^{-\gamma_h}\chi_l q_l &\leq 0 \\
\frac{-\chi_h q_h + \chi_l q_l + e^{-\gamma_h}}{2} &\leq P_l \\
\chi_h q_h - \chi_l q_l &= P_h - P_l
\end{aligned}
$$

$$\chi_h + \chi_l = 1,\ P_h > q_h,\ P_l > q_l,\ 0 \leq \chi_h, \chi_l \leq 1$$

This problem is convex and can be solved by numerical methods.

$\gamma_l = \gamma_{hl} < \gamma_h, \; \chi_h = 0$

We have $P_l = e^{-\gamma_l}$, $P_h = e^{-\gamma_h}(1 - q_l e^{\gamma_l})$ and we have to:

$$\min_{\gamma_h, \gamma_l} w_h \frac{1 - e^{-\gamma_h}(1 - q_l e^{\gamma_l})}{e^{-\gamma_h}(1 - q_l e^{\gamma_l}) - q_h} T_s + w_l \frac{1 - e^{-\gamma_l}}{e^{-\gamma_l} - q_l} T_s$$

subject to

$$\begin{aligned}
q_h e^{\gamma_h} E_i(\gamma_h) + q_l e^{\gamma_l} E_i(\gamma_l) &= \frac{\rho T_S}{\rho_0} \\
\gamma_l &< log(\frac{1}{q_l}) \\
q_h e^{\gamma_h} + q_l e^{\gamma_l} - 1 &< 0 \\
\gamma_h, \gamma_l &> 0
\end{aligned}$$

This problem is convex and can be solved by numerical methods.

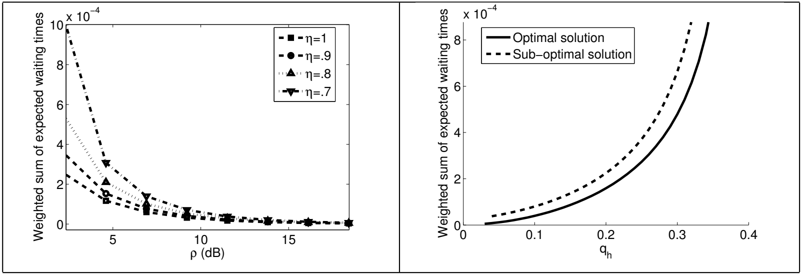
Final notes

- It is not necessary to obtain solutions for all the sub-problems.
- Since each sub-problem is convex and for the objective function L, $\frac{dL}{dP_l}$ is continuous across sub-problems, it is sufficient to evaluate $\frac{dL}{dP_L}$ at $\chi_h = 0$ and $\chi_l = 0$ when $\gamma_h = \gamma_{hl} = \gamma_l$ to find which contains the optimal solution.
- Considering a non-ideal battery (efficiency η) used by the node when $\rho T_s < \frac{\rho_0}{\gamma}$, we can derive similar solutions by replacing $\rho_0 E_i(\gamma)$ by $\int_{\gamma}^{\infty} e^{-\gamma} g(\gamma) d\gamma$ where:

$$g(\gamma) = \begin{cases} \frac{\rho_0}{\gamma} & \gamma > \gamma_p \\ \rho T_s + \frac{1}{\eta}(\frac{\rho_0}{\gamma} - \rho T_s) & \gamma \leq \gamma_p \end{cases}$$

and $\gamma_p = \frac{\rho_0}{\rho T_s}$

Simulations



Conclusions

- Proposed an energy allocation problem in EHS with packets of different priorities
- Under the decoupling approximation, the optimal solution is of the form of Truncated Channel Inversion
- Demonstrated how to find this optimal solution
- Future work: Relax the Decoupling Approximation, Other profiles of power harvesting and packet arrival