Journal watch
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- A vector $x \in \mathbb{R}^M$ is $k-$**sparse** if it has k nonzero coordinates. That is, $\|x\|_0 := |\{i \mid x_i \neq 0\}| = k < M$
- One of the central problems in CS is that of reconstructing an unknown sparse vector $x \in \mathbb{R}^M$ from the linear measurements $y = (\langle x, \phi_1 \rangle, \ldots, \langle x, \phi_M \rangle) \in \mathbb{R}^m$
- There are many greedy algorithms to solve the above problem. Among all, OMP is most popular algorithm
- Tropp and Gilbert showed that when the measurement matrix Φ is generated i.i.d. at random, and the measurement size is on the order of $K \log M$, OMP ensures the accurate recovery of every fixed K-sparse signal with overwhelming probability
- Davenport and Wakin showed that OMP ensures exact reconstruction of any $K-$sparse signal under $\delta_{k+1} < \frac{1}{3\sqrt{k}}$

- Recently, it has been shown by Zhang that OMP recovers any $K-$sparse signal with $30K$ iterations under $\delta_{31k} < \frac{1}{3}$

Theorem

Let $x \in \mathbb{R}^M$ be any $K-$sparse signal and let Φ be the measurement matrix satisfying the RIP of order $\lfloor (c+1)K \rfloor$. Then if c satisfies

$$c \geq -\frac{4(1+\delta)}{1-\delta} \log\left(\frac{1}{2} - \sqrt{\frac{\delta}{2+2\delta}}\right), \textit{where } \delta = \delta_{\lfloor (c+1)K \rfloor},$$

OMP_{ck} perfectly recovers the signal x from the measurements $y = \Phi x$.

- it requires $30K$ iterations to recover $K-$sparse signals with $\delta_{31k} = \frac{1}{3}$ in Zhang's result. Whereas, it requires only $\lceil 15.4K \rceil$ iterations in the present result
- With high probability, OMP_{ck} can recover $K-$sparse signals in $\lceil 2.8K \rceil$ iterations when the number of random Gaussian measurements is on the order of $k \log(\frac{M}{k})$

- A vector $x = [x_1^T, x_2^T, \ldots, x_p^T]^T \in \mathbb{R}^{np}$, with i^{th} block $x_i \in \mathbb{R}^n$, is called block $k-$sparse if x_i has nonzero Euclidean norm for at most k indices i
- Block-Sparse Basis Pursuit (BS-BP):

$$\min_{z \in \mathbb{R}^{np}} \|z\|_{b1} \ s.t. \ \|y - Az\|_2 \leq \epsilon$$

- Block-Sparse Dantzig selector (BS-DS):

$$\min_{z \in \mathbb{R}^{np}} \|z\|_{b1} \ s.t. \ \|A^T(y - Az)\|_{b\infty} \leq \mu$$

- Block-Sparse LASSO estimator (BS-LASSO):

$$\min_{z \in \mathbb{R}^{np}} \frac{1}{2}\|y - Az\|_2^2 + \mu\|z\|_{b1}$$

Definition

For $s \in [1, p]$ and matrix $A \in \mathbb{R}^{m \times np}$, define

$$w_\gamma(Q, s) = \min_{z:\ \|z\|_{b1}/\|z\|_{b\infty} \le s} \frac{\|Qz\|_\gamma}{\|z\|_{b\infty}},\ \text{where } Q \text{ is either } A \text{ or } A^T A.$$

Theorem

Suppose x is $k-$block sparse satisfying $y = Ax + w$, and the noise w satisfies $\|w\|_2 \le \epsilon$, $\|A^T w\|_{b\infty} \le \mu$, and $\|A^T w\|_\infty \le k\mu, k \in (0, 1)$, for the BS-BP, the BS-DS, and the BS-LASSO, respectively. We have,

$$\|\hat{x} - x\|_{b\infty} \le \frac{2\epsilon}{w_2(A, 2k)}, \quad \text{for the BS-BP,}$$

$$\|\hat{x} - x\|_{b\infty} \le \frac{2\mu}{w_{b\infty}(A^T A, 2k)}, \quad \text{for the BS-DS, and,}$$

$$\|\hat{x} - x\|_{b\infty} \le \frac{(1+k)\mu}{w_{b\infty}(A^T A, \frac{2k}{1-k})}, \quad \text{for the BS-LASSO.}$$

On Sparse Vector Recovery Performance in Structurally Orthogonal Matrices via LASSO

Chao-Kai Wen, Jun Zhang, Kai-Kit Wong, Jung-Chieh Chen and Chau Yuen

- The authors have considered the following sparse recovery problem in the case of noisy measurements:

$$y = \Phi x + \sigma w$$

  where w is assumed to be the standard complex Gaussian noise vector and σ is a noise magnitude
- The measurement matrix was constructed by concatenating several randomly orthogonal bases, which they refer to as structurally orthogonal matrices
- The following LASSO algorithm used for signal estimation:

$$x' = \operatorname{argmin}_{x \in \mathbb{C}^{\mathbb{M}}} \left\{ \frac{1}{\lambda} \|y - \Phi x\|_2^2 + \|x\|_1 \right\}$$

- Analytically they proved that the structurally orthogonal matrices are at least as good as their i.i.d. Gaussian counterparts

Distributed Compressive Sensing: A Deep Learning Approach

Hamid Palangi, Rabab Ward and Li Deng

- One of the central problem in CS is finding the sparse solution vectors for multiple measurement vectors (MMV)
- In this paper, the authors relaxed the joint sparsity condition and assumed that the sparse vectors are depend on each other, but this dependency is assumed unknown
- They proposed the two step greed reconstruction algorithm for finding the dependencies between the sparse vectors and update their nonzero locations
- They showed that the proposed method significantly outperforms the general MMV solver (the Simultaneous Orthogonal Matching Pursuit (SOMP)) and a number of the model-based Bayesian methods
- The proposed method is a data-driven method, it is only applicable when training data is available

Other Interesting Papers

- Deterministic Cram er-Rao Bound for Strictly Non-Circular Sources and Analytical Analysis of the Achievable Gains by J. Steinwandt, et.al.
- New Sparse-Promoting Prior for the Estimation of a Radar Scene with Weak and Strong Targets by M. Lasserre, et.al.
- Bayesian Learning of Degenerate Linear Gaussian State Space Models Using Markov Chain Monte Carlo by P. Bunch, et.al.

Thank you