

# Journal Watch

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# Data Recovery and Subspace Clustering from Quantized and Corrupted Measurements - Ren Wang et al

## Goal

- To solve the problem of data recovery and data clustering from quantized and partially corrupted measurements when the data satisfies the union of subspaces (UoS) model

## Contributions

- Sparse alternative proximal algorithm (Sparse-APA) to solve the nonconvex constrained maximum log-likelihood problem
- Theoretical analysis of the proposed data recovery method

$$Y = \mathcal{Q} \left( \begin{array}{c} L^* \\ E^* \\ N \end{array} \right)$$

Multiple low dimensional subspaces      Sparse      Independent noise

## Problem Statement

- Given observations  $Y$ , known boundaries  $\omega_0 < \omega_1 < \dots < \omega_K$  and noise distribution  $\Phi$ , recover  $L^*$  and cluster the data into the corresponding subspaces  $S_i$ 's simultaneously

- Nonconvex optimization problem

$$\min_{L, E \in \mathbb{R}^{m \times n}, C \in \mathbb{R}^{n \times n}} - \sum_{i=1}^m \sum_{j=1}^n \sum_{l=1}^K \mathbb{1}_{[Y_{ij}=l]} \log(f_l(L_{ij} + E_{ij})) \quad (1)$$

$$\text{s.t. } (L, E, C) \in S_f \quad (2)$$

where  $S_f = \{(L, E, C) : L = LC, \text{rank}(L) \leq r, \|L\|_\infty \leq \alpha_1, \|E\|_\infty \leq \alpha_2, \|E\|_0 \leq s, \|c_i\|_0 \leq d, C_{ii} = 0, \forall i \in [n]\}$  and  $f_l(X_{ij}) = P(Y_{ij} = l | X_{ij}) = \Phi(\omega_l - X_{ij}) - \Phi(\omega_{l-1} - X_{ij})$

- Proximal map associated to  $\kappa$  is defined as

$$\text{prox}^\kappa(x) = \arg \min_u \left\{ \kappa(u) + \frac{1}{2} \|u - x\|_F^2 \right\}$$

- Proximal gradient method applied to get the solution
  - Revised objective function is Lipschitz differentiable
  - Sum of the objective function and other functions in the problem (refer the paper for exact details) satisfies the Kurdyka-Lojasiewicz (KL) property
  - Global convergence to a critical point
- Spectral clustering to obtain the final group labels
- Theoretical analyses of the algorithms
  - Probabilistic guarantees

# Nonparametric Composite Hypothesis Testing in an Asymptotic Regime - Qunwei Li et al

## Goal

- To investigate the nonparametric, composite hypothesis testing (NP-CHT) problem for arbitrary unknown distributions in the asymptotic regime

## Contributions

- Asymptotic viewpoint to understand the NP-CHT problem when the number of hypotheses scales exponentially in the sample size
- Maximum mean discrepancy (MMD) and Kolmogorov-Smirnov (KS) distance approaches to solve the NP-CHT problem
  - Derive the error exponents and the discrimination rates (analogous to channel coding rate)
  - Achievability and converse proofs

## Problem Statement

- $M$  hypotheses and each hypothesis corresponds to a set  $\mathcal{P}_m$  of unknown distributions  $\{p_{m,1}, \dots, p_{m,M_m}\}$  for  $m = 1, 2, \dots, M$
- For each set  $m$ , we observe training sequences  $\mathbf{X}_m = [\mathbf{x}_{m,1}, \dots, \mathbf{x}_{m,M_m}] \in \mathbb{R}^{n \times M_m}$
- Determine the hypothesis that the test sequence  $\mathbf{y}$  belongs to
- Regime when  $M = 2^{nD}$ , where  $D$  is the discrimination rate

## Connection to the Channel Coding Problem

- Total number of hypotheses corresponds to the total number of messages in channel coding
- Discrimination rate  $D$  corresponds to the channel coding rate  $R$

## Main Results

- MMD-Based Test

$$\delta_{MMD} \left( \{\mathbf{X}_m\}_{m=1}^M, \mathbf{y} \right) = \arg \min_{m, i_m} \text{MMD}^2 (\mathbf{x}_{m, i_m}, \mathbf{y}) \quad (3)$$

where

$$\text{MMD}^2(\mathbf{x}, \mathbf{y}) = \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j \neq i}^n k(x_i, x_j) + \frac{1}{m(m-1)} \sum_{i=1}^m \sum_{j \neq i}^m k(y_i, y_j) - \frac{2}{nm} \sum_{i=1}^n \sum_{j=1}^m k(x_i, y_j)$$

## Theorem

Suppose the MMD-based test in (3) is applied to the NP-CHT problem under an assumption (refer (2) in the paper), where the kernel satisfies  $0 \leq k(x, y) \leq \mathcal{K}$  for all  $(x, y)$ . Then the average probability of error is upper bounded as

$$P_e \leq 2^{nD} \exp \left( -\frac{n(D_O - D_I)^2}{96\mathcal{K}^2} \right). \quad (4)$$

Thus, the achievable discrimination rate is

$$D = \frac{\log e}{96\mathcal{K}^2} \liminf_{M \rightarrow \infty} \min_{1 \leq i, j \leq M} \text{MMD}^4(p_i, p_j) \quad (5)$$

- Kolmogorov-Smirnov (KS) Test

$$\delta \left( \{\mathbf{X}\}_{m=1}^M, \mathbf{y} \right) = \arg \min_{m, i_m} D_{KS}(\mathbf{x}_{m, i_m}, \mathbf{y}) \quad (6)$$

where  $D_{KS}(\mathbf{x}, \mathbf{y}) = \sup_{a \in \mathbb{R}} |F_{\mathbf{x}}(a) - F_{\mathbf{y}}(a)|$ , and  $F_{\mathbf{x}}(a) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{[-\infty, a]}(x_i)$

### Theorem

Suppose the KS-based test in (6) is applied to the NP-CHT problem under an assumption (refer (2) in the paper), where the kernel satisfies  $0 \leq k(x, y) \leq \mathcal{K}$  for all  $(x, y)$ . Then the average probability of error is upper bounded as

$$P_e \leq 6 \times 2^{nD} \exp \left( -\frac{n(D_O - D_I)^2}{8} \right). \quad (7)$$

Thus, the achievable discrimination rate is

$$D = \frac{\log e}{8} (D_O - D_I)^2 \quad (8)$$

- Converse proof on the discrimination capacity
  - Based on Fano's inequality

# Joint Channel Estimation and Multiuser Detection for Uplink Grant-Free NOMA - Yang Du et al

## Goal

- Joint channel estimation (CE) and multiuser detection (MUD) in a frame based multiuser transmission scenario

## Contributions

- Proposed a novel joint active user detection (AUD), CE, and MUD framework for UL grant free NOMA systems
- Multiple measurement vector-compressive sensing (MMV-CS) problem transferred to a block sparse single measurement vector compressive sensing (BS-SMV-CS) problem
- Block sparsity adaptive subspace pursuit (BSASP) algorithm is proposed to solve the above problem

## System Model

- $K$  single antenna UEs out of which sparse set of UEs are active,  $N$  antenna BS

$$\mathbf{y}_p = \sum_{k \in \Gamma} h_k \mathbf{a}_k x_{p,k} + \mathbf{n}_p = \mathbf{A} \mathbf{h} + \mathbf{n}_p, \quad (9)$$

$$\mathbf{y}_d^{[j]} = \sum_{k \in \Gamma} h_k \mathbf{a}_k x_{d,k}^{[j]} + \mathbf{n}_d^{[j]} = \mathbf{A} \text{diag}(\mathbf{h}) \mathbf{x}_d^{[j]} + \mathbf{n}_d^{[j]}. \quad (10)$$

## Problem Formulation

- Channel and data share the same support

$$\mathbf{y}_d^{[j]} = \mathbf{A}\mathbf{s}_d^{[j]} + \mathbf{n}_d^{[j]}. \quad (11)$$

- Received pilot and data are combined as

$$\begin{bmatrix} \mathbf{y}_p \\ \mathbf{y}_d^{[j]} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{h} \\ \mathbf{s}_d^{[j]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_p \\ \mathbf{n}_d^{[j]} \end{bmatrix}, j = 1, 2, \dots, J, \quad (12)$$

- The received signal matrix from (12) is vectorized to form a BS-SMV-CS model
- Solved using BSASP algorithm
  - Exploits the block sparsity structure to choose blocks at a time instead of random sparse indices
- Computational complexity analysis



## Other Interesting Papers

- 1 Performance analysis of approximate message passing for distributed compressed sensing
- 2 On geometric analysis of sparse subspace clustering
- 3 Optimal detection and error exponents for hidden semi-Markov models
- 4 Bayesian Nonparametric Causal Inference: Information Rates and Learning Algorithms
- 5 Near-Optimal Noisy Group Testing via Separate Decoding of Items
- 6 Maximum entropy low-rank matrix recovery
- 7 Community Detection with Side Information: Exact Recovery under the Stochastic Block Model