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Lekshmi Ramesh

Signal Processing for Communications Lab
IISc, Bangalore

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- Goal: Outlier support identification in linear regression
- Background
  - Linear regression: find $\mathbf{x}$ given $\{\mathbf{a}_i^\top\}, \mathbf{y}$

$$y_i = \mathbf{a}_i^\top x_i + \eta_i + e_i$$

  - Overdetermined system, no freedom to design $\{\mathbf{a}_i^\top\}$
  - Ordinary least squares sensitive to outliers
  - Dealing with outliers using sparse signal recovery methods "Projection method"

$$F\mathbf{y} = F\mathbf{e} + F\boldsymbol{\eta}$$

    where rows of F form an orthobasis for colsp(A)

- Model:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} + \mathbf{e},$$

  where $\mathbf{A} \in \mathbb{R}^{m \times n}$
  $\boldsymbol{\eta}$: inlier noise, small in magnitude
  $\mathbf{e}$: outliers, large in magnitude, sparse

- Proposed cost:

$$J(\mathbf{x}, \mathbf{e}) = \|\mathbf{y} - \mathbf{A}\mathbf{x} - \mathbf{e}\|_1 + \alpha\|\mathbf{e}\|_0$$

- Contributions
  - Algorithm for outlier identification
  - Convergence analysis, theoretical guarantees for proposed algorithm

Algorithm for Robust Outlier Support Identification (AROSI)

Input: $y, A, \alpha > 0$

Initialization: $k = 0, e^{(0)} = 0, \boldsymbol{S}_0 = \{1, \ldots, m\}$

While $J(x, e)$ not converged DO:

  Iteration $k + 1$

  Step 1 (update x): $x^{(k+1)} = \arg\min_x \left\| y_{S_k} - A_{S_k} x \right\|_1$;

    If $\left\| y_{S_k} - A_{S_k} x^{(k+1)} \right\|_1 = \left\| y_{S_k} - A_{S_k} x^{(k)} \right\|_1$,

    further update $x^{(k+1)} = x^{(k)}$.

  Step 2 (update e and $\boldsymbol{S}$): $e_i^{(k+1)} = \begin{cases} 0, & \left|(y - Ax^{(k+1)})_i\right| \leq \alpha \\ (y - Ax^{(k+1)})_i, & otherwise \end{cases}$

    $\boldsymbol{S}_{k+1} := \{i : e_i^{(k+1)} = 0\}$

  $k := k + 1$

End While

Output: solution $\tilde{x}$

- Goal: Solve detection problems using compressed data
- Background:
  - Obtain multimodal data from sensors, perform inference/detection tasks by fusing data
  - Challenging because of complex inter/intra modal dependencies
  - Two previously known approaches:
    - Copula theory
    - “Product approach”
- Contribution:
  - Detection schemes based on compressed data in the presence of inter modal dependencies

- Binary hypothesis testing problem; obtain compressed data:

$$\mathbf{y}_j = \mathbf{A}_j \mathbf{x}_j,\ j \in [L]$$

  Under $\mathcal{H}_0$: temporal and spatial independence
  Under $\mathcal{H}_1$: temporal independence and spatial dependence

- Assumptions:
  - Observations are noiseless
  - Each $\mathbf{A}_j$ is an orthoprojector

$$\mathbf{A}_j \mathbf{A}_j^\top = \mathbf{I}$$

Two schemes

- LR based detection using compressed data:
  - Gaussian approximation-based
  - First and second order statistics of $\mathbf{x}$ under the hypotheses is assumed to be known
- A covariance based test statistic in the compressed domain
  - Estimate statistics of $\mathbf{x}$ from $\mathbf{y}$, then form test statistic
  - Test statistic T

$$T = \frac{\sum_{i,j} |D_x(i,j)|}{\sum_i |D_x(i,i)|}$$

    Tests for D_x diagonal ($T = 1$) vs. D_x non diagonal ($T > 1$)
  - D_x estimated using $\mathbf{y}_i$

- Goal: DOA estimation in the presence of outliers
- Setup: K sources falling on an M-element array

$$\mathbf{Y} = \mathbf{A}_{\hat{\theta}}\mathbf{S} + \mathbf{E} + \mathbf{W}$$

$\{\hat{\theta}_1, \ldots, \hat{\theta}_{\hat{K}}\}$: fixed sampling grid
$\mathbf{Y} : M \times T$
$\mathbf{A}_{\hat{\theta}} : M \times \hat{K}$
$\mathbf{S} : \hat{K} \times T$
$\mathbf{E} : M \times T$ outlier matrix
$\mathbf{W} : M \times T$ dense noise matrix

- SBL formulation

$$p(\hat{\mathbf{S}}|\delta) = \prod_{t=1}^{T} \mathcal{CN}(\hat{\mathbf{s}}_t|\mathbf{0}, \mathrm{diag}(\delta))$$

$$p(\mathbf{E}|\Gamma) = \prod_{i=1}^{M} \prod_{j=1}^{T} \mathcal{CN}(e_{i,j}|0, \gamma_{i,j})$$

  - Gamma prior on hyperparameters
  - $(\hat{\mathbf{S}}, \mathbf{E})$ treated as hidden variable, and $p(\delta, \Gamma|\mathbf{Y})$ maximized via EM
- Technique to handle off grid gap also mentioned

Other interesting papers

- Fully Decomposable Compressive Sampling With Joint Optimization for Multidimensional Sparse Representation *W. Dai, Y. Li, J. Zou, H. Xiong, and Y. F. Zheng*
- Performance Analysis of Linear Receivers for Uplink Massive MIMO FBMC-OQAM Systems *F. Rottenberg, X. Mestre, F. Horlin, and J. Louveaux*
- Statistical Anomaly Detection via Composite Hypothesis Testing for Markov Models *J. Zhang and I. Ch. Paschalidis*
- Optimal Energy-Efficient Source and Relay Precoder Design for Cooperative MIMO-AF Systems *F. Heliot and R. Tafazolli*