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- Goal: Support recovery of jointly k-sparse signals from multiple measurements:

$$\mathbf{y}_i = A\mathbf{x}_i \quad i \in [L]$$

where $A \in \mathbb{R}^{m \times N}$.

- Contributions
  - Analysis of the exhaustive ML-based decoder
    Probability of error for this decoder is at most δ provided

$$k \leq \frac{\text{krank}(A \odot A)}{2}$$

$$L \geq \frac{1}{\gamma}\left(\log \frac{1}{\delta} + k \log \frac{N}{k}\right)$$

Parameter γ depends on (N, m, k)

- A covariance identifiability condition
  For $p \in \mathbb{R}_+^k$ and $A \in \mathbb{R}^{m \times N}$, we have covariance identifiability when

$$R_i = R_j \text{ if and only if } S_i = S_j \ \forall\, i, j \in [q]$$

  where $q = \binom{N}{k}$ and $R_i = A_{S_i} \mathrm{diag}(p) A_{S_i}^\top$.

- For matrices with iid entries from a continuous distribution, covariance identifiability violated w.h.p. if $k \geq m^2 + m + 2$ and $n \geq 2k$

- Goal: Automated selection of number of clusters in graph clustering problems
- Contributions
  - Phase transition of spectral clustering on the Random Interconnection Model (RIM)
  - A model order selection method based on the phase transition threshold

- Random Interconnection Model
  - Describes a graph on n nodes with k clusters, with the i^{th} cluster having size n_i. The adjacency matrix has the following form

$$A = \begin{bmatrix} A_1 & C_{12} & \cdots & C_{1k} \\ C_{21} & A_2 & \cdots & C_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & A_k \end{bmatrix}$$

    where
    A_i : $n_i \times n_i$ represents within cluster connections
    C_{ij} : $n_i \times n_j$ represents connections between clusters i and j.

    Under RIM: A_i are arbitrary, C_{ij} are mutually independent and have $\text{Ber}(p_{ij})$ entries.
  - The popular stochastic block model is a special case of RIM where A_i have iid $\text{Ber}(p)$ entries

- Setup

$$\underset{X \in \mathbb{R}^{n \times m}}{\text{minimize}} \ f(X)$$
$$\text{s.t. } \operatorname{rank}(X) \leq r$$

  f is smooth

- Factorization approach: decompose X in terms of two smaller matrices as $UV^\top$– reduces computational complexity, introduces non convexity in objective

- Key result: Under certain conditions on f, factored problem has no spurious local minima

- Simple algorithms like gradient descent can provably solve the factored problem with global convergence
- Key assumption on f
  - f is restricted strongly convex and smooth, i.e.,

$$\alpha\|G\|_F^2 \leq [\nabla^2 f(X)](G, G) \leq \beta\|G\|_F^2 \tag{1}$$

for all $n \times m$ X and G with rank at most r

- Goal: Analysis of support recovery performance of weighted ℓ_1 minimization from compressive measurements

$$\mathbf{y} = A\mathbf{x} + \mathbf{z}$$

  $A \in \mathbb{R}^{m \times N}$ with iid standard normal entries, $\mathbf{x}$ is k-sparse, $k < m < N$

- Weighted ℓ_1

$$\arg\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \sum_{i=1}^{N} w_i x_i$$

  Weights can incorporate prior information about $\mathbf{x}$

- Contributions
  - Exact support recovery via weighted ℓ_1 minimization if $x_{\min}$ large enough and
$$m \geq 2\eta k \log(N-k) \tag{2}$$
where η is a function of the weights
  - An algorithm for support recovery based on iterative weighted ℓ_1 minimization

Other interesting papers

- Efficient Analysis and Synthesis Using a New Factorization of the Gabor Frame Matrix. *S. M. -Picot, F. J. Ferri, M. A.-Herráez, W. D.-Villanueva*
- Spatio-Temporal Structured Sparse Regression With Hierarchical Gaussian Process Priors. *D. Kuzin, O. Isupova, L. Mihaylova*
- Joint Detection and Localization of an Unknown Number of Sources Using the Algebraic Structure of the Noise Subspace. *M. W. Morency, S. A. Vorobyov, G. Leus*
- Mitigating Quantization Effects on Distributed Sensor Fusion: A Least Squares Approach. *S. Zhu, C. Chen, J. Xu, X. Guan, L. Xie, K. H. Johansson*
- A Model Selection Criterion for High-Dimensional Linear Regression. *A. Owrang, M. Jansson*