Journal Watch
IEEE Transactions on Signal Processing (December 01, 2016)

Lekshmi Ramesh

Signal Processing for Communications Lab
IISc, Bangalore

December 3, 2016

- Goal: Joint sparse recovery with statistically related supports
- Setup: $\Phi_k \in \mathbb{R}^{n_k \times m}$, $w_k \sim \mathcal{N}(0, I)$

$$y_k = \Phi_k x_k + w_k, \quad k \in [L].$$

Structured sparsity models

- Convex methods (by relaxing the following formulations)

$$\min_{X} \sum_{k=1}^{L} \|\Phi_k x_k - y_k\|_2^2 + \alpha_1 \sum_{k=1}^{L} \|x_k\|_0 + \alpha_2 \|X\|_{0,\text{row}} \quad \text{SSM-1}$$

$$\min_{C,S} \sum_{k=1}^{L} \|\Phi_k (c_k + s_k) - y_k\|_2^2 + \beta_1 \sum_{k=1}^{L} \|s_k\|_0 + \beta_2 \|C\|_{0,\text{row}} \quad \text{SSM-2}$$

- Contributions
  - SBL for structured sparsity models using a dual-space view of the convex penalties. For e.g.,

$$\|x_k\|_1 = \min_{\gamma_{kj}^a \geq 0} \frac{1}{2} \sum_j \frac{x_{kj}^2}{\gamma_{kj}^a} + \gamma_{kj}^a.$$

  - Centralized and decentralized algorithms for solving the SSM recovery problem

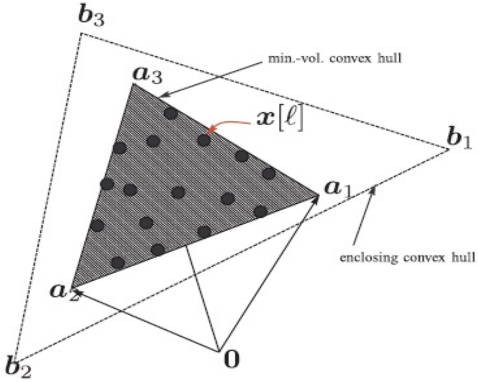
- Goal: Structured matrix factorization (SMF) using volume minimization
- Background, Setup
  - Data matrix=Basis matrix × Coefficient matrix
    Columns of one factor matrix lie on the unit simplex
  - Applications in document clustering, remote sensing
  - Key question: Identifiability of factor matrices
  - Model:
    $$x_i = As_i + v_i, \quad i \in [L]$$
    with $A \in \mathbb{R}^{M \times K}$, noise v_i, $s_i \geq 0$, and $\mathbf{1}^\top s_i = 1$.

- SMF: Factor X into A and S.
  VolMin criterion:

$$
\begin{aligned}
(A, s_i) = \arg\min_{B, c_i} \operatorname{vol}(B) \\
\text{s.t.} \;\; x_i = B c_i, \\
\mathbf{1}^\top c_i = 1, \; c_i \geq 0.
\end{aligned}
$$

  Find minimum-volume simplex that encloses all columns of data matrix

- Contributions
  - Proposed an algorithm based on alternating minimization to solve VolMin
  - Showed equivalence of two previously known conditions for identifiability


b_3
min.-vol. convex hull
a_3
$x[\ell]$
b_1
a_1
enclosing convex hull
a_2
b_2
0

- Goal: Estimate sparse eigenvectors of a symmetric matrix
- Setup: Orthogonal sparse PCA problem
  - Data matrix $A \in \mathbb{R}^{n \times m}$, sample covariance $S = A^\top A$

$$\begin{aligned} \underset{u}{\text{maximize}} \quad & u^\top S u - \rho \|u\|_0 \\ \text{subject to} \quad & u^\top u = 1 \end{aligned}$$

To extract multiple eigenvectors,

$$\begin{aligned} \underset{U}{\text{maximize}} \, & \mathrm{Tr}(U^\top S U D) - \sum_{i=1}^{q} \rho \|u_i\|_0 \\ \text{subject to} \quad & U^\top U = I_q \end{aligned}$$

- Non convex objective, non convex feasible set
    - Convexify objective, use minorization-maximization
    - Solve a sequence of Procustes problems

$$\begin{aligned} \underset{X}{\text{minimize}} \; & \|X - Y\|_F^2 \\ \text{subject to} \; & X^\top X = I \end{aligned}$$

- Contributions
    - New method to estimate dominant sparse eigenvectors, under orthogonality constraint
    - An algorithm for estimating the covariance matrix when its eigenvectors are known to be sparse

Other interesting papers

- Noisy Compressive Sampling Based on Block-Sparse Tensors: Performance Limits and Beamforming Techniques. *R. Boyer and M. Haardt*
- Learning Laplacian Matrix in Smooth Graph Signal Representations. *X. Dong, D. Thanou, P. Frossard, and P. Vandergheynst*
- Proximal Multitask Learning Over Networks With Sparsity-Inducing Coregularization. *R. Nassif, C. Richard, A. Ferrari, and A. H. Sayed*