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# f-Divergence Inequalities

- Igal Sasson, Sergio Verdu
- f-Divergence measures the dissimilarity between two probability measures. Special cases include relative entropy, chi-squared divergence, Hellinger distance, Jeffrey's divergence, total variation distance etc etc
- Definition: P, Q probability measures

$$D_f(P\|Q) = \int f\left(\frac{\mathrm{d}P}{\mathrm{d}Q}\right)\mathrm{d}Q$$

- The paper derives several inequalities bounding one kind of divergence with another (35 Theorems!)

# The Shannon Lower Bound is Asymptotically Tight

- Tobias Koch
- Rate Distortion Function
- Shannon lower bound

$$R(D) = \inf_{P_{\hat{\mathbf{X}}|\mathbf{X}}:\, \mathsf{E}\left[\|\mathbf{X}-\hat{\mathbf{X}}\|^r\right] \le D} I\left(\mathbf{X};\hat{\mathbf{X}}\right)$$

$$R_{\mathrm{SLB}}(D) = h(\mathbf{X}) + \frac{d}{r}\log\frac{1}{D} - \frac{d}{r}\log\left(\frac{r}{d}\left(V_d\Gamma(1+d/r)\right)^{r/d}e\right)$$

- Theorem 2: Suppose the d-dimensional source has a PDF. Assume that H(floor(X)) < infinity and |h(X)| < infinity. Then,

$$\lim_{D\downarrow 0}\left\{R(D) - R_{\mathrm{SLB}}(D)\right\} = 0.$$

- Theorem 3: For every distortion level D > 0, the rate distortion function R(D) of the d-dimensional real valued source X is finite if and only if H(floor(X)) < infinity
- Together, they give necessary and sufficient conditions for asymptotic tightness of the SLB

# Estimating the Directed Mutual Information and Testing for Causality

- Ioannis Kontoyiannis and Maria Skoularidou
- Estimating the directed information between $\{X_n\}$ and $\{Y_n\}$ using the plug-in (ML) estimator

$$I(X_1^n \to Y_1^n) = H(Y_1^n) - \sum_{i=1}^{n} H(Y_i | Y_1^{i-1}, X_1^i)$$

$$I(X \to Y) = \lim_{n \to \infty} \frac{1}{n} I(X_1^n \to Y_1^n)$$

- If $\{(X_n, Y_n)\}$ is a Markov chain, the plug-in estimator is asymptotically Gaussian and converges at the optimal O(1/sqrt(n)) rate
- Connection drawn between directed mutual information and testing for causality. Null hypothesis: absence of causality.
- It is shown that the test statistic is chi-square distributed under the null hypothesis, and the plug-in converges at the faster rate O(1/n)

# State Amplification Subject to Masking Constraints

- Onur Ozan Koyluoglu, Rajiv Soundararajan, Sriram Vishawanath
- State dependent BC with 1 tx (Alice) and 2 rx (Bob and Eve)
- Goal: “amplify” channel state sequence to Bob while masking it from Eve
- Characterize the tradeoff region between state amplification and state leakage
- Derive achievable rates and outer bounds for secure state amplification rates
- Degraded Gaussian channel: gap is less than 0.5 bits