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Rectified Gaussian Scale Mixtures and the Sparse Non-Negative Least Squares Problem

A. Nalci, Igor Fedorov, Maher Al-Shoukairi, Thomas T. Liu, and Bhaskar D. Rao

- Considers the problem of Sparse Non-Negative Least Squares (S-NNLS), i.e.,

$$\min_{\mathbf{x}\geq\mathbf{0},\mathbf{y}=\phi\mathbf{x}} \|\mathbf{x}\|_0$$

- They propose a Bayesian framework for sparse recovery
- Choose Rectified Gaussian hyperprior on $\mathbf{x}$, i.e.,

$$p(x_i|\gamma_i) = \mathcal{N}^R(x_i;\mu,\gamma_i) = \sqrt{\frac{2}{\pi\gamma_i}}\frac{e^{-\frac{(x-\mu)^2}{2\gamma_i}}u(x)}{erfc(\frac{-\mu}{2\gamma_i})}$$

- Type-II estimation using expectation-maximization algorithm (Rectified-SBL)

- In M-step, compute $\mathbb{E}[x_i^2]$, no closed form expression
- Proposed numerical methods such as MCMC-EM and GAMP
- GAMP has computational complexity of $\mathcal{O}(MN)$

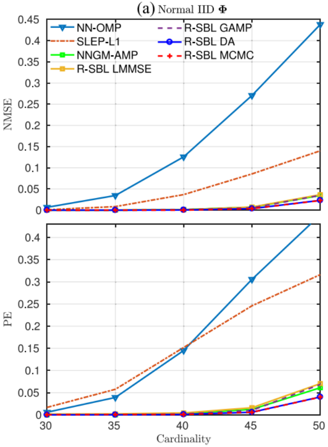

Figure 1 : $\Phi \in \mathbb{R}^{100\times 400}$

Recovery of Structured Signals With Prior Information via Maximizing Correlation

Xu Zhang , Wei Cui , and Yulong Liu

Objective: Recover structured signals in presence of prior information ϕ by solving,

$$\min_{\mathbf{x}} \|\mathbf{x}\|_{sig} - \lambda \langle \mathbf{x}, \phi \rangle$$

$$s.t. \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \delta$$

where, $\|\cdot\|_{sig}$ is an appropriate norm

- Structure considered - Sparsity, Block Sparsity and Low rank
- Performance guarantees under sub-Gaussian measurements, precisely $m = \mathcal{O}(\gamma^2(\mathcal{T}_f \cap \mathbb{S}^{n-1}))$
- $\mathcal{T}_f$ - convex cone induced by $f = \|\mathbf{x}\|_{sig} - \lambda \langle \mathbf{x}, \phi \rangle$
- $\gamma(\xi) = \mathbb{E}\sup_{x \in \xi} |\langle \mathbf{x}, \mathbf{g} \rangle|$, $\mathbf{g} \sim \mathcal{N}(0, 1) \rightarrow$ Gaussian complexity

Let $\mathbf{x}^*$ and $\hat{\mathbf{x}}$ be the true and estimated sparse signal
Define,

- $v_1 := \max_{\mathbf{w} \in \partial ||\mathbf{x}^*|| - \lambda\phi} ||\mathbf{w}||^2$
- $u_1 := ||sign(\mathbf{x}^*) - \lambda\phi||^2$
- v_1 and u_1 are used to characterize the Gaussian complexity of $\mathcal{T}_f \cap \mathbb{S}^{n-1}$

Theorem: Let $\mathbf{A}$ be an $m \times n$ matrix whose rows are independent, centered, isotropic and sub-Gaussian random vectors and $\mathbf{x}^* \in \mathbb{R}^n$ be an s-sparse vector. If,

$$\sqrt{m} \geq CK^2 \min \left\{ \sqrt{n \cdot \left(1 - \frac{n}{v_1} \cdot \frac{2}{\pi}\left(1 - \frac{s}{n}\right)^2\right)}, \sqrt{s + (n-s)u_1} \right\} + \epsilon$$

then with probability $1 - o(1)$,

$$||\mathbf{x}^* - \hat{\mathbf{x}}||_2 \leq \frac{2\delta}{\epsilon}$$

Remark: $v_1 = \sum_{i \in I} (sign(\mathbf{x_i}^*) - \lambda \phi_i)^2 + \sum_{i \in I^c} (1 + |\lambda \phi_i|)^2 \geq n - s$ hence,

- $n \cdot \left(1 - \frac{n}{v_1} \cdot \frac{2}{\pi} \left(1 - \frac{s}{n}\right)^2\right) \geq n - \frac{2}{\pi}(n - s)$
- In extreme sparsity, $n - \frac{2}{\pi}(n - s) \gg s$
- Suitable prior information can lead to $u_1 \to 0$
- Hence, second term dominates, leading $m = \mathcal{O}(s)$

Minimum Data Length for Integer Period Estimation

Srikanth Venkata Tenneti and Palghat P. Vaidyanathan

Objective: Derive minimum number of samples required to estimate period of a signal independent of algorithms

- Common intuition 2P samples, if P is the true period
- Can do better if some additional information present

Theorem: Let $x(n)$ be a periodic signal, whose period is known to lie in the integer set $P = \{P_1, P_2, ..., P_K\}$. To estimate the period using L consecutive samples, it is both necessary and sufficient that:

$$L \geq L_{min} = \max_{P_i, P_j \in P} P_i + P_j - \langle P_i, P_j \rangle$$

where, $\langle \cdot, \cdot \rangle$ is the greatest common divisor

Consider a sequence with period either 4 or 10. $L \geq 10 + 4 - 2 = 12$

Period 4: AGAT AGAT AGA

Period 10: AGATAGATAG A

Undecided, until 12th element is not shown.

If $x(12) = T$, $P = 4$

If $x(12) = G$, $P = 10$

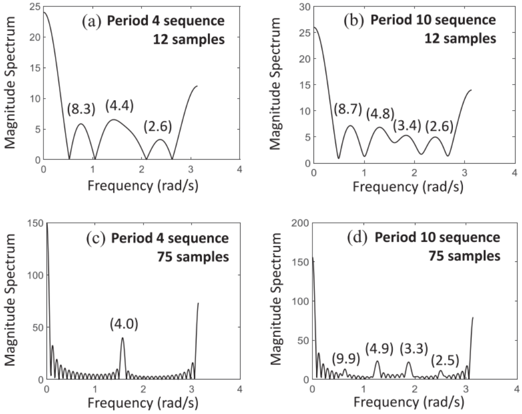

Figure 2 : DFT spectra with $A = 1$, $T = 2$, $C = 3$, $G = 4$

Uniform Recovery Bounds for Structured Random Matrices in Corrupted Compressed Sensing

Goal

- Uniform recovery guarantee for the following problem:

$$\mathbf{y} = \mathbf{A}\mathbf{x}^* + \mathbf{z}^* + \mathbf{w}$$

where, $\mathbf{x}^*$ and $\mathbf{z}^*$ are unknown sparse vectors and $\mathbf{w}$ is dense noise with bounded energy

- They provide RIP constants for structured matrix $\mathbf{A}$
- $\mathbf{A} = \mathbf{UDB}$, where $\mathbf{U}$ is a unit-norm tight frame, $\mathbf{D}$ is a diagonal matrix with independent zero-mean and unit variance sub-gaussian entries and, $\mathbf{B}$ is column-wise orthonormal matrix i.e., $\mathbf{B}^*\mathbf{B} = \mathbb{I}$
- Such structured matrix is used in designing mask for double random phase encoding
- It encompasses structured matrices like partial random circulant matrices and random probing

Theorem: Suppose $\mathbf{y} = \mathbf{A}\mathbf{x}^* + \mathbf{z}^* + \mathbf{w}$ with $\boldsymbol{\Theta} = [\mathbf{A} \;\; \mathbf{I}]$, $\mathbf{A} = \mathbf{UDB}$, $||\mathbf{x}^*||_0 \leq s$ and $||\mathbf{z}^*||_0 \leq k$. If, for $\delta \in (0, 1)$

$$m \geq c_1 \delta^{-2} s \; \hat{n} \mu^2(\mathbf{B}) \log^2 s \log^2 \hat{n}$$

$$m \geq c_1 \delta^{-2} k \log^2 k \log^2 \hat{n}$$

then with probability at least $1 - 2\hat{n}^{-\log^2 s \log \hat{n}}$, $\delta_{s,k} \leq \delta$

Other interesting papers

- M. Wang, Z. Zhang, and A. Nehorai, Jr.," **Performance Analysis of Coarray-Based MUSIC in the Presence of Sensor Location Errors**"
- X. Shen and Y. Gu, " **Nonconvex Sparse Logistic Regression With Weakly Convex Regularization**"
- H. Fu and Y. Chi, " **Quantized Spectral Compressed Sensing: Cramer?Rao Bounds and Recovery Algorithms**"
- S. Zhang, S. Liu, V. Sharma, and P. K. Varshney, " **Optimal Sensor Collaboration for Parameter Tracking Using Energy Harvesting Sensors**"
- Z. You, R. Raich, X. Z. Fern, and J. Kim, " **Weakly Supervised Dictionary Learning**"
- D. Spano, M. Alodeh, S. Chatzinotas, and B. Ottersten, " **Symbol-Level Precoding for the Nonlinear Multiuser MISO Downlink Channel**"